Predicting the Performance of a Shell Structure

Bend and Break Group 6
Jan Karsemeijer, 5364523
Joris Reijn, 5161096
Pieter van der Werf, 5526353
TU Delft, 2023

Introduction

Shell structures are very efficient structures in carrying loads to the foundation. Shell structures come in different shapes and are made from different materials, which each have their advantages. In this report, a shell structure model will be designed, built and tested to obtain more knowledge on the performance of shell structures. The goal of this report is to predict the performance of the shell structure as well as possible. In addition, research will be performed on the real-life application of the model.

To achieve this goal different alternatives have been considered to come up with a design. Subsequently, the performance of the design was predicted using hand calculations and the results of the test were analyzed.

Table of contents

H1 Design

1.1 Material

We have chosen to make our design out of wood for a couple of reasons. First of all, we were all three personally interested in exploring the properties of wood when used in a shell. Wood is also from a future perspective interesting to work more with and learn about. Second of all, wood is easier to experiment with than concrete. Concrete needs curing en special tests to make sure it gives the results you want. Wood is bendable when dry and even more bendable when soaked in water. This makes it perfect for creating the curves needed in a shell structure. And last, there are a lot more different options, patterns and kinds of shells to explore using wood instead of concrete. A concrete shell is mainly a dome and this wasn’t really in our interest.

The wooden slats that are used in the design are 3 x 7 mm and vary in length. These dimensions are based on some prior experimenting and the final dimensions of the framework.

1.2 Pattern

There are a lot of different patterns possible when working with wood. These are the patterns we looked further into:

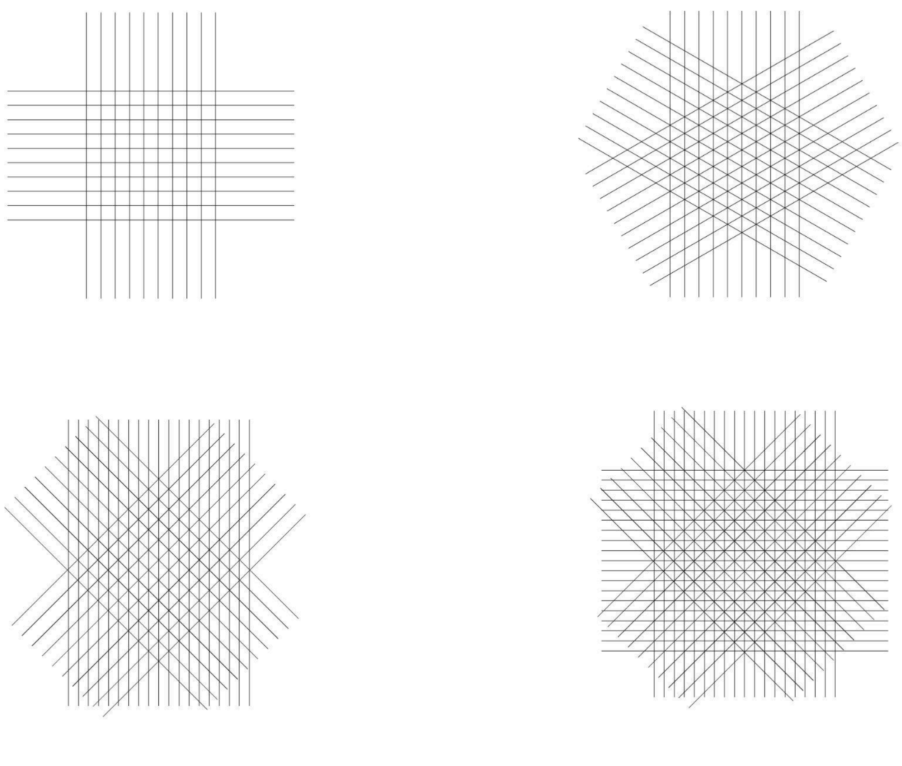

Figure 1: Grid patterns alternatives

All patterns could make a great shell structure, but we chose to go with the first pattern because we needed to be able to fabricate it within a short period of time and in such a case the simple design is most often the best design. By using the somewhat simple design we could also make sure that all the wooden slats fit perfectly within the space available to connect them and everything is secured well.

