SHELL STRUCTURES

Bend & Break

Group 5
Larrissa Hart
Thant Sin Oo
Falco Kal

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Introduction

The assignment of the Shell Structures course is to design and build a shell that can, at a minimum, carry it's own weight plus an added weight in snow. The shell is to consist out of either timber or concrete and be built at scale to test its carrying capacity. Before testing, a prediction is made on how at what load the shell will fail, and in what way.

Design process

Design concept

As a group we decided we decided to challenge ourselves in the design by creating a wooden shell that is unique in shape. Our fundamental design concept was to let go of the standard shell design, being a shell that bends in a single direction (a dome being the most simplistic example). Our idea was to design a shell that would be bent in both directions, while maintaining structural efficiency.

Design development

We developed our design by starting with the most basic shell shape possible: a dome. Because we were planning to bend the members both ways, a 'half dome' was chosen, a dome that is half as high as it is wide (Figure 1).

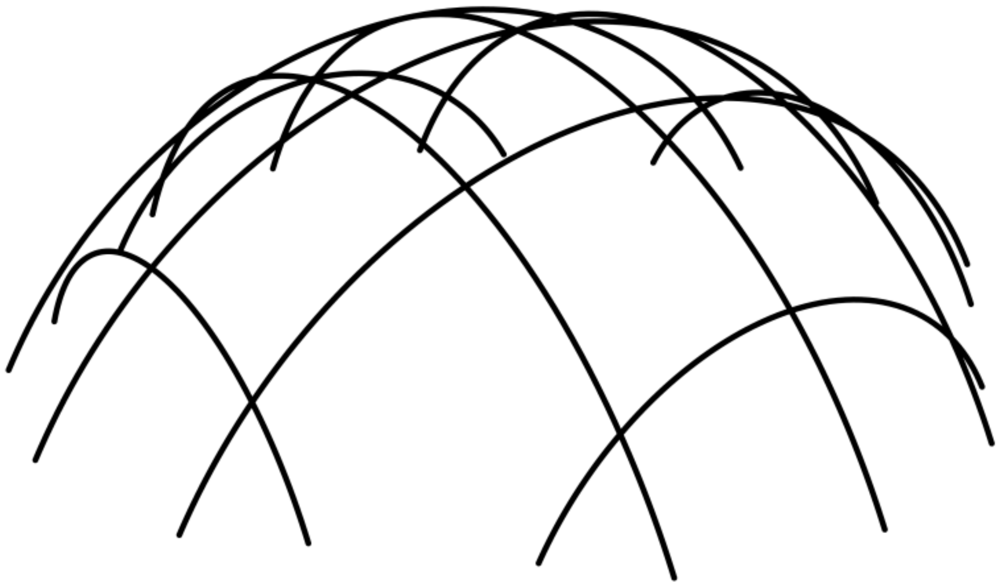


Figure 1: Dome

The members along one of the axis were then flipped (Figure 2) and repositioned so they touch the other members at their center point (Figure 3).

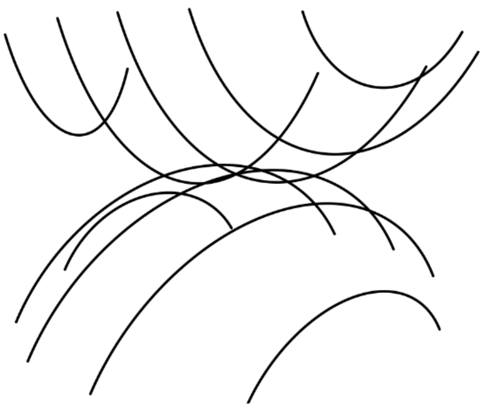


Figure 2: Flipped axis

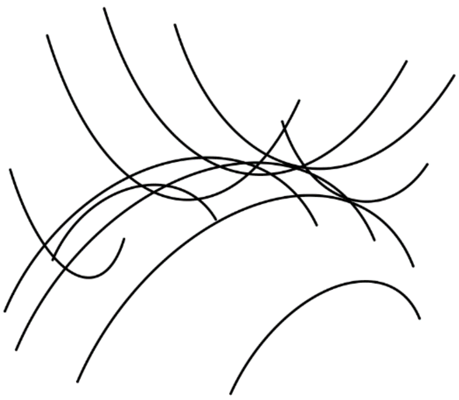


Figure 3: Repositioned along first axis

This process was then repeated for the members of the dome below, creating the fundamental shape of the design (Figure 4). To finalize the design a frame is added (Figure 5), to create the border of the surface and to allow attachment of the members.

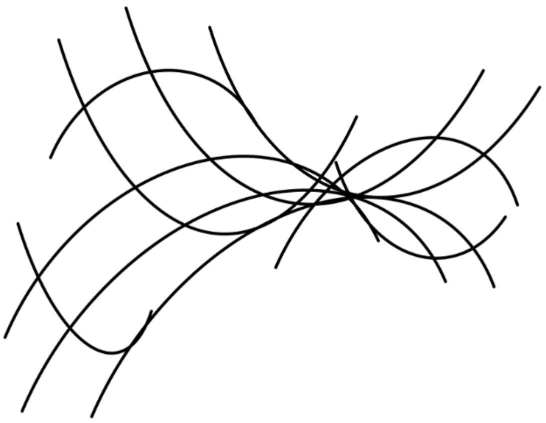


Figure 4: Repositioned along second axis

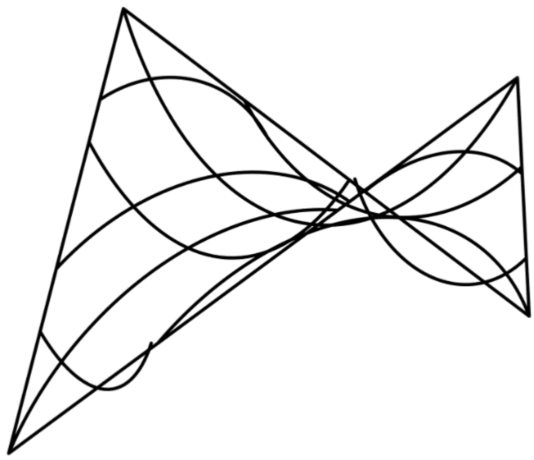


Figure 5: Frame added

We then expanded upon the design by creating a concept of how it could be constructed. This concept is pictured below (Figure 6). The members are to be made of wood, the frame will also be made of wood, but of thicker wooden beams. The shell is imagined to be attached to a concrete foundation.

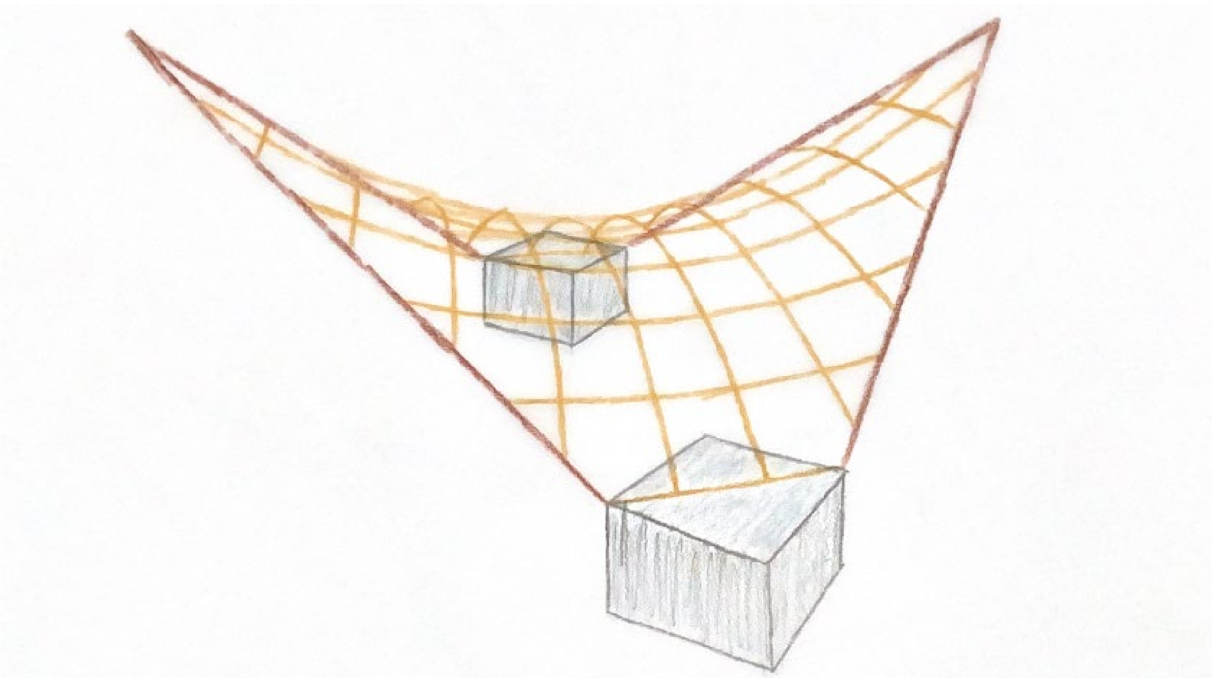


Figure 6: Design concept

Design realization

In realizing the design we initially had to decide on the dimensions. We decided to build the structure on a 0.5m by 0.5m base. Also, we had to decide on the dimensions of the members, as these had to be ordered. Initially, we decided on 2cm x 1cm members, these however turned out to be too stiff and were cut into 1 cm x 0.5 cm members.

To bend the members we designed a bending board as pictured below (Figure 7). This bending board allowed us to bend the members to the exact required extent, using the flexible board as a mold.

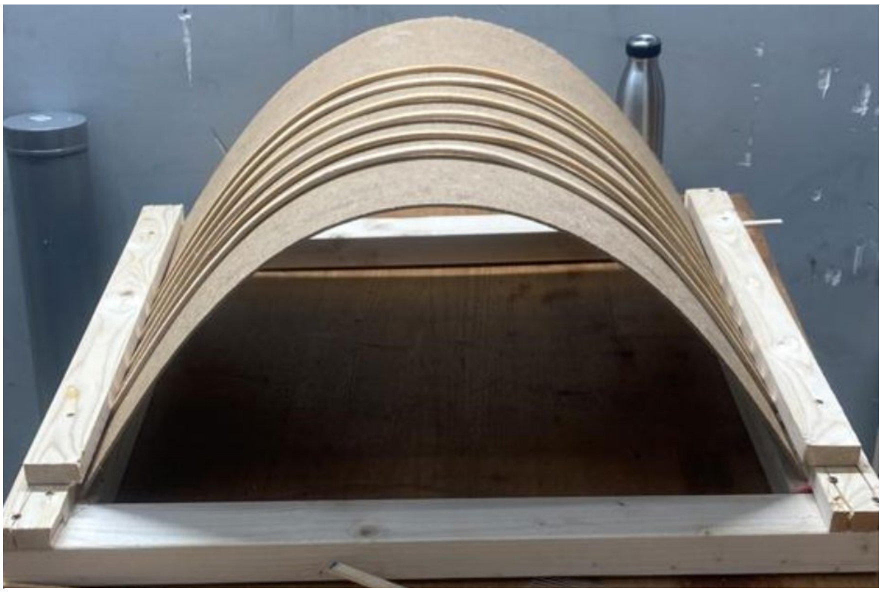


Figure 7: Bending board

As for the frame, we used the thickest beams available. The reason for this is the attachment of the members. To attach the members we drilled holes into the frame, the ends of the members are placed in these holes, using the members tension to hold them into place. The holes were then filed with glue to ensure proper attachment. The frame used is therefore not representative of the dimensions it would have if the shell was to be realized in reality, the dimensions were chosen to ensure the structural integrity of the model.

