Design Tool for Computation of Safety Factors

Pierre C. J. HOOGENBOOM
Researcher
Delft University of Technology
Delft, The Netherlands

Pierre Hoogenboom, born 1965, received a M.Sc. and Ph.D. degree in civil engineering from Delft University of Technology. His research interests include reinforced concrete modelling, performance-based design and software development.

Summary

This paper presents a software tool for computation of partial safety factors and load combinations. The implemented theory is summarised. Due to numerical algorithms few approximations show to be needed compared to manual calculation of safety factors. In addition, an algorithm is proposed to compute the optimum level of reliability. Computed safety factors can supplement safety factors in codes of practice for special projects or exceptional safety requirements.

Keywords: design software; partial factors; safety factors; load and resistance factor design; load combinations.

1. Introduction

To date, reliability theory is sufficiently developed for computation of failure probabilities of both structural elements and structural systems. In the near future personal computers will be sufficiently fast to process reliability analyses in everyday engineering practice. This is a most promising development because it is commonly expected that reliability-based design will be more economical than load and resistance factor design or partial factor design.

However, reliability-based design might not be suitable in the first cycle of a design process because it does not show how much structural dimensions should be improved. In other words, the convergence of a strictly reliability-based design process can be slow. Instead, typical of traditional design procedures is that the first – and often only – design cycle already gives quite acceptable dimensions. Therefore, a future design procedure might start with traditional safety factors in the first design cycles and use reliability analysis in the last cycle to validate or optimise a structural design. So, it can be expected that safety factors will be used for a long time to come.

Of course, safety factors and load combinations are included in codes of practice (standards). However, codified factors and combinations do not cover design of special projects with exceptional safety and durability requirements. For such a project a structural designer needs to calculate safety factors and load combinations himself. The theory and data to do so is often available [1] but what is lacking is support by user-friendly software. Therefore, a simple software tool is being developed for computing safety factors and load combinations.

2. Computation of Safety Factors

In this section the safety factors are derived in two steps applying well know probability theory. In Section 2.2 stochastic processes are approximated as combinations of stochastic variables. In Section 2.3 each combination of stochastic variables is approximated as combinations of deterministic values.

2.1 Stochastic Processes

In this paper a load case is defined as a set of correlated data that presents a hazard to a structure. It can include forces, support displacements, temperature increases, material shrinkage, prestress and absent structural components. For example all actions related to a storm can be conveniently assembled in one load case. Fluctuation of the load can be described as a load case multiplied by a factor λ. When λ equals 1, the structure carries the characteristic or nominal load. When λ equals a load factor γ , the structure carries the design load. Due to uncertainty of future loading, the factor λ is best described as a stochastic process in time. Uncertainty of future strength can be described in the same way. The characteristic strength is multiplied by a stochastic factor λ. When λ equals 1, the strength equals the characteristic or nominal strength. When λ equals the strength reduction factor ϕ, the strength equals the design strength.

2.2 Combinations of Stochastic Load Cases

Usually more than one load case can act on a structure, however, it is not feasible to simulate every minute of the live of a structure. Therefore the combination rule of Ferry Borges [2] was implemented to approximate the most demanding moments. The rule uses periods in which the largest magnitude of a load case is approximately constant. Each of the load cases in turn is considered leading except for dead load D and strengths R. The leading load has its largest magnitude during a reference period or the design live t. Load cases with larger periods than the leading load case have usual magnitudes. The load cases with smaller periods have largest magnitudes in the period of the load case with a just longer period. Thus the simultaneous stochastic processes are reduced to combinations of stochastic variables.

As an example, suppose that wind load W is leading; thus a storm. Dead load D is always present and is just added. Live load L is also added because it changes in a longer period than wind. Clearly, a large earthquake is very unlikely in the small period in which the storm is extreme but a small earthquake load E cannot be excluded. So, the maximum earthquake over this period is added. This combination is shown in the third row of the list below.