1.3 Shape & dimensions

For our design we started with a square border with dimensions 0.5m x 0.5m and tried to fit the chosen pattern above within these borders. The ideal height of the shell followed out of the handout and was set to be 150 mm. Based on these dimensions and the chosen pattern we created our final design:

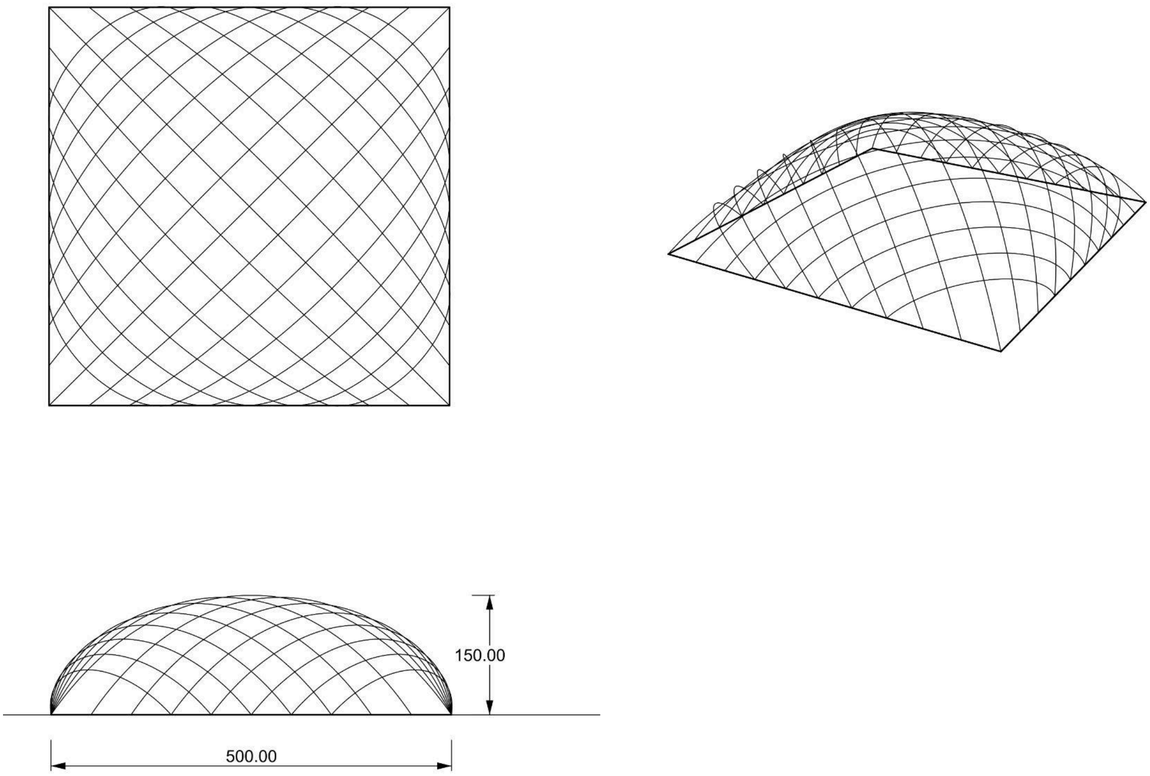

Figure 2: Design shell structure

1.4 Connections between elements and supports

The connections between the wooden slats are provided through a thin layer of wood glue. To make sure that the glue could adhere the elements were pressed together using iron wire. The ends of the slats are wedged into the wooden border of the frame by first drilling a big enough hole and later on filling the empty space with wood glue. In figure 3 the connection between the wooden slats and the support beam is displayed.

Figure 3: Connection between elements and supports

H2 Hand Calculations

In this chapter, different failure mechanisms will be analysed to determine at which load the shell structure will fail and to determine what mechanism will cause failure of the shell structure.

2.1 Shell Buckling

The first failure mechanism that is considered is shell buckling. This mechanism causes sudden deformation of the whole shell structure. The buckling force of the shell structure can be estimated with the following formula:

$$n_{cr} \approx -0.6\frac{Et^2}{a}$$

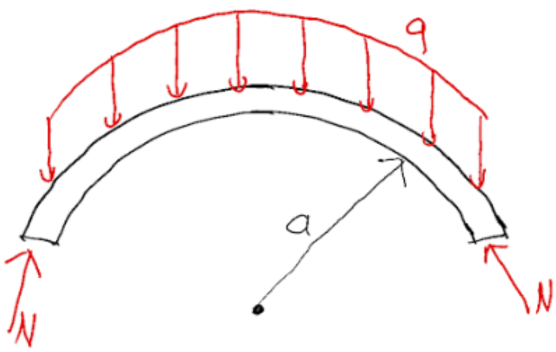

Figure 4: Sketch of buckling problem formulation

Since this formula is for a solid shell structure, as opposed to an open grid shell structure, an equivalent thickness needs to be used. The d in this formula is the distance between elements.

