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Performance-Based Design of Reinforced Concrete Panels on the WWW

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ABSTRACT

This paper presents a dedicated computer program for analysis of reinforced concrete panels loaded in plane stress. The program can accurately predict the performance of parts of reinforced concrete walls and deep beams. It has been specially developed for routine use by structural designers. It can be used next to code formulae to show that critical parts of a structure fulfil all performance requirements. The material model is based on the modified compression field theory (MCFT). The program has been built in the Java language and is conveniently available on the WWW.

KEY WORDS: Reinforced Concrete Design; Performance Based Design; Design Tool; Panel; Disk; WWW; Java Applet; MCFT

INTRODUCTION

Code formulae provide conservative approximations of the required reinforcement in concrete walls and deep beams. Regularly it proofs difficult to meet these code regulations perfectly. Examples are congestion of reinforcement and rehabilitation of existing structures. In these situations expensive design adaptations are made. However, this is not necessary when can be shown that the structure nonetheless fulfils all performance criteria. To this end a dedicated computer program has been developed that can simulate the material performance in parts of reinforced concrete walls and deep beams. The program is easily available on the WWW as a Java applet at mechanics.citg.tudelft.nl/rc (See appendix).

In 1999 E.C. Bentz developed the program Membrane 2000 for analysis of reinforced concrete panels (Bentz 2000)(Bentz 2003a). This program shows many if not all details of a panel loaded in plane stress. It includes the strains and stresses of the materials in the reinforcement directions and principle directions, the shear stress in the cracks and Mohr's circles of the strain and stress states for any loading stage. Due to its elaborate features, Membrane 2000 requires considerable study effort to be understood. Unfortunately, this makes it unsuitable for routine checks of structural performance. In addition to Membrane 2000 a spreadsheet program is available on the analysis of reinforced concrete panels (Bentz 2003b). However, this program is not valid for all loading conditions and needs careful operation to obtain reliable results.

The applet presented in this paper does not have these drawbacks. It has been specially developed for frequent checks of structural performance. Therefore, it displays only the essential information that a structural designer needs to make decisions. The applet input is panel dimensions, materials and loading. The output is crack width, material stresses and ultimate load. A structural designer can obtain the input loading from a linear-elastic finite element analysis of the structure or also from a strut-and-tie analysis or stringer-panel analysis.

It is noted that the applet cannot be used to predict buckling that might occur in slender walls. Also it cannot be used for walls loaded predominantly in bending. It can be used to predict cracking and stresses in a reinforced concrete wall part under any plane stress loading.

MATERIAL MODEL

The modified compression field theory (MCFT) is a constitutive model for reinforced concrete subjected to plane stress static loading. In this paper the theory is formulated such that it can correctly handle tension and cracking in two directions, both positive and negative shear loading.

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \overset{(1)}{\Rightarrow} \begin{matrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{bmatrix} \overset{(2)}{\Rightarrow} \begin{bmatrix} f_{sx}\\ f_{sy}\end{bmatrix} \\ \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \theta\end{bmatrix} \overset{(3)}{\Rightarrow} \begin{bmatrix} f_{c1}\\ f_{c2}\\ \theta\end{bmatrix}\end{matrix} \overset{(4)}{\Rightarrow} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}$$

Figure 1. Structure of the MCFT

Input of the MCFT consists of the strains ε_{xx} , ε_{yy} and γ_{xy} (Fig 1.). The elementary length over which the strains are computed is approximately equal to the crack distance (Fig. 2). As a consequence the local deformation in the cracks is evenly distributed over the surface. In step 1 the strains are used to compute the principle strains ε_1 and ε_2 and the principle strain direction θ. The equations follow from Mohr's Circle.

