Shell structures practical

Final report

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Preface

This report was written by two students at Delft University of Technology. For the course 'Shell structures' of the minor 'Bend and Break', we made a shell out of concrete that will be tested at the end of the course. We designed and constructed the shell ourselves.

While reading this report we assumed that the reader has basic knowledge of shell structures and concrete as a building material.

Readers that are particularly interested in how the shell was designed can find this in chapter 1. Readers that are more interested in the work process of making our shell can find this in chapter 2. The calculations used for the hypothesis can be found in chapters 3 and 4. If only the test results are of interest, these can be found in Chapters 5 and 6.

We would like to thank dr. ir. P.C.J. Hoogenboom for his advice and help during the course as well as the technicians who helped cut the timber for the test set-up.

Delft, February 1, 2024
Eline Blommert
Sam Albracht

Summary

This report focuses on the design and testing of a shell structure. The shell was required to withstand a real-life load, meaning it needed to support its own weight and an additional load, such as that caused by a layer of snow when built to scale. The second objective was to estimate the load at which the shell would fail as accurately as possible.
The design chosen for the shell was a dome with a span of 688 mm and a sagita of 285 mm. It was constructed using a paper mache mold, which was covered with iron mesh and next with concrete. The shell was made of concrete because it was easy to mold into the desired diameter. Wooden laths for a grid shell were deemed too fragile unless treated, and the use of concrete simplified the calculations for predicting the failure load. The iron mesh made it possible to apply the concrete to the steep surface at the bottom of the dome.

Figure 1: First test set-up

The shell would be tested using a set-up depicted in figure 1. In this set-up, bricks were placed on a loading platform. This load was through a contraption of strings and wooden blocks transferred to apply a distributed load on the shell. In figure 1 the dome shaped shell can also be seen.

Calculations indicated that the dome was likely too strong to fail when loaded under a distributed load. It was determined that the first thing to fail would be the load platform at a load of 6 kN. There was a risk mentioned as well of the knots in the strings being pulled through the distribution blocks, but the load at which this would happen could not be predicted.
The test results confirmed this hypothesis; the ropes broke at a load of 1,72 kN, but the dome remained intact. Therefore, a decision was made to apply a point load instead of a distributed load. According to calculated predictions, this construction was expected to fail at 2.5 kN, which closely aligned with the actual failure load of 2.54 kN. At this load, a hole was punched in the dome, as shown in figure 2.

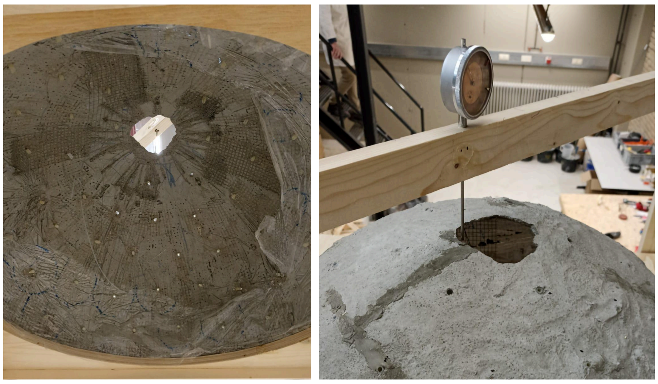

Figure 2: Failed shell (inside and top view

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Introduction

Shell structures derive their strength from their shape and are characterized by their efficient use of material and beautiful shapes (Khan, n.d.). These structures can be found in various forms and are commonly used in architecture and engineering. Some of the world's most famous buildings are in fact shell structures: the Pantheon, Taipei 101, the Sagrada Familia and the Sydney Opera House are just a few examples.

Exploring the principles behind these kinds of structures is a central focus of the course 'Shell Structures' within the minor 'Bend & Break.' In this course, each student group has to perform an experiment on a shell structure that they are tasked to build themselves. **The assignment consists of two goals.The first goal of the assignment is to construct a shell structure that can sustain a real-life load.** A real-life load means that it would be able to carry its own weight and an additional load, that can be caused by a layer of snow, when it is built on scale.**The second goal is to estimate the load at which the shell will fail as close as possible.** At the end of the course the shell structure will be tested and afterwards the load at which it failed will be compared to the failure load that was calculated. The shell will be tested by applying a distributed weight load on the structure. This load will be applied by using strings that pull on the structure as seen in figure 0.1. All strings will eventually be connected by wooden distribution blocks until they come together in one distribution block. A force is then applied to that piece of wood by hanging a platform with sand-lime bricks weighing approximately 2 kg each from it.

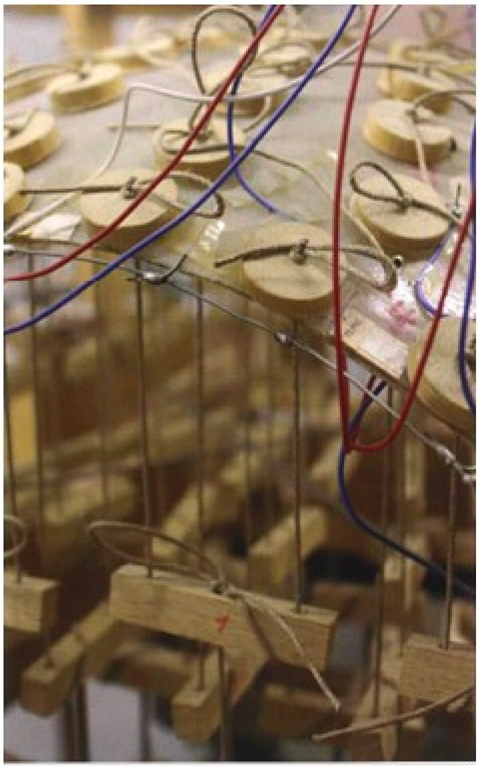

Figure 0.1: Test set-up with distribution blocks

This report will present the steps that were taken to design our shell and is structured as follows. In chapter 1, various designs and construction possibilities were explored. Chapter 2 provides a description of how the shell was made. Following that, chapter 3 presents the results of the concrete tests conducted. Chapter 4 then details the calculations performed to determine the failure load. Subsequently, chapters 5 and 6 describe the tests on the shell along with their results. Finally, chapter 7 includes an analysis of the tests along with a few recommendations.

1 Design phase

First of all it is needed to decide on a design for the shell structure. As almost everything is possible, it is needed to inventorize the possibilities of different shapes, types and materials to make our shell. Therefore in section 1.1 some inspiration is first to be gained. In section 1.2 the shape and material are chosen. With the shape and material it is possible to make some calculations that can be found in section 1.3. Also a SCIA model is made to estimate the shell's performance in section 1.4.

1.1 Shells

Shell structures come in all different shapes and sizes. From rather complicated looking superstructures to relatively small and simple looking shells. They can be found in places all over the world. Some famous examples are King Cross station and the Millenium Dome in London, the Pantheon in Rome and the Lotus temple in Delhi (Figures 1.1.1 to 1.1.4).

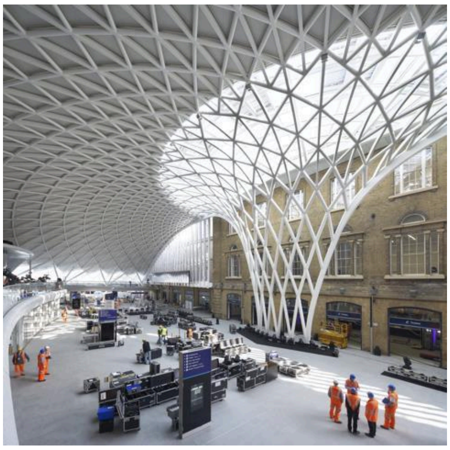

Figure 1.1.1: King Cross station (Khan, n.d.)

Figure 1.1.2: Millennium Dome (Goyal, 2023)

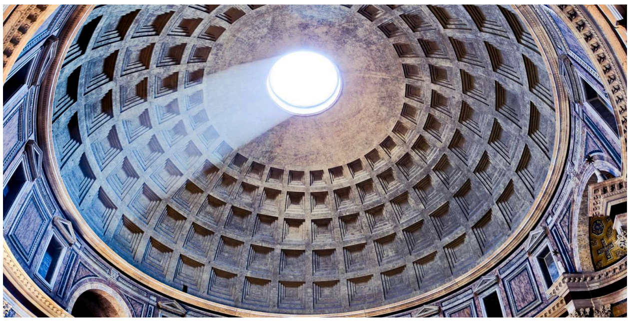

Figure 1.1.3: Pantheon (Wikimedia, 2016)

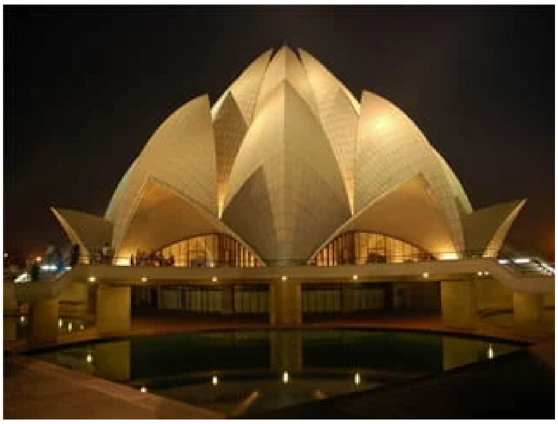

Figure 1.1.4: Lotus temple (Paul, 2017)

These are just a few examples out of many. From these few buildings it can also be seen what the main construction types of shells are. Mainly there are three types: concrete shell structures (Lotus temple and Pantheon), grid shell structures (King Cross station) and membrane shell structures (Millennium Dome). The materials that can be used to make shell structures are almost as numerous as the shapes that can be made. Some shells make use of steel and fabrics whereas others use wood and acrylic or even concrete and marble. From this research it can be concluded that with time and money every kind of shell structure can be made.

1.2 Shape and material

Unfortunately for this course, time and money are not unlimited. Therefore choices had to be made about what shape and material could be realized during the four weeks of the course. To begin with, an inventorization was made about the advantages and disadvantages of the available materials. The materials provided were wooden slats and concrete mortar.

As we had not yet worked with timber before, this would be an interesting material to get to know. The slats would be ideal to make a grid shell structure. This grid could be formed by using glue, nails or steel wire to bind the slats together or by weaving the slats in a way they would support themselves. In the nodes, formed by crossing slats, wires could be tied to be able to apply the distributed load for testing. We were warned however that this type of structure would be difficult to model in the Finite Element Software we are to use in our calculations. As we started to test the materials for the first time in the laboratory, we noticed that the arch radius that could be archived by bending the slats was fairly limited. The slats broke quickly when they were bent too far. There were a lot of imperfections present in the slats as well, these form weak spots in the material, where the slats are even more prone to breaking.
Concrete mortar is quite strong if it has enough time to harden. This material would be perfect for solid shells. This material is easier to model and the arches that could be realized with this material are far steeper. There are some challenges however. The first of these is the consistency of the mortar. If it is too dry, it cannot be shaped easily, but if it is too liquid, it will flow down the structure before it gets time to harden. Next some sort of formwork has to be constructed to act as a support for the wet mortar. For this bendable wooden boards were provided. But these could only bend in one direction at a time, which we found a bit simple. If concrete were to be chosen, a better way would have to be found to construct the formwork. Lastly a solution would have to be found for the attachment of the wires for the test load. Holes could be drilled in the concrete, but this might compromise the shell. Other options would be to install straws or wooden cubes, through which could be drilled without damaging the concrete shell.