Another adjustment that was made from the concept to the actual design was the attachment of the shell to the platform. Instead of using concrete for the attachment is was simple constrained between two wooden elements, as seen in the pictures of the definitive shell below (Figure 8&9).

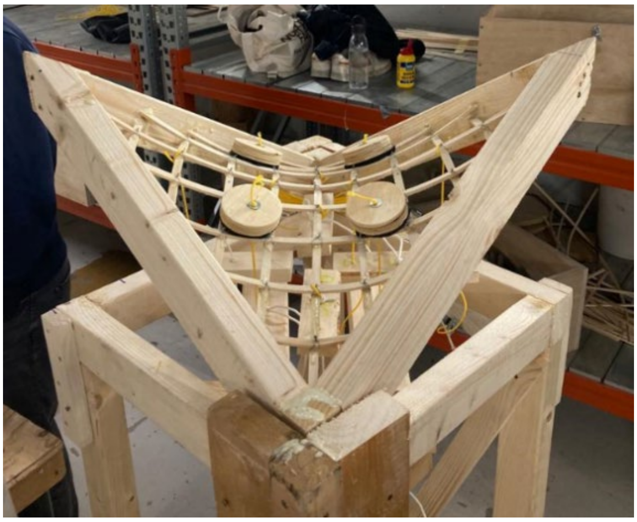


Figure 8: Definitive shell 1

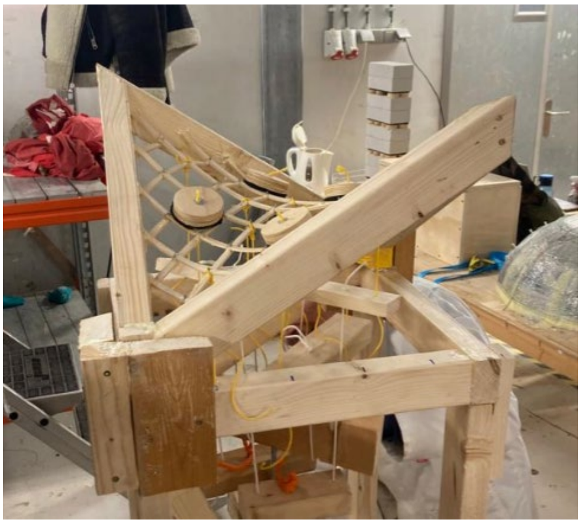


Figure 9: Definitive shell 2

Simulation and results

Manual calculations

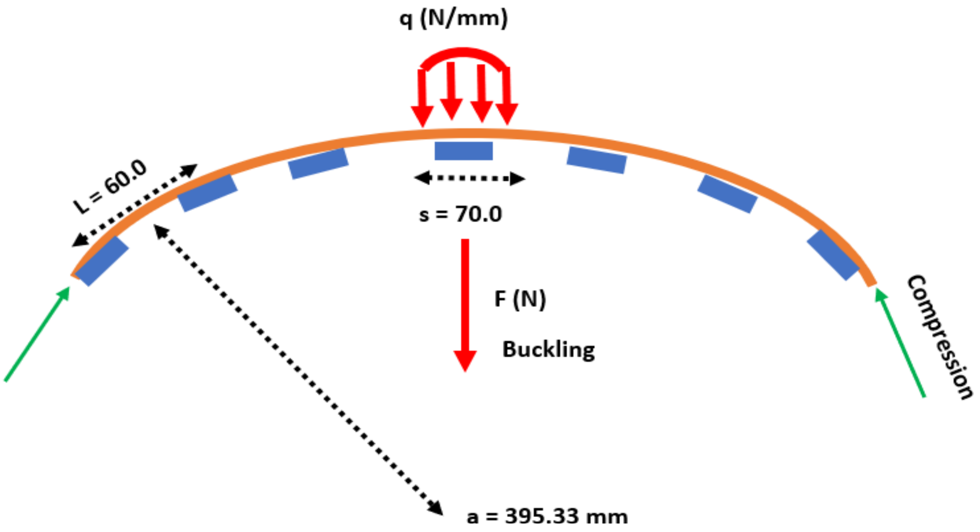


Figure 10: Schematic view of construction

First buckling load will be calculated. Calculations are as follows:

N_e = N (buckling load at which our construction will collapse)
E = 10000 N/mm^2 (approximated Young Modulus of timber)
I = 1/12*b*h^3 mm^4 (Moment of inertia of Timber lath)
b = 10 mm (width of Timbe lath)
h = 4.5 mm (height of Timbe lath)
L = 60.0 mm (buckling length or distance between two nodes)

$$N_e = \frac{\pi^2 * E * I}{L^2} = \frac{\pi^2 * 10000 * \frac{1}{12} * 10 * 4.5}{60.0^2} = 2081.87\ N$$

$$F = m * a$$

$$m_b = \frac{F}{a} = \frac{2081.87}{9.81} = 212.22\ kg$$

Compression load according to collapse load of Timber is as follows:

N = N (compression force at which our construction will collapse)
q = N/mm (shear flow)
σ = 51 N/mm^2 (compression strength of timber as stated in assignment)
s = 70.0 mm (diameter of wooden disk used to exert force on laths)
A = b*h mm^2 (surface cross section of lath)
a = 395.33 mm (radius of middlemost lath)
d = 706.3 mm (total length of the central lath)

$$\sigma = \frac{N}{A}$$

$$N = \sigma * b * h = 2295.0 \text{ N}$$

$$q = \frac{N}{a} = \frac{2295.0}{395.33} = 5.81 \frac{N}{mm}$$

$$F_d = q * d = 4100.228\ N$$
$$or$$
$$F_s = q * 2 * s * 4 = 3250.95\ N$$

$$m = \frac{F}{g}$$
$$m_d = 418.308\ kg\ or\ m_s = 331.39\ kg$$

Calculations above are done using the compression strength of timber. This leads to certain N which can be compared to N_e from buckling load. It is clear that $N_e < N$, therefore our construction will collapse due to buckling load before reaching maximum compression load. So our buckling load will be the decisive factor for failure. When using compression load there are two values of maximum weight. F_d is done assuming that the load is distributed over the whole lath which is not really the case. On the other hand, F_s is closer to real setup where four wooden disk are used to exert equal force on two laths, hence 2*s*4. The latter gives us smaller weight and the value is more plausible for thin wooden laths. However, as stated before, these compression load cannot be reached and some calculation has to be done for weight limit at buckling load.

$$F = m_b * a = 212.22 * 9.81 = 2081.87\ N$$

$$q = \frac{F}{2 * s} = 14.87 \frac{N}{mm}$$

$$N_b = q * a = 5878.75\ N$$

$$\sigma = \frac{N_b}{A} = \frac{N_b}{b * h} = 130.64\ \frac{N}{mm^2}$$

The calculation above shows that $N_e < N_b$, which means even for the same weight our construction is likely to collapse due to buckling. Of course, working with N_b will give us bigger σ for compression strength which is unlikely for a thin timber. However a lot of assumption were made during the calculations which can affect the outcomes and these calculations will become more complicated as accuracy increases.

For example, when calculating F_s, it is assumed that all four wooden disk are equally distributed on the middlemost lath and then they exert forces on whole structure from these positions. Naturally, this is very simplified assumption and correct calculations require exact measurements of the length of every lath where wooden disks are resting upon. This could be further complicated by combining the surface areas of the laths and force of individual members. But all the complex calculations fall outside the program of this cursus.

Additionally when calculating buckling load, only the buckling length of middlemost lath is taken into account. It is assumed that buckling will take place in the middle. But it is totally possible that buckling length could be different for each nodes.

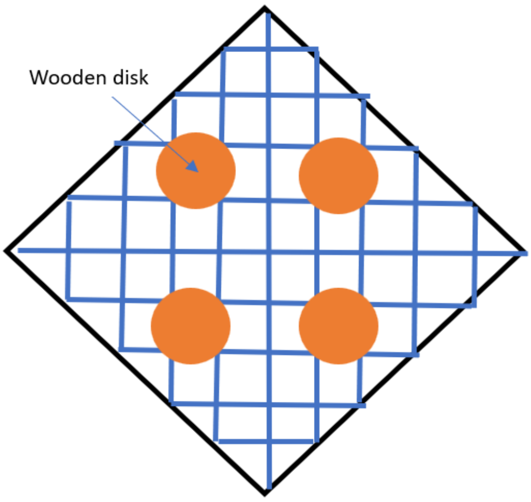


Figure 11: Correct position of wooden disks

Thus according to buckling calculation our construction will buckle at around 212 kg, which is taken as 200 kg for prediction.

If the construction does not collapse for buckling, a point load will be applied as shown in figure 12. A calculation of point load is provided here in case buckling load does not work. Point load can be easily obtained by adjusting F_s. Instead of four wooden disks, only one disk will be acting on the whole construction in the center, hence point load.

$$F_p = \frac{F_s}{4} = 812.73\ N$$

$$m_s = \frac{F_p}{9.81} = 82.85\ kg$$

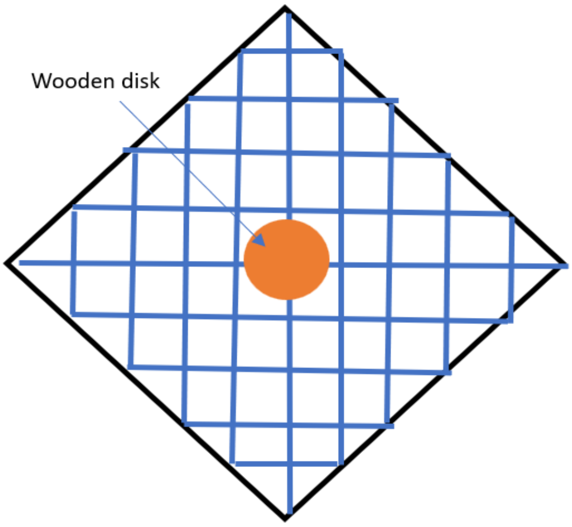


Figure 12: Schematic setup for point load

Our construction will collapse at point mass of 82.85 kg which is lower than buckling mass. This value is acceptable given that all the forces are located at one point instead of being distributed over the whole construction.

SCIA simulations

In addition to manual calculations, 3D models are also analyzed using an engineering software called SCIA. Different combination with different type loads are simulated for this analysis. The smaller beams have cross section of 10x4 mm and thicker beams have cross section of 40x65 mm.

