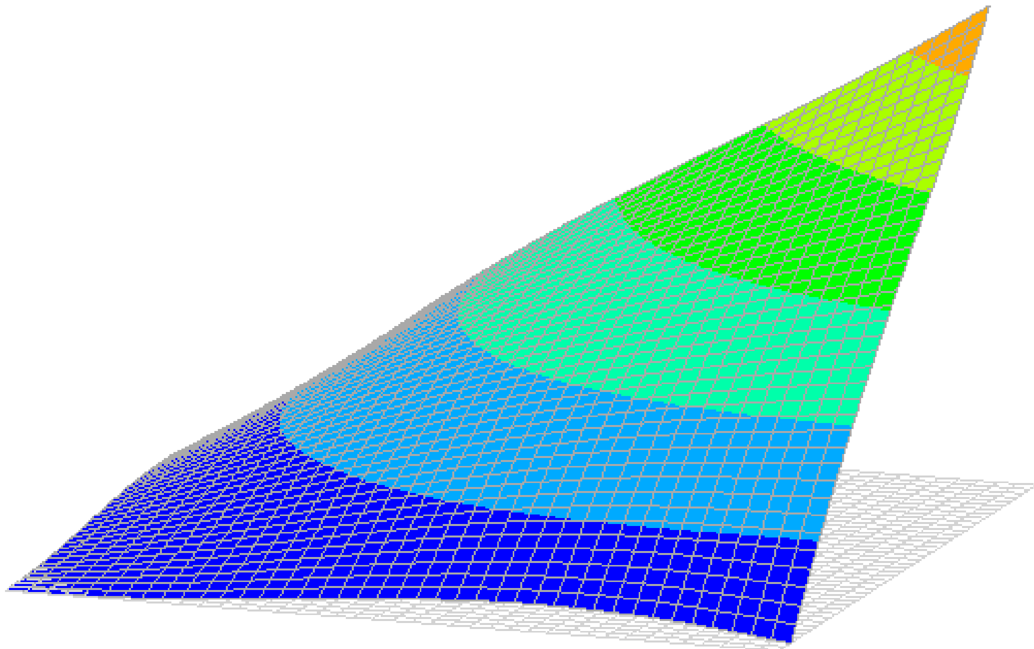
Twisted plate behaviour

CIE4143 – Shell Analysis



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Introduction

For the course, CIEM5301 Shell Structures, an assignment is conducted focusing on the behaviour of twisted plates and their effects on membrane forces, bending moments, and Gaussian curvature. This assignment specifically investigates the effects of membrane forces. The primary objective is to determine a more accurate scaling factor for the formulas used to calculate membrane forces. The analysis is performed using Diana 10.8, which includes non-linear finite element analysis capabilities.

State of the art

For a flat plate, the Gaussian curvature is zero. When the plate is twisted, it exhibits negative Gaussian curvature. As the plate transitions from a flat to a twisted state, the Gaussian curvature changes, indicating an in-extensional deformation where membrane forces can develop. When one of the corners is displaced out of the plane, significant compressive membrane forces develop in the middle of the plate, while tension occurs at the edges.

The compressive forces in a flat plate increase substantially when one of the corners is displaced out of the plane to the extent that the plate "buckles." This is due to the transition from significant Gaussian curvature to nearly zero. The membrane forces at this stage in the middle of the plate are given by the formula:

$$n_1 = n_2 = \frac{1}{64} E t b^2 k_G \tag{1}$$

The membrane forces at the edge can be described with the following formulas:

$$n_1 = -\frac{1}{32} E t b^2 k_G, \quad n_2 \approx 0 \tag{2}$$

Assignment description

The task involves creating a model of a square thin plate. Three corners have a hinged support and the 4th corner is displaced until failure. A plate is considered thin when $8 \ldots 10 < \frac{a}{h} < 80 \ldots 100$. Where a is the plate's dimension and h is the thickness of the plate. A geometrical non-linear analysis is performed to investigate the behaviour of the plate. The goal is to define more accurate coefficients for $-\frac{1}{32}$ and $\frac{1}{64}$ shown in formulas (1) and (2).

Model assumptions

The plate dimensions are 1000 mm by 1000 mm, and three different plate thicknesses (10 mm, 20 mm, and 30 mm) are analysed to determine if there is any variation in results. The corner points of the plate are supported with hinged conditions: the support at the opposite corner of the one subjected to displacement has its translations set to zero in the x, y, and z directions. One of the other supports has translations set to zero in the x and z directions, and the third support has translation set to zero only in the z-direction. All rotations for all supports are free. A displacement of 1000 mm is applied to the last corner point to ensure failure.

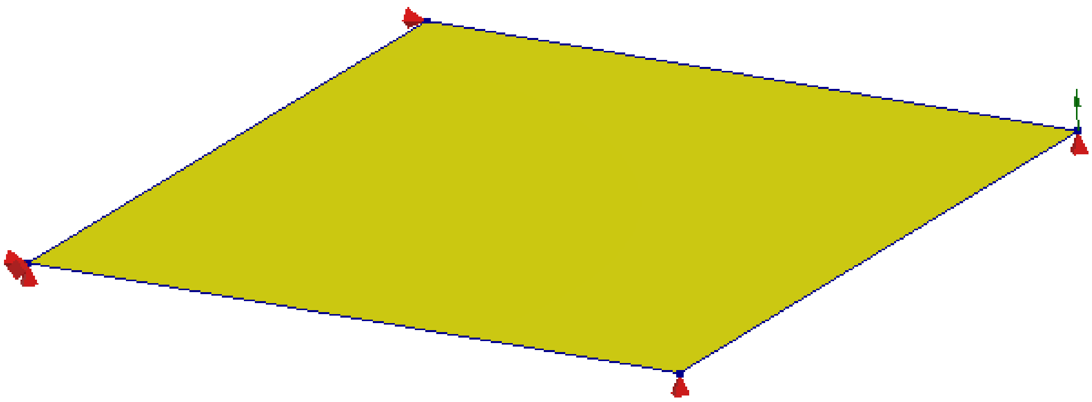

Figure 1: Model boundary conditions

The Newton-Raphson method is used for the analysis, with convergence norms set for both force and displacement with tolerances of 0.01, and an abort criterion set at 10,000. The material properties used for the analysis are provided in Table 1.