With all this in mind about the materials, the brainstorming about the shape began. The first idea was to make a kind of dome with notches which can be seen in figure 1.2.1.

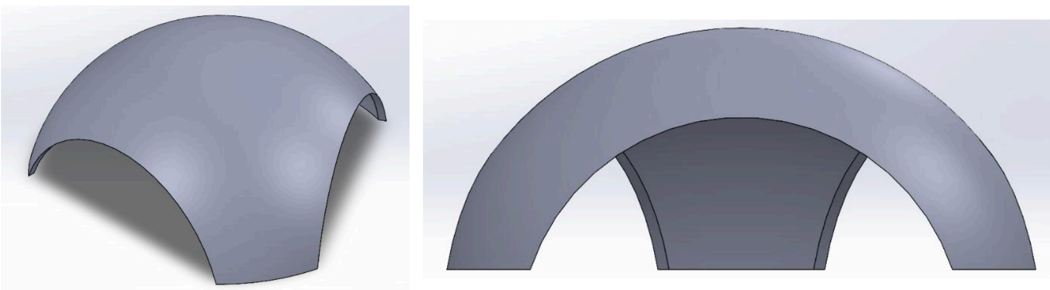

Figure 1.2.1: Design one: Dome with notches

The model of this dome would have a diameter of 700 mm. According to us, the dome should be made either from a wooden grid structure (like figure 1.2.2) or completely out of concrete (like figure 1.2.3). A grid structure was in our opinion more ecstatic so an attempt was made to first make an arch out of wooden slats with the dome's diameter. This turned out to be impossible with the slats that were provided. They could only reach a diameter of about 1 meter without any sort of treatment (steaming for example). Therefore the idea was to make the shell out of concrete just like the pantheon after all. Beside the fact that concrete can be poured into just about any shape, using concrete also has some advantages when modeling the dome in SCIA since it makes for a solid object with a uniformly behaving material.

Figure 1.2.2: Grid structured dome (Von Bülow, 2023)

Figure 1.2.3: Concrete dome (Khan, n.d.)

For making the dome out of concrete we tried using an idea of Heinz Isler which involves hanging fabrics (Figure 1.2.4). His studies involving fabric are particularly intriguing due to the interplay between the fabric's tensile capacity and concrete's compressive strength. The small-scale models, crafted from draped fabric, delineate optimal structural curvatures, with the material predominantly experiencing tension – a quality at which fabric excels. In order to apply this same curvature to concrete, the model is "frozen" with epoxy resins and then flipped 180 degrees, thereby putting the material into compression – a strong characteristic of concrete. This geometric approach is scalable to any desired size (Build LLC, 2009). The straightforward yet elegant connections between geometry and material characteristics captivate our interest so we gave it a try ourselves.

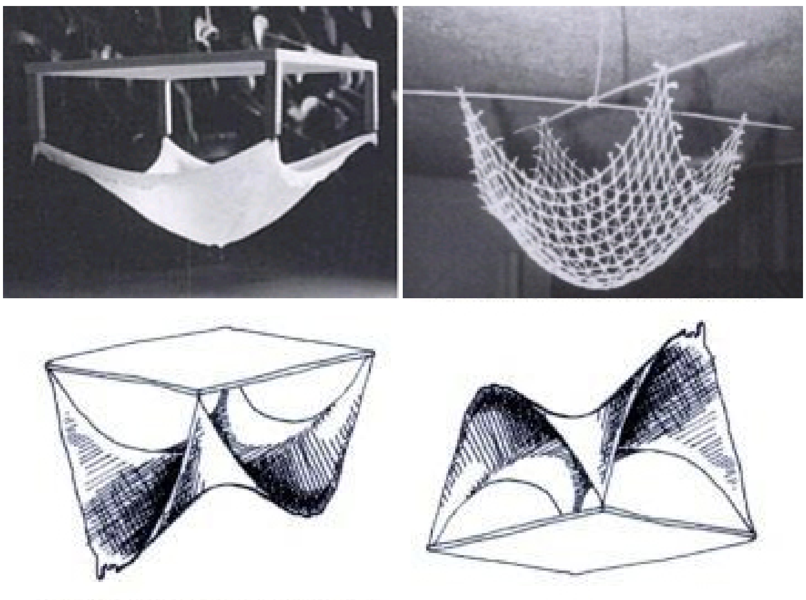

Figure 1.2.4: Heinz Isler's hanging fabrics (Build LLC, 2009)

We tried hanging an old sheet on a triangular frame and covering it with concrete. We soon found out that using Islers idea was not as easy as it seemed. It can be seen that using a sheet will result in an uneven surface full of ridges and folds. Also, when attempting to smear the concrete against the sheet it wouldn't stick at all. It all slid back down to the lowest point. Another problem that occurred was that the sheet allowed water to pass through. This resulted in a very dry concrete mixture inside the sheet. Pictures of this test attempt can be seen in figure 1.2.5.

Figure 1.2.5: Testing of Heinz Isler's hanging fabrics

Due to the problems that occurred using Islers hanging fabric idea, a new approach was needed. Before coming up with something, we first wanted to test if it was possible to apply the concrete on surfaces with steep angles and found out that some support was needed. In figure 1.2.6.a it can be seen that we used a small iron mesh to deliver the needed support for pouring the concrete onto the mold. For the top of this test mold, this worked very well. For the sides there were some complications. Since the shape of this test mold was not very stable, it deformed when smearing the concrete onto it. This deformation can clearly be seen in figure 1.2.7 where the shape is slightly pushed from the side. The deformation causes the iron mesh to rise out of the concrete that was smeared over it (Figure 1.2.6.b). Leaving an uneven coverage as seen in figure 1.2.6.c.

Figure 1.2.6: Testing of pouring concrete on a steep surface. a) Iron mesh. b) Iron mesh visible through concrete. c) Difference in results on the top and on the sides.

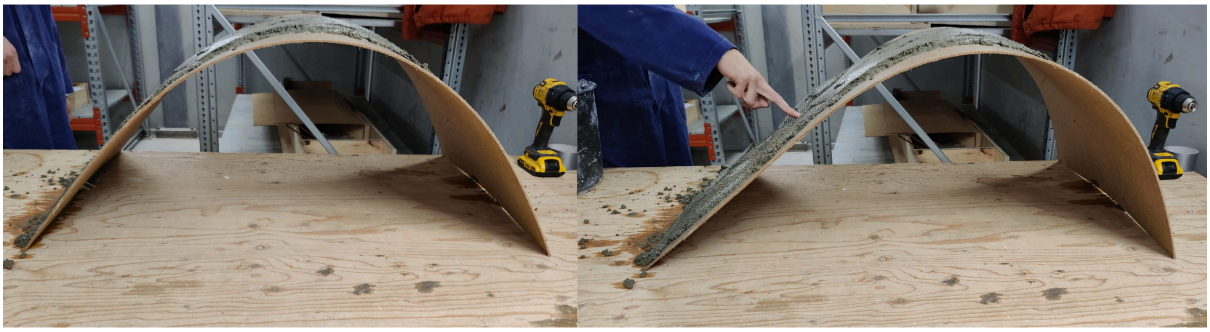

Figure 1.2.7: Deformation of test mold

The conclusions that could be drawn from this test are the following: With a layer of iron mesh as support, concrete can be applied to almost vertical surfaces and the formwork needs to be firm in order to produce a proper layer of concrete that's covering the iron mesh.

With this in mind we started thinking of new ideas for formworks. Paper mache came to mind to be used. To test this idea, we blew up a balloon, tore some newspapers and made some glue from water and flour (in a 1 to 1 ratio). We covered the balloon in several layers of paper mache and let it dry for several days. The result can be seen in figure 1.2.8. This paper mache dome formed the base shape where the concrete will be poured upon. It is much more stable than the board used for the previous test, so the problem of deforming while the concrete is hardening will probably not occur.

Figure 1.2.8: Paper mache mold of a balloon

Before putting the iron mesh over the paper mache dome, the dome is first covered in plastic foil. This is done for two reasons. First reason is so the paper mache dome can easily be removed from the concrete dome since the plastic and concrete won't stick together. The second reason is so the paper mache doesn't get soggy because of the water that's in the concrete. If the paper mache gets wet when smearing the concrete, it might collapse under the weight of the concrete. This also prevents the concrete mixture from drying out as it had in the test with the hanging fabric.

The concrete that will be made consist of four ingredients (Figure 1.2.9): Cement (CEM I 42,5 N), sieve sand (size 0,125-0,250 mm), sieve sand (size 1-2 mm) and water. From the first three ingredients the ratio 1-2-1 will be used respectively. Water will then be added until the right concrete consistency is met. The concrete must not be too wet because then it will slide down. Too dry and the concrete wont get hard. The right consistency can be seen in figure 1.2.10.

Figure 1.2.9: Ingredients concrete mixture

Figure 1.2.10: Concrete mixture used

After the paper mache was wrapped in foil, it was covered in iron mesh (so the concrete will not slide off the steep edges) and then covered with concrete. After letting the concrete dry for a couple of days the dome was hard enough for the paper mache to be removed. This could be done pretty easily. It is to be noted that the concrete test-dome does not look like the dome we wanted to build which was seen in figure 1.2.1. In fact, because we put the concrete leftovers on top of the dome, it looks more like a hat. The result of our test-dome can be seen in figure 1.2.11. From this test it can be concluded that making a dome using a paper mache formwork is possible.

Figure 1.2.11: Results test-dome

Because it seemed very hard to make the three notches of the original design the design will be changed. There is relatively little time to make a concrete dome and there is no time to make a second shell in case the first shell doesn't work. For this reason the original design will be brought back to just a dome of about 12,5 kg (Figure 1.2.12), which will probably take all the available time to build. The ‘real’ dome will be scaled 1:50. Giving it a diameter of 35 meters.
This decision will also make it easier to predict the load that the dome can take, which was one of the goals of the assignment. For a dome there are only moments on the edges whereas the dome with the notches probably has moments everywhere since it resembles more of an arch. This causes the old and new design to behave very differently (Hoogenboom, 2023). Also, for a regular dome more formulas were covered in class and hence it will be easier to predict.