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy} + r) \qquad r = \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy} - r)$$

$$\cos^2\theta = \frac{\varepsilon_{yy} - \varepsilon_{xx}}{r}$$

$$\sin^2\theta = \frac{\gamma_{xy}}{r}$$

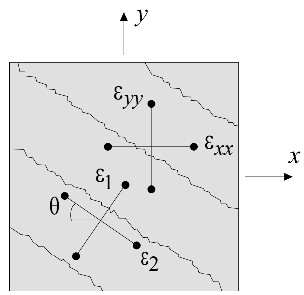

Figure 2. Strains in Reinforced Concrete

The reinforcing bars are directed in the x and y directions. In step 2 the stresses in the reinforcement are computed (Fig. 3). These stresses can be interpreted as an average over the length of the bars. Hardening, breaking and buckling of the bars are not modelled.

$$f_s(\varepsilon) = \begin{cases} f_y & \text{if} \quad \varepsilon > \varepsilon_y \\ E_s\,\varepsilon & \text{if} \quad -\varepsilon_y \le \varepsilon \le \varepsilon_y \\ -f_y & \text{if} \quad \varepsilon < -\varepsilon_y \end{cases}$$

where $\varepsilon_y = \dfrac{f_y}{E_s}$, E_s is the steel Yong's modulus and f_y is the steel yield stress.

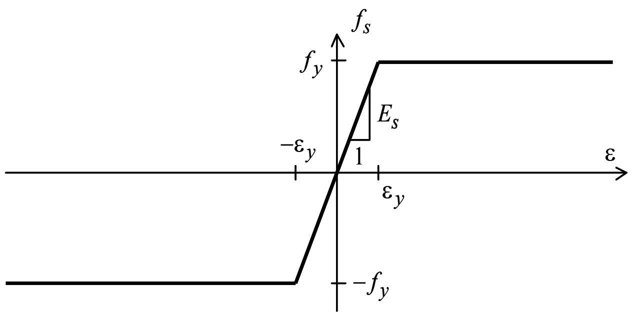

Figure 3. Stress-strain diagram of reinforcing steel in the MCFT

In step 3 the principle stresses in the concrete are computed (Fig. 4.). These equations have been derived from experiments on 30 reinforced concrete panels (Vecchio 1986). The behaviour of the compressed concrete is modelled by a parabola. The compressive strength $f_{c\max}$ is reduced by the strain ε_t in the lateral direction. The behaviour of the tensioned concrete is linear until it cracks. The concrete between the cracks is tensioned because it is extended by the enclosed reinforcement (tension-stiffening). Therefore also after cracking an average concrete stress occurs. The principle stress direction equals the principle strain direction (co-axiality).

$$f_c(\varepsilon,\varepsilon_t) = \begin{cases} 0 & \text{if} \quad \varepsilon < 2\varepsilon'_c \\ f_{c\max}\left(2\dfrac{\varepsilon}{\varepsilon'_c} - \left(\dfrac{\varepsilon}{\varepsilon'_c}\right)^2\right) & \text{if} \quad 2\varepsilon'_c \le \varepsilon \le 0 \\ \dfrac{f_{cr}}{\varepsilon_{cr}}\varepsilon & \text{if} \quad 0 \le \varepsilon \le \varepsilon_{cr} \\ \dfrac{f_{cr}}{1+\sqrt{200\varepsilon}} & \text{if} \quad \varepsilon > \varepsilon_{cr} \end{cases}$$

$$f_{c\max} = \frac{f'_c}{0.8 - 170\varepsilon_t} \le f'_c \qquad \varepsilon'_c = 2\frac{f'_c}{E_c} \qquad \varepsilon_{cr} = \frac{f_{cr}}{E_c}$$

where E_c is the concrete Yong's modulus, f'_c is the compressive strength (negative value), f_{cr} is the tensile strength and ε_t is the strain in the lateral direction.