Concrete C25/30	
Aggregate type	Quartzite
Cement type	Class N
Young's modulus	31475.8 N/mm^2
Poisson ratio	0.2

Table 1: Material properties

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Analyses and results

Three analyses are performed to examine the influence of thickness on the results. As determined by Staaks, buckling of the plate occurs at a displacement of $16.8t$ (Staaks, 2003). Therefore, the initial assumption is that for all analyses with different plate thicknesses, buckling will occur at this displacement. To verify this, a graph is generated of N_{xx} or N_{yy} in the mid node of the plate (N_{xx} and N_{yy} give the same results but in the other directions).

To achieve convergence, different step size and the number of maximum iterations is used. An overview of the settings is shown in Table 2.

	Plate thickness [mm]	Load steps	Load increment [mm]	Maximum iterations
Analysis 1	10	1000	1	500
Analysis 2	20	500	2	250
Analysis 3	30	500	2	200

Table 2: Non-linear properties per analysis

Analysis 1

Figure 2 illustrates a graph of the first 300 load steps, corresponding to 300mm displacement. The buckling is expected at occur at $16.8t$ (168 mm), indicated with a blue line. The results of the other load steps are not considered as the plate has already buckled. The plate does not exactly buckle at $16.8t$ but at $16.4t$. For the determination of the coefficients, this point will be considered.

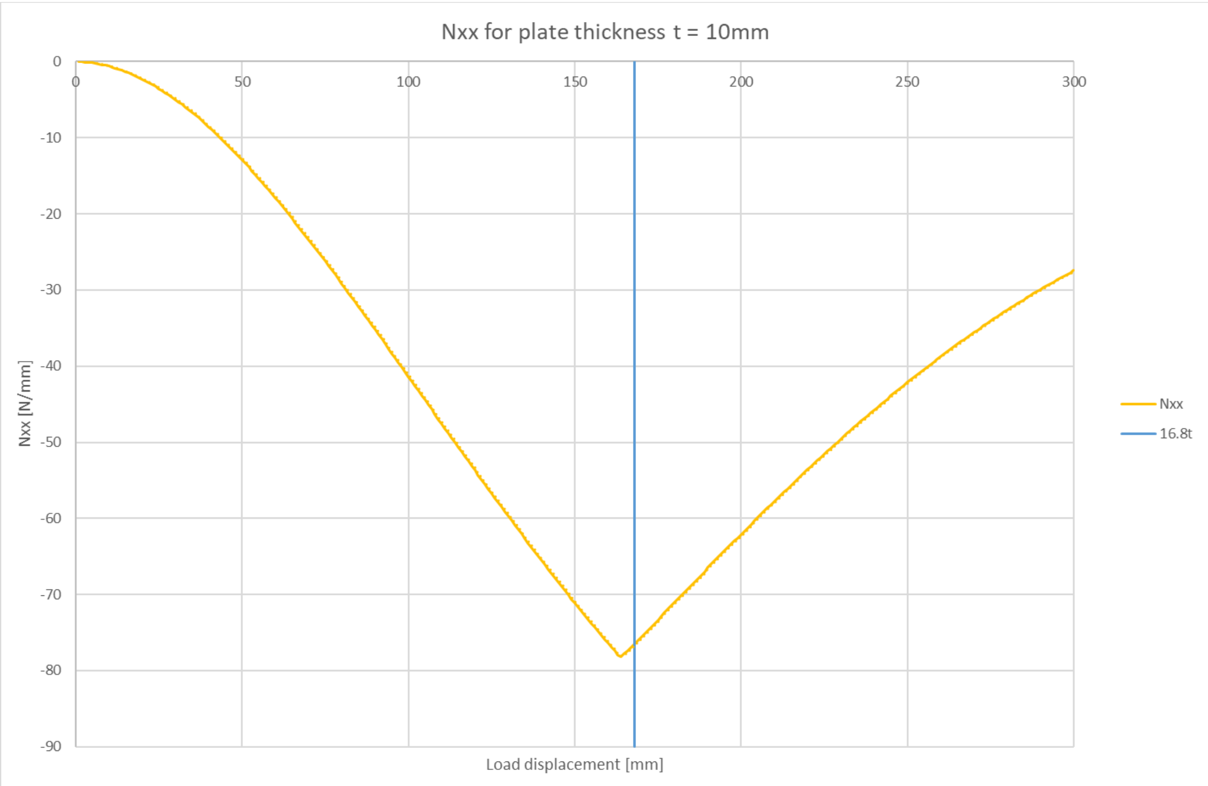

Figure 2: Nxx for t = 10mm, buckling point check

The deformation plot is shown in Figure 3. At $1t$ a small displacement is observed. At $10t$, deformation has developed and a curve is visible over the diagonal of the plate. Also called Gaussian curvature. This means that the plate has not buckled yet. At $16.4t$, the observed buckling point, the curve is still visible, and the edge of the plate is not straight anymore which indicates that stresses are developing. At $20t$, post-buckling, it is visible that the curve becomes straight with slight curvature only at the left and right points.

