**Shell response derived from a scale model**

dr.ir. P.C.J. Hoogenboom, 16 November 2023

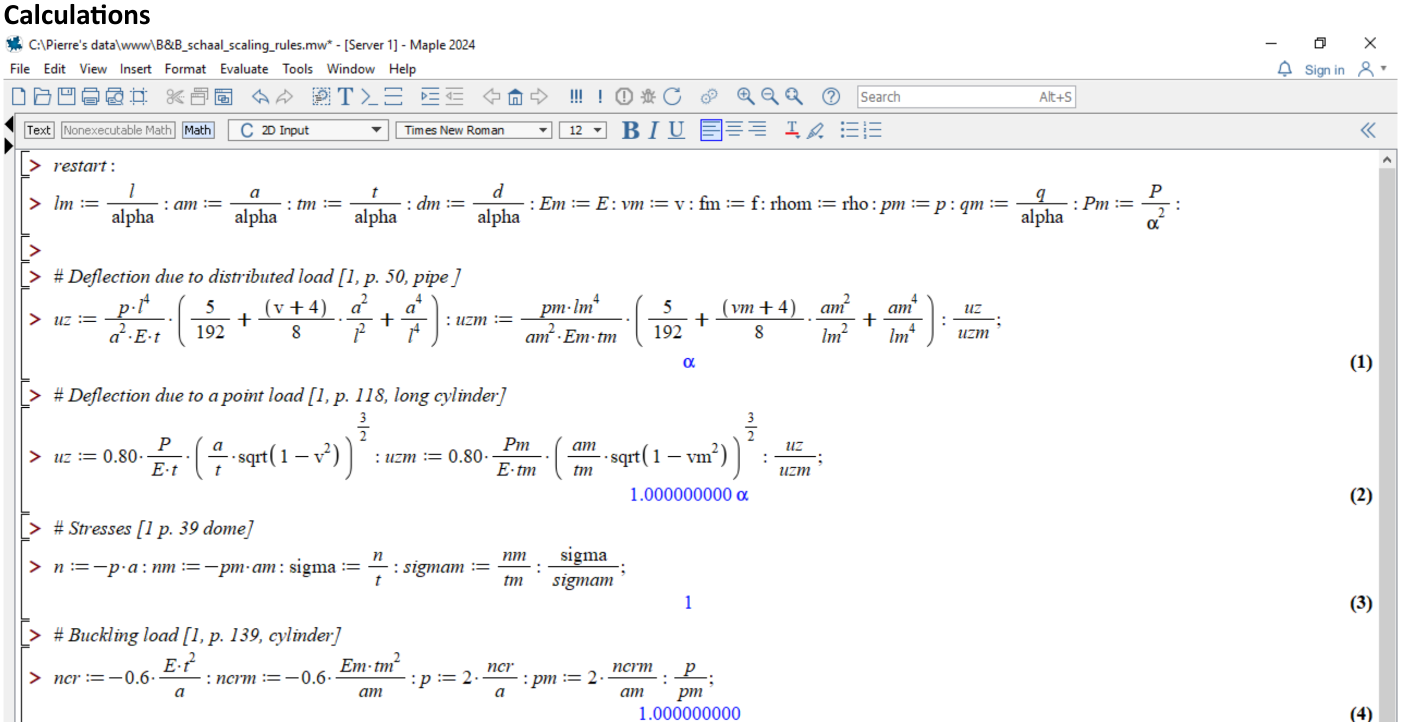
This text gives the relation between the response of a shell structure and that of a scaled down model of the shell structure.

**Input**

| | real shell | scaled-down model | |
|---|---|---|---|
| span | *l* | *l* / α | m |
| radius of curvature | *a* | *a* / α | m |
| thickness | *t* | *t* / α | m |
| reinforcement diameter | *d* | *d* / α | m |
| reinforcement spacing | *s* | *s* / α | m |
| imperfection length | *l* | *l* / α | m |
| imperfection amplitude | *a* | *a* / α | m |
| Young's modulus | *E* | *E* | $N/mm^2$ |
| Poisson's ratio | ν | ν | – |
| yield strength | *f* | *f* | $N/mm^2$ |
| mass density * | ρ | ρ | $kg/m^3$ |
| damping | ξ | ξ | – |
| line support stiffness | *k* | *k* | $kN/m^2$ |
| distributed load | *p* | *p* | $kN/m^2$ |
| line load | *q* | *q* / α | kN/m |
| point load | *P* | $P / \alpha^2$ | kN |

* This mass density is the inertia in dynamic calculations. Self-weight is applied as a distributed load on the shell model.