$$A = t \cdot d = 3 \cdot 7 = 21\, mm^2$$

$t = \frac{21}{d}$, with $d = 44.5\, mm$, $t = 0.4719\, mm$

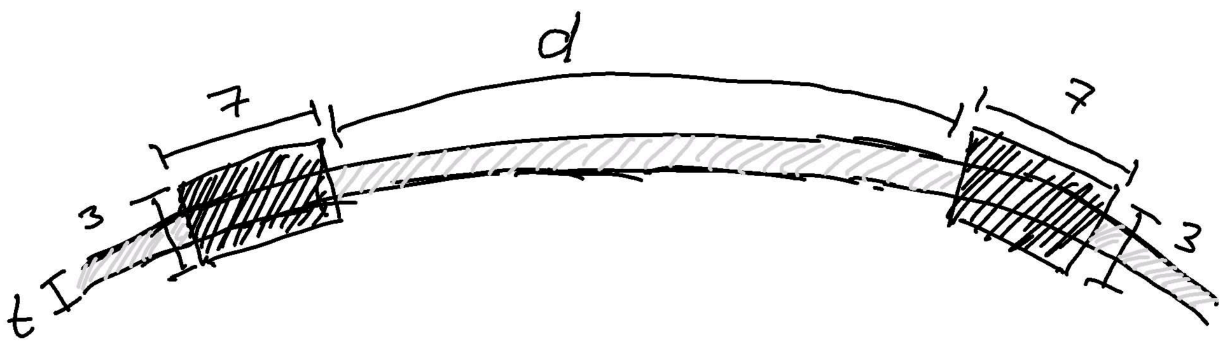

Figure 5: Sketch for calculating supplementary thickness

In the formula the following values are used:

$E = 10000\, N/mm^2$

$a = 0.5s + 0.125\frac{l^2}{s}$, with $s = 150\, mm$, $l = 500\sqrt{2}\, mm$.

Since this design has the same curvature in both directions, only one radius follows:

$a = 491.667\, mm.$

This gives:

$n_{cr} \approx -2.72\, N/mm$

By using the following formula the internal normal force in an element can be calculated.

$N = d \cdot n_{cr}$

$N = 44.5 \cdot 2.72$

$N = 120.94\, N$

From here the distributed load can be calculated by dividing this internal normal force by the radius a.

$q = N/a$

$q = 120.94/ 491.667$

$q = 0.246\, N/mm$

F is the force acting on the shell by one rope.

$F = qd$

$F = 0.246 \cdot 44.5 = 10.95\, N$

By multiplying F by the number of ropes, i.e. 114 ropes, the total force acting on the shell by the weights can be found. This is divided by the gravitational acceleration to find the respective mass.

$W = 114 \cdot F/9.81 = 127.2\, kg$

The mass where shell buckling occurs is calculated to be 127.2 kg.

2.2 Element Buckling

The second failure mechanism that is considered is element buckling. The buckling force of a single element in the shell structure can be calculated using the Euler buckling formula:

$$N_k = \frac{\pi^2 EI}{L_k^2}$$

A single element in the shell structure is assumed to be restricted by two rotating supports as shown in figure 6.

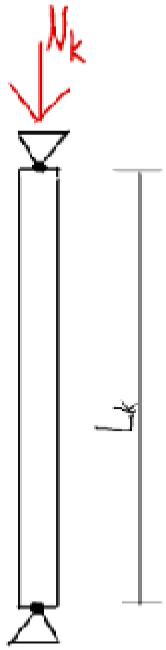

Figure 6: Sketch element buckling length

In the formula the following values are used:

$$I = \frac{1}{12}bh^3 = \frac{1}{12} * 7 * 3^3 = 15.75\ mm^4\ (in\ weakest\ direction)$$

$$L_k = L_{element} = 44.5\ mm$$

$$E = 10000\ N/mm^2$$

Filling in these values gives that the critical buckling force in the elements is equal to 785.0 N. To calculate the load that corresponds to this internal force the following formula is used:

$$q = \frac{N}{a} = \frac{F}{d}$$

$$F = \frac{N*d}{a} = \frac{785.0*44.5}{491.667} = 71.05\ N$$

This is the load that each support node can withstand. The corresponding weight that the total shell structure can support is obtained by multiplying this load by the number of nodes in the shell structure which is equal to 114.