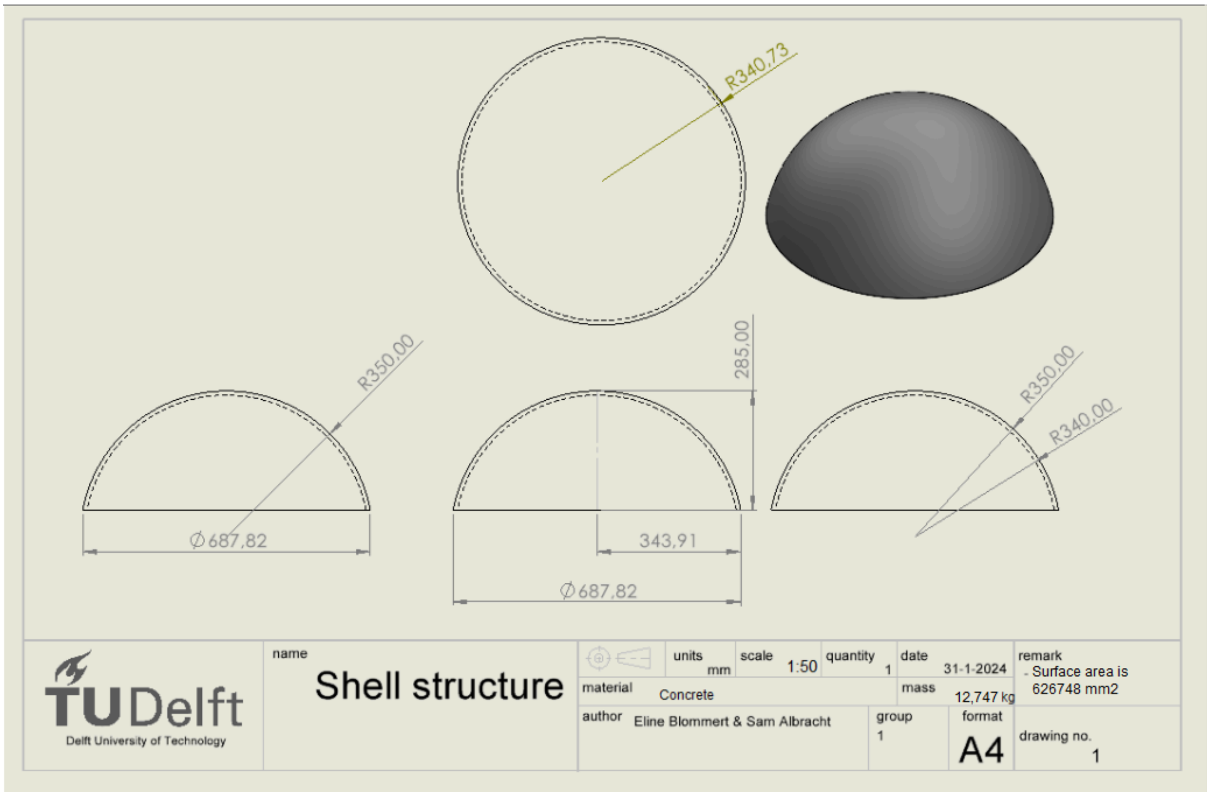

Figure 1.2.12: Technical drawing final design dome

2 Building process

Now the design phase is completed the shell structure can be made. First the paper mache mold was made (Section 2.1). Secondly the reinforcement was added (Section 2.2). THen the concrete was smeared onto the reinforced mold (Section 2.3). Finally the test set-up was made (Section 2.4).

2.1 Paper mache

To make the paper mache mold we first needed to find a spherical object. For our test mold we used a balloon which could be deflated and then removed. This worked well and for our scale model we are therefore using an inflatable yoga ball with a diameter of 700 mm (Figure 2.1.1).

Figure 2.1.1: Inflatable yoga ball

Figure 2.1.2: Paper mache covering

This ball was then covered with strips of newspaper and the same glue as used for the test mold consisting of flour and water (Figure 2.1.2). After this was left to dry for two days the yoga ball was deflated and removed from the paper mache. The dome that was left can be seen in figure 2.1.3. It was made 0,5 cm thick so it would for sure be able to hold the concrete that had to be casted over it.

Figure 2.1.3: Paper mache mold

2.2 Reinforcement

The dome of figure 2.1.3 was cut off to the right height and wrapped in plastic foil (Figure 2.2.1). By wrapping it in foil the paper mache won't get soggy and can easily be removed since the concrete won’t stick to it as mentioned in chapter one.
After this the dome was covered with a fine iron mesh as seen in figure 2.2.2. This made it easier to smear the concrete onto the steeper edges without it sliding off. A positive side effect is that the mesh might help resist the hoop forces since iron is better in withstanding tension than concrete.
At one point the mesh is tied together with iron wire to close the hoop and hence badly connected. For this reason the calculations will only use the concrete strength to predict failure under tension and not take the iron mesh into account. Additionally the mesh cannot be compared with normal reinforcement steel since it has a very low quality. Therefore the effects of adding this material are also neglected.

Figure 2.2.1: Mold covered in plastic foil

Figure 2.2.2: Iron mesh covering

2.3 Concrete

Because of how the shell will be tested, there need to be holes in the structure to put the strings through. Since we are using concrete as a construction material, we would rather not drill holes because concrete is a brittle material. Therefore we put tubes in through the paper mache in such a way the tubes will form openings in the concrete after it is cast.
The tubes have to be evenly distributed over the shell. A cup was used to divide the shell in small even areas. Each area represents a force because of the shell's weight and this force will be applied through the strings.
The marked dome and the tubes that were added can be seen in figure 2.3.1. In the end 78 tubes were placed. The ends of the tubes are covered in tape so no concrete will get in.

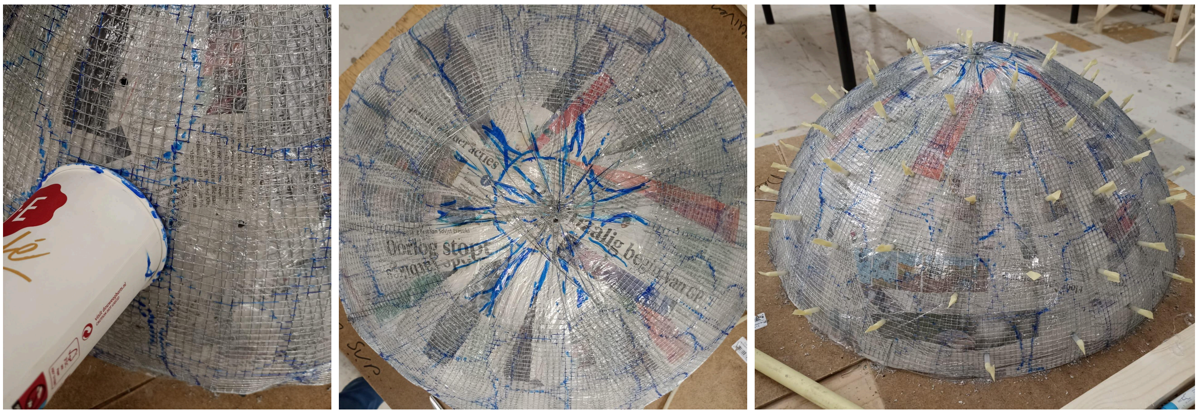

Figure 2.3.1: a) Using a cup to measure even areas. b) Shell mold divided in small areas. c) Tubes placed on shell mold

After covering the paper mache with iron mesh and tubes, the concrete has to be smeared on. The concrete will again consist of four ingredients (Figure 2.3.2): Cement (CEM I 42,5 N), sieve sand (size 0,125-0,250 mm), sieve sand (size 1-2 mm) and water. From the first three ingredients the ratio 1-2-1 will be used respectively and water will then be added until the right concrete consistency is met (Figure 2.3.3).

Figure 2.3.3: Consistency concrete

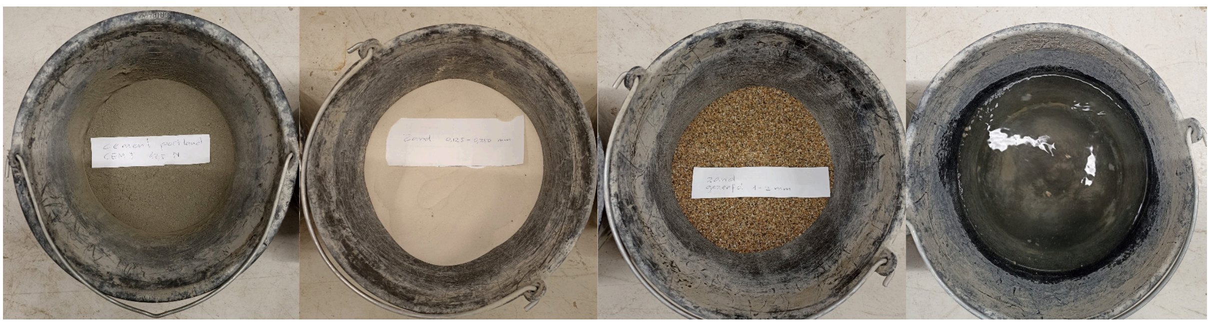

Figure 2.3.2: Concrete components

The finished concrete layer was about 1 cm thick. The just covered dome can be seen in figure 2.3.4.a. Unfortunately, while curing, some cracks started to form because the concrete was shrinking (Figure 2.3.4.b). The cracks might have been formed because the paper mache mold did not allow for shrinkage. The paper mache does not shrink while the concrete does. This might have caused some tension resulting in small cracks.
Another culprit could be that the concrete is more a kind of mortar since there are no ‘big’ aggregates in it. The larger aggregates normally help reduce shrinkage. Without them more volume changes will occur when drying. This will eventually result in more shrinkage forces and thus cracks (Ozerkan, 2023).
We tried to patch the cracks up by smearing a cement and water mixture over them. These dark, patched up areas can be seen in figure 2.3.4.c. In this figure some strings for the test set-up are already made. However, the way they are tied here, they will immediately be pulled through the tubes. This will be solved during the construction of the test set-up.
In addition to the dome itself, samples of the concrete have also been made. These will be tested to determine the concrete strength. From this, it will be determined which compressive and tensile forces the self-made concrete can withstand.

Figure 2.3.4: a) Dome covered in wet concrete. b) Crack in dried concrete. c) Patched up cracks

2.4 Test set-up

As mentioned in the introduction the load will be applied by hanging sand-lime bricks from the construction. In order to do this a frame needs to be built on which the shell can be placed. Underneath the shell there needs to be room for all the strings, distribution blocks and bricks. First the frame needed to be built (2.4.1). After that the string-distribution block testing contraption followed (2.4.2).

2.4.1 Frame

The frame needed to support the shell and had to provide enough space for the string-distribution block contraption and the loading plate with bricks. Therefore the frame has to have the following dimensions: 800x800x1200 mm (LxWxH). First, all the needed wood was cut to length (Figure 2.4.1). Then the two sides were assembled separately (Figure 2.4.2) after which they were attached together in one piece.

Figure 2.4.1: Timber used for frame

Figure 2.4.2: One side of the frame

2.4.2 Strings

The shell was placed on top of the frame on a wooden plate with a round hole in it. A series of strings were pulled through the tubes. To prevent the strings from being pulled back through the tubes again and to distribute the load, a knot is tied after using a wooden disk and a sheet metal ring as can be seen in figure 2.4.3.a. The disk also has a rubber layer which makes for a smoother transfer of forces onto the shell (Figure 2.4.3.b).
Every set of two strings was then connected to a distribution block with a new string in the middle, which had to evenly transfer the force of the lower string into the two following strings above it. These connections were made until the strings ended up in one distribution block (Figure 2.4.3.c).
Since the lower strings have to carry more force they have to be thicker and thus stronger then the strings that are pulled through the shell on top. Therefore three kinds of rope were used, yellow, white and orange colored, all with different specified strengths. For the same reason the lower distribution blocks have to be thicker, so they do not bend or in the worst case break.
The final test set-up can be seen in figure 2.4.3.d. Here the shell stands on the frame and the string contraption is attached. During the test a load platform will be attached to the final distribution block by means of a steel rod. This plateau will be loaded with sand-lime bricks to generate a force on the shell.

Figure 2.4.3: a) Distribution disk on the shell. b) Side view distribution disk. c) String-distribution block testing contraption. d) Final test set-up

The steel rod will be threaded. This means it can be attached to the last distribution block by means of a nut at the top. The hole in the distribution block has been made in the shape of a cone. This way the rod can stay vertical in case the platform is loaded unevenly. The force on the shell will stay evenly distributed as can be seen in figure 2.4.4.