$$
\begin{array}{lllll}
D\ \mathrm{inst}(\lambda_D) & & & & \&\ R\ \mathrm{inst}(\lambda_R) \\
D\ \mathrm{inst}(\lambda_D) & \&\ L\ \mathrm{extr}(\lambda_L, t) & \&\ W\ \mathrm{extr}(\lambda_W, t_L) & \&\ E\ \mathrm{extr}(\lambda_E, t_W) & \&\ R\ \mathrm{inst}(\lambda_R) \\
D\ \mathrm{inst}(\lambda_D) & \&\ L\ \mathrm{inst}(\lambda_L) & \&\ W\ \mathrm{extr}(\lambda_W, t) & \&\ E\ \mathrm{extr}(\lambda_E, t_W) & \&\ R\ \mathrm{inst}(\lambda_R) \\
D\ \mathrm{inst}(\lambda_D) & \&\ L\ \mathrm{inst}(\lambda_L) & \&\ W\ \mathrm{inst}(\lambda_W) & \&\ E\ \mathrm{extr}(\lambda_E, t) & \&\ R\ \mathrm{inst}(\lambda_R)
\end{array}
\tag{1}
$$

where $\mathrm{inst}(\lambda)$ is the instantaneous value of load factor λ and $\mathrm{extr}(\lambda, t)$ is the extreme value of λ over period t. The distribution functions of the extreme values can be derived as

$$
P\{\mathrm{extr}(\lambda_x, t_y) \le z\} = [F_x(z)]^{\frac{t_y}{t_x}} \tag{2}
$$

where F_x is the extreme distribution function of load factor λ_x over reference period t_x .

2.3 Combinations of Deterministic Load Cases

Clearly, a structure needs to have sufficient safety against each of the combinations of stochastic load cases in the previous section. This is fulfilled if the limit state boundary – plotted in the space of load factors λ – is at sufficient distance from the origin. To this end, combinations of deterministic load cases (i.e. conservatively estimated design points) are selected for each combination of stochastic load cases. A structure will be safe if designed such that the considered limit state shall not occur for any of these deterministic combinations.

As usual, the safety factors γ can be related to sensitivity factors α according to

$$
[F_x(\gamma)]^{\frac{t_y}{t_x}} = \Phi(\alpha\beta) \tag{3}
$$

where Φ is the standard normal distribution function and β is the target reliability index of the considered limit state over a reference period. When α has been selected, the safety factor γ in Equation 3 can be quickly computed as a root by iteratively dividing the feasible interval [3]. It is noted that there is no need for further approximations because closed form expressions of γ are not needed.

The following algorithm is adopted to generate sensitivity factors α [1]. The sensitivity factor of a leading load case is 0.70 while sensitivity factors of all other load cases are 0.28 or -0.28. Non leading load cases are also included with α = 0.70 whilst for the leading load cases α = 0.28 and for the others α = 0.28 or α = -0.28. The sensitivity factor of the leading material is -0.80 and -0.32 for the other materials. If ductility is essential (capacity-based design) the other materials should also be included with α = 0.32. For example if three load cases and two materials (concrete and reinforcing steel) are present, the α values of Table 1 would be used when live load *L* is leading. Thus, each row of the table provides the safety factors of a deterministic load combination.

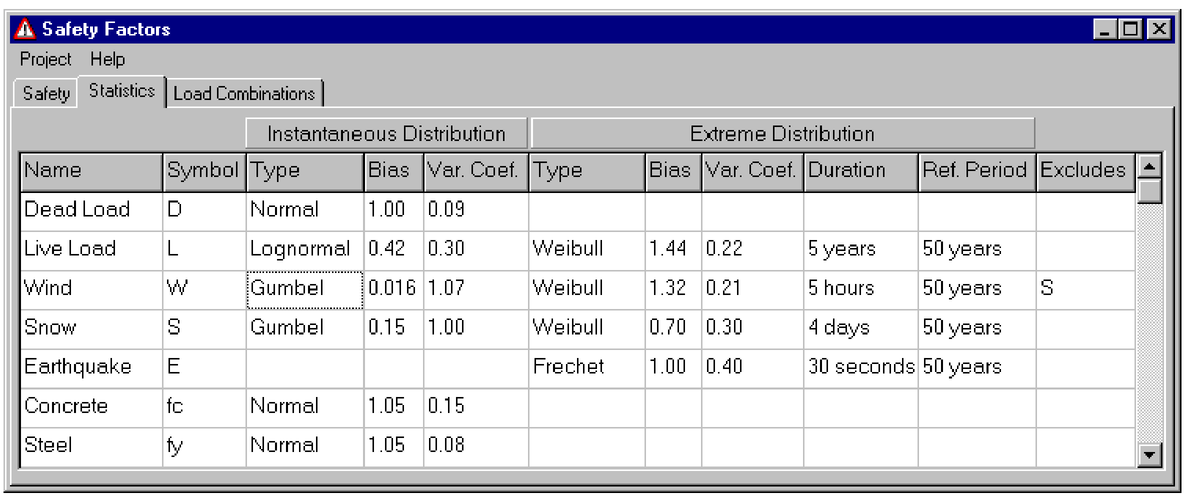
The number of deterministic combinations that result from Table 1 is considerably larger than adopted for calculation of safety factors in many codes of practice. However, these codified factors require considerable judgement as to how load cases contribute to section forces. It would therefore be difficult to implement this in software, especially if the structural model is nonlinear. A more convenient way to program structural design software is systematically processing a large number of combinations and subsequently to determine the envelope of section forces.