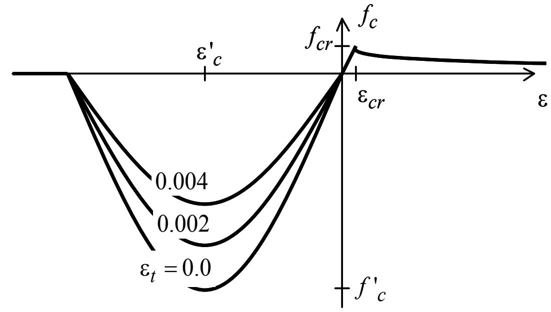

Figure 4. Stress-strain diagram of concrete in de MCFT

In step 4, equilibrium in the cracks is checked. When necessary the average concrete stress $|f_{c1}|$ is reduced. This is explained in the next section. Subsequently, the concrete stresses are rotated to the x-y reference frame using Mohr's circle.

$$f_{cx} = a - b\cos^2\theta \qquad a = \frac{1}{2}(f_{c1} + f_{c2})$$

$$f_{cy} = a + b\cos^2\theta \qquad b = \frac{1}{2}(f_{c1} - f_{c2})$$

$$v_c = b\sin^2\theta$$

Finally the stresses in the concrete and steel are averaged.

$$\sigma_{xx} = \rho_x f_{sx} + (1-\rho_x) f_{cx}$$

$$\sigma_{yy} = \rho_y f_{sy} + (1-\rho_y) f_{cy}$$

$$\sigma_{xy} = v_c$$

EQUILIBRIUM IN A CRACK

An essential part of the modified compression field theory (MCFT) is an equilibrium check of the stresses in the cracks. When the computed stresses cannot be carried trough the cracks the concrete stress f_{c1} is reduced. Unfortunately, the MCFT is not clear on how this equilibrium check should be carried out. Many algorithms are used next to each other. In this paper we adopted a systematic approach using the lower bound theorem of plasticity theory. According to this theorem every statically admissible equilibrium system gives a safe approximation of the strength. The applet selects the equilibrium system that provides the largest concrete stress f_{c1}.

In a crack the following stresses occur (Fig. 5.).

f_{sxcr} normal stress in the reinforcing bars in the x-direction in a crack
f_{sycr} normal stress in the reinforcing bars in the y-direction in a crack
v_{ci} shear stress in a crack
f_{ci} compression stress in a crack

The maximum value of the shear stress is (Walraven 1981)

$$v_{cimax} = \frac{\sqrt{-f'_c}}{0.31 + 24\, w/(a+16)}$$

where w is the crack width, a the diameter of the largest aggregate in mm and f'_c the compressive strength of the concrete in MPa. The compressive stress f_{ci} in a crack follows from the shear stress v_{ci}.

$$f_{ci} = \begin{cases} v_{ci\max}\left(1 - \sqrt{1.22\,(1 - |v_{ci}|/v_{ci\max})}\right) & \text{if } |v_{ci}| \geq 0.180\, v_{ci\max} \\ 0 & \text{if } |v_{ci}| < 0.180\, v_{ci\max} \end{cases}$$

The stresses in Figure 5 are averages over the surface of the panel. These are computed using the constitutive equations. The stresses in Figure 6 occur in a crack. The average stresses and stresses in the crack need to be in equilibrium. Two equilibrium equations can be formulated, for the x direction and the y direction.

$$f_{sxcr} \sin\theta\ \rho_x - v_{ci} \cos\theta - f_{ci} \sin\theta = f_{sx} \sin\theta\, \rho_x + f_{c1} \sin\theta$$
$$f_{sycr} \cos\theta\, \rho_y + v_{ci} \sin\theta - f_{ci} \cos\theta = f_{sy} \cos\theta\, \rho_y + f_{c1} \cos\theta$$

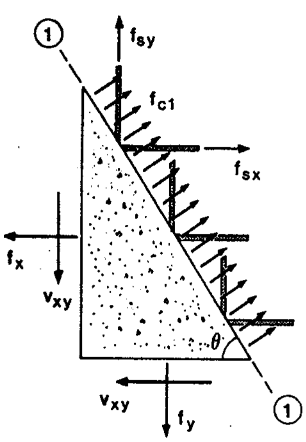

Figure 5. Average Stresses in a Section (Vecchio 1986)