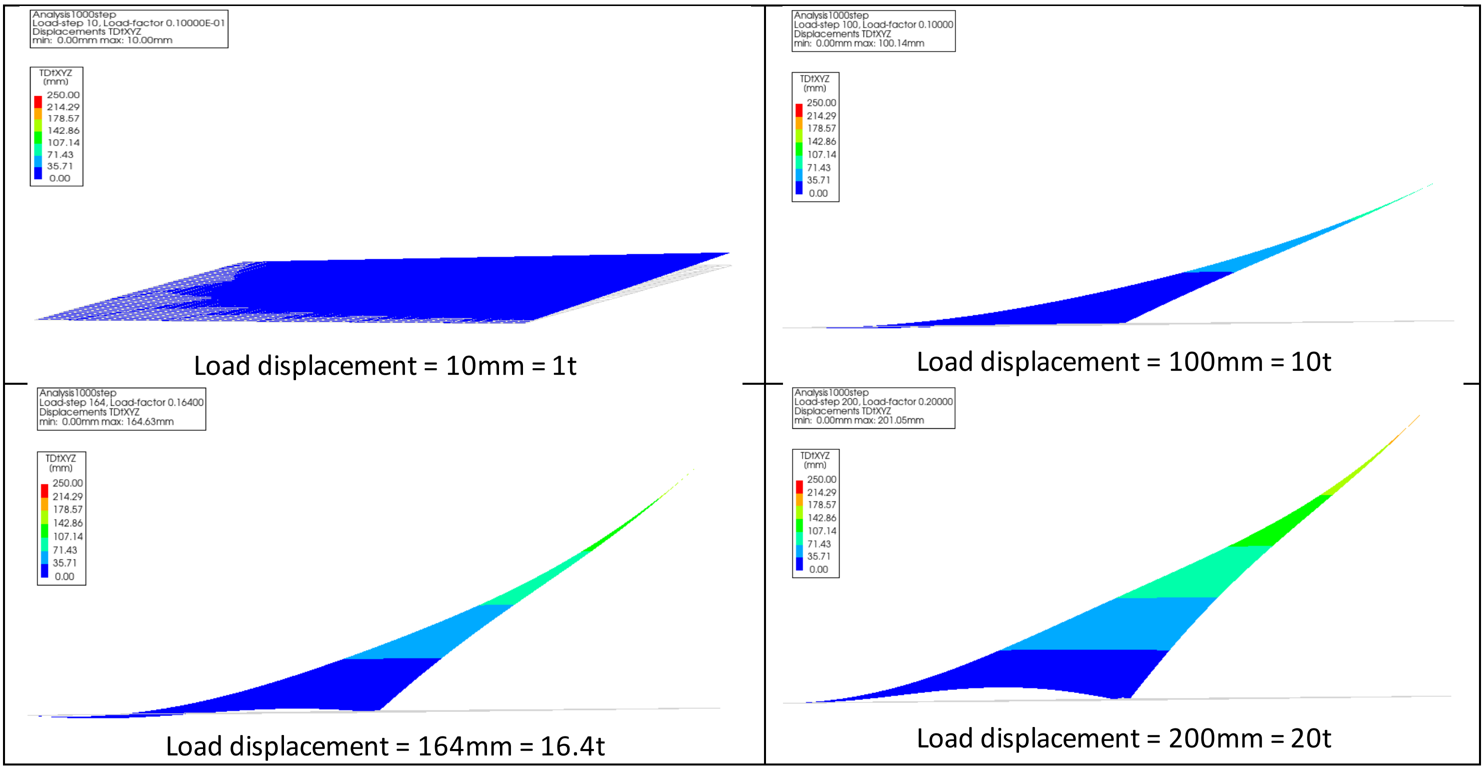

Figure 3: Displacement graphs for t = 10mm at different load steps, display scale = 3

Figure 4 shows the development of membrane forces. At $10t$, forces start to develop and are visible at the buckling point. Large compressive stresses are observed in the middle of the plate, with tension at the edges. At $20t$, the plate is past the buckling point, compressive forces are developing towards the corner points as there is no shell behaviour anymore.