Table 1: Sensitivity factors in case of three load cases and two materials

α_D	α_L	α_W	α_{fc}	α_{fy}
0.28	0.70	0.28	-0.32	-0.80
0.28	0.70	0.28	-0.80	-0.32
-0.28	0.70	0.28	-0.32	-0.80
-0.28	0.70	0.28	-0.80	-0.32
0.28	0.70	-0.28	-0.32	-0.80
0.28	0.70	-0.28	-0.80	-0.32
-0.28	0.70	-0.28	-0.32	-0.80
-0.28	0.70	-0.28	-0.80	-0.32
0.70	0.28	0.28	-0.32	-0.80
0.70	0.28	0.28	-0.80	-0.32
0.70	0.28	-0.28	-0.32	-0.80
0.70	0.28	-0.28	-0.80	-0.32
0.28	0.28	0.70	-0.32	-0.80
0.28	0.28	0.70	-0.80	-0.32
-0.28	0.28	0.70	-0.32	-0.80
-0.28	0.28	0.70	-0.80	-0.32

3. Software Tool

A small software program has been developed based on the discussed theory (Fig. 1.). Basically, the program allows a structural designer to enter the statistics of the loading on the structure and select its target reliability. Subsequently, it generates the safety factors of the load combinations.


Safety Factors
Project Help
Safety Statistics Load Combinations

		Instantaneous Distribution			Extreme Distribution					
Name	Symbol	Type	Bias	Var. Coef.	Type	Bias	Var. Coef.	Duration	Ref. Period	Excludes
Dead Load	D	Normal	1.00	0.09						
Live Load	L	Lognormal	0.42	0.30	Weibull	1.44	0.22	5 years	50 years	
Wind	W	Gumbel	0.016	1.07	Weibull	1.32	0.21	5 hours	50 years	S
Snow	S	Gumbel	0.15	1.00	Weibull	0.70	0.30	4 days	50 years	
Earthquake	E				Frechet	1.00	0.40	30 seconds	50 years	
Concrete	fc	Normal	1.05	0.15						
Steel	fy	Normal	1.05	0.08						

Fig. 1: Software tool for computation of safety factors and load combinations

To improve the usability of the program, it needs to include an extensive database of load data for locations all over the world. Substantial maintenance will be needed to provide a designer with the last data available. This suggests that the best way to make the tool available is on the World Wide Web (WWW). An additional advantage of a WWW application is that designers do not need to install software but can start the tool in a few mouse clicks, provided that WWW access is available. It is planned to build a WWW version of the program if potential users show to be interested (Fig. 2.).

Table 2: Statistics of the load factors and strength factors

		Bias Inst.	Var. Coef. Inst.	Bias Ext.	Var. Coef. Ext.
D	Dead Load	1.05	0.07		
L	Live Load (Imposed Load)	0.30	0.40	1.00	0.40
W	Wind Load	0.15	1.10	1.20	0.20
fc	Concrete Crushing Strength	1.10	0.15		
fy	Steel Yield Strength	1.15	0.08		

As an example, safety factors are computed for design of a reinforced concrete structure. The data (Table 2) could represent a location at the North Sea for design of an offshore structure. All distributions are normal and Turkstra combinations are used so that the results can be verified manually. The target reliability index β is 3.0. The result is shown in Table 3.