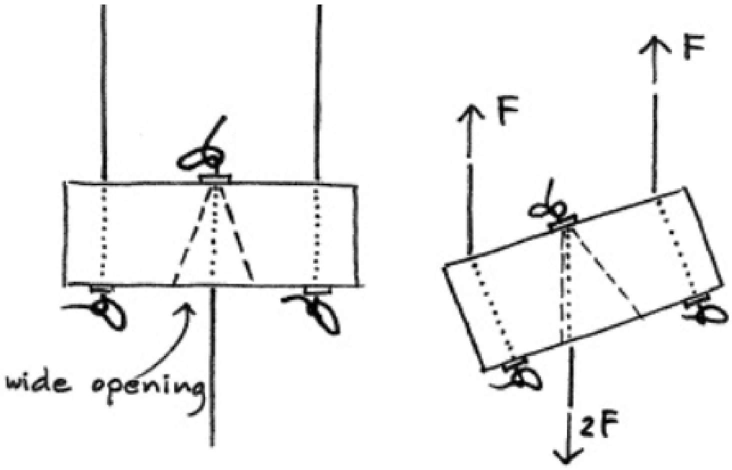

Figure 2.4.3: Working of cone shaped hole (Hoogenboom, 2023)

3 Analyzing of cube tests

The concrete needs to be tested so it is known at which force the concrete will collapse, either due to compression or tension. This chapter will explain how the test for compression (3.1) and how the test for tension (3.2) was executed and what the results were.

3.1 Compression

First some samples were tested in compression. The samples are loaded and compressed by a hydraulic press as seen in figure 3.1.1. The force through the concrete increases until the sample fails and cracks. The maximum compression force can then be told from the instruments on the press.
This test was done three times. Since the samples were not completely smooth on the compression surfaces, two more tests were executed where the sample was placed between two plates of wood. The wooden plates will transfer the load more evenly on the sample surface.
From the tests the force at failure is known and from the shape of the sample the smallest diameter can be measured. (The smaller diameter will have the highest stress and is therefore used in the calculations.) This diameter was 3 cm wide. The measured forces and therefrom calculated stresses can be found in table 3.1.1.

Figure 3.1.1: Compression tests

Table 3.1.1: Results compression test

Test	**Force [kN]**	**Stress [kN/m2]**
1	6,7	9483
2	2,9	4105
3	5	7077
4 (with wood)	5,6	7926
5 (with wood)	8,4	11890
Average	**5,72**	**8096**

3.2 Tension

After the compression tests were done, some samples were tested for tension. This was done by doing a brazilian splitting test. By putting a sample on its side in the hydraulic press the tension in the sample can be calculated if the dimensions of the sample and the cracking force are measured. The way the sample is placed can be seen in figure 3.2.1. The formula to calculate the stress due to tension is as follows:

$$\sigma = \frac{2 \cdot F}{\pi \cdot D \cdot t}$$

Where:
σ = stress [kN/m2]
F = compression force [kN]
D = diameter of the cylinder (here 3 cm)
t = length of cylinder (here 3,2 cm)

Figure 3.2.1: Brazilian splitting test

The measured splitting forces and the calculated stresses can be found in table 3.2.1.

Table 3.2.1: Results brazilian splitting test

Test	**Force [kN]**	**Stress [kN/m2]**
1	2,5	1659
2	2,4	1592
3	1,8	1194
Average	**2,2**	**1482**

From these tests can be concluded that the shell will probably fail if compression stresses reach about 8,1 MPa or if tensile stresses reach about 1,5 MPa.

4 Calculations

To estimate the load at which the shell will fail as close as possible, it is necessary to calculate the failure load of a few components. It needs to be calculated which component, shell, rope or frame, will probably fail first and under what load. Therefore this chapter will cover the calculations that were performed to estimate the load at which the structure will fail. First some hand calculations are covered in section 4.1 and after that a SCIA simulation will indicate if the hand calculations came down to the right conclusions (4.2).

4.1 Hand calculations

4.1.1 Shell crushing

If the shell crushes, this will most likely happen at the bottom of the dome, as the forces are largest here. To calculate the force the shell can withstand before it crushes, the area of the shell base has to be multiplied with the crushing strength of the concrete.
The area of the shell base is:

$Area_{base} = \pi l t = \pi \cdot 688 \cdot 10 = 21\,614\, mm^2$

Where:
l = span [mm] (Figure 4.1.1)
t = thickness [mm]

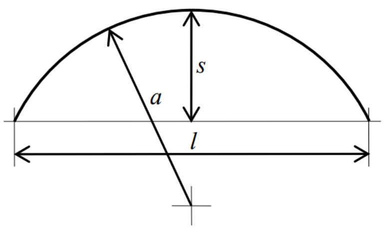

Figure 4.1.1: Dimensions dome (Hoogenboom, 2023)

The span has previously (chapter 1.2) been calculated using the formula:

$a = \frac{1}{2} \cdot s + \frac{1}{8} \cdot \frac{l^2}{s}$

The crushing strength of the concrete (as determined in chapter 3) is 8096 kN/m^2 or 8,1 MPa. This means the force at which the shell will crush is:

$F_{crushing} = Area_{base} \cdot \sigma_{crushing} = 21\,614 \cdot 8,1 = 173\,129\, N = 173\, kN$

4.1.2 Shell buckling

The formula used to calculate the force per length-unit the shell can withstand before buckling is the following:

$n_{buckling} = -\ 0,6 \frac{Et^2}{a}$

Where:
E = modulus of elasticity of the concrete [MPa]
a = radius of dome arch [mm] (Figure 4.1.1)

To use this formula, first the modulus of elasticity of the concrete has to be calculated.

$$E = 22\,000\left(\frac{f_{cm}}{10}\right)^{0,3} = 22\,000\left(\frac{f_{ck}+8}{10}\right)^{0,3} = 22\,000\left(\frac{8,1+8}{10}\right)^{0,3} = 25\,379\ MPa$$

Where:

f_{ck} = concrete strength (chapter 3.1)

Using this in the buckling formula gives:

$$n_{buckling} = -\ 0,6\frac{25\,379 \cdot 10^{2}}{350} = -\ 4351\ N/mm = -\ 4,35\ kN/mm$$

To get to the buckling force, this has to be multiplied with the circumference of the dome.

$$F_{buckling} = n_{buckling} \cdot circumference = n_{buckling} \cdot \pi l = -\ 4,35 \cdot \pi \cdot 688 = -\ 9404\ kN$$

In reality, structures usually buckle before reaching this buckling force, due to imperfections.To take this into account, the buckling force is reduced, using a knockdown factor of ⅙.

$$F_{buckling,\,cr} = \frac{1}{6} \cdot F_{buckling} = \frac{1}{6} \cdot -\ 9404 = -\ 1567\ kN$$

4.1.3 Shell fracture

The shell can fracture due to the tensile forces in the dome. These occur at the top of the dome and at the bottom (Figure 4.1.2), but those at the bottom are largest. For this calculation we disregard the reinforcement of the concrete, which is not that strong anyway, and focus solely on the tension the concrete itself can suffer. This means the actual fracture force may be higher than the one calculated. How much higher depends on the strength of the reinforcement and on how well it is connected. The tensile strength of the concrete (as determined in chapter 3) is 1482 kN/m^2 or about 1,5 MPa. The fracture stress is calculated as follows:

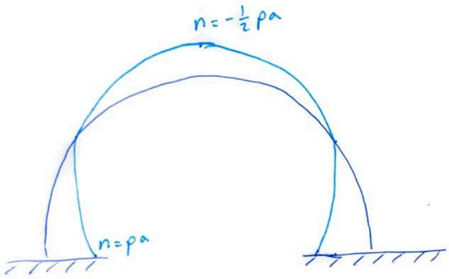

Figure 4.1.2: Tensile line forces in dome

$$p_{fracture} = \frac{n}{a} = \frac{\sigma_{tensile} \cdot t}{a} = \frac{1,5 \cdot 10}{350} = 0,043\ N/mm^{2}$$

The fracture force can be calculated by multiplying this with the area of the shell.

$$Area_{shell} = 2\pi as = 2\pi \cdot 350 \cdot 285 = 626\,748\ mm^{2}$$

Where:

s = sagita of dome [mm] (Figure 4.1.1)

$$F_{fracture} = p_{fracture} \cdot Area_{shell} = 0,043 \cdot 626\,748 = 26\,861\ N = 27\ kN$$

4.1.4 String

Three kinds of rope are used in the testing set-up: yellow, white and orange. These all have different strengths. Starting with the yellow rope, this can bear at most 0,5 kN. In the lowest layer of distribution wood, where this type of rope is used, twenty strands carry the load. This means the structure can carry:

$$F_{rope,\,yellow} = 20 \cdot 0,5 = 10\ kN$$

As for the white rope, the least amount of threads carrying the load here is 10. One string can hold 1,7 kN, thus:

$$F_{rope,\,white} = 10 \cdot 1,7 = 17\ kN$$

The orange rope carries at most ⅘ of the load and one string can carry 7,5 kN.

$$F_{rope,\,orange} = \frac{5}{4} \cdot 7,5 = 9\ kN$$

An additional risk regarding the rope construction is that the knots in the strings could be pulled through the distribution blocks.

4.1.5 Load platform

The load platform has been constructed by dr. ir. P.C.J. Hoogenboom. He has indicated that it could hold 6 kN before it would break. This means:

$$F_{load\ platform} = 6\ kN$$

4.1.6 Steel bar

The strength of the steel is 600 MPa, and the bar has a diameter of 10 mm. The force the bar should withstand is therefore:

$$F_{bar} = \frac{1}{4}\pi d^2 \sigma = \frac{1}{4}\pi \cdot 10^2 \cdot 600 = 47\,124\ N = 47\ kN$$

Where:
d = diameter of bar [mm]
σ = steel strength [MPa]

This force might be slightly lower, as the bar is threaded. This can decrease the strength of the bar. The failure load of the steel bar is however far higher than the lowest failure load so far, so it is unlikely that it will fail first, even if the failure load is lower due to the thread.

4.1.7 Distribution wood

Up until now the lowest failure load is 6 kN. For the distribution wood, the deflection at this load will be calculated to see if it will fail before a load of 6 kN is reached. The dimensions of the blocks are 4,8x2,1x14,5 cm (bxhxl).

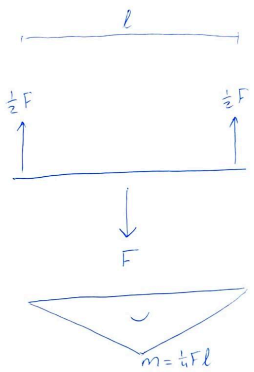

Figure 4.1.3: Distribution block

$$I = \frac{1}{12}bh^3 = \frac{1}{12} \cdot 48 \cdot 21^3 = 37\,044\ mm^4$$

$$E_{wood} = 7000\ MPa$$

$M = \frac{1}{4}Fl = \frac{1}{4} \cdot 6000 \cdot 145 = 217\,500\ Nmm$

$u = \frac{1}{12} \cdot \frac{Ml^2}{EI} = \frac{1}{12} \cdot \frac{217500 \cdot 145^2}{7000 \cdot 37044} = 1,5\ mm$

This deflection is very small. Therefore it is not plausible the wood will fail before a load of 6 kN is reached.

4.1.8 Hypothesis

From all hand calculations a prediction can be made as to at what load and at what point the structure will fail. If the shell itself will fail, this will probably happen due to tension, leading to shell fracture. This will happen at a load of 27 kN. More probable however is that the test set-up will fail before the shell. The load platform is expected to fail at a load of 6 kN and the orange rope at 9 kN. An overview of the failure loads of the different components can be found in table 4.1.1.