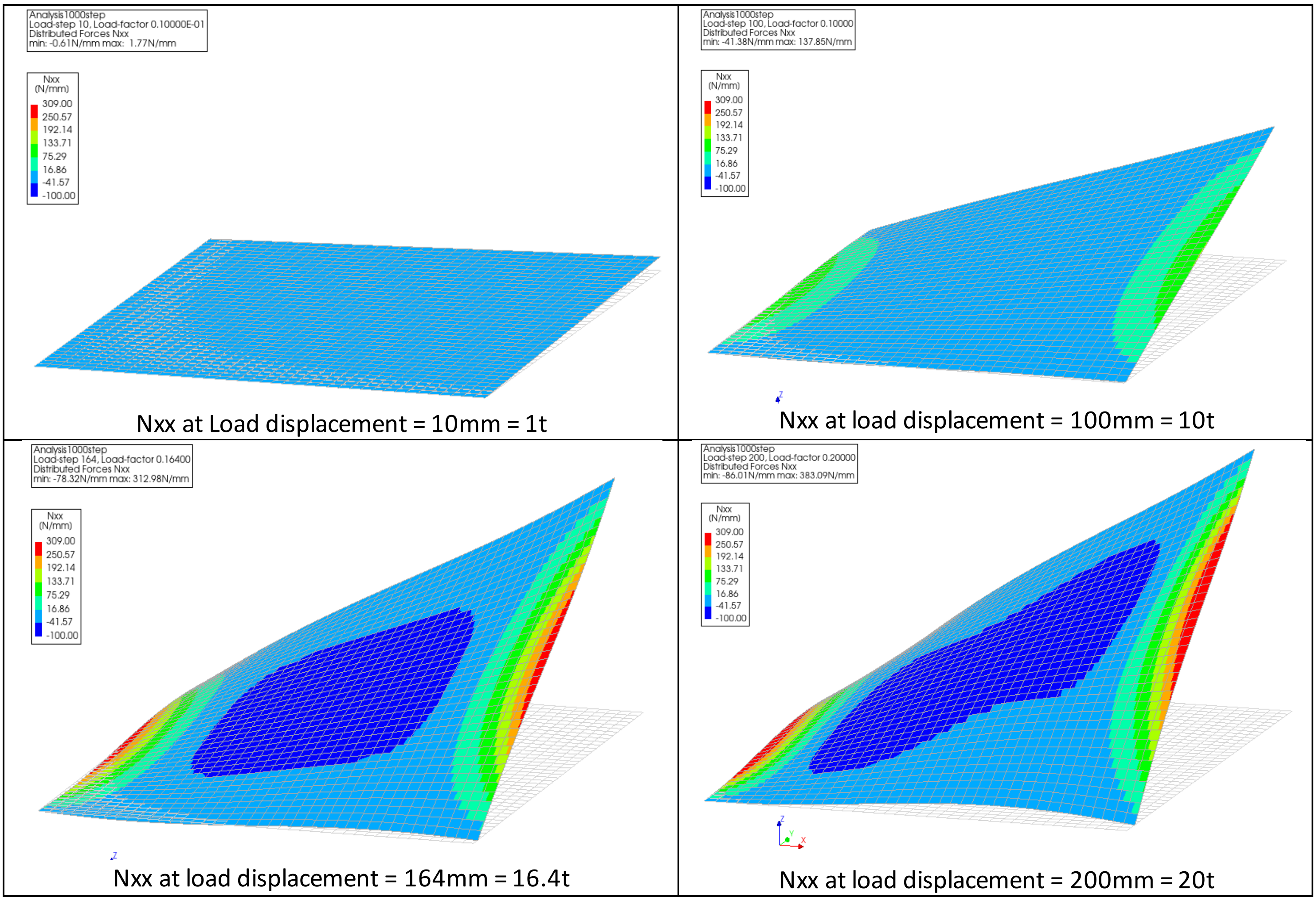

Figure 4: Nxx contour plot for t = 10mm at different load steps. Display scale = 3

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Analysis 2

In Figure 2 a graph is shown of the first 300 load steps which also equals 600 mm displacement. The buckling should occur at $16.8t$ (336 mm), marked by a blue line. The results of the other load steps are excluded as the plate is already buckled. The plate does not buckle at exactly $16.8t$ but this time at $15.8t$. For the coefficient determination, this point is used.

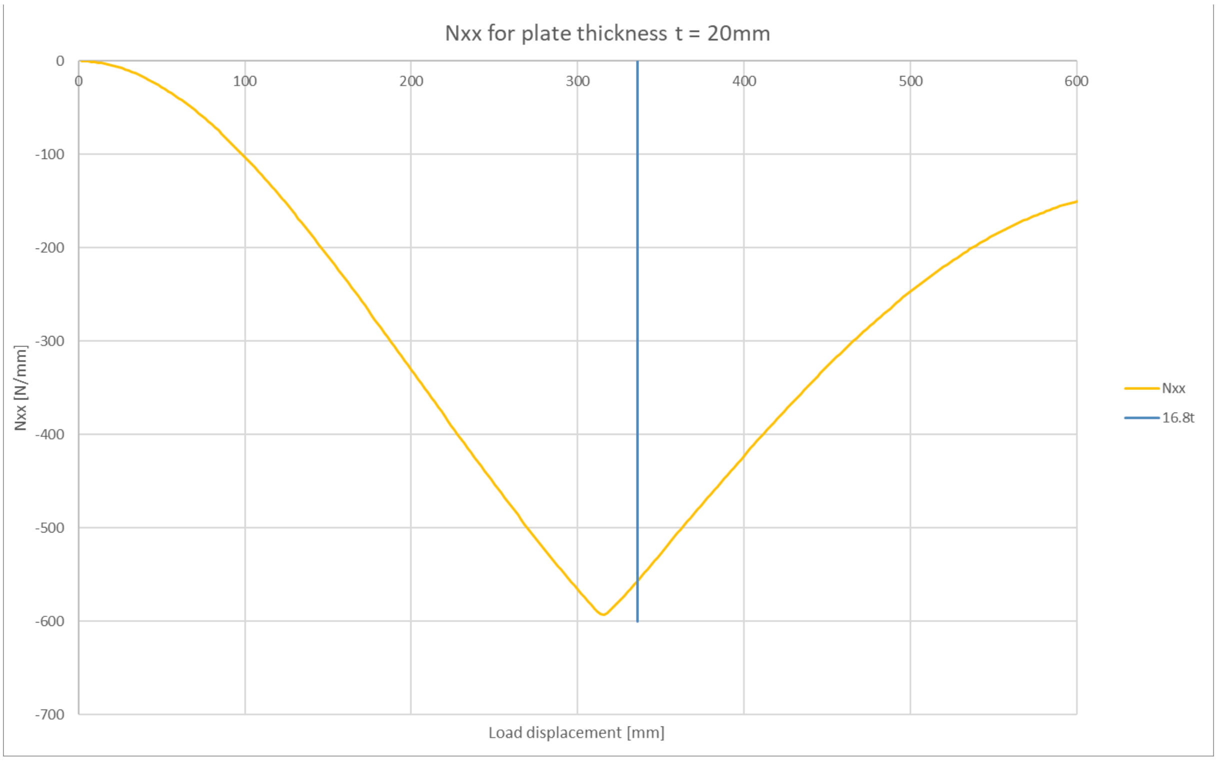

Figure 5: Nxx for t = 20mm, buckling point check

The deformation plot is presented in Figure 6Figure 3. The same behaviour as the plate with $t = 10\ mm$ is observed. At the buckling point of $15.8t$, some curvature remains, and the plate's edge is no longer straight, showing stress development. At $20t$, post-buckling, it is visible that the centre of the plate becomes straight while the left and right points are slightly curved.