Table 3: Computed safety factors for design of an offshore structure

1.20 *D*,			0.94 *fc*,	0.93 *fy*
1.20 *D*,			0.70 *fc*,	1.06 *fy*
1.11 *D*,	1.84 *L*,	0.29 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	1.84 *L*,	0.29 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	1.84 *L*,	0.29 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	1.84 *L*,	0.29 *W*,	0.70 *fc*,	1.06 *fy*
1.11 *D*,	1.84 *L*,	0.01 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	1.84 *L*,	0.01 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	1.84 *L*,	0.01 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	1.84 *L*,	0.01 *W*,	0.70 *fc*,	1.06 *fy*
1.20 *D*,	1.34 *L*,	0.29 *W*,	0.94 *fc*,	0.93 *fy*
1.20 *D*,	1.34 *L*,	0.29 *W*,	0.70 *fc*,	1.06 *fy*
1.20 *D*,	1.34 *L*,	0.01 *W*,	0.94 *fc*,	0.93 *fy*
1.20 *D*,	1.34 *L*,	0.01 *W*,	0.70 *fc*,	1.06 *fy*
1.11 *D*,	1.34 *L*,	0.50 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	1.34 *L*,	0.50 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	1.34 *L*,	0.50 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	1.34 *L*,	0.50 *W*,	0.70 *fc*,	1.06 *fy*
1.11 *D*,	0.40 *L*,	1.70 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	0.40 *L*,	1.70 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	0.40 *L*,	1.70 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	0.40 *L*,	1.70 *W*,	0.70 *fc*,	1.06 *fy*
1.11 *D*,	0.20 *L*,	1.70 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	0.20 *L*,	1.70 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	0.20 *L*,	1.70 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	0.20 *L*,	1.70 *W*,	0.70 *fc*,	1.06 *fy*
1.20 *D*,	0.40 *L*,	1.40 *W*,	0.94 *fc*,	0.93 *fy*
1.20 *D*,	0.40 *L*,	1.40 *W*,	0.70 *fc*,	1.06 *fy*
1.20 *D*,	0.20 *L*,	1.40 *W*,	0.94 *fc*,	0.93 *fy*
1.20 *D*,	0.20 *L*,	1.40 *W*,	0.70 *fc*,	1.06 *fy*
1.11 *D*,	0.55 *L*,	1.40 *W*,	0.94 *fc*,	0.93 *fy*
1.11 *D*,	0.55 *L*,	1.40 *W*,	0.70 *fc*,	1.06 *fy*
0.99 *D*,	0.55 *L*,	1.40 *W*,	0.94 *fc*,	0.93 *fy*
0.99 *D*,	0.55 *L*,	1.40 *W*,	0.70 *fc*,	1.06 *fy*

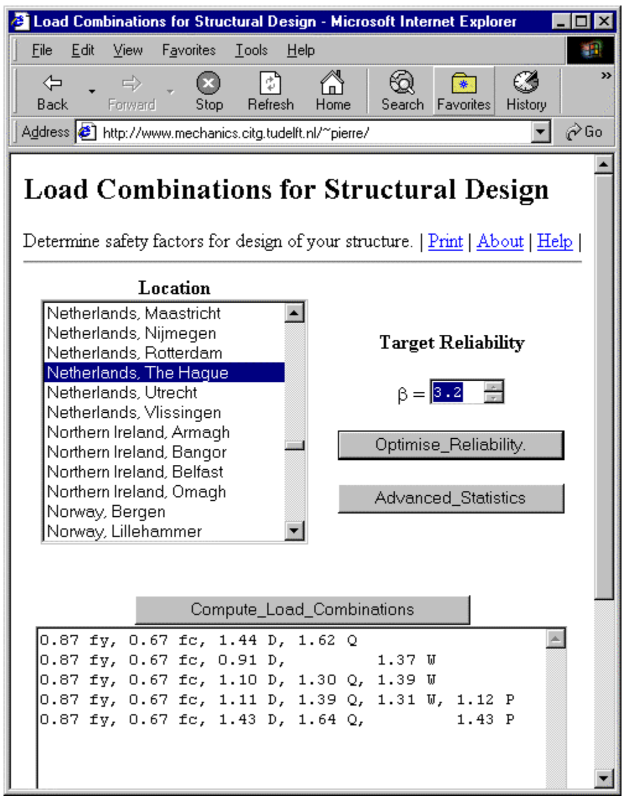


Fig. 2: Concept of a WWW applet for computing safety factors

4. Optimum Reliability

An expression for optimum reliability has been derived in [4]. This expression considers normally distributed strength R and load S and assumes a limit state function $Z = R - S$ in the optimisation process. However, this elegant expression is restricted to just two variables. Closed form formulae for optimum reliability are derived in [5]. These expressions include just one loading, which is very suitable for Japanese practice but less for Western design. (Typical for Japanese design is that all load cases in a combination are factored by a single safety factor.) Therefore, in this section an expression is derived for optimum reliability including multiple load cases and strengths.