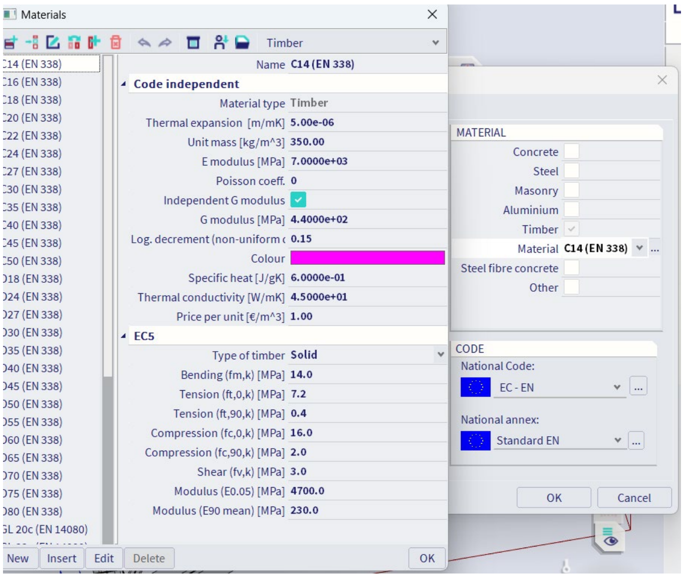


Figure 13: Properties of material

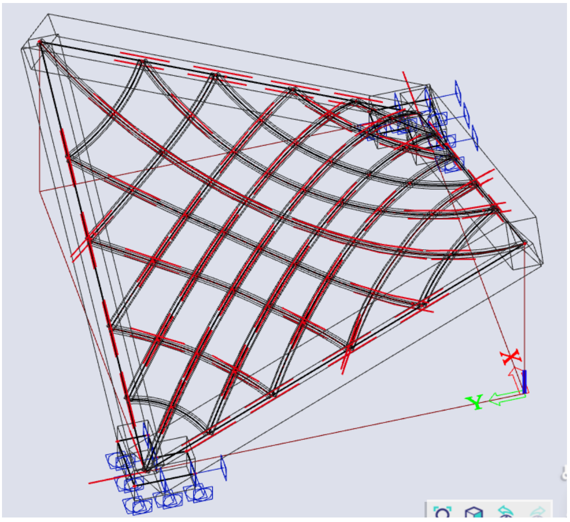


Figure 14: General view of 3D model

Blue markers indicate fixed points of the structure and red lines indicate rigid connections at the nodes.

The simulation are as follows:

- Self-weight is put on the whole structure.
- Self-weight is put as an external line load on the small beams, intentionally putting no weight on the thicker beams at the edges. Weight of the snow is also included.
- Buckling load of 5.27 kN/m is applied as external line load on the small laths.
- These situations can be done in both linear and non-linear combinations.

Buckling load of 5.27 kN/m is derived on N_e. The construction is supposed to buckle at around 2081.87 N but external line load has to be assigned in kN/m. This can be done by using the radius of longest lath right in the middle, which the buckling weight is based on.

$$\frac{N_e}{a} = \frac{2081.87}{395.33} = 5.27\frac{N}{mm} = 5.27\frac{kN}{m}$$

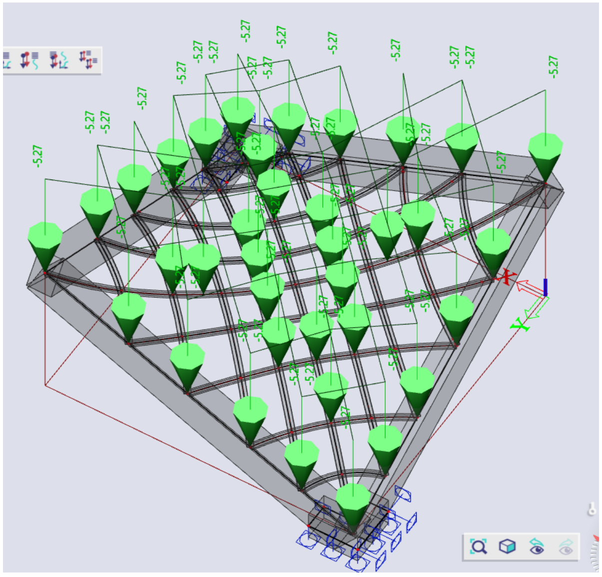


Figure 15: Application of external line load

As shown in figure 15 external loads can be seen as green arrows pointing downward. These loads are distributed over small beams and no other individual loads are applied on thicker beams at the edges. But such line loads still have to be applied at nodes connecting the both type beams, which is why nodes of the thicker beams are still loaded.

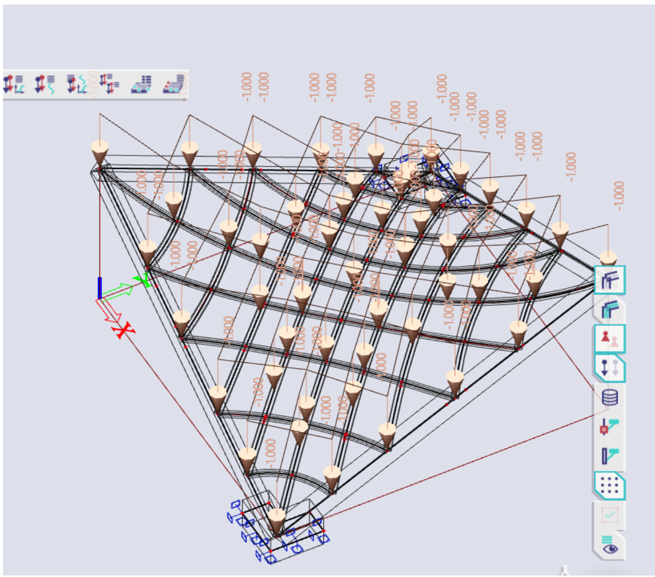
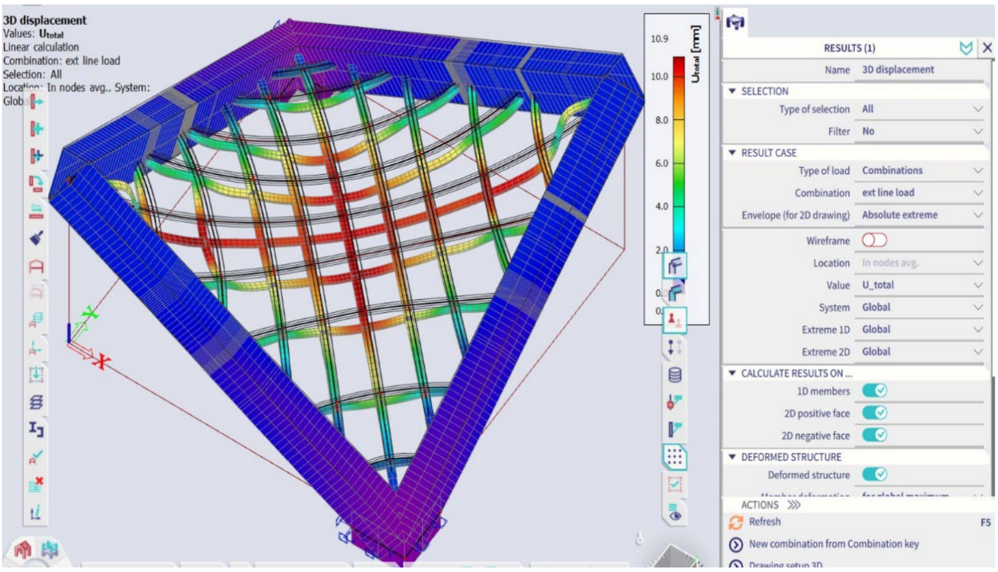


Figure 16: Application of self-weight as external line load

Self-weight is applied at scale of 1. But this can be manually increased in menu setting.

Linear combinations

In figure 17. External line load of 5.27 kN/m is simulated. The structure has a maximum displacement of 10.9 mm. This structure is scaled on our model.



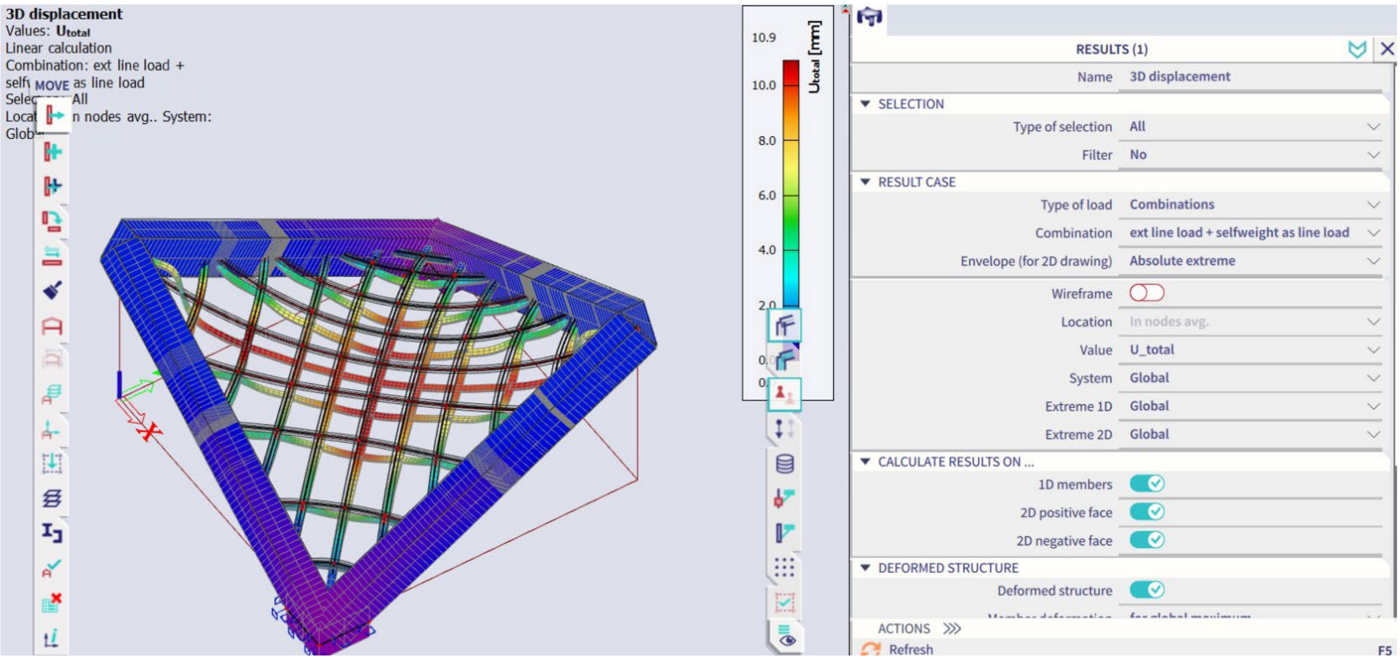


Figure 17: Both external load of 5.27 kN/m and self-weight are simulated as line loads

The structure has a maximum displacement of 10.9 mm. This structure is scaled on our model. This simulation indicates that for buckling self-weight does not contribute much when used it as external load.