Equilibrium System 1

In equilibrium system 1, it is assumed that the reinforcement yields in both directions. Therefore $f_{sxcr} = f_{yx}$ and $f_{sycr} = f_{yy}$. The equilibrium equations can be used to derive the shear stress v_{ci} in the crack and the average concrete stress f_{c1}.

$$v_{ci} = \left(\rho_x (f_{yx} - f_{sx}) - \rho_y (f_{yy} - f_{sy})\right) \sin\theta \cos\theta$$
$$f_{c1} = \rho_x (f_{yx} - f_{sx}) \sin^2\theta + \rho_y (f_{yy} - f_{sy}) \cos^2\theta - f_{ci}$$

If the computed shear stress $|v_{ci}|$ is larger than the maximum shear stress $v_{ci\max}$ this equilibrium system cannot occur.

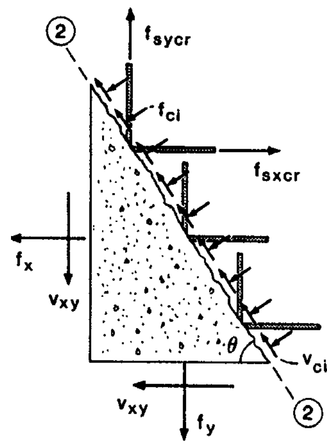

Figure 6. Stresses in a Crack (Vecchio 1986)

Equilibrium System 2

In equilibrium system 2, it is assumed that the reinforcing bars in the x direction yield and the crack starts to crush.

$$f_{sxcr} = f_{yx}$$
$$v_{ci} = v_{ci\max}$$
$$f_{sycr} = \frac{1}{\rho_y}\left(\rho_x (f_{yx} - f_{sx}) - \frac{v_{ci\max}}{\sin\theta\cos\theta}\right) + f_{sy} \leq f_{yy}$$
$$f_{c1} = \rho_x (f_{yx} - f_{sx}) - v_{ci\max}\left(\frac{1}{\tan\theta} + 1\right)$$

Equilibrium System 3

In equilibrium system 3, it is assumed that the reinforcing bars in the x direction yield and the crack starts to crush due to shear in the opposite direction as in equilibrium system 2.

$$f_{sxcr} = f_{yx}$$
$$v_{ci} = -v_{ci\max}$$
$$f_{sycr} = \frac{1}{\rho_y}\left(\rho_x (f_{yx} - f_{sx}) + \frac{v_{ci\max}}{\sin\theta\cos\theta}\right) + f_{sy} \leq f_{yy}$$
$$f_{c1} = \rho_x (f_{yx} - f_{sx}) + v_{ci\max}\left(\frac{1}{\tan\theta} - 1\right)$$

Equilibrium System 4

In equilibrium system 4, it is assumed that the reinforcement in the y direction yields and the crack starts to crush.

$$f_{sycr} = f_{yy}$$
$$v_{ci} = v_{ci\max}$$
$$f_{sxcr} = \frac{1}{\rho_x}\left(\rho_y (f_{yy} - f_{sy}) + \frac{v_{ci\max}}{\sin\theta\cos\theta}\right) + f_{sx} \le f_{yx}$$
$$f_{c1} = \rho_y (f_{yy} - f_{sy}) + v_{ci\max}(\tan\theta - 1)$$

Equilibrium System 5

In equilibrium system 5, it is assumed that the reinforcement in the y direction yields and the crack starts to crush due to shear in the opposite direction as equilibrium system 4.

$$f_{sycr} = f_{yy}$$
$$v_{ci} = -v_{ci\max}$$
$$f_{sxcr} = \frac{1}{\rho_x}\left(\rho_y (f_{yy} - f_{sy}) - \frac{v_{ci\max}}{\sin\theta\cos\theta}\right) + f_{sx} \le f_{yx}$$
$$f_{c1} = \rho_y (f_{yy} - f_{sy}) - v_{ci\max}(\tan\theta + 1)$$

COMPUTATION OF CRACK WIDTHS

Localised Cracking

When the first crack occurs the reinforcement carries the force through this crack. A second crack can only occur if this force becomes larger. This will not happen if there is so little reinforcement that this yields directly after cracking of the concrete. In this case all deformation is localised in the first and only crack. The reinforcement ratio for which this happens is called minimum ratio or critical ratio. Often, codes of practice allow smaller ratios than the critical ratio. This does not need to give problems because even when all deformation localises in one crack the crack width can still be acceptable.