$$W = 114 \cdot 71.05/9.81 = 825.66\ kg$$

2.3 Compressive stress failure elements

The shell structure could also possibly fail due to a lack of compressive resistance of the wooden elements in the structure. The load at which this failure occurs can be calculated with the following simple formula:

$$N = \sigma_{max} * A$$

Where the following values are used:

$$\sigma_{max} = 51\ N/mm^2$$

$$A = 3 * 7 = 21\ mm^2$$

Filling in these values results in a maximum internal force of 1071 N. This internal force corresponds to a load of:

$$F = \frac{N*d}{a} = \frac{1071*44.5}{491.667} = 96.9\ N$$

As the load is distributed over 114 nodes the total weight is equal to:

$$W = 114 * 96.9/9.81 = 1126.5\ kg$$

2.4 Compressive stress failure foundation columns

The load at which the foundation columns fail due to a lack of compressive strength can be calculated in the same way as the compressive failure of the shell structure elements:

$$N = \sigma_{max} * A$$

Where the following values are used:

$$\sigma_{max} = 51\ N/mm^2$$

$$A = 68 * 43 = 2924\ mm^2$$

Filling in these values results in a maximum internal force of 149.124 kN. As the shell structure is supported by 4 columns, the total weight at which the columns will fail is equal to:

$$W = 4 * 149124/9.81 = 60805\ kg$$

2.5 Failure metal bar

The failure load of the metal bar can also be calculated with the same formula. The bar is made of aluminium.

$$N = \sigma_{max} * A$$

Where the following values are used:

$$\sigma_{max} = 310\ N/mm^2$$

$$A = \frac{1}{4}\pi d^2 = \frac{1}{4} * \pi * 10^2 = 78.54\ mm^2$$

This results in a maximum tensile force of 24347 N. The failure load would then be;

$$W = 24347/9.81 = 2481.9\ kg$$

2.6 Failure ropes

Since the yellow rope broke in a test at a load of 500 N and the layer with the least yellow ropes contains 57 ropes. A maximum weight this layer could hold would be;

$$W = 57 \cdot 500/9.81 = 2905.2\, kg$$

The white rope has a strength of 1700 N and the layer with the least white ropes contains 13 ropes. A maximum weight this layer could hold would be;

$$W = 13 \cdot 1700/9.81 = 2252.8\, kg$$

The red rope has a strength of 7500 N and the layer with the least white ropes contains 2 ropes. A maximum weight this layer could hold would be;

$$W = 2 \cdot 7500/9.81 = 1529.1\, kg$$

2.7 Failure distribution blocks

To calculate the load at which the distribution blocks will fail the following formulas are used:

$$\sigma = \frac{M*I}{z} \rightarrow M = \frac{\sigma*z}{I}$$

$$M_{max} = \frac{1}{4}FL$$

Combining these formulas gives the following formula for the maximum load:

$$F = \frac{4*\sigma*z}{I*L}$$

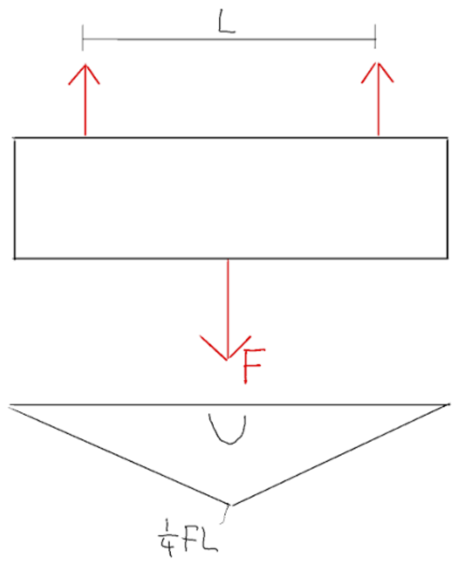

Figure 7: Sketch bending moments in distribution blocks

As each layer of distribution blocks has different values the following Excel sheet was created:

Layer	L (mm)	h (mm)	b (mm)	z (mm)	I (mm^4)	Sigma (N/mm^2)	Number of ropes below	F (N)	Gewicht (kg)
1	48	21	46	10,5	35501	51	57	14369,25	1437
2	67	21	46	10,5	35501	51	28	10294,39	1029
3	100	21	46	10,5	35501	51	14	6897,24	690
4	160	43	68	21,5	450540	51	6	26718,05	2672
(Triangle) 4	100	17	85	8,5	34800	51	6	8352,10	835
(Triangle) 5	285	34	120	17	393040	51	2	16549,05	1655
6	265	71	71	35,5	2117640	51	1	45920,66	4592

Table 1: Failure loads for different layers of distribution blocks

From this table it can be concluded that the distribution blocks in layer 3 will fail first at a weight of 690 kg.