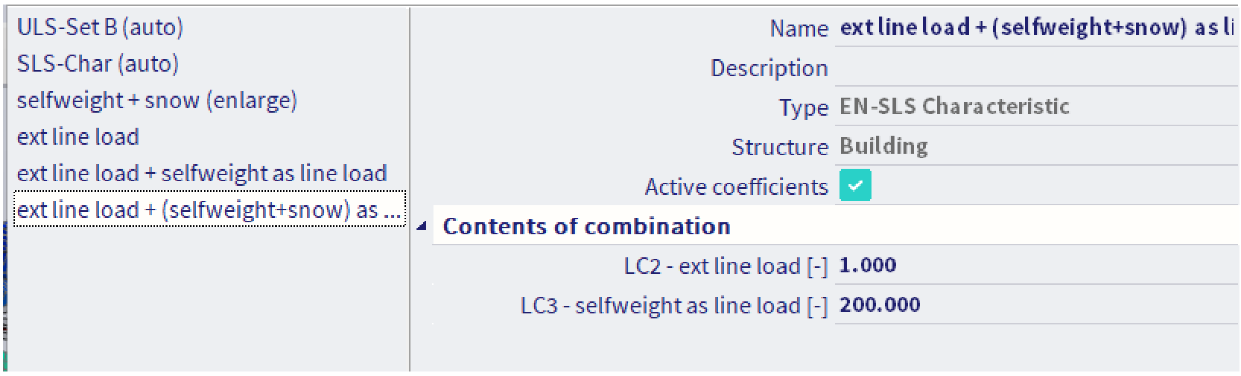


Figure 18: Configuration of enlarged model. Self-weight is increased.

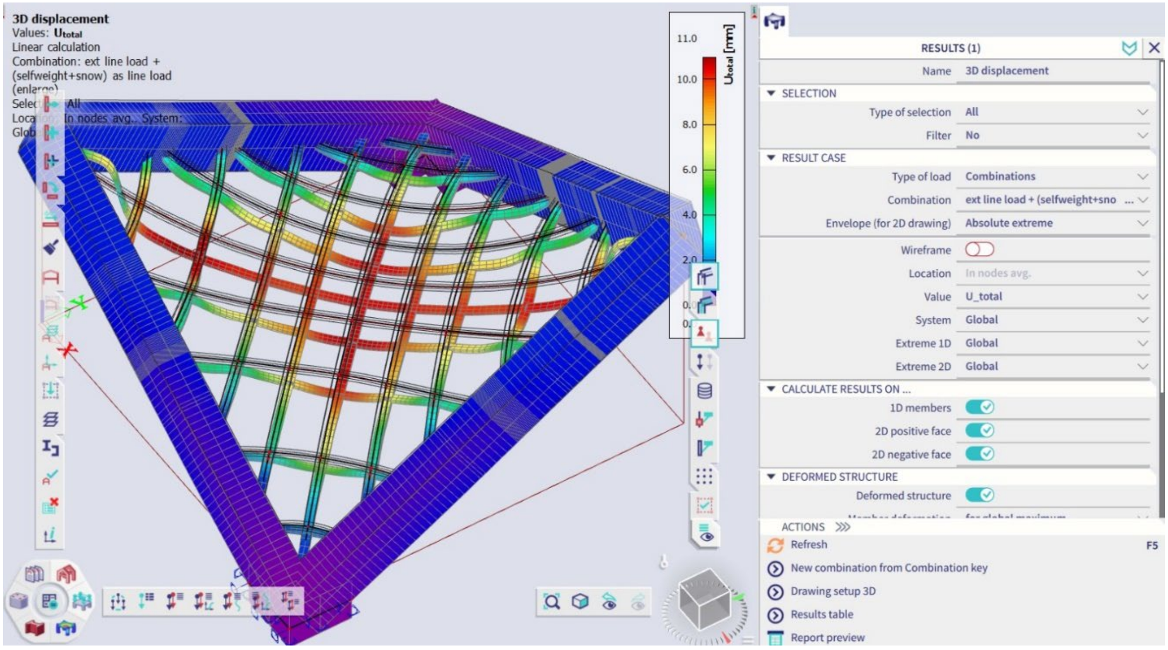


Figure 19: Both external load of 5.27 kN/m and self-weight are simulated as line loads

This structure has a maximum displacement of 10.9 mm. Figure above is based on real world application. The model has to match real scenario and it is simply assumed that model is 100 times bigger than our miniature construction. Therefore it is enlarged by a factor is 200 (self-weight + snow). This simulation indicates that for buckling self-weight does matter when building a big realistic model.

Non-linear combinations

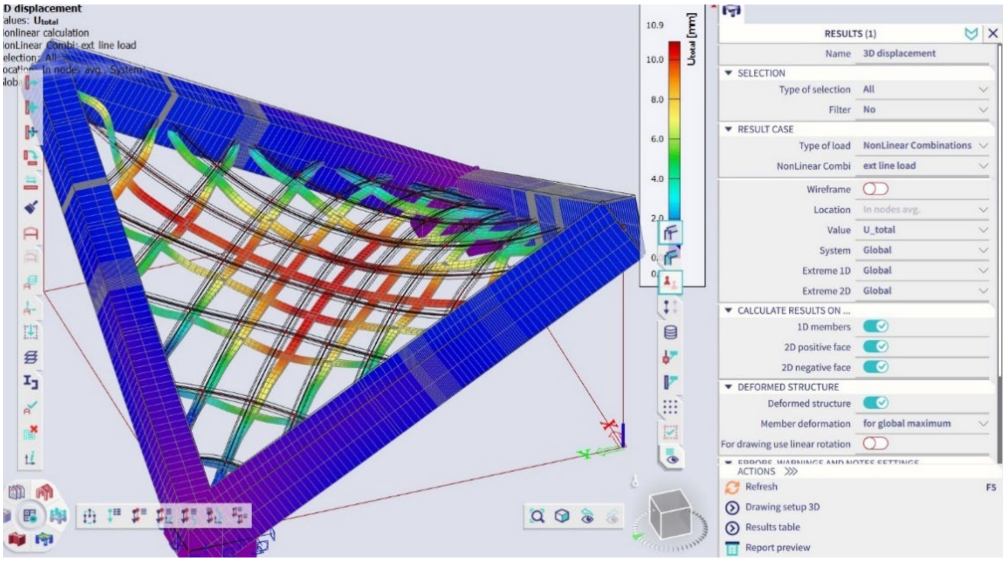


Figure 20: External line load of 5.27 kN/m is simulated

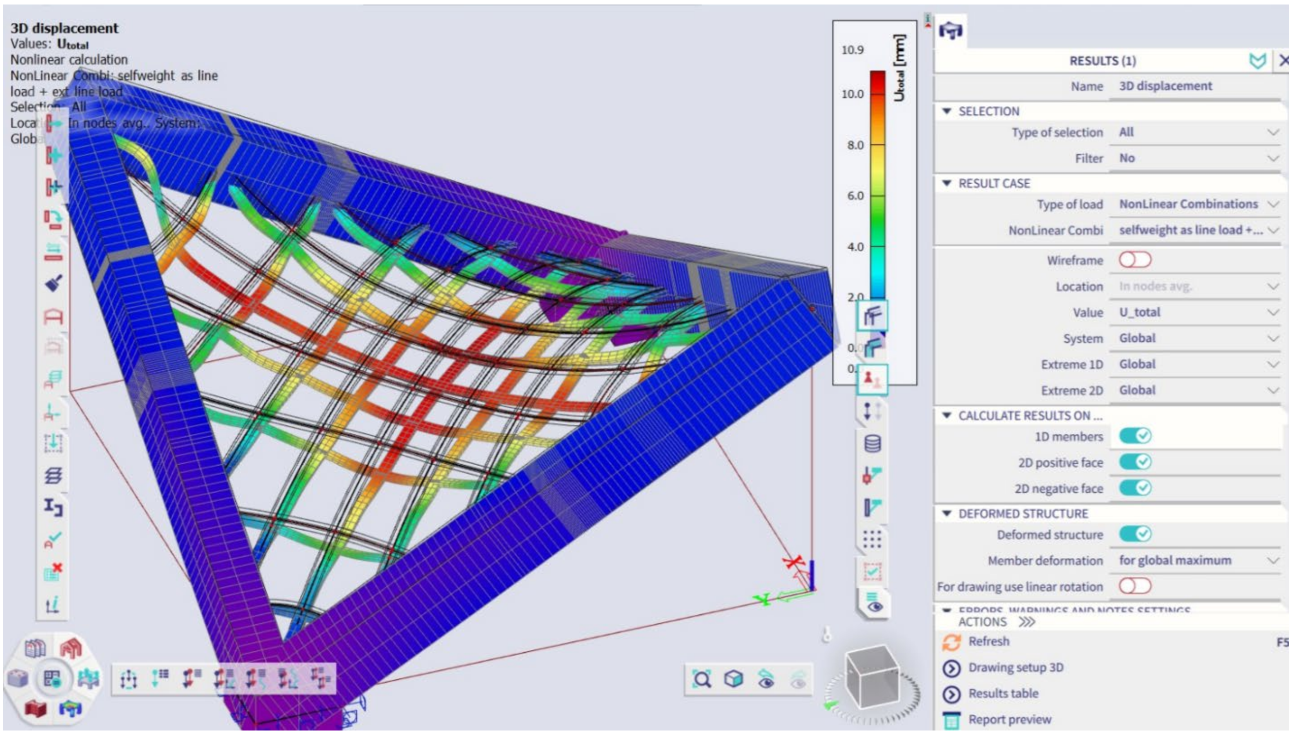


Figure 21: Combinations of external load and self-weight as line loads

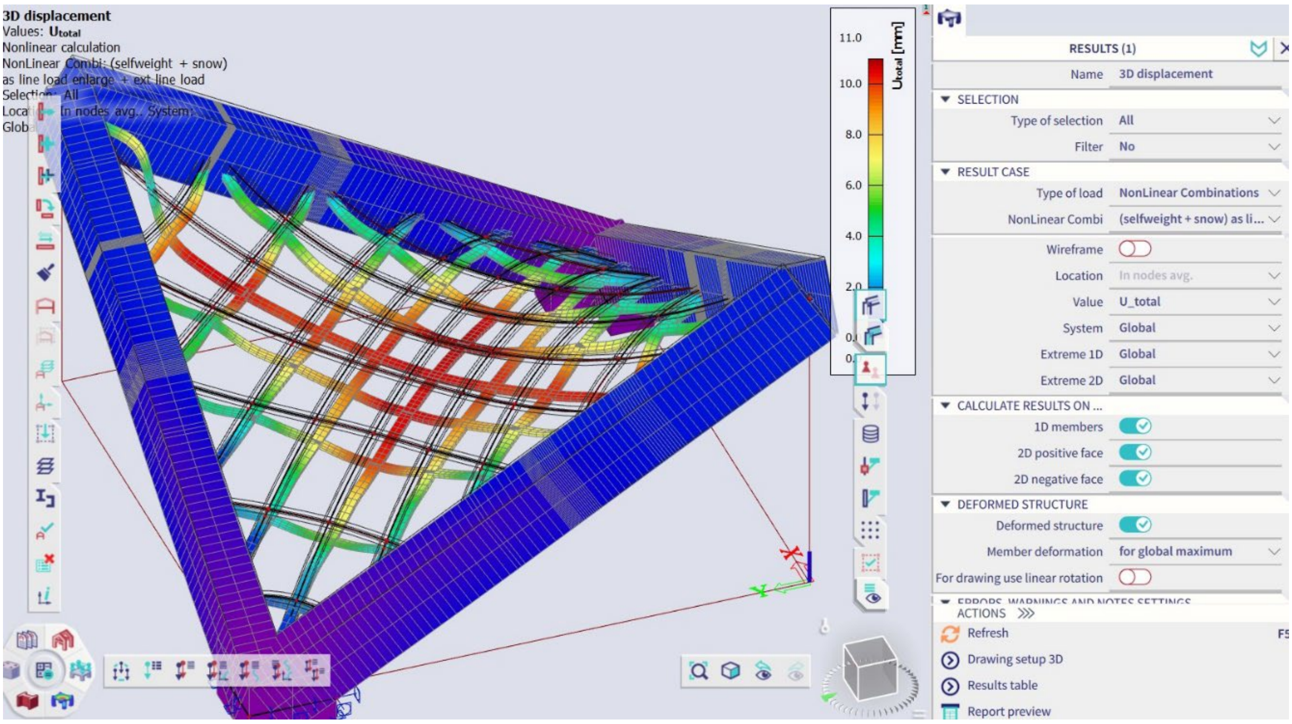


Figure 22: Both external load of 5.27 kN/m and self-weight are simulated as line loads. Based on real world application