In case of localised cracking the applet uses a crack spacing s = 800 mm which is equal to width of the considered panel. In reality this crack spacing depends on the force flow in the structural element. Often the crack spacing for localised cracking will be substantially larger than that in the case of a distributed crack pattern. Therefore crack widths predicted by the applet need to be interpreted carefully when just one crack occurs.

Distributed Crack Pattern

In case of a distributed crack pattern the crack spacing s_x for loading in the x direction only is (ENV 1992)

$$s_x = \frac{2}{3}\frac{d_x}{3.6\rho_x}$$

where d_x is the bar diameter and ρ_x the reinforcement ratio. The crack spacing s_x for a loading in the y direction only is.

$$s_y = \frac{2}{3}\frac{d_y}{3.6\rho_y}$$

The crack spacing s perpendicular to the crack direction θ is

$$s = \frac{1}{\frac{|\sin\theta|}{s_x} + \frac{|\cos\theta|}{s_y}} .$$

The crack width w is

$$w = s\,\varepsilon_1 ,$$

where ε_1 is the largest principle strain.

COMPUTATIONAL ALGORITHM

The constitutive model provides the stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ that result from the strains $\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$. The applet uses the modified Newton-Raphson method to inverse this relation and compute the strains from imposed stresses.

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}_{\text{new}} = \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}_{\text{old}} + K^{-1}\left(\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}_{\text{imposed}} - \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}_{\text{old}}\right)$$

For every iteration the inverse of the initial stiffness matrix K is used, which can be derived as

$$K^{-1} = \begin{bmatrix}\frac{1}{E_c + \rho_x E_s} & 0 & 0\\ 0 & \frac{1}{E_c + \rho_y E_s} & 0\\ 0 & 0 & \frac{2}{E_c}\end{bmatrix}$$

This algorithm proves to be very robust and sufficiently fast for real-time computations. The iterations start from zero strain and continue until sufficient convergence. The following termination criterion has been implemented.

$$\left\| \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}_{\text{imposed}} - \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}_{\text{new}} \right\| < 0.000001$$

If 10000 iterations have occurred without satisfying the termination criterion the panel is assumed to have failed.

The load-deformation behaviour is computed using load control. The ultimate load is found with the bisection method. For plotting the load-deformation graph the loading is increased proportionally from zero in 100 steps until the ultimate load. The panel behaviour beyond the ultimate load is not computed because the MCFT is not suitable to predict the panel ductility. Moreover, this aspect of reinforced concrete behaviour is often of minor interest to structural designers.

COMPARISON TO EXPERIMENTS

Strength

As an example the strength of panel PV20 from the experimental program of Vecchio and Collins (1986) has been compared to the strength predicted by the applet. The panel has a thickness of 70 mm. The concrete compressive strength is –19.6 MPa and the tensile strength is 1.47 MPa. The maximum aggregate size is 6 mm. The horizontal reinforcement consists of two layers of bars with a diameter of 4.5 mm, a spacing of 25.4 mm and a yield strength of 460 MPa. The vertical reinforcement consists of two layers of bars with a diameter of 2.1 mm, a spacing of 11.1 mm and a yield strength of 297 MPa. Young's modulus of all bars is assumed to be 210000 MPa. Consequently, the reinforcement ratios are 0.0179 and 0.0089 in the x and y direction respectively. The loading is pure shear. The resulting crack spacing is 47 mm and 44 mm in the x and y direction respectively. In the experiment an ultimate load of 4.26 MPa was found. The program Membrane 2000 predicts an ultimate load of 4.63 MPa. The applet predicts an ultimate load of 4.30 MPa, which is 1 % too large.