Table 4.1.1: Overview failure loads of different components

Component	Failure load [kN]
Shell crushing	173
Shell buckling	1567
Shell fracture	27
String	9
Load platform	6
Steel bar	47

4.2 Scia calculations

The real shell with a span of 35 meters was analyzed using a FEM program called SCIA Engineer. Since the stresses at failure are the same for the full scale shell and the small model shell, failure of the big shell will help estimate failure of our scale model.

To start with, the self weight of the structure will be calculated. Knowing the thickness and radius of the shell it can be calculated how much the shell weighs. From the weight and the surface of the sphere it can be calculated how much the weight load per surface area is. Since a computer model of the sphere was already made, the weight and surface area can be obtained from the model after giving a value for the density. Given a density of 2100 kg/m^3, the weight of the shell is 1593 tons with a surface area of 1566 m^2. Since the concrete was not compacted a relatively low density was chosen. From these weight and surface value's it is calculated that the force per area will be:

$$\frac{1593396*9.81}{1566,869} = 9976 \approx 10\ kN/m^2$$

For the first model, two types of load were added on the surface of the shell. The previously calculated 10 kN/m^2 self weight load and a 1 kN/m^2 snow load. This load combination will be analyzed using the Newton Rapson method since this method is more accurate than linear elastic methods. From the analysis report of this simulation it can be seen that the concrete dome will easily hold this load with little deflection, compression or tension.

The figures holding the results of the SCIA results can also be found in appendix 1. Here they are enlarged to make it easier to read them.

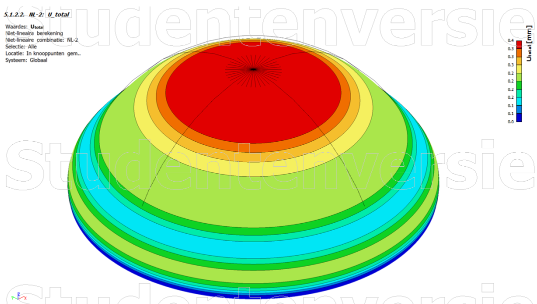

Figure 4.2.1: Deformation at a load of 11 kN/m^2

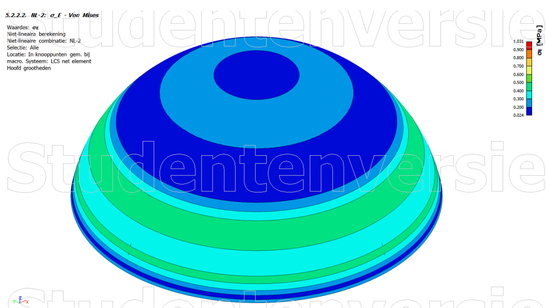

Figure 4.2.2: Von Mises stresses at 11 kN/m^2

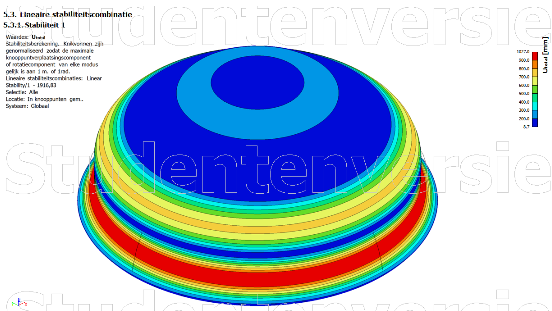

Figure 4.2.3: Buckling analysis at 11 kN/m^2

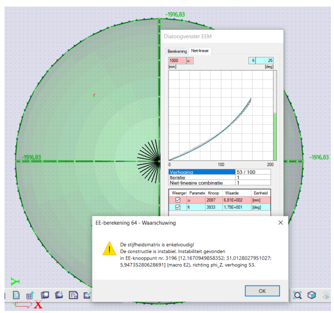

Figure 4.2.4: Diversion of Newton Raphson method

From figure 4.2.1 it can be seen that the maximum deformation with this load combination will be 0,4 mm. It is also shown that the Von Mises stress is only 1,031 MPa at most (Figure 4.2.2). This stress can not clearly be seen in the figure and will probably occur at one of the edges (edge disturbance). The last figure (4.2.3) tells at what load the shell will buckle. This happens at a load 1916,83 times the current load.

The load will be multiplied by this factor to see if the non-linear method can be computed. It turned out that the method diverges at load increment 53 (Figure 4.2.4). Therefore the load factor has to be multiplied by $\frac{53}{100}$. This gives the load that is often maximum (Hoogenboom, 2023) and which will be used for a next evaluation. The results of this analysis can be found below.

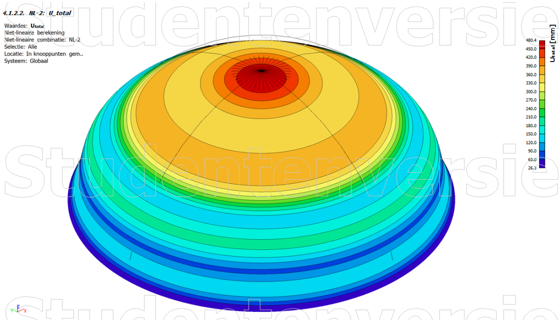

Figure 4.2.5: Deformation at a load of 11175 kN/m^2

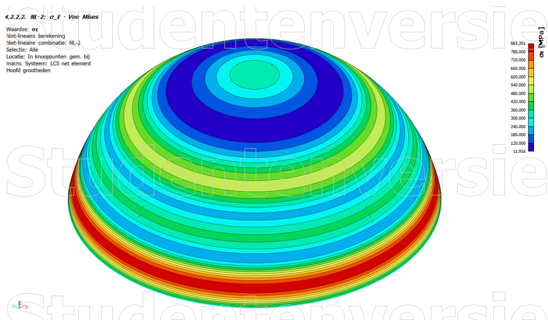

Figure 4.2.6: Von Mises stresses at 11 175 kN/m^2

It can be seen in figure 4.2.5 that the deformation is now 480,4 mm at most. This already seems pretty much for a concrete dome. In figure 4.2.6 it is shown that the Von Mises stress is now 883,351 MPa. This is very high since the concrete already crushes at 8,1 MPa and tears at 1,5 MPa (chapter 3). Therefore it is concluded that the shell will not buckle before it crushes or tears. This can also be seen in figure 4.2.7 which shows a strong tension ring at about half the height of the dome. This is more likely to be a spot where the concrete dome will fail.

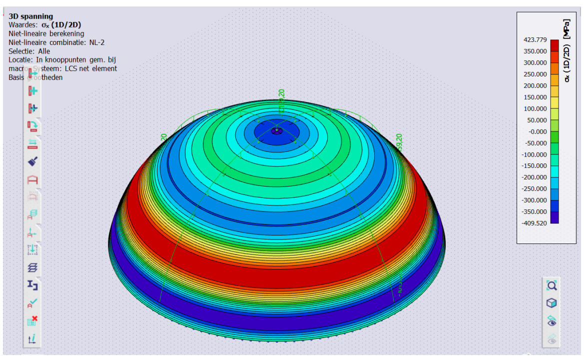

Figure 4.2.7: Stresses in dome at a load of 11175 kN/m^2

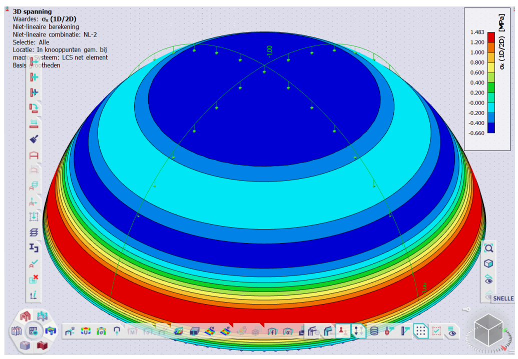

Figure 4.2.8: Stresses in dome at a load of 35 kN/m^2

After another simulation with a combined load of 35 kN/m^2 it can be seen in figure 4.2.8 that the tension ring has a stress of 1,483 MPa. This is very close to the limit of 1,5 MPa the concrete can withstand. It can also be seen from this figure that the compression stresses are very low (at most 0,66 MPa) compared to the 8,1 MPa that the dome can carry.
The failure load of the dome as calculated from the SCIA model is thus 35 kN/m^2. It has to be noted that this situation is relatively close to what was calculated in section 4.1.3. There it was said the dome would fail at 27 kN/m^2 due to tension. This is very close to the 35 kN/m2 of the simulation. Nevertheless, both 27 and 35 kN/m^2 are still way more than the rope or loading platform can carry. These still are most likely to be the points of failure.

5 Distributed load test

In this chapter the testing of the shell is discussed. The shell is subjected to a distributed load in two set-ups. The set-ups, and the execution of the tests are explained and the results of the tests are given.

In chapter 4 it was predicted that the structure will fail at 6 kN, due to failure of the load platform. This means it will fail when 296 bricks are stacked on the platform, as the platform itself weighs 9 kg. This is all under the condition that the knots in the ropes do not get pulled through the holes of the distribution blocks, which was mentioned to be a risk in chapter 4.1.4.

5.1 First set-up

For the first test, the set-up described in chapter 2.4 is used as can be seen in figure 5.1.1. A device to measure the deflection in the top of the dome was added to the set-up.
Before the start of the test it was concluded that 296 bricks probably require too much space under the shell. Therefore this amount can not be loaded on the structure in this set-up and the structure will probably not fail this way.
In the first attempt at loading, one layer of bricks was added to the platform. At that point however, the bottom of the platform had hit the floor due to the strain in the ropes. This was tried to be fixed by increasing the initial height of the load platform to the floor. Yet, if this was increased too much, there would not be any space left to stack the bricks. With the lifted platform two layers of bricks could be added before the platform hit the ground again (Figure 5.1.2).
In this set-up the structure was loaded with 56 bricks (112 kg) and the load platform (9 kg). The total load was thus 1,2 kN at which the structure did not fail, as predicted.

Figure 5.1.1: First test set-up

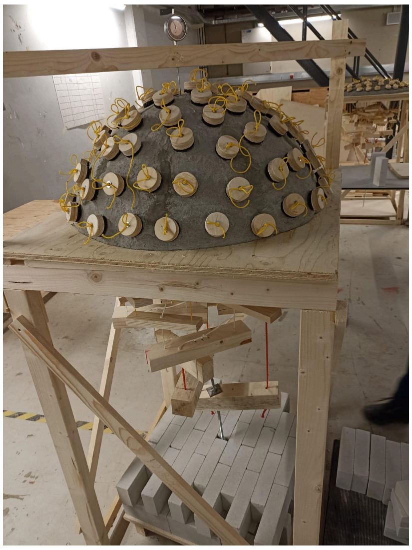

Figure 5.1.2: Shell loaded with 56 bricks

5.2 Second set-up

Figure 5.2.1: Second test set-up

As the original set-up could not accommodate enough bricks to reach the failure load, it was raised by 1,2 meters by placing it on steel beams as can be seen in figure 5.2.1. The loading platform was altered so more bricks would fit on the platform (the new weight was 10 kg) and the test was executed again. When three layers of bricks were stacked on the platform, the deflection was read from the device. The load at that moment consisted of 60 bricks and the platform and was thus 1,3 kN. The deflection at that load read 1,63 mm as can be seen in figure 5.2.2.

After this reading, the structure was loaded further. At a load of 1,72 kN it failed. First a few of the knots in the yellow ropes were pulled through the distribution blocks (Figure 5.2.3.a), next the orange rope at the bottom of the structure snapped (Figure 5.2.4.b). One of the other orange strings can be seen in figure 5.2.3.c and 5.2.3.d. It has been partly pulled through the distribution block and metal body ring. This ring was starting to cut into the rope. This was probably the reason the other string snapped.
The failure load was far lower than predicted, but the risk of the knots being pulled through the blocks was acknowledged. It could not be predicted at what load this would have happened. Now it is known that this is at around 1,72 kN.