As can be seen in figures, both linear and non-linear have same displacements when loads are applied. A possible reason could be the result of neglecting initial imperfections. Initial imperfections are not taken into account and it is hard to predict such imperfections for massive structure. Another reason might be ignoring plasticity of timber.

Linear stability combinations

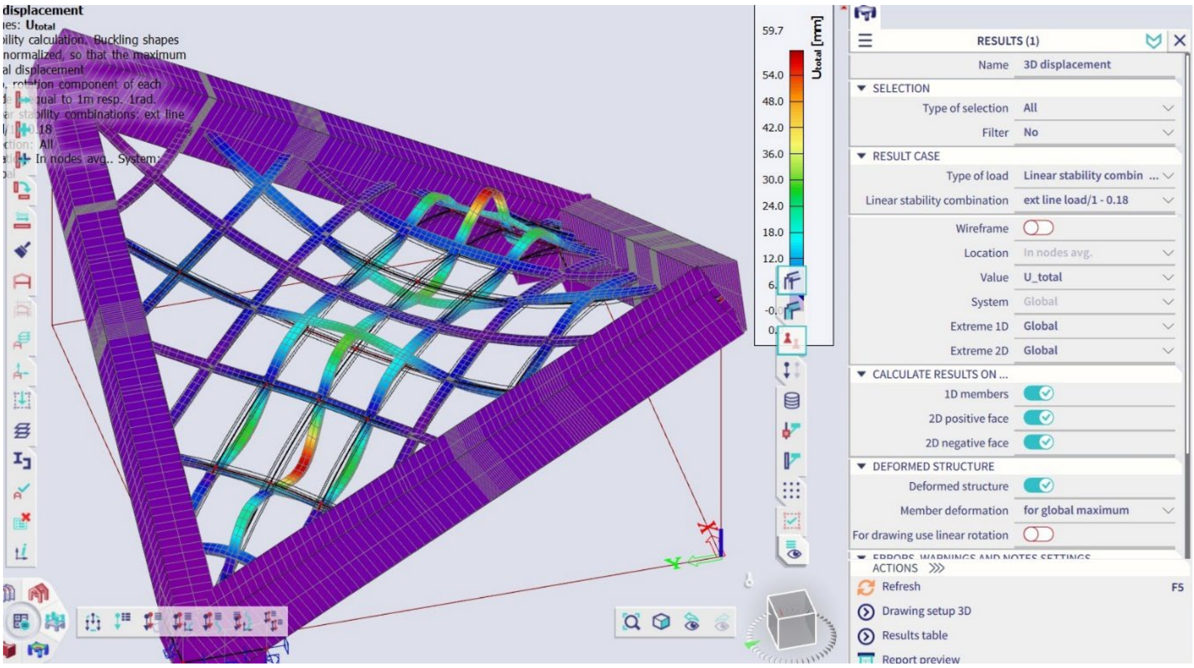


Figure 23: External line load of 5.27 kN/m is simulated

The simulation indicates that our miniature model can hold 0.18 of 5.27 kN/m before experiencing any deformations. Maximum deformation is 59.7 mm as stated above.

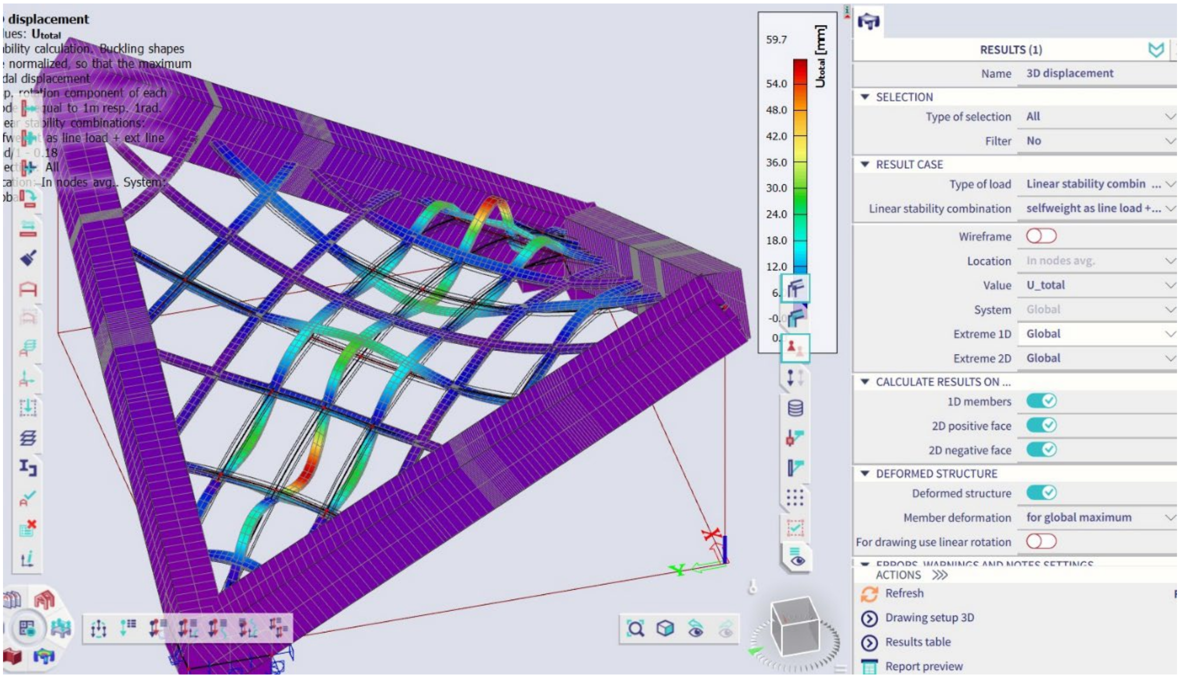


Figure 24: Both external load of 5.27 kN/m and self-weight are simulated as line loads

The simulation also indicates that our miniature model can hold 0.18 of 5.27 kN/m before experiencing any deformations. Maximum deformation is same as before. It becomes clear that self-weight does not really matter when the structure is small and loads are not enlarged.

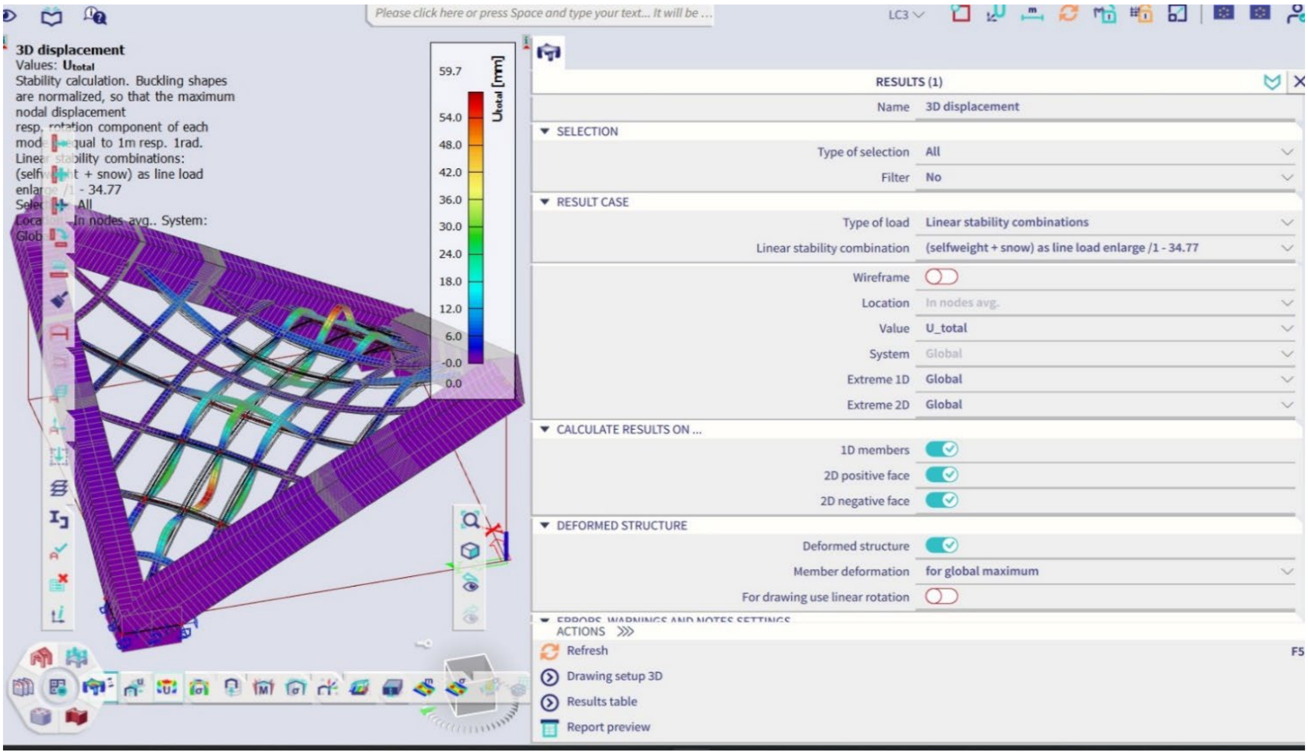


Figure 25: Both external load of 5.27 kN/m and self-weight are simulated as line loads. Based on real world application

On larger scale our structure van hold much bigger load which is expected given size of timbers also increases. For practical application, our structure can hold 34.77 times its own weight, snow combined with the external load of 5.27 kN/m before the structure comes unstable. However the displacement is still the same for any cases which leads to proper investigation if this has to be built for practical use.