**Calculations**



> # Buckling length [1, p. 139, cylinder]

> $lcr := 1.7 \cdot \mathrm{sqrt}(a \cdot t) : lcrm := 1.7 \cdot \mathrm{sqrt}(am \cdot tm) : \frac{lcr}{lcrm};$

$$\frac{1.000000000 \sqrt{a\,t}}{\sqrt{\frac{a\,t}{\alpha^2}}} \tag{5}$$

> # Vibration [1, p. 163, curved plate]

> $fn := \mathrm{sqrt}\left(\frac{\mathrm{Pi}^2}{12 \cdot (1 - \mathrm{v}^2)} \cdot \frac{E \cdot t^2}{\mathrm{rho} \cdot l^4}\right) : fnm := \mathrm{sqrt}\left(\frac{\mathrm{Pi}^2}{12 \cdot (1 - \mathrm{vm}^2)} \cdot \frac{Em \cdot tm^2}{rhom \cdot lm^4}\right) : \frac{fn}{fnm};$

$$\frac{\sqrt{\frac{E\,t^2}{(-12\,\nu^2 + 12)\,\rho\,l^4}}}{\sqrt{\frac{E\,t^2\,\alpha^2}{(-12\,\nu^2 + 12)\,\rho\,l^4}}} \tag{6}$$

**Output**

| | real shell | scaled-down model | |
|---|---|---|---|
| deflection | $u$ | $u / \alpha$ | mm |
| stresses | $\sigma$ | $\sigma$ | N/mm$^2$ |
| membrane force | $n$ | $n / \alpha$ | kN/m |
| moment | $m$ | $m / \alpha^2$ | kNm/m |
| collapse load factor (plastic mechanism) | $\lambda$ | $\lambda$ | – |
| buckling load factor | $\lambda$ | $\lambda$ | – |
| buckling length | $l$ | $l / \alpha$ | m |
| natural frequencies | $f$ | $\alpha\, f$ | Hz |

**Comments**

The formulas in the calculations are for specific shell shapes, small displacements and linear elastic material behaviour. However, it is believed that the conclusions can be generalized to any shell shape. A mathematical proof is not part of this text.

The rules have been verified in a bachelor end project [2]. In this project finite element models of large and small structures have been compared in linear, geometrically nonlinear and physically nonlinear behaviour. The result was that no deviations from the rules were found.

**Size effect**

Some materials break in a brittle way, for example, glass, unreinforced concrete and masonry. It is known that structures made of these materials fail at a stress that depends on the size of the structure. Large structures fail at a smaller stress than small structures. This is called the *size effect* of structural strength [3].

Size effect is caused by the internal structure of the material, for example, the bricks in a masonry beam. Head joints in masonry are not strong and failure starts by fracture of the brick above the bottom middle head joint (Fig. 1). If we calculate the stresses in both beams linear elastically, the large beam fails at a smaller stress.

*Exercise:* Derive a formula for the beam linear elastic failure stress as function of the beam depth, brick height and material fracture stress.

Consequently, when we build a scaled-down masonry shell model, we need to use scaled-down bricks too.

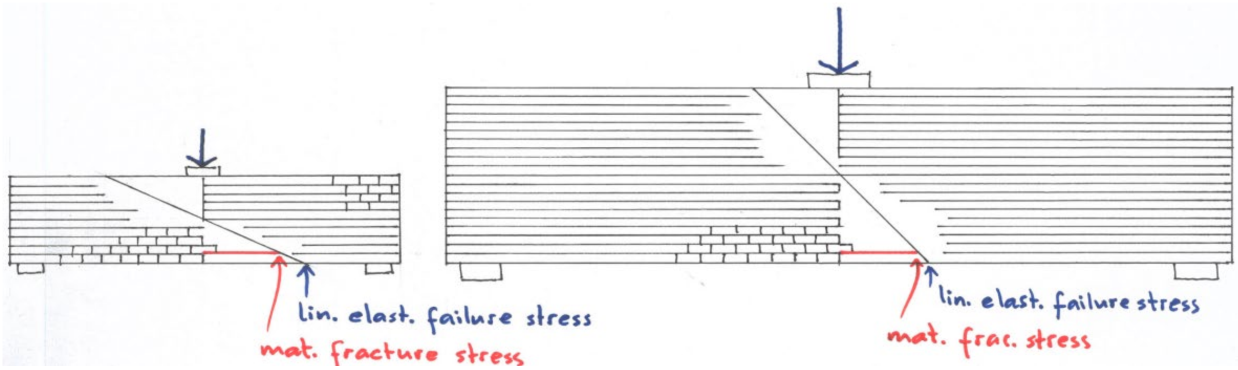


Figure 1. Small and large masonry beams. The material fracture stresses are the same. The linear elastic failure stresses are different.

**Literature**

1. P.C.J. Hoogenboom, Notes on Shell Structures, Reader CIEM5301 Delft University of Technology, Civil Engineering and Geosciences, 2023, online: http://phoogenboom.nl/b17_schedule.html

2. A. Alami, Finite element analysis of structures; Comparing large and small-scale structures in linear and non-linear behaviour, Bachelor end project, Delft University of Technology, Faculty of Civil Engineering and Geosciences, June 2024, online: https://phoogenboom.nl/BSc_projects/eindrapport_alami.pdf

3. Z.P. Bazant, Size effect on structural strength, Wikipedia, retrieved 3 July 2024