As the test structure was the component that failed first and the shell itself was still whole, it was thought to be interesting to see if it would fail if a point load was applied instead of a distributed load. This test will be discussed in chapter 6.

Figure 5.2.2: Deflection reading

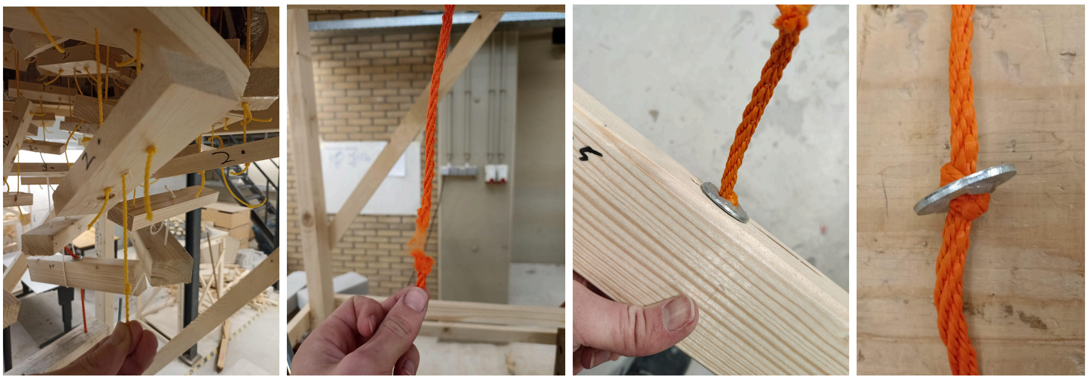

Figure 5.2.3: a) Yellow rope pulled through distribution block. b) Snapped orange rope. c) Knot in orange rope pulled through distribution block. d) Knot in orange rope pulled through metal ring

6 Point load test

As discussed in chapter 5, the shell did not fail due to the distributed load applied to it. As it is essential for the shell to fail to draw conclusions about its strength, the shell has been loaded with a point load next. First a new hypothesis for the failure load is calculated (6.1), then the new test set-up is described (6.2) and lastly the test itself and its results are discussed (6.3).

6.1 New hypothesis

The formula used to calculate the maximum stresses in the dome due to a point load at the top is the following:

$\sigma = \frac{n}{t}$

$n = -\frac{\sqrt{3}}{8} \cdot \frac{F_{point}}{t} \cdot \sqrt{1 - \nu^2}$

Where:

t = thickness [mm]

F_{point} = point load [N]

ν = poisson ratio [-]

The maximum stress the concrete can withstand is 8,1 MPa. This leads to the following:

$n = \sigma \cdot t = - 8,1 \cdot 10 = - 81\, N/mm$

$F_{point} = -\frac{8}{\sqrt{3}} \cdot \frac{nt}{\sqrt{1-\nu^2}} = -\frac{8}{\sqrt{3}} \cdot \frac{-81 \cdot 10}{\sqrt{1-0,15^2}} = 3784\, N = 3,8\, kN$

However, we wanted to take imperfections into account, because of all the holes added to accommodate the first set-up. Therefore we used a knockdown factor of ⅔ . This resulted in a predicted failure load of 2,5 kN.

6.2 New test set-up

For the new test set-up the string-distribution block testing contraption and the distribution discs were removed from the structure. This way only the shell, the frame and the load platform with the steel bar were left. A hole had to be drilled in the top of the dome. The platform was through the steel bar directly attached to the shell. Between the shell and the nut keeping the bar in place, one distribution disc was added (Figure 6.2.1). This makes for a smoother transfer of the force. The finished set-up can be seen in figure 6.2.2.

Figure 6.2.1: Steel bar attached to top dome

Figure 6.2.2: Finished second test set-up

6.3 Results

The deflection was read at two points during the test. First when the platform was loaded with 60 bricks, resulting in a load of 1,3 kN. At that moment the deflection was 1,65 mm (Figure 6.3.1.b). The second reading was at a load of 2,2 kN (105 bricks and the platform), resulting in a deflection of 2,79 mm (Figure 6.3.1.c).

Figure 6.3.1: a) Reference at start of test. b) Deflection at 1,3 kN. c) Deflection at 2,2 kN

The shell failed when it was loaded with 2,54 kN (122 bricks and the platform). The distribution disc was pulled clear through the shell, resulting in a hole in the top of the dome. Beside this hole the shell was still intact. It has thus failed due to crushing and punching shear (Figure 6.3.2).

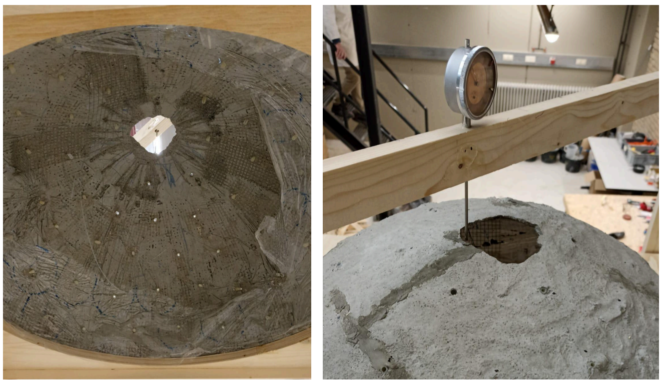

Figure 6.3.2: Failed shell (inside and top view)

7 After testing

Now the tests have been performed, conclusions can be drawn. These can be found in section 7.1. In section 7.2 recommendations will be given.

7.1 Conclusions of the results of the test

The technical drawing of the shell constructed for this practical can be seen in figure 7.1.1. It will have a span of 688 mm, a sagita of 285 mm and a thickness of 10 mm. The finished shell is shown in figure 7.1.2.

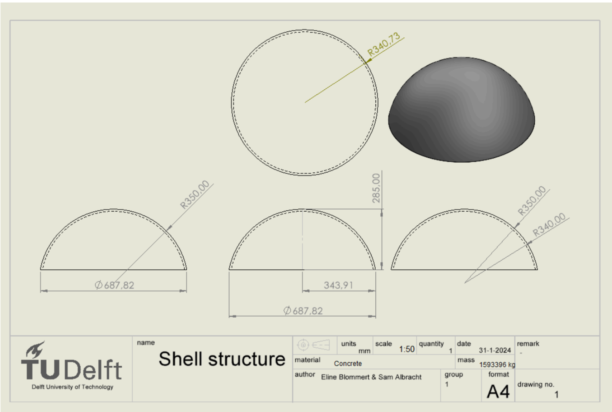

Figure 7.1.1: Technical drawing shell

Figure 7.1.2: Finished shell

The shell is made of hand-made concrete. A few samples of this concrete have been tested in a compression test and a brazilian splitting test. From these tests it was concluded that the average crushing strength of the concrete was 8,1 MPa and the average tensile strength was 1,5 MPa.
With the information gained from the concrete tests a hypothesis could be stated regarding the failure load and mode of the construction. For all possible failure mechanisms the failure load was calculated (Table 7.1.1). The structure was predicted to fail at a load of 6 kN due to failure of the load platform. Furthermore there was a risk of the knots in the strings of the testing set-up to be pulled through the wooden distribution blocks. However, the load at which this would happen could not be predicted.

Table 7.1.1: Overview failure loads of different components

Component	**Failure load [kN]**
Shell crushing	173
Shell buckling	1567
Shell fracture	27
String	9
Load platform	6
Steel bar	47

When testing the shell under a distributed load, the structure failed at 1,72 kN. At this load a few of the knots in the yellow ropes were pulled through the distribution blocks (Figure 7.1.3.a) and the orange rope at the bottom of the structure snapped because the metal distribution ring had cut into the rope (Figure 7.1.3.b & 7.1.3.c).
This failure load was far lower than the predicted 6 kN. The risk of the knots being pulled through the blocks was however previously acknowledged. It only could not be predicted at what load this would have happened.

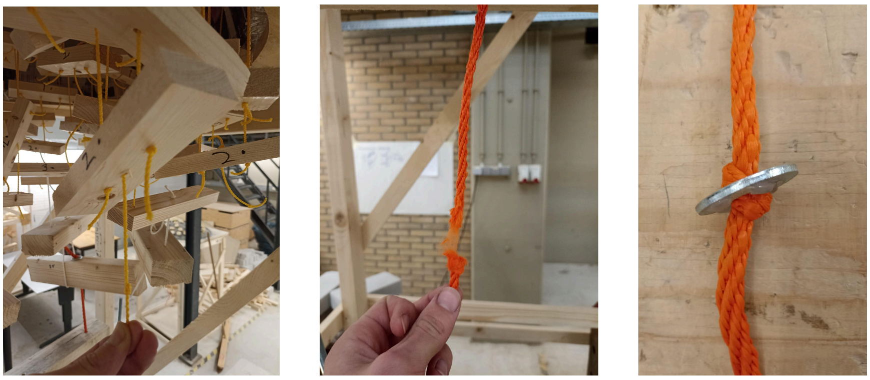

Figure 7.1.3: a) Yellow rope pulled through distribution block. b) Snapped orange rope. c) Knot in orange rope pulled through metal ring cutting in it.

Next the shell was tested under a point load. The predicted collapse load for this loading condition was 2,5 kN. The structure failed at 2,54 kN due to crushing and punching shear (Figure 7.1.4), making this an accurate prediction.

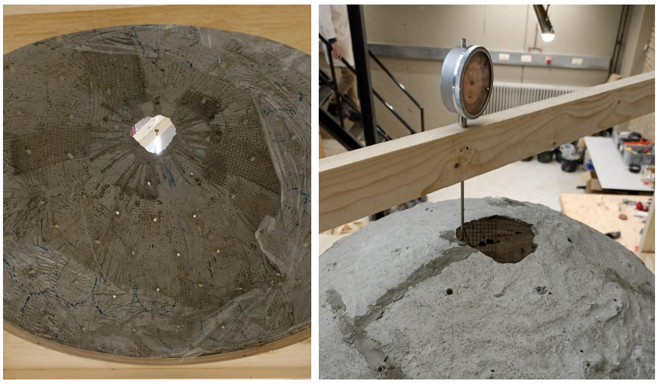

Figure 7.1.4: Shell failed due to punching of point load

From the test with the distributed load it can be concluded that the model shell can at least carry 1,72 kN, which translates to 2,74 kN/m^2. The full scale shell should therefore be able to carry a load of 2,74 kN/m^2 as well. The self weight of the full scale shell is however about 10 kN/m^2. From this experiment unfortunately no conclusions can be drawn as to whether or not the full size shell will be able to withstand this force.
From the test with the point load it can be concluded that the full scale shell will fail at a load of 6350 kN, as these forces behave according to the following relation:

$$F_{model} = \frac{1}{scale^2} \cdot F_{full\ size} = \frac{1}{50^2} \cdot F_{full\ size}$$

The deflection of the shell was measured at a load of 1,3 and of 2,2 kN. At these loads the deflection was respectively 1,65 and 2,79 mm. This means the full scale shell will deflect with 16,5 mm at a point load of 3250 kN and with 27,9 mm at a point load of 5500 kN. These deflections are fairly low and should be acceptable.