Non-Linear stability combinations

Non-linear stability combinations cannot be done using local nonlinearity of the structure. SCIA found a singularity in a node and no results were displayed. But this could be a strong indication that our structure could be locally too unstable for practical use. However, for a simple analysis this option is intentionally omitted.

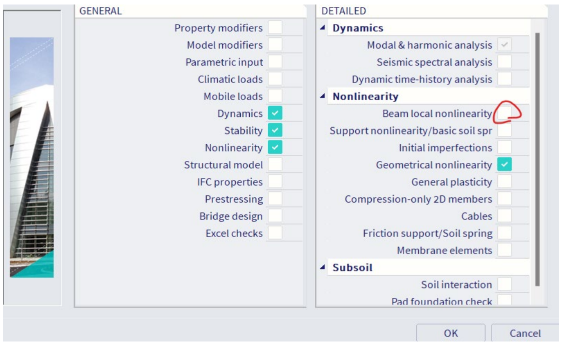


Figure 26: Configuration of SCIA setting

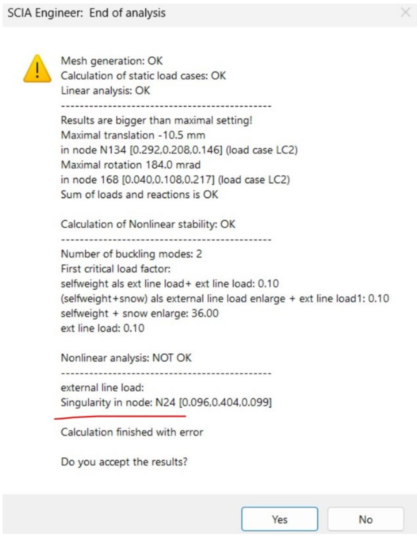


Figure 27: Error message

Non-linear stability simulations take very long time complete and therefore mesh size has to be adjusted to prevent the system from crashing.

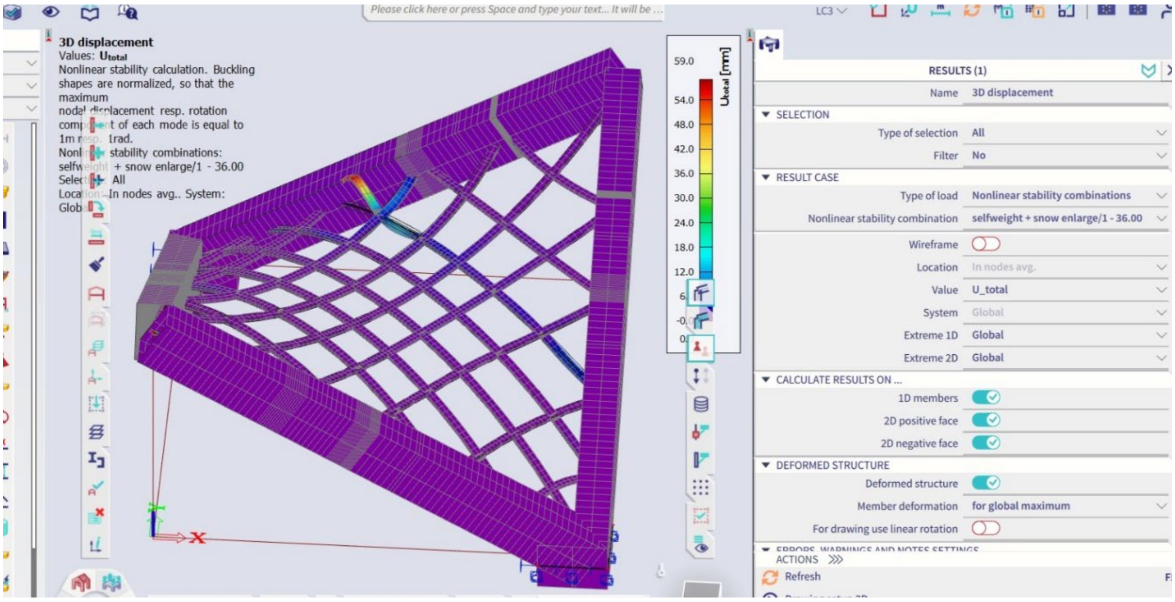


Figure 28: Self-weight is simulated on every beam. Based on real world application

This simulation indicates that our structure can hold 36 times its own weight and snow. Displacement is 59 mm. The model also shows the beam at which buckling will take place. However accuracy of this model might be off given that all the weight and snow are now lying on the whole including thicker beams at the edges. In reality, there would be less snow on the edges and more weight will be concentrated on the smaller beams, inside the outer beams. Despite the less accuracy, this simulation is still a good indication of how much our structure can hold.

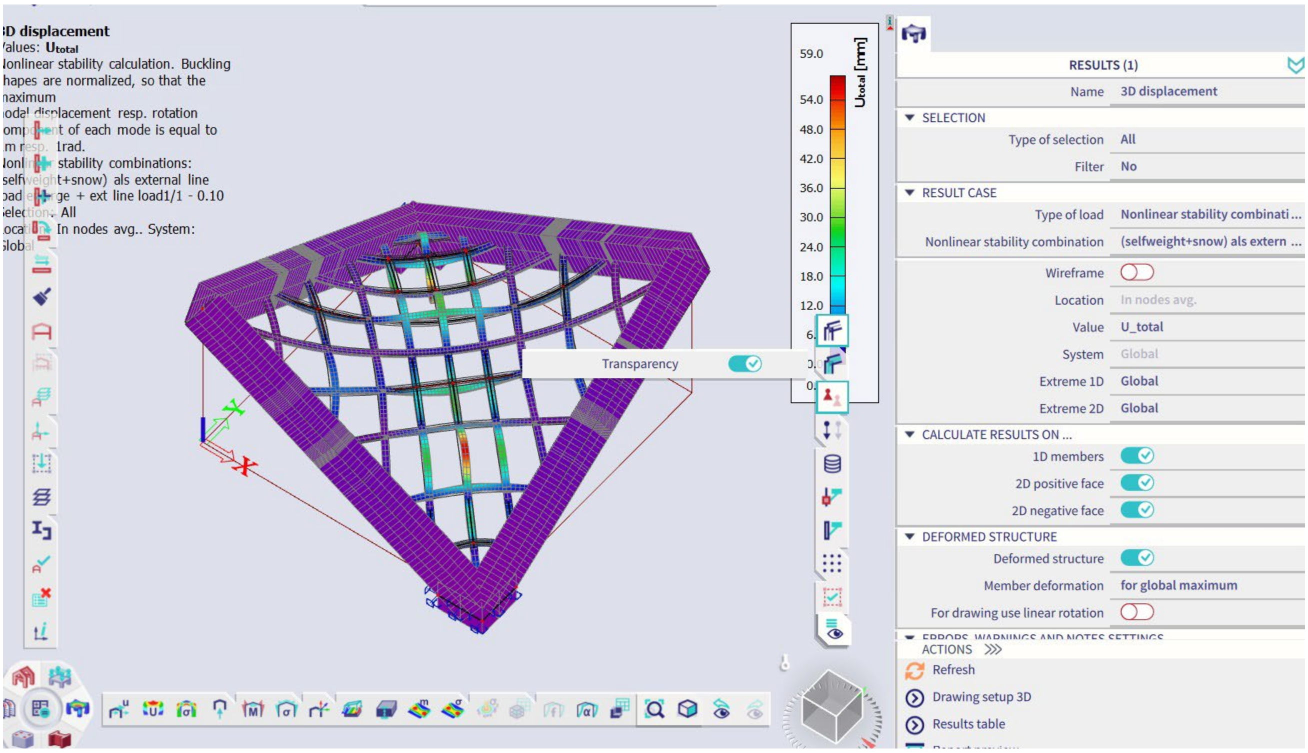


Figure 29: Both external load of 5.27 kN/m and self-weight are simulated as line loads. Based on real world application

Simulation above shows a better representation of load distribution. Self-weight, snow and buckling load are assigned as line loads. But this model can only hold 0.10 of combined line load before displaying any instabilities. This limit is extremely low and require further adjustment for practical use. When model is enlarged and line loads also increases with each m^2. This means that for such massive structure line loads could become too heavy and different buckling load is required for proper testing.

Figure 30: Different combination of loads

As can be seen in figure 30 most combinations give load limit of 0.10 which is not very realistic even for miniature model, such as first and last combination types.

In reality a massive beam of timber is cumbersome and extremely heavy to work around. There is a chance that these massive beams at edges will collapse under its own weight. Therefore for new simulation these beams are replaces with smaller beam of 0.5 of its own size. All the simulation are done for practical applications. Because our miniature model does not have thin laths at edges, simulations for experimental model are omitted.