2.8 Results Hand Calculation

In the table below the failure load of each failure mechanism is displayed:

Failure mechanism	***Critical Load (kg)***
Shell buckling	127
Element buckling	826
Compressive stress failure element	1127
Compressive stress failure columns	60805
Failure metal bar	2482
Failure ropes	1529
Failure distribution blocks	690

Table 2: Failure loads per mechanism of failure

From the results of the hand calculations it can be concluded that the shell structure will fail due to shell buckling with a predicted load of 127 kg. Since the other groups' predictions were about half the load of the actual failure load, it was decided to up the prediction to 170 kg.

H3 Building process

The first step in the process was designing the shell. After this was done, the dimensions of the wooden slats needed to be selected. These dimensions were based off of the dimensions of the slats that we got to experiment with and the dimensions of our framework. The next step in the process was building the framework and drilling the holes in the top border so we could easily connect the wooden slats later on.

After the framework was completely done we could start to build the shell structure itself. To guide the first main diagonals a column was placed in the middle of the framework as is shown in figure 9. These first diagonals form the guidance for the next wooden slats that need to be placed. The used wooden slats are quite small and because of this already pretty bendable. This meant that the slats in the middle with the highest span didn't need to be soaked in water first and could easily be connected to the frame. The other slats did need to be soaked in hot water first but could then be bent into the predrilled holes with ease. To give the wooden slats the correct length the slats were bent into the right position before the cut was made.

After the slats were fully dried they could be glued to each other. To make sure that the glue adhered well to the slats, the slats were connected using iron wire (figure 8), which was removed once the glue was completely dry. Lastly, the shell structure was secured to the framework by filling the drilled holes with wood glue.

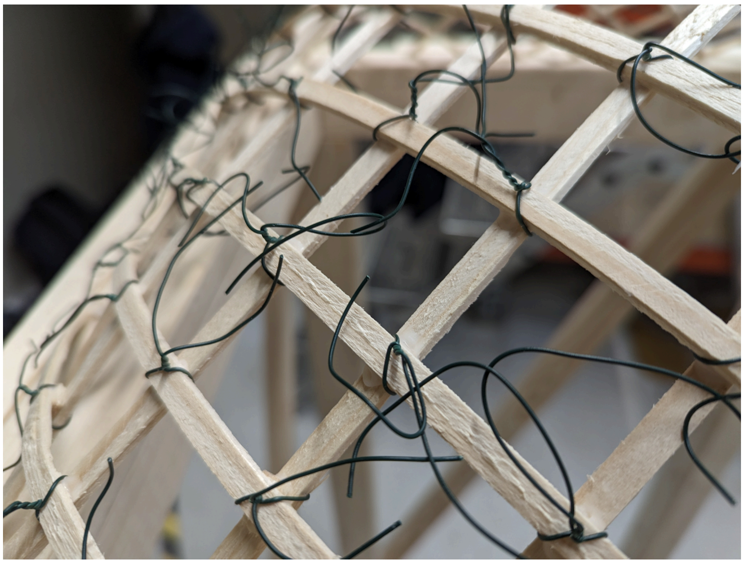

Figure 8: Temporary connections using iron wire

Figure 9: Guidance column

To make sure that the load was distributed well over all the nodes in the framework, almost every node was connected to a rope. These ropes were connected to distribution blocks, which divided the number of ropes by 2. In the end the force of all ropes comes together in one metal bar which carries a wooden plate, on which the load can be applied. This mechanism ensures that the load on the shell structure is well divided between the nodes.

By placing bricks with an average weight of 2.1 kg on the bottom plate the load on the shell structure was slowly increased. The deformation of the shell structure was measured and noted for different loads. The load was gradually increased until the shell structure collapsed.

Figure 10: Distribution blocks and ropes

Figure 11: Loading of the shell

H4 Test results

Based on the hand calculations that were made on the shell structure the shell should fail due to a load of 170 kg. The predicted failure mechanism that corresponds to this load is shell buckling. The plateau was loaded with bricks until the point of failure. The shell failed in the form of buckling, according to the prediction. In the pictures in figure 11, the shell structure after failure is displayed. From these pictures, it becomes visible that the entire top of the shell structure broke.

The shell structure failed when 126 bricks were placed on the wooden plateau. Since the bricks weigh approximately 2.1 kg each. This corresponds to a load of 264.6 kg, excluding the weight of the test setup and the self weight of the structure. The plateau weighed 7.8 kg and the structure’s self weight including the weight of the test setup was 8 kg. This led to the total load of failure of 280.4 kg. This means the prediction was off by 110.4 kg i.e. 65%.