7.2 Recommendations

If the shell were to be constructed and tested again, there are several recommendations to consider. Firstly, careful attention should be paid to ensure that the shell is not overbuilt. Since our shell was much stronger than the maximum load that could be applied using the initial test set-up, adjustments had to be made to this set-up. This added extra work and negated a significant amount of previous effort. It would have been more efficient if the testing had initially been designed for a point load, and the construction had been prepared accordingly. In that case, only one hole in the dome and one distribution disc would have needed to be constructed, while also eliminating the need for distribution blocks.

If a test with a distributed load is executed, using ropes and distribution woods after all, it is advised to make sure the ropes can not be pulled through the woods. This might be done by tying a better/bigger knot or perhaps by putting something in the knot that makes it impossible for it to slip through.

From the calculations it could be concluded that the shell could be constructed with a slightly lower thickness. The theoretical failure load is about 3 times higher than the calculated self weight. It is recommended to look into this and to determine the thickness necessary to carry the self weight and a small additional load. It has to be noted that it is probably very hard to make a thinner shell model while making sure the iron mesh gets covered properly. This is also the reason why our shell was 1 cm thick. If a thinner shell is required, maybe another building technique has to be considered.

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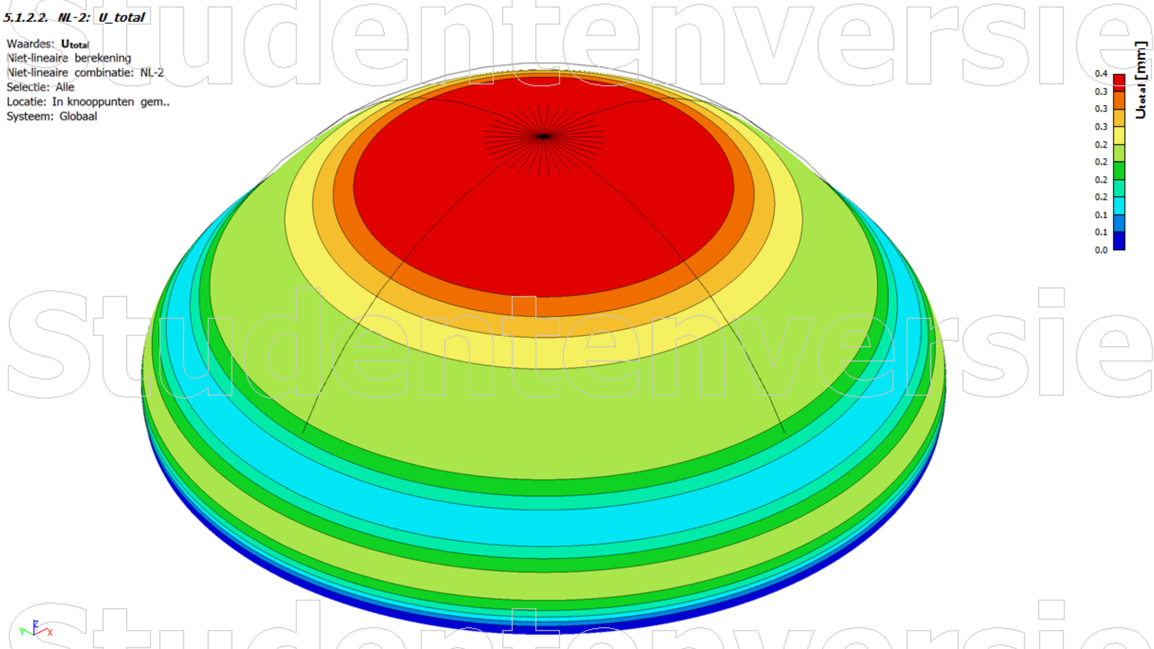
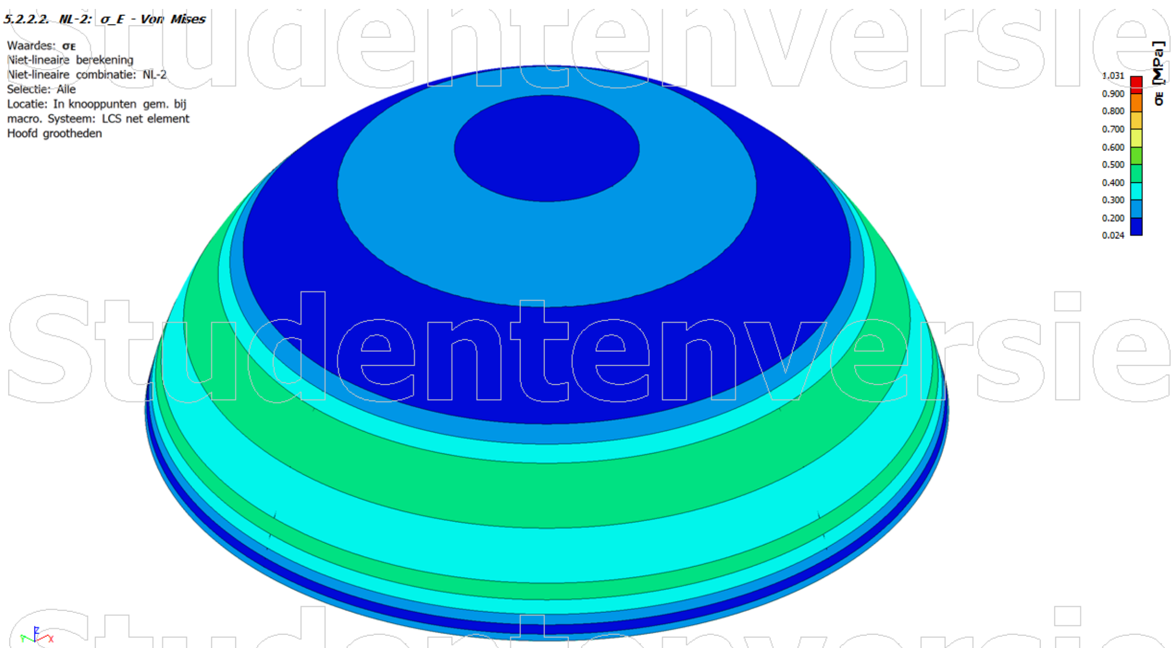
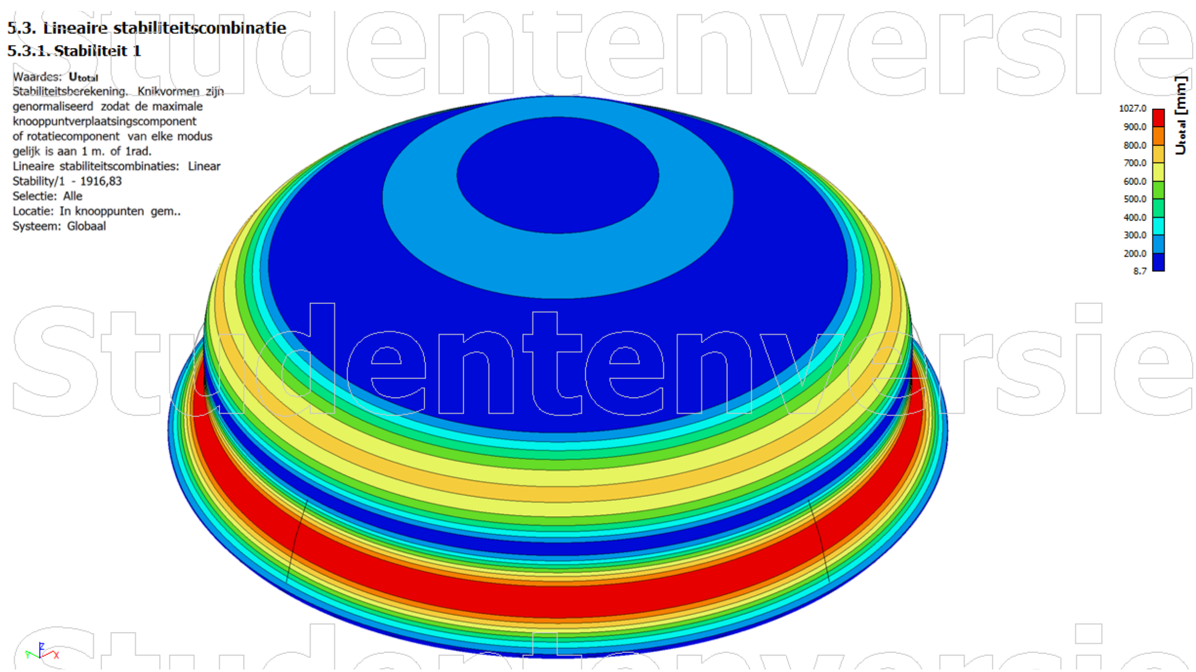
Appendix 1: Enlarged SCIA results