The results of the other panels of this experimental program are shown in Table 1. The last three columns of the table show the strength found in the experiments, the strength computed with the program Membrane 2000 and de strength computed with the applet. The symbol > in the table indicates that the test needed to be aborted before the ultimate load was obtained.

All but one prediction are less than 20% too large. The exception is panel PV2 for which both Membrane 2000 and the WWW applet predict almost 40% more strength than that found in the experiment. It seems that large deviations can occur for panels with little reinforcement.

The quotient of the strength predicted by the applet and the experimental strength has an average value of 1.01 and a standard deviation of 0.20.

Membrane 2000 and the applet are both based on the modified compression field theory. Nonetheless Table 1 and 2 show differences in the computation results of the programs. Clearly, some aspects of the MCFT have been implemented differently.

Crack Width

Pang and Hsu have performed tests on ten orthogonally reinforced concrete panels loaded in pure shear (Pang 1995). The specimens were 1397 by 1397 mm and 178 mm thick (Table 2.). The data used here has been obtained from (Christiansen 2000). Young's modulus of the bars is assumed to be 210 GPa. Young's modulus of the concrete is assumed to be 30 GPa. The maximum aggregate size a is assumed to be 30 mm. The concrete tensile strengths have been measured from the panel load displacement curves. The loadings σ_{xy} at which the crack widths w have been measured is approximately half the ultimate loading.

The last column presents the quotient of the crack width of the applet and the test. This ratio can be interpreted as a model factor. The average of this ratio is 0.98 and the standard deviation is 0.30.

CONCLUSIONS AND RECOMMENDATIONS

The modified compression field theory (MCFT) seems to be less accurate for concrete with little reinforcement. Comparison with additional experimental data needs to be performed to determine the accuracy of the MCFT.

The implementation of the MCFT in Membrane 2000 and the WWW applet need to be compared in order to determine the cause of the differences in the predicted behaviour.

WWW applets are very suitable to disclose expert knowledge in universities and research institutes to practicing engineers. Many useful design tools can be devised based on models that are larger than a design formula and smaller than a finite element program.

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Table 1. Comparison of experimental and predicted strengths