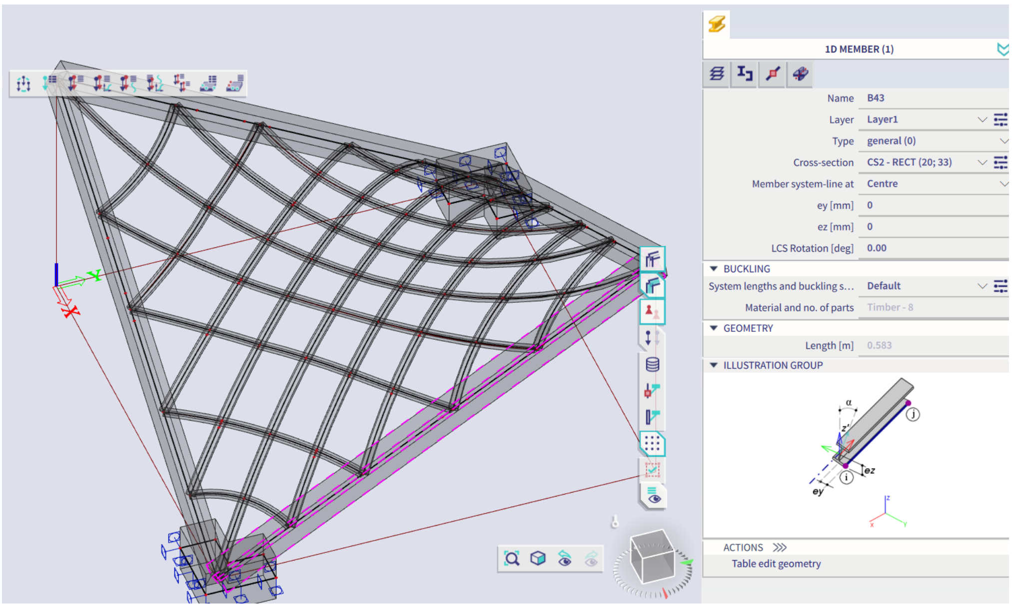


Figure 31: Outer beams with cross section of 20x33

Linear combinations

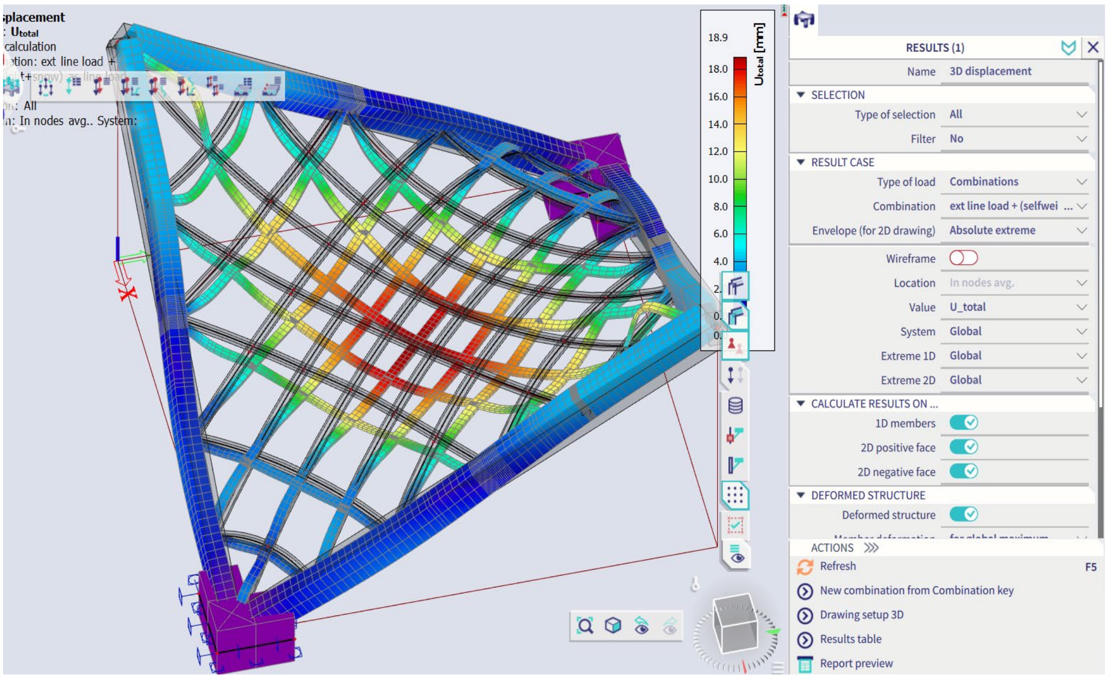


Figure 32: Both external load of 5.27 kN/m and self-weight are simulated as line loads

Non-linear combinations

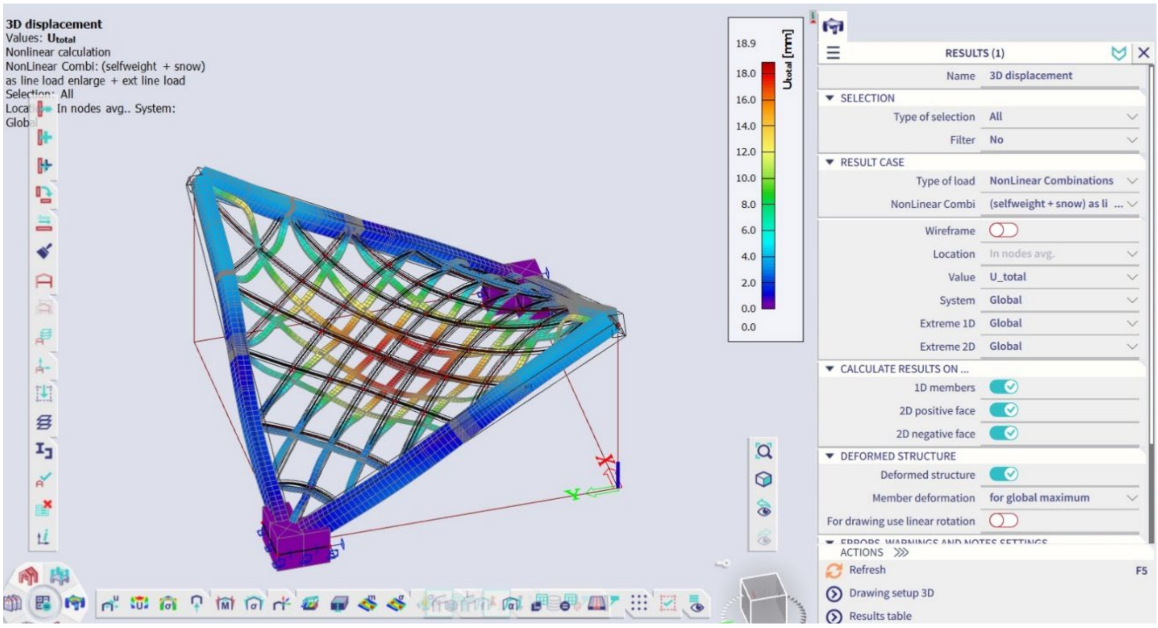


Figure 33: Both external load of 5.27 kN/m and self-weight are simulated as line loads

Linear stability combinations

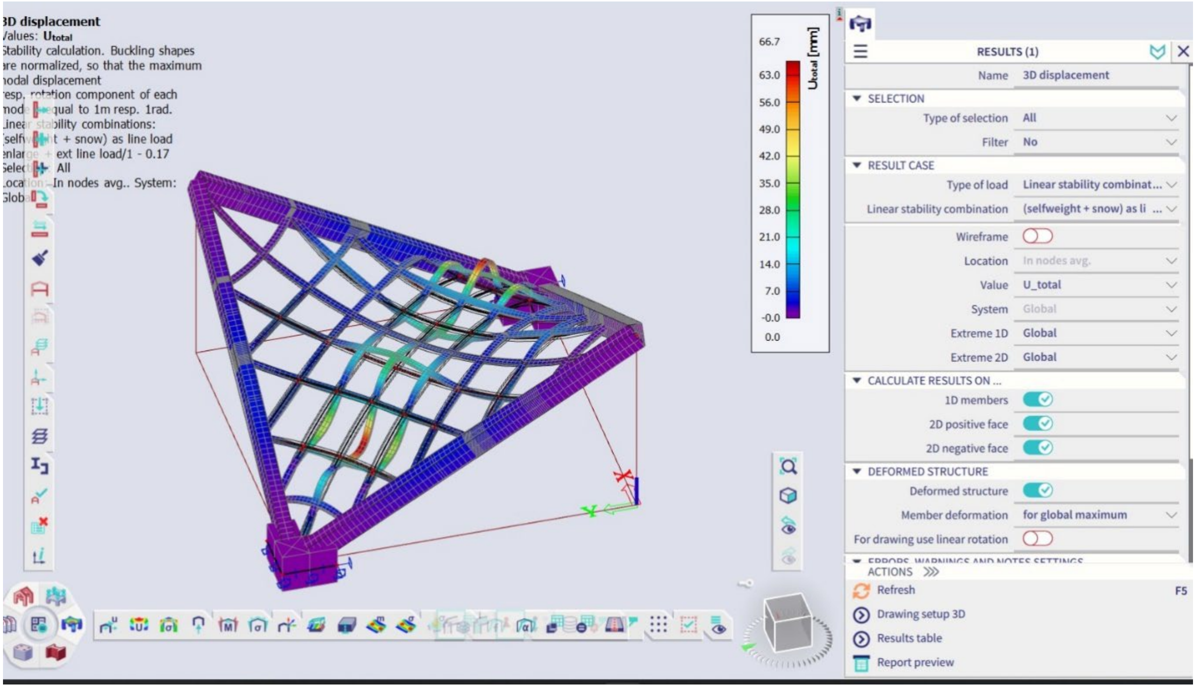


Figure 34: Both external load of 5.27 kN/m and self-weight are simulated as line loads

Non-Linear stability combinations

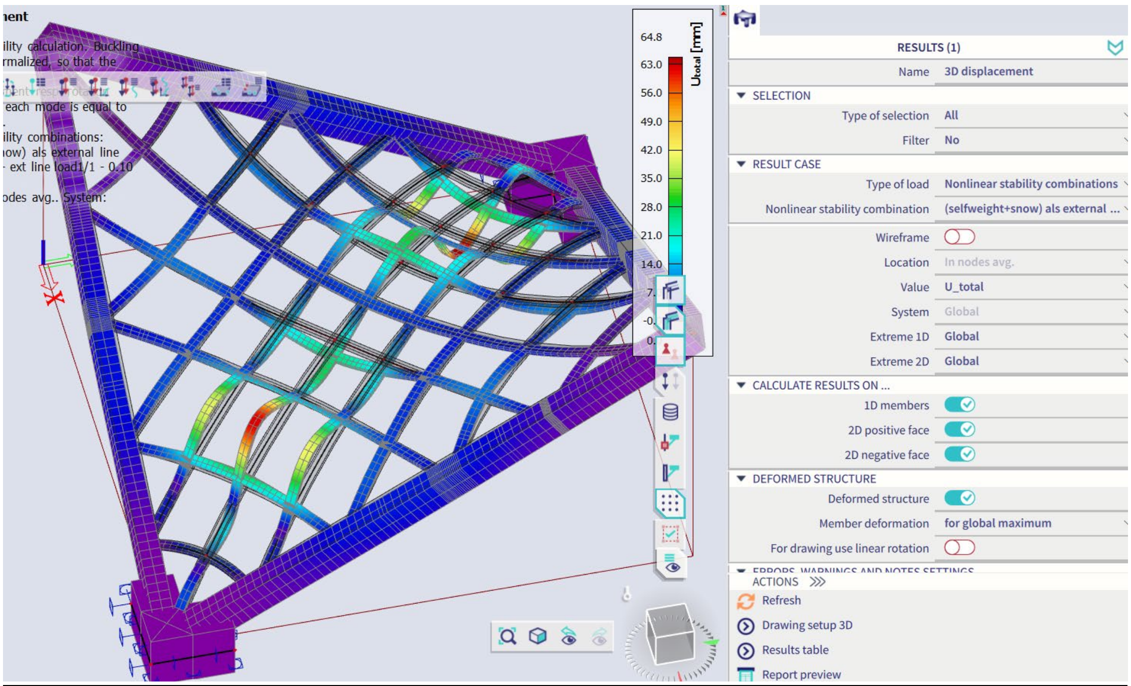


Figure 35: Both external load of 5.27 kN/m and self-weight are simulated as line loads

There is also possible combination where self-weight is exerted on whole structure including thicker beams at edges. However these situations are not simulated given that neither our experiment or real scenario deals with such situation. It is unlikely that buckling loads and snow will be exerting noticeable forces at the edges.

As explained above, some simulations seem a bit unrealistic. The explanations are stated below:

- SCIA model is not very accurate and beams are drawn as crossing each other instead of laying on each other.
- Geometry of nodes. In our model node at the thicker laths are bounded by glue and some laths are not in full contact with each other. In SCIA everything is rigid and fully connected.
- Geometrical accuracy of both models. It is not plausible to draw exact model with correct measurement.
- Applications of line loads. In reality any loads could be applied in different manners.
- Elastic modulus of timber. Property of materials will be different in reality.

However it should be noted that for most simulation, stress concentrations can be mostly found in center beams which backs up the hypothesis of buckling in the longest and middlemost element.