4.1 Minimum Expected Cost

The total expected cost of a project C_e consists of the initial costs C_p and the risk of structural failure. The risk of failure is defined as the probability of failure P_f during the design live times the costs of failure C_f. So,

$$C_e = C_p + P_f C_f \tag{4}$$

In this formulation maintenance cost and interest on investments have been neglected. The initial project cost is a function of the safety factors used in design of the structure. $C_p = C_p(\gamma_2, \gamma_2, \gamma_3, \ldots)$. This includes all safety factors of one load combination and applies to each load combination of a limit state. The equation can be approximated by a Taylor expansion about the values that are being used in current practice for similar structures.

$$C_p = C_o\left(1 + \sum_{i=1}^{n} k_i(\gamma_i - \gamma_{oi})\right) \tag{5}$$

where n is the number of safety factors. In this C_o is the initial project cost when common safety factors are used. The variables γ_{oi} are common safety factors and γ_i are the optimised safety factors to be calculated. The factors k_i are cost ratios, which are defined as

$$k_i = \frac{1}{C_o}\frac{\partial C_p}{\partial \gamma_i} \tag{6}$$

The failure costs are expressed in the initial project costs C_o and cost ratio g.

$$C_f = gC_o \tag{7}$$

The total cost needs to be minimised with respect to the safety index β.

$$\frac{dC_e}{d\beta} = 0 \tag{8}$$

Substitution of Equations (4) (5) and (7) in (8) gives

$$g\frac{dP_f}{d\beta} + \sum_{i=1}^{n} k_i \frac{d\gamma_i}{d\beta} = 0 \tag{9}$$

Both the failure probability P_f and the safety factors γ_i depend on β.

$$P_f = \Phi(-\beta) \qquad F_i(\gamma_i) = \Phi(\alpha_i \beta) \tag{10}$$

where Φ is the standard normal probability distribution and F_i the distribution function of the load or strength parameter λ_i . Substitution of Equations (10) into (9) gives the following relation.

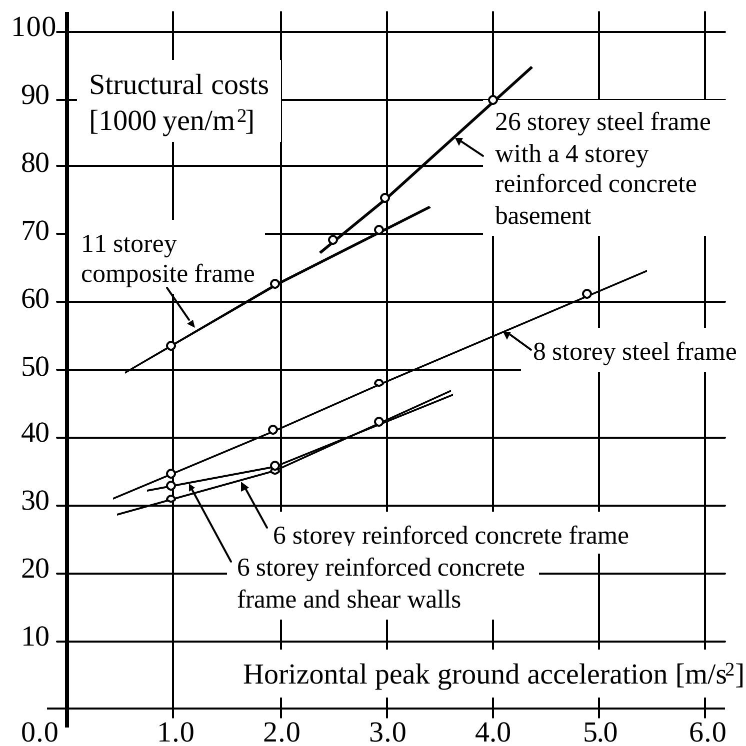


Fig. 3: Structural costs of different high-rise buildings for varying earthquake loads used in design (Assembled from data in [7]). The costs are per square meter of floor plus roof.