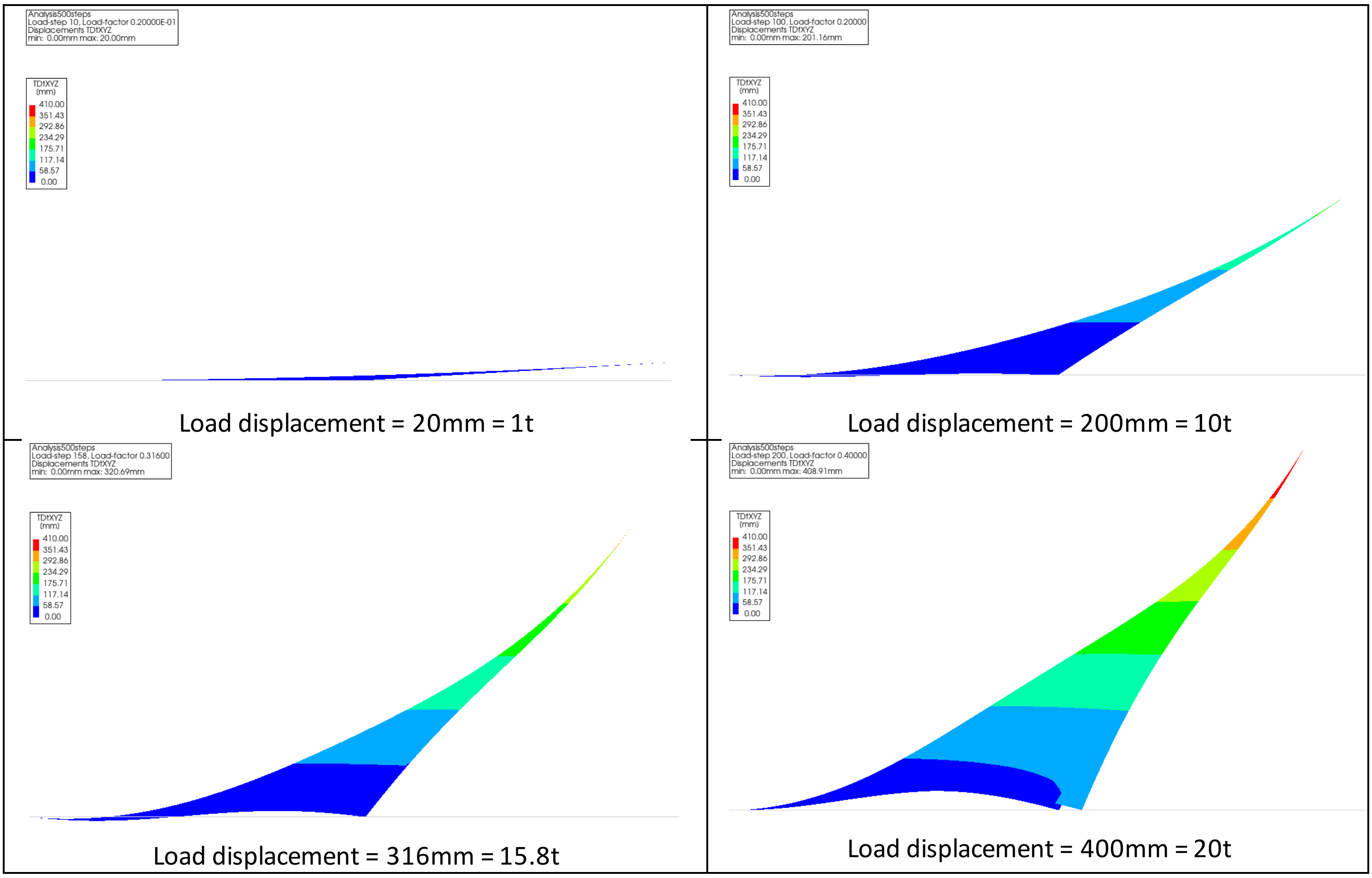

Figure 6: Displacement graphs for t = 20mm at different load steps, display scale = 2

Figure 7 illustrates the membrane forces, showing similar behaviour to the plate with thickness of 10 mm. After buckling at $20t$, the compressive forces develop a different pattern due to the buckled shape of the plate.