Test set-up

Introduction

The original set-up was a wooden frame with the same surface as the dimensions of the shell structure, so 50cm by 50cm, and a height of around 1.20m.

The shell structure itself was loaded with a semi-distributed load. The ropes that connected the structure to the wooden beams and therefore the platform, were placed in the following parts of the grid shown in figure 35.

These locations were chosen to distribute the load as much as possible over the whole grid. The reason the ropes were not placed at every knot, is because of the small structure and the high possibility of other wooden pieces touching each other, which makes the effort unnecessary.

The problem with this frame was that it was too low to load enough bricks on the platform to achieve structural failure. So the set-up shown on in figure 36 was made.

One brick has a weight of 2.09 kilograms. The wooden beams have a total weight of 3.35 kg and the platform for the bricks is 8 kg.

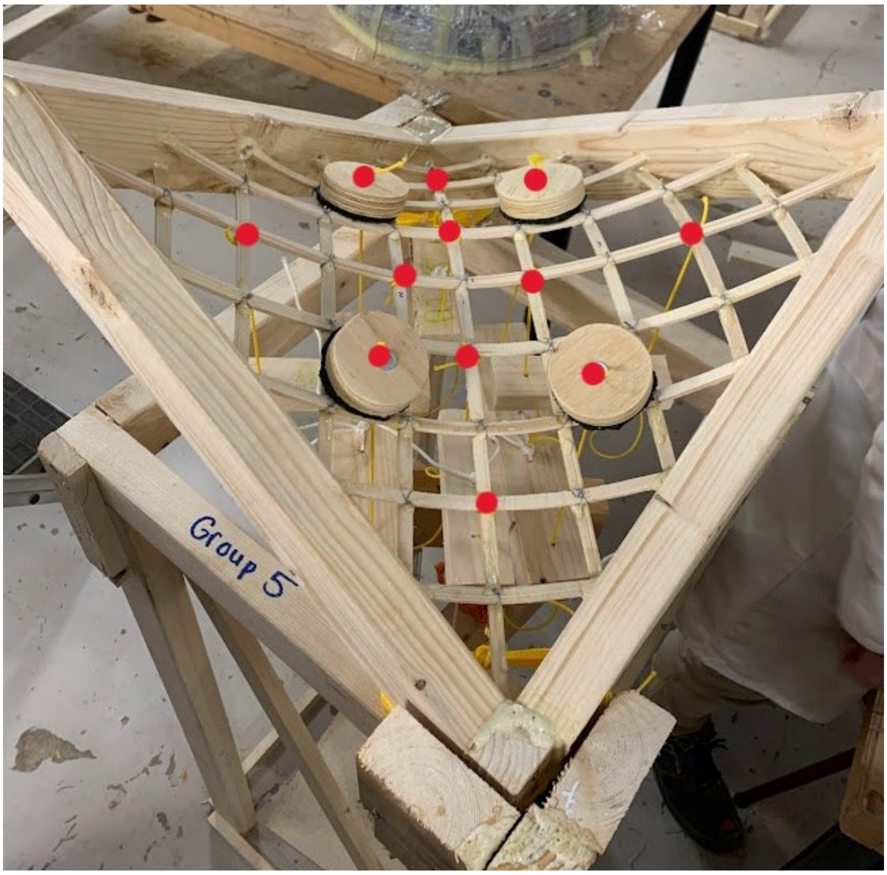


Figure 36: Shell loading

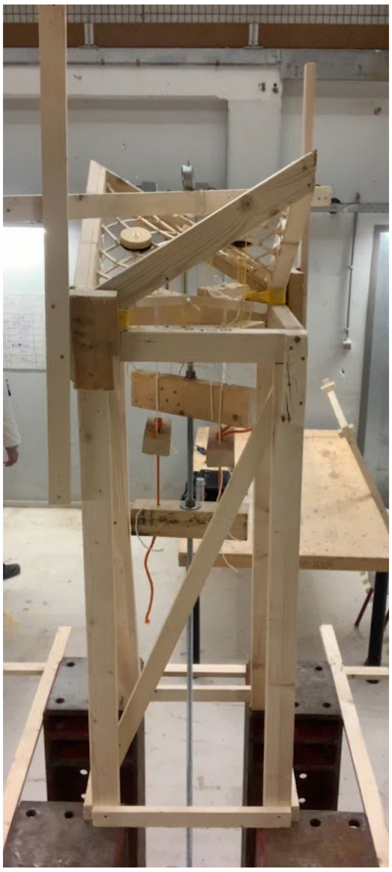


Figure 37: Second testing setup

Results

Displacement

The construction was “analyzed” every 10 bricks. This means that every 10 bricks the displacement is measured and we checked for small failures. These results are shown in the table below (Figure 38).

Bricks	Weight (kg)	Load (N)	Failure	Displacement
0	11.35	111.3435	-	0
10	32.25	316.3725	-	0
20	53.15	521.4015	-	0
30	74.05	726.4305	-	0
40	94.95	931.4595	-	0
50	115.85	1136.489	-	1
60	136.75	1341.518	-	1
70	157.65	1546.547	-	2
80	178.55	1751.576	noice	2.5
90	199.45	1956.605	two failure points	4

Figure 38: Displacement table

Failures

The point of total failure was a complete break in the central wooden element of the shell, pictured in figure 339 and 40.

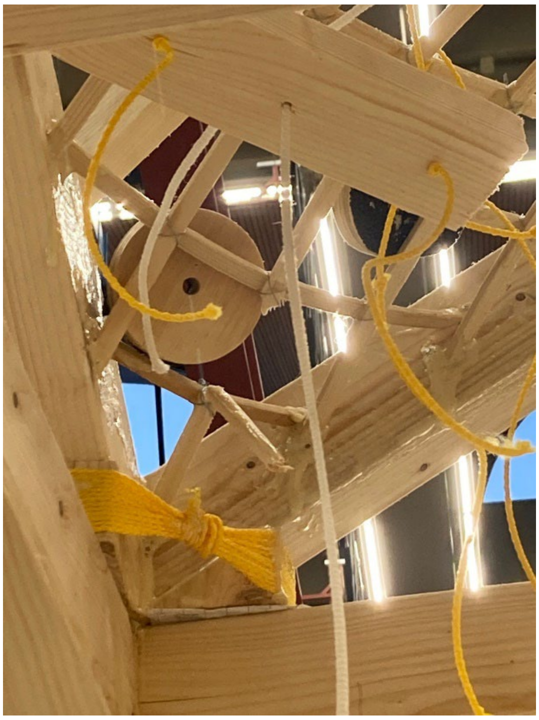


Figure 39: Point of failure 1

Figure 40: Point of failure 2

A secondary point of failure was to be found in the exact same location on the other side of the shell. In this location a break formed, but had not caused complete failure yet.

Discussion & Conclusion

Given the structure failed due to buckling at 199.45kg, our final prediction of failure due to buckling at 200kg was incredibly close. We can however not conclude our calculation to be perfect given the way the structure buckled.

The buckling failure was a break of the wooden element exactly at the connection point with the metal wire. We can therefore assume that the wire slowly cut through the wood because of the added weight, so at the failure load the wood broke because of the cut, not because of traditional buckling.

Therefore we can conclude that excluding the metal wire, the structure would be able to bare more load. Another shortcoming of the test is the distribution of the load. The load was not distributed across all nodes, but across a select number of nodes due to the size of the shell. This indicates that the shell be able to bear more load if the load were to be perfectly distributed.

An extra possibility, not mentioned in the chapters before, is that the ropes broke before the element broke. This shock could be leading to the failure of the wood. Since it happens so fast and in the video it is not clear as well, this theory could not be proven. The other conclusions mentioned before have a high probability of occurring, so we assume that to be the failure cause. Nevertheless it was needed to mention this probability.

Appendix

Python script

```
import pandas as pd
import math as math
import openpyxl
import numpy as np

#Length [m]
L1 = 0.78911937910611
L2 = 0.78911937910611
L3 = 0.636161191857668
L4 = 0.636161191857668
L5 = 0.549374271950669
L6 = 0.549374271950669
L7 = 0.493318658668926
L8 = 0.493318658668926
L9 = 0.395453358995482
L10 = 0.395453358995481
L11 = 0.332990378141224
L12 = 0.332824704631076
L13 = 0.239534313100565
L14 = 0.239534313100565
L15 = 0.193656722383534
L16 = 0.193656722383534
L17 = 0.122105496021115
L18 = 0.122105496021115

d = 706.3 #curve lengte in mm
s = 70.0 #afstand van gewicht rondje diameter in mm
b = 10 #breedte in mm
h = 4.5 #hoogte in mm
L = 60 #afstand verbinding in mm

E = 10000 #E modulus
g = 212.22 #gewicht in kg
F = 9.81 * g #geschatte kracht in N
A = b*h #opp in mm^2

print ('gewicht = ', g, 'in kg')
#print('y = -1.2(x-sqrt(2)/4)^2 + 1.5')
print('knik lengte = 60cm')
straal = 1/2*0.15 + (1/8)*(0.62**2)/0.15
a = straal * 1000 #in mm
print('straal = ', a, 'in mm')
print()

#70 mm
q_punt = F/(2*s) #in N/mm
print('F_last = ', F, 'in N')
print('q_last = ', q_punt, 'N/mm')
N_punt = q_punt * a  # in N, #kracht loodrecht op een lat
print('N_last = ', N_punt, 'N')
sigma_punt = N_punt/A #in N/mm
print('sigma_last = ',sigma_punt, 'N/mm^2')
print()

#hele lat
```

```
print('sigma_hout = 51 N/mm')
N_hout = 51 * (b*h)
print('toelaatbare N_hout = ',N_hout, 'N')
q_hout = (N_hout)/a
print('toelaatbare q_hout = ',q_hout, 'N/mm')
F_hout = (q_hout)*(d)
print("toelaatbare F_hout = ",F_hout, 'N')
print()

print('sigma_hout = 51 N/mm^2')
N_hout = 51 * (b*h)
print('toelaatbare N_hout = ',N_hout, 'N')
q_hout = (N_hout)/a
print('toelaatbare q_hout = ',q_hout, 'N/mm')
F_hout = (q_hout)*(2*s*4)
print("toelaatbare F_hout = ",F_hout, 'N')
print()

I = (b*(h**3))/12
Ne = (((math.pi)**2)*E*I)/((L)**2) #knik belasting in N, kleiner dan buckling
print ('knikbelasting = ', Ne, 'N')

knik = Ne/9.81
print ('knik gewicht = ', knik, 'in kg')
print()

#point load
print('sigma_hout = 51 N/mm^2')
print()
N_point = 51 * b*h
print('N_heel = ',N_point, 'N')
q_point = (N_point)/a
print('q_heel = ',q_point, 'N/mm')
F_point = (q_point)*(2*70)
#F_heel =
(q_heel)*((L1+L2+L3+L4+L5+L6+L7+L8+L9+L10+L11+L12+L13+L14+L15+L16+L17+L18)*0.9*1000
)
print("F_heel = ",F_point, 'N')
gewicht_point = F_point / 9.81
print('gewicht_heel = ', gewicht_point, 'in kg')
```