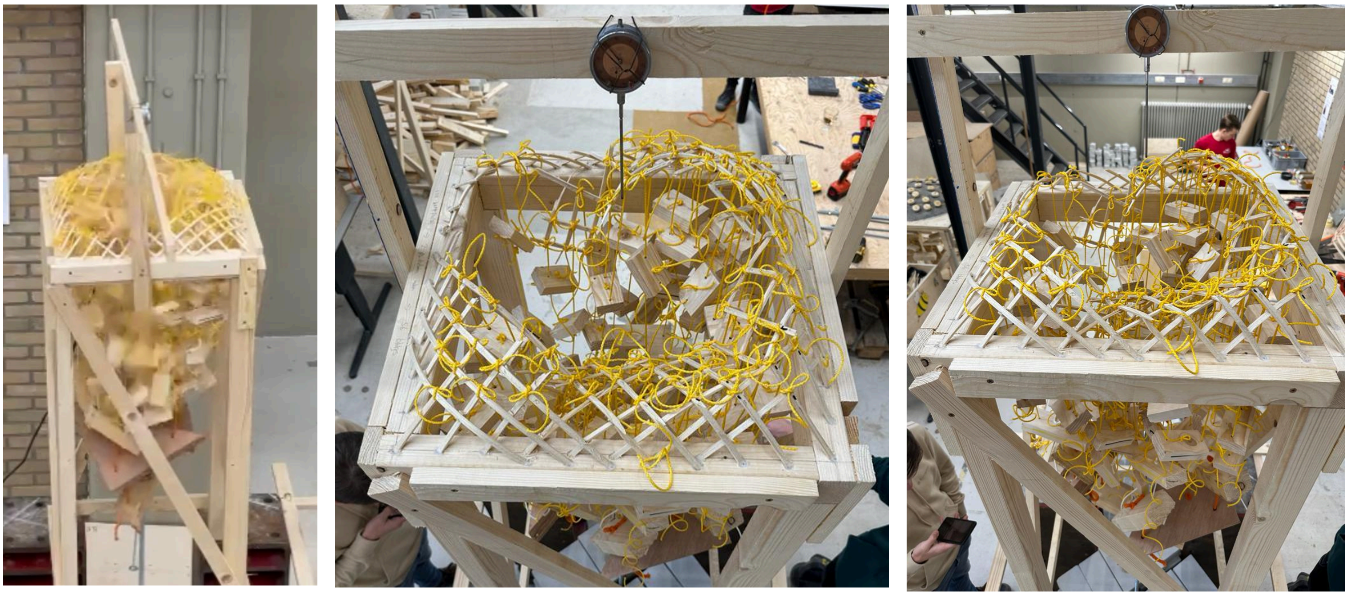

Figure 11: Shell structure during (most left picture) and after failure.

In figure 12, the displacement of the top of the shell structure is displayed for an increasing load. From this graph the deformation of the shell structure due to the snow load can be estimated. The weight of the snow load is equal to 1 kN/m^2. This corresponds to a deflection of 0.13 mm of the model. This corresponds to a deflection of 1.3 mm in real-life, when the shell structure was scaled up with a 1:10 ratio.