	ρx	ρy	$f'c$ MPa	ft MPa	Ec MPa	fyx MPa	fyy MPa	Loading Ratio's σxy	σxx	σyy	Test MPa	Membrane MPa	Applet MPa	Applet / Test
PV1	0.0179	0.0168	-34.5	1.94	31400	483	483	1	0	0	> 8.02	8.40	8.30	
PV2	0.0018	0.0018	-23.5	1.60	20400	428	428	1	0	0	1.16	1.56	1.60	1.38
PV3	0.0048	0.0048	-26.6	1.70	23100	662	662	1	0	0	3.07	3.20	2.94	0.96
PV4	0.0106	0.0106	-26.6	1.70	21300	242	242	1	0	0	2.89	2.58	2.57	0.89
PV5	0.0074	0.0074	-28.3	1.76	22600	621	621	1	0	0	> 4.24	4.60	4.13	
PV6	0.0179	0.0179	-29.8	1.80	23800	266	266	1	0	0	4.55	4.80	4.77	1.05
PV7	0.0179	0.0179	-31.0	1.84	24800	453	453	1	0	0	> 6.81	8.20	8.20	
PV8	0.0262	0.0262	-29.8	1.80	23800	462	462	1	0	0	> 6.67	9.35	11.00	
PV9	0.0179	0.0179	-11.6	1.12	8290	455	455	1	0	0	> 3.74	4.30	5.90	
PV10	0.0179	0.0100	-14.5	1.26	10700	276	276	1	0	0	3.97	3.80	3.80	0.96
PV11	0.0179	0.0131	-15.6	1.30	12000	235	235	1	0	0	3.56	3.60	3.60	1.01
PV12	0.0179	0.0045	-16.0	1.32	12800	469	469	1	0	0	3.13	3.69	3.69	1.18
PV13	0.0179	0	-18.2	1.41	13500	248		1	0	0	2.01	1.40	1.40	0.70
PV14	0.0179	0.0179	-20.4	1.49	18500	455	455	1	0	0	> 5.24	6.50	7.60	
PV15	0.0074	0.0074	-21.7	1.54	21700	255	255	0	-1	0	>19.60	error	23.50	
PV16	0.0074	0.0074	-21.7	1.54	21700	255	255	1	0	0	4.12	1.90	1.90	0.46
PV17	0.0074	0.0074	-18.6	1.42	18600	255	255	0	-1	0	21.30	error	20.50	0.96
PV18	0.0179	0.0032	-19.5	1.46	17700	431	412	1	0	0	> 3.04	2.84	2.94	
PV19	0.0179	0.0071	-19.0	1.44	17300	458	299	1	0	0	3.95	4.50	4.30	1.09
PV20	0.0179	0.0089	-19.6	1.46	21800	460	297	1	0	0	4.26	4.63	4.30	1.01
PV21	0.0179	0.0130	-19.5	1.46	21700	458	302	1	0	0	5.03	5.40	5.40	1.07
PV22	0.0179	0.0152	-19.6	1.46	19600	458	420	1	0	0	6.07	6.20	7.10	1.17
PV23	0.0179	0.0179	-20.5	1.49	20500	518	518	1	-0.39	-0.39	8.87	7.40	10.20	1.15
PV24	0.0179	0.0179	-23.8	1.61	25100	492	492	1	-0.83	-0.83	> 7.94	10.60	12.20	
PV25	0.0179	0.0179	-19.2	1.45	21300	466	466	1	-0.69	-0.69	9.12	8.10	9.90	1.09
PV26	0.0179	0.0101	-21.3	1.52	22400	456	463	1	0	0	5.41	6.00	6.00	1.11
PV27	0.0179	0.0179	-20.5	1.49	21600	442	442	1	0	0	6.35	6.60	7.60	1.20
PV28	0.0179	0.0179	-19.0	1.44	20000	483	483	1	0.32	0.32	5.80	5.82	6.00	1.03
PV29	0.0179	0.0089	-21.7	1.54	24100	441	324	1	-0.29	-0.29	5.87	error	4.80	0.82
PV30	0.0179	0.0101	-19.1	1.44	20100	437	472	1	0	0	> 5.13	5.70	6.00	

Table 2. Comparison of experimental and predicted crack widths

	dx mm	sbx mm	dy mm	sby mm	$f'c$ MPa	ft MPa	fyx MPa	fyy MPa	σxy MPa	Test w mm	Membrane w mm	Applet w mm	Applet / Test
A1	10	148	10	148	-42.2	1.47	444	444	1.14	0.00	0.00	0.00	1.00
A2	15	166	15	166	-41.2	1.33	462	462	2.69	0.38	0.39	0.28	0.74
A3	20	197	20	197	-41.6	2.06	446	446	3.83	0.20	0.45	0.24	1.20
A4	25	185	25	185	-42.4	1.67	469	469	5.66	0.20	0.42	0.20	1.00
B1	15	166	10	148	-45.2	1.67	462	444	2.00	0.25	0.33	0.24	0.96
B2	20	197	15	166	-44.0	1.67	446	462	3.03	0.15	0.37	0.24	1.60
B3	20	197	10	148	-44.9	1.77	446	444	2.18	0.38	0.33	0.21	0.55
B4	25	185	10	148	-44.7	1.81	469	444	2.53	0.28	0.32	0.18	0.64
B5	25	185	15	166	-42.8	1.87	469	462	3.58	0.20	0.35	0.20	1.00
B6	25	185	20	197	-42.9	1.31	469	446	4.57	0.20	0.45	0.23	1.15

APPENDIX

Screen shot of the WWW program for predicting the performance of reinforced concrete

Performance of Reinforced Concrete