$$g - \sum_{i=1}^{n} k_i \alpha_i \frac{e^{\frac{1}{2}\beta_{opt}^2(1-\alpha_i^2)}}{f_i(\gamma_i)} = 0 \tag{11}$$

were f_i is the probability density function of load or strength parameter λ_i . A solution of β_{opt} can be computed quickly by the secant algorithm [3]. It is noted that it can differ for different load combinations. The α_i values in (11) are the same as in the previous section. (It can be argued that the α_i values should be normalised to a unit length because in this situation the approximation should not be conservative). The γ_i values are computed with Equation (10). In the distributions the design live is used instead of a reference period. The cost ratios k_i and g are discussed in the subsequent section.

For normally distributed safety factors, equation (11) can be written in closed form

$$\beta_{\text{opt}} = \sqrt{-2\ln\left[\frac{\sqrt{2\pi}}{g}\sum_{i=1}^{n} k_i\,\alpha_i\,\sigma_i\right]} \tag{12}$$

For $n = 1$ this expression has been derived in [5]. Note that σ_i is the standard deviation of load factor λ_i . As a small example consider the third load combination of Table 3. When we select $g = 6$, $k_i = 0.2, 0.2, 0.1, -0.1, -0.2$ respectively then the optimal reliability index is computed as $\beta_{\text{opt}} = 2.6$ in the design live of the structure.

4.2 Economical Parameters

Failure cost ratios g for different structures are available in literature [5][6]. For example for houses $g = 2$ for tall office buildings $g = 7$ and for hospitals $g = 30$. They should be used as indication only because of varying local situations. For an individual project, actual values can be calculated or judiciously estimated.

Unfortunately, cost ratios k are not constants. They depend not only on the type of structure but also on the costs of non-structural components and on the magnitude of the load. However, using Equation 8 we can write k as

$$k_i = \frac{L_i}{C_o} K_i \qquad \text{where} \quad K_i = \frac{\partial C_p}{\partial(\gamma_i L_i)} \tag{13}$$

Symbol C_o represents the usual initial project costs while L_i is the nominal load, which can be selected arbitrarily as long as it is consistently used. There is evidence that the factors K_i are fairly constant. In Figure 3 the structural costs of several high-rise buildings are plotted for varying magnitude of a design earthquake load. It shows that almost linear relations exist between design load and cost. In this figure, K is interpreted as the slope of a line. (The design load has little influence on non-structural costs of a building. So, non-structural costs can be neglected in calculating K.)

5. Conclusions

Safety factors and load combinations can be computed with few approximations when numerical algorithms on modern personal computers are used. Computed safety factors can supplement safety factors in codes of practice for special projects or exceptional safety requirements. For automatic processing more load combinations are needed than for manual calculations. However, advantages include 1) nonlinear analysis in design 2) possible economies in materials and 3) fewer design errors.

6. References

[1] ISO 2394, "General principles on reliability for structures", International Organisation for Standardisation, Geneva, 1998

[2] Ferry Borges, J., "Quality Assurance and Reliability of Concrete Structures", in Conceptional Preparation of Future Codes", *Comite Euro-International du Beton* (CEB), Bulletin 147, January 1982.

[3] Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., "*Numerical Recipes in Pascal*, *The Art of Scientific Computing*", Cambridge University Press, October 1989.

[4] Rosenblueth, E., "Towards Optimum Design through Building Codes", *J. Struct. Div.*, 102 (3) 1976, pp. 591-607.

[5] Kanda, J., Ellingwood, B., "Formulation of Load Factors Based on Optimum Reliability", *Structural Safety*, Vol. 9, 1991, pp. 197-210.

[6] Kanda, J., Shah, H., "Engineering Role in Failure Cost Evaluation for Buildings", *Structural Safety*, Vol. 19, No. 1, 1997a, pp. 79-90.

[7] Kanda, J., et al., "Evaluation of the Effect of Economic Factors on Optimum Reliability", Report 9313 *University of Tokyo*, 1994, (in Japanese).