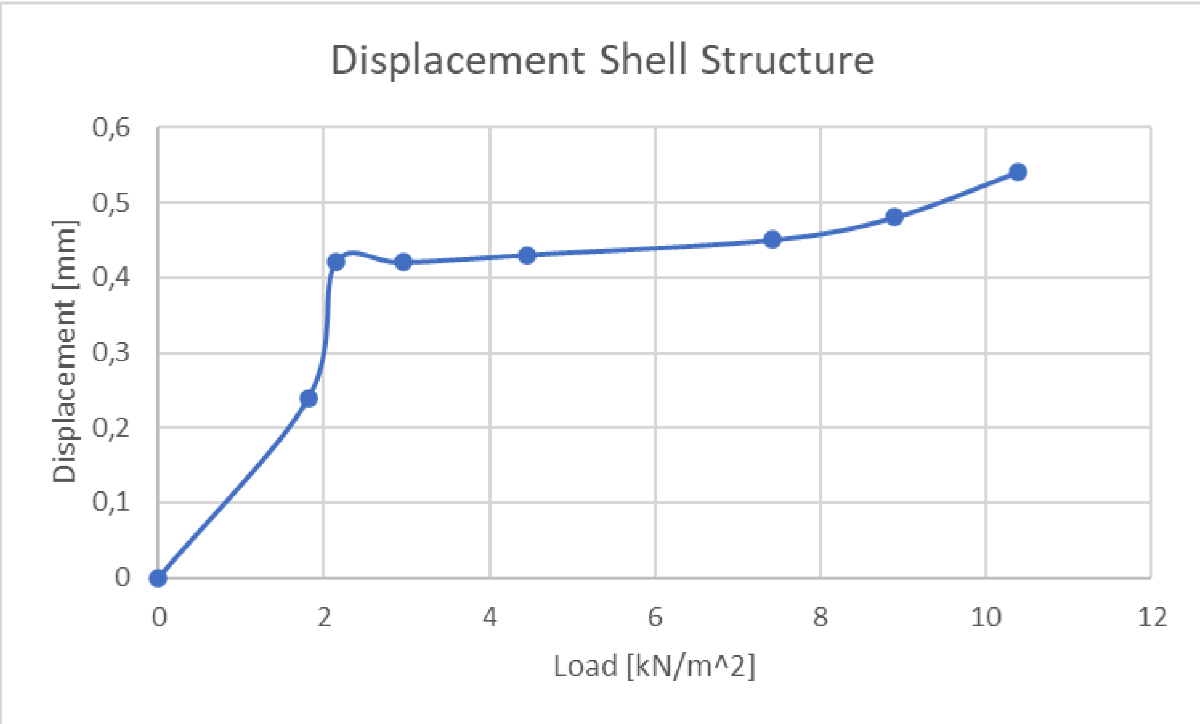

Figure 12: Displacement of the top of the shell for different loads

The measured deflection of the top of the shell structure is considered to be low. This raises questions about the correctness of the measurements. Based on the video footage of the loading of the shell structure the shell structure does not deflect visibly, which suggests very low deflections. Only the ropes below the shell structure showed visible deformation. In addition, the stiffness of the shell structure seemed to be very high when people tried to push down the shell before the test. However, the correctness of the measurement cannot be guaranteed. It is possible that the measurement device was not placed correctly or that the measurement device malfunctioned. It is believed that no errors were made during the reading of the measurements.

H5 Real-size shell structure

The model was built on a 1:10 scale. This means that the foundation of the real-life shell structure has dimensions of 5 by 5 meters and the height of the structure will be 1.5 meters.

The biggest span in our shell structure is about 700 mm. The model is scale 1:10 so that would mean a span of about 7 meters in real life. It is possible to make whole, solid, wooden beams of these dimensions, but in reality these would probably be made out of two parts and connected by finger-joints.

In figure 13 an example of a finger-joint is displayed. Finger-joints are a connection technique by which two pieces of wood are connected by a series of pointing outwards and pointing inwards ‘fingers’. These overlapping fingers fit perfectly together and make a strong, reliable and durable wood connection by creating an enlarged surface area for the wood glue. Bigger spans are possible by using these joints, because a few small beams can be connected to form one large beam. By using a few smaller beams instead of one large beam, a lot of waste can be prevented. The seamless finger-joints are also aesthetically great to use in shell structures.

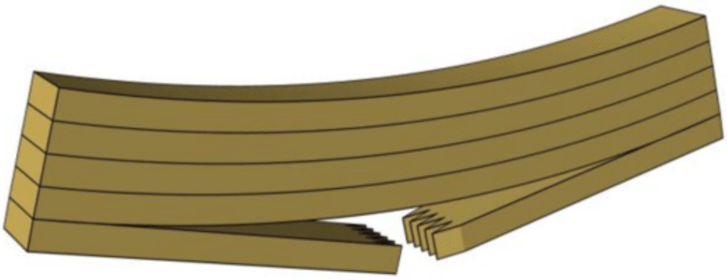

Figure 13: Example of finger joints

In the model, the beams are connected with holes in the foundation. To place the beams in these holes, they need to be further bent than their final shape. As this could be hard in real life, it is preferred to connect the beams to the foundation with bolted connections.

The model failed when a load of 280 kg was applied. As the area of the structure was equal to 0.25 m^2, this corresponds to a load of 1120 kg/m^2 = 11.2 kN/m^2. The self-weight of the shell structure was estimated by multiplying the length of all the elements with the area of the elements and the density of wood:

$$Length\ elements\ =\ 18095\ mm$$
$$Area\ cross\ section\ =\ 3\ *\ 7\ =\ 21\ mm^2$$
$$Total\ volume\ wood\ =\ 18095\ *\ 21\ =\ 379995\ mm^3$$
$$Density\ wood\ =\ 600\ kg/m^3$$
$$Total\ weight\ =\ 600\ *\ 379995\ *\ 10^{-9}\ =\ 0.228\ kg\ =\ 2.237\ N$$
$$Corresponding\ load\ =\ (2.237\ *\ 10^{-3})\ /\ 0.25\ =\ 0.0089\ kN/m^2$$

In real-life the weight of the shell structure is equal to 0.228 * 10^3 = 228 kg. The corresponding load from the self-weight of the shell structure can be calculated by dividing the weight by the area of the structure in real-life. This results in a load of (228 * 9.81 * 10^-3) / 25 = 0.0895 kN/m^2. This means that the scale factor of the load due to self-weigth is equal to the scalefactor of the dimensions (scale 1:10).