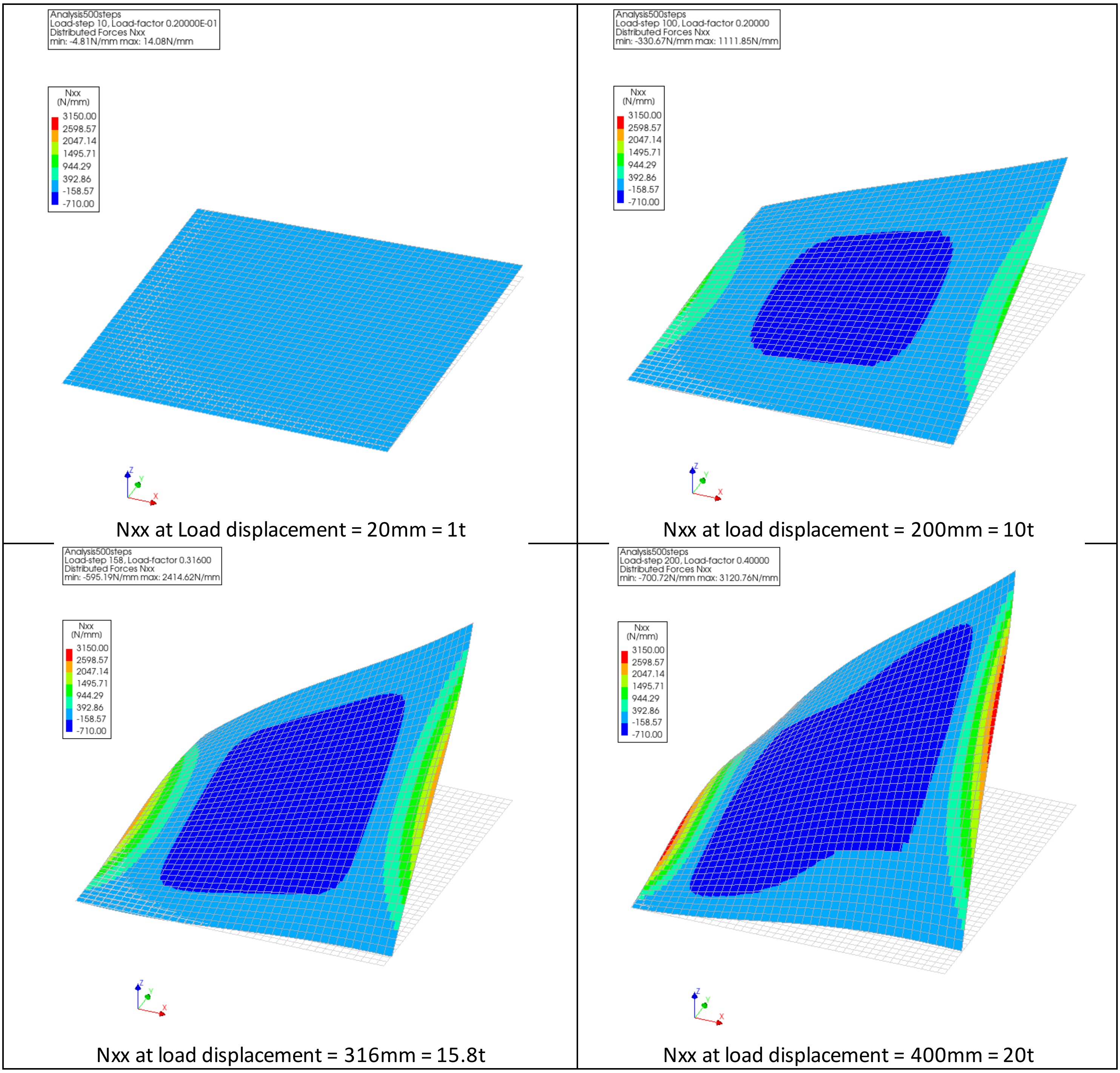

Figure 7: Nxx contour plot for t = 20mm at different load steps. Display scale = 3

Analysis 3

Figure 9 shows a graph of the first 333 load steps which also equals 700 mm displacement. The buckling should occur at $16.8t$, 168mm, displayed as a blue line. The results from the remaining load steps are not considered, as the plate has already buckled. The plate does not exactly buckle at $16.8t$ but at $14.9t$. For the determination of the coefficients, this point will be considered.

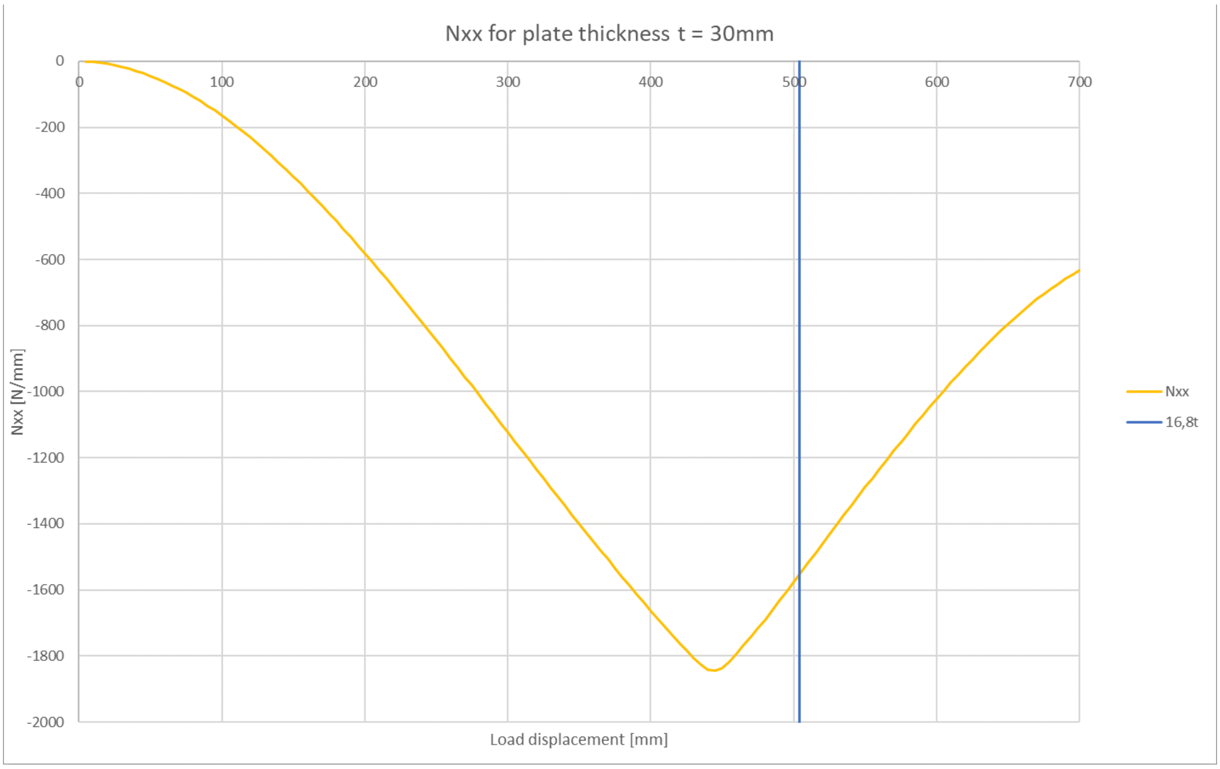

Figure 8: Nxx for t = 30mm, buckling point check

Figure 9 and Figure 10 show similar behaviour of the plate as with the plate thicknesses. The only difference is the buckling point relative to the thickness.