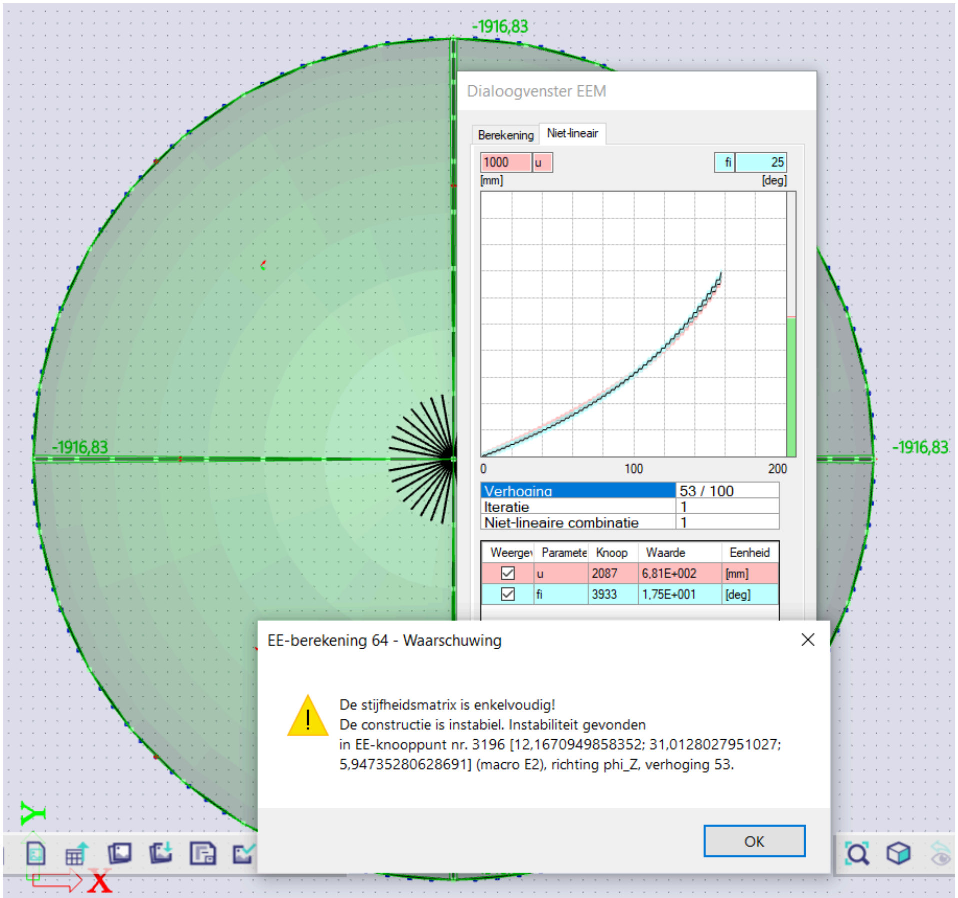
Deformation at a load of 11 kN/m²



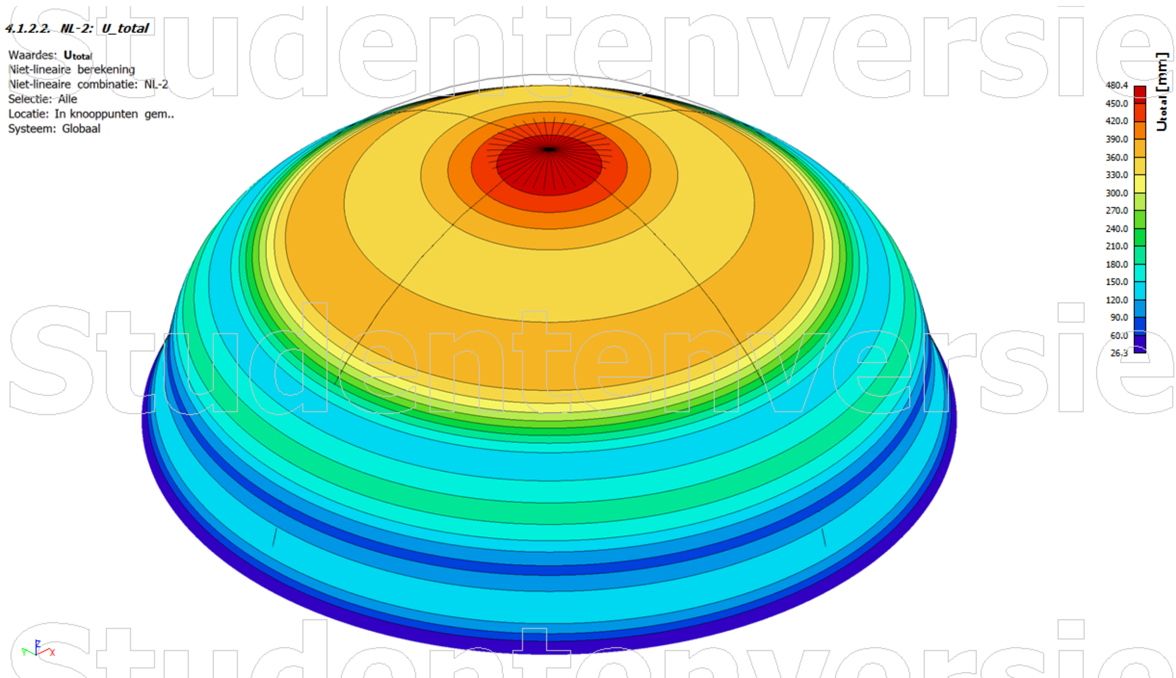
Von Mises stresses at a load of 11 kN/m²



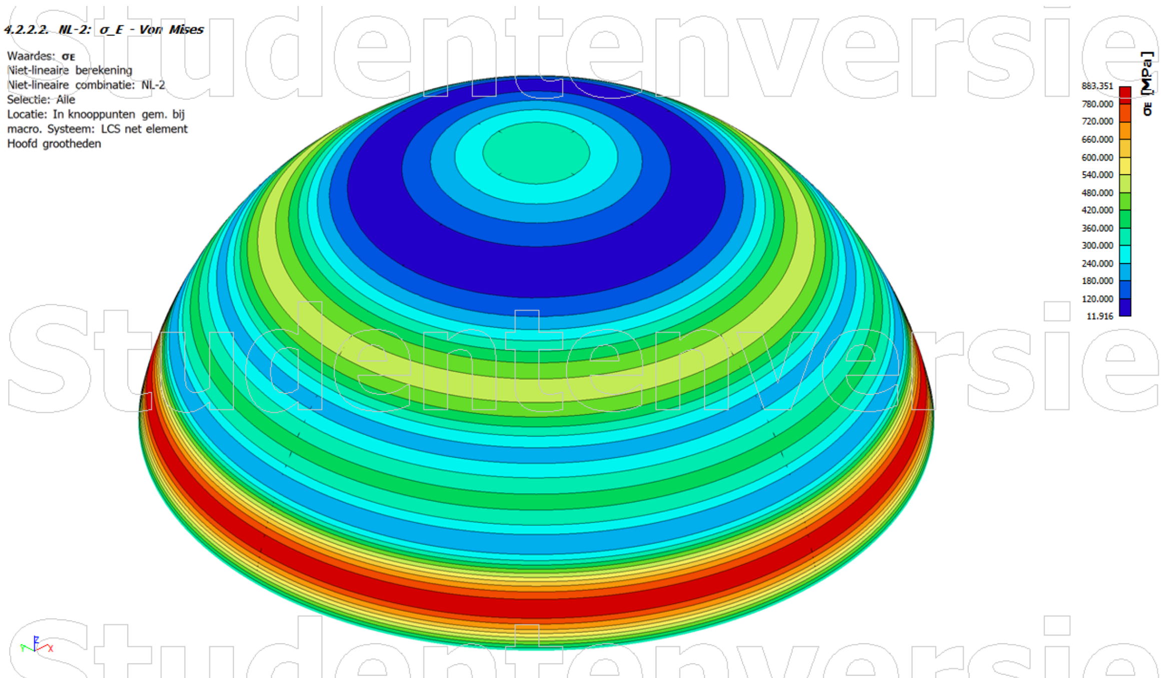
Buckling analysis at a load of 11 kN/m²



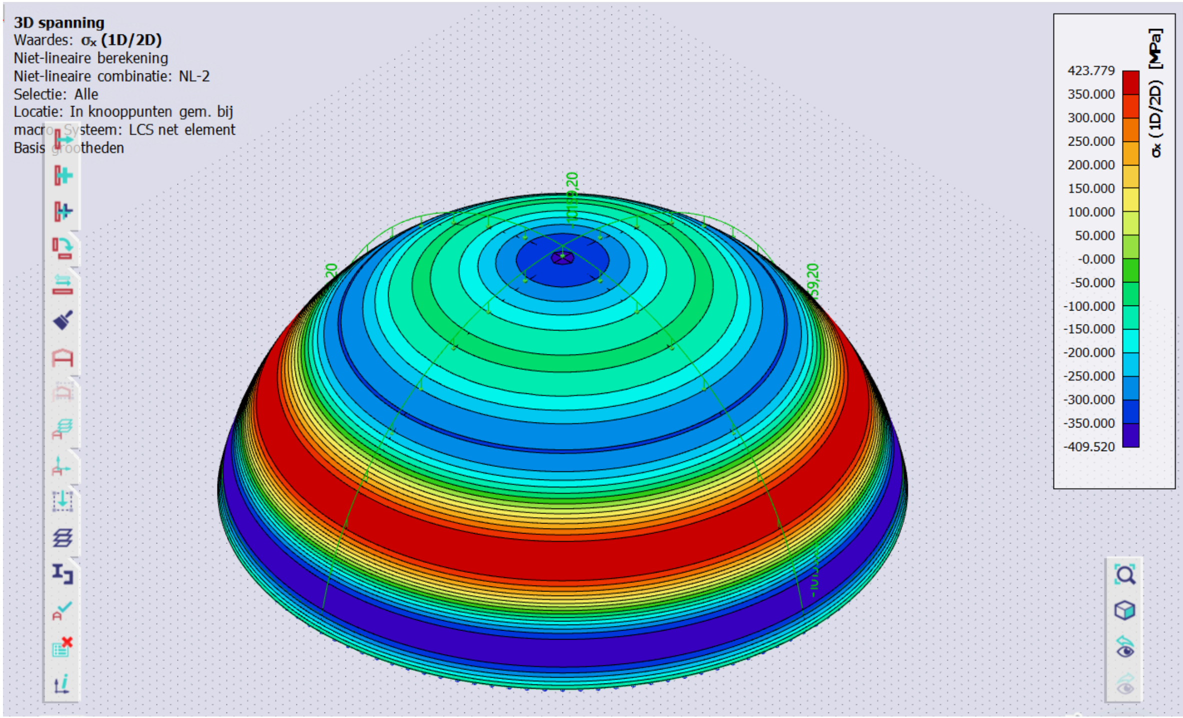
Diversion of Newton Raphson method



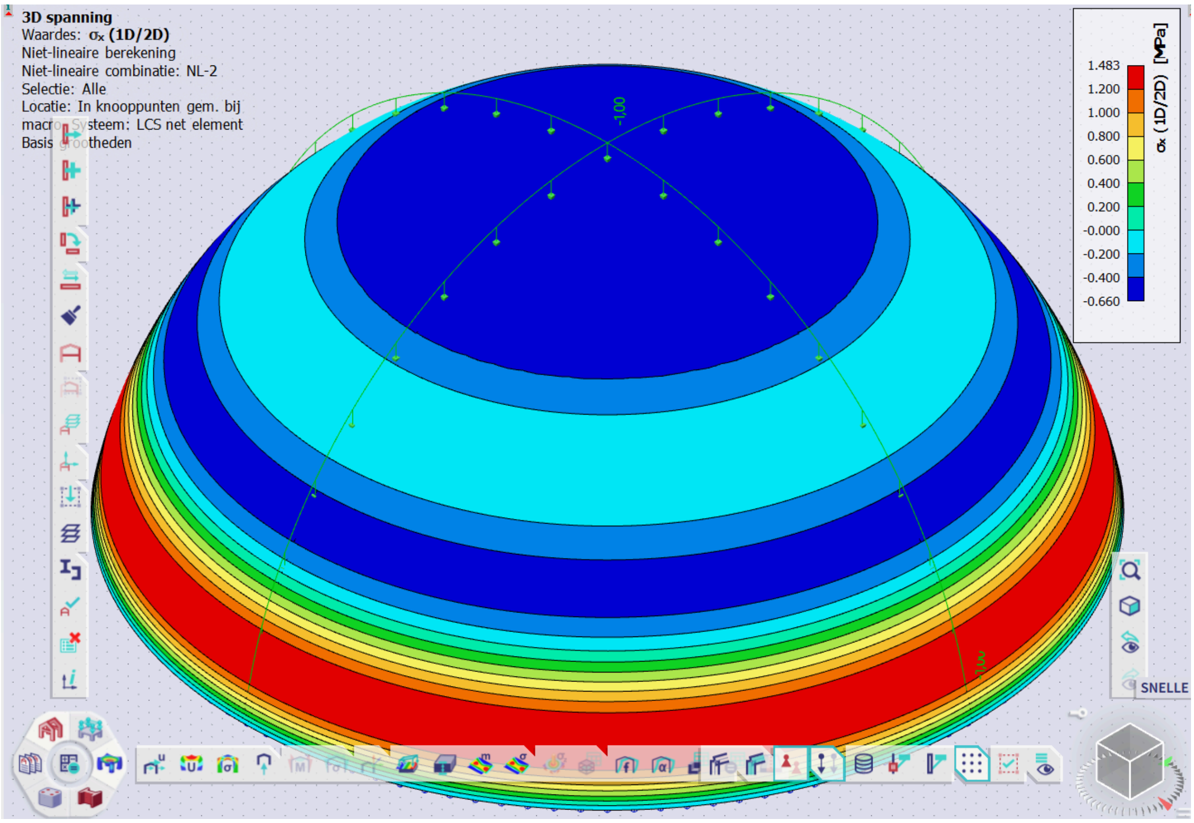
Deformation at a load of 11175 kN/m²



Von Mises stresses at a load of 11175 kN/m²



Stresses in dome at a load of 11175 kN/m^2



Stresses in dome at a load of 35 kN/m^2