The snow load on the shell structure is equal to 1 kN/m^2. This means that the total load on the structure in real-life is equal to 1.09 kN/m^2. The failure load of the structure in real-life is equal to 11.2 kN/m^2. Therefore it can be concluded that the shell structure in real-life can easily withstand the snow load.

When the shell structure is subjected to the snow-load, the deflection on the model of the shell structure is equal to 0.13 mm. This corresponds to a deflection of 1.3 mm in real-life, when the shell structure was scaled up with a 1:10 ratio. This deflection is very small and will therefore not influence the serviceability of the shell structure.

This shell is designed with a scale factor of ten in mind. The maximum allowable scale factor follows from the equation below;

$$\frac{m_{self} \cdot g \cdot S^3}{A \cdot S^2} + \frac{F_{snow}}{A \cdot S^2} = \frac{m_{failure} \cdot g}{A \cdot S^2}$$

with m_{self}: self weight of the model; 0.228 kg, A: area of the model; $0.25\ m^2$, g: gravitational acceleration; $9.81\ \frac{m}{s^2}$, $m_{failure}$: weight at which the model collapsed; 280 kg, S: scaling factor and F_{snow}: load that acts on the shell.

The load of the snow remains 1 kN so $\frac{F_{snow}}{A} = 1000$, since A=0.25 then $F_{snow} = 250\ N$.

Using these values the Scaling factor can be determined.

$$0.228 \cdot 9.81 \cdot S^3/(0.25 \cdot S^2) + 250/(0.25 \cdot S^2) = 280 \cdot 9.81/(0.25 \cdot S^2)$$

$$0.228 \cdot 9.81 \cdot S^3 + 250 = 280 \cdot 9.81$$

$$0.228 \cdot 9.81 \cdot S^3 = 280 \cdot 9.81 - 250$$

$$S^3 = \frac{2496.8}{0.228 \cdot 9.81}$$

$$S = 10.3735$$

The maximum allowable scale factor is determined to be 10.37, this means the biggest this model could be made is 5.18675 by 5.18675 meters.

H6 Discussion

The prediction was off by 110.4 kg, it would be a lot closer if the initial idea of doubling the calculated load was maintained. This idea results from the results of previous groups where the failure load turned out to be approximately twice the predicted load. Too few tests were performed to prove the existence and correctness of this relationship. Using the idea would’ve led to a prediction of 254 kg, only off from the actual load by 26.4 kg i.e. 10.4%. This incorrect prediction follows from a formula to calculate a solid shell. The assumption was made that this would be rectified by using a supplementary thickness, this turned out to not be enough to make an accurate prediction. Although the predicted load of the shell structure differed from the actual failure load, the failure mechanism was predicted correctly.

The building process of the shell structure can be considered to be successful. No major mistakes were made during the design process. Based on visual observations, failure did not occur due to production errors.

In a follow-up study, more detailed calculations on the shell structure could be made to estimate the performance of the shell structure better. In addition, a finite-element program, such as SCIA Engineer could be used. Making a model wherein all the elements are correctly connected in Scia Engineer turned out to be too difficult due to the relatively complex shape of the design. The use of finger joints is not tested in this report, so the effectiveness of the use in this application would need to be tested in further research.

H7 Conclusion

Concluded in this report is that despite the incorrect prediction of the load of failure, the shell structure is strong enough to support its self-weight as well as the additional 1 kN/m^2 of snow. The top of the model of the shell structure deformed with 0,13 mm which corresponds to a deflection of 1,3 mm in the real-life structure, which is not a problem for the serviceability of the structure. As predicted the shell structure will fail first due to shell buckling with a corresponding failure load of 10.4 kN/m^2. The design is certainly achievable on a real life scale, with the use of finger joints and bolted connections to the foundation.

H8 References

CT3280 Shell Roofs. (2023). phoogendoorn.nl. retrieved in 2023, from: https://phoogenboom.nl/B&B_schaal_0.html