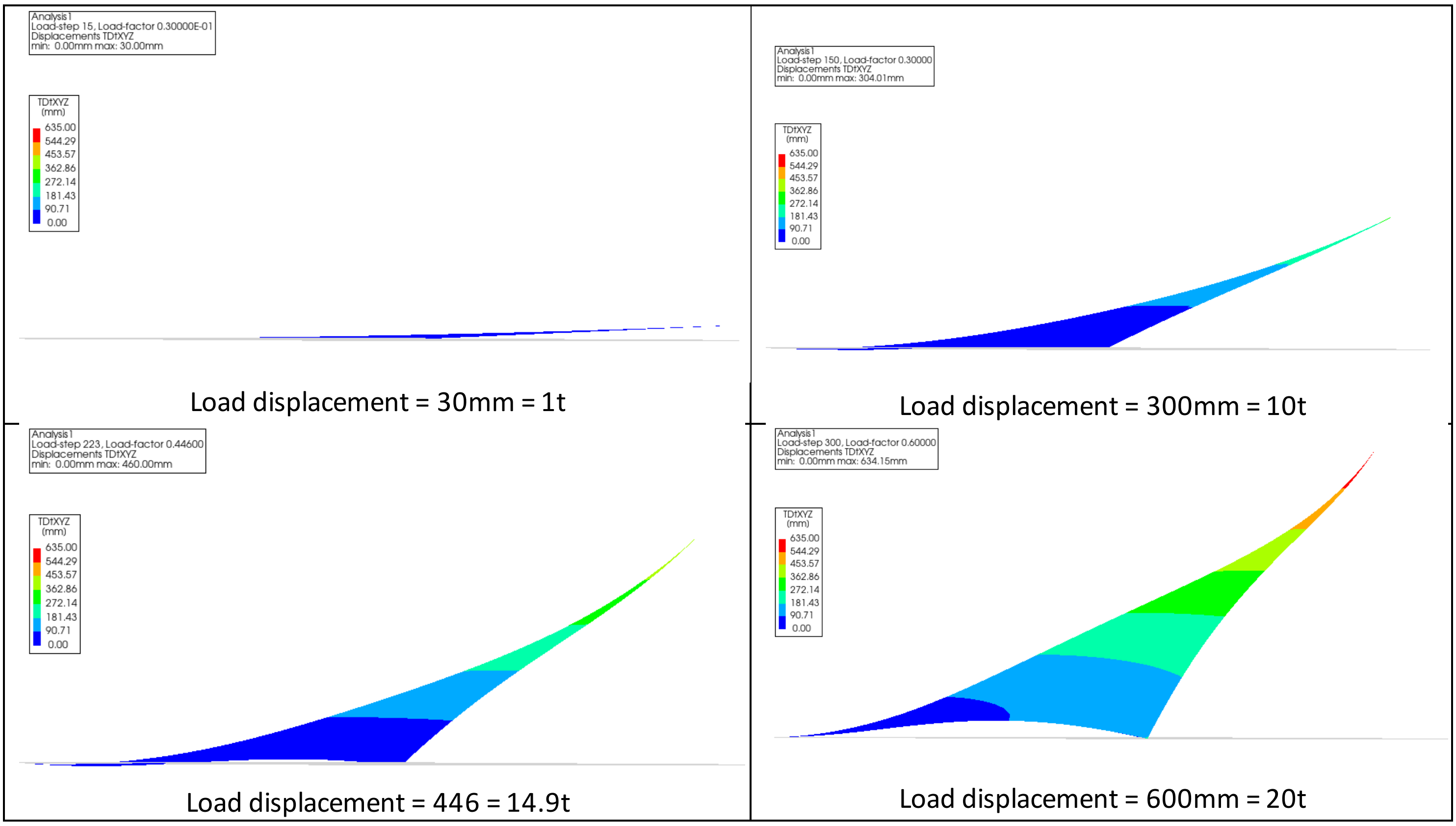

Figure 9: Displacement graphs for t = 30mm at different load steps, display scale = 1

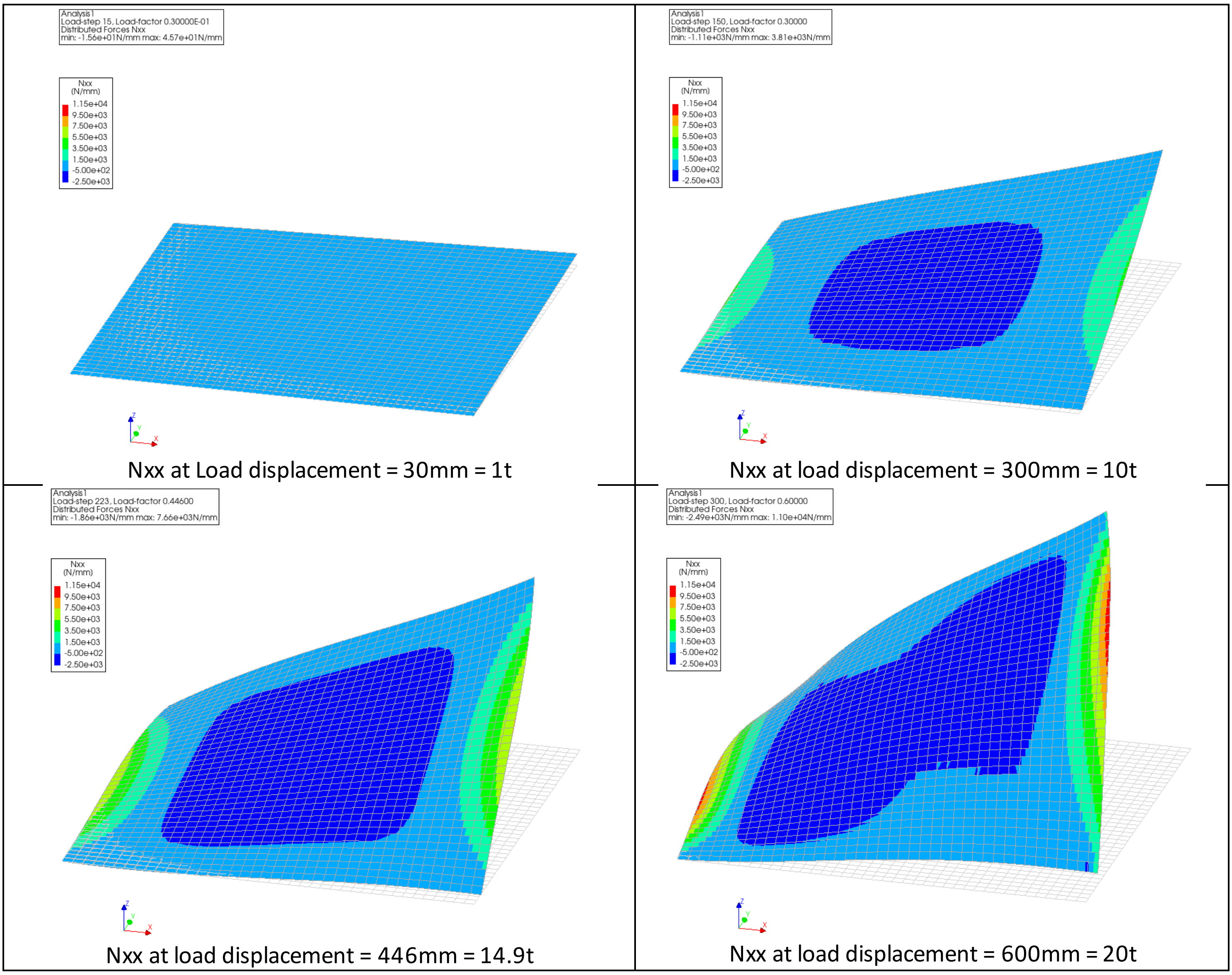

Figure 10: Nxx contour plot for t = 30mm at different load steps. Display scale = 1

Coefficients

The coefficients are calculated based on the data at the buckling points rather than at $16.8t$. First, a general n is calculated using the formula:

$$n = Etb^2k_G \tag{3}$$

With:

$$k_G = -\frac{u^2}{b^4} \tag{4}$$

This is then divided by the output from Diana for the mid and edge nodes to determine the coefficients.

$$Coefficients = \frac{n}{n_{Diana\,output}} \tag{5}$$

The calculated coefficients are provided in Table 3.

	Mid node		Edge node	
	n_1	n_2	n_1	n_2
t = 10mm	$\frac{1}{106}$	$\frac{1}{106}$	$-\frac{1}{26}$	$\frac{1}{192}$
t = 20mm	$\frac{1}{104}$	$\frac{1}{104}$	$-\frac{1}{25}$	$\frac{1}{186}$
t = 30mm	$\frac{1}{100}$	$\frac{1}{100}$	$-\frac{1}{23}$	$\frac{1}{178}$

Table 3: Coefficients per plate thickness for mid and edge nodes

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Conclusions

The results from the non-linear finite element analyses show that the plate's buckling does not occur precisely at the theoretical displacement of $16.8t$ (Staaks, 2003), but rather slightly before this point, varying with plate thickness. For plates with thicknesses of 10 mm, 20 mm, and 30 mm, the buckling occurs at approximately $16.4t$, $15.8t$, and $14.9t$ respectively. This indicates that the actual buckling behaviour is influenced by the plate's thickness. In this assignment, the displacement force is applied vertically, which may not represent the correct assumption for the force direction. Since the force direction is always vertical, it has a component along the plate's plane, which could influence the buckling behaviour for different thicknesses.

The calculated coefficients for membrane forces at both mid and edge nodes indicate variability depending on the plate's thickness or the assumption of the force. However, they lie very close together such that based on this analysis, the coefficients can be generalised. The coefficients at the mid node (for n_1 and n_2) decrease slightly as thickness increases (e.g., $\frac{1}{106}$ for 10 mm and $\frac{1}{100}$ for 30 mm), while the coefficients at the edge node show more significant changes (e.g., from $-\frac{1}{26}$ for 10 mm to $-\frac{1}{23}$ for 30 mm for n_1).

For the mid node the following coefficient is suggested for n_1 and n_2:

$$n_1 = n_2 = \frac{1}{103} Etb^2 k_G$$

The suggestion for the edge node is the following:

$$n_1 = -\frac{1}{25} Etb^2 k_G, \quad n_2 = \frac{1}{185} Etb^2 k_G \approx 0$$

References

1. Hoogenboom, P. (2024). *Notes on Shell Structures.* Delft University of Technology.
2. Staaks, D. (2003). *Koud torderen van glaspanelen in blobs.* Eindhoven University of Technology.