Concrete column support failure

Modelling the capacity of a support column in relation to bearing distance of a beam

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Challenge the future

CONTENTS

Introduction

Current models used for determining a minimum distance from a column edge to a supported beam are not sufficient to make an accurate design. It does not give enough information to have a completely safe design which will not cause any damage. These damages do occur, but there is no model developed to give an accurate calculation of the capacity and thus the safety.

To compute such a model, the simplest alternative was chosen: using the equilibrium conditions. The question is whether this basic method will work for such a complicated problem. In order to research this, reference models for concrete detailing computation will be used. References to literature about these models will be made later in the following chapters.

By stating and researching four main questions, a solution to the problem defined above, might be found. When a sufficient result is obtained, some recommendations can be given.
The following questions will be stated and discussed in the next chapters:

- Can there be a simply defined model and what will the corresponding assumptions and boundary conditions be?
- What are the current standards?
- How to determine the support's ultimate capacity?
- Which recommendations can now be made?

The first question is actually the input of the created model. Here, assumptions and boundary conditions for input parameters will be stated. Calculations will be made in a two- dimensional plane and are supported by sketches to define the parameters visually. These parameters are now subjected to a python script (displayed in the Appendix), which will generate (perhaps valuable) output.

Secondly, the current standards have to be researched. Which standard could be applied in theory and what disadvantage(s) does it have? Answers can only be obtained by searching literature.

To determine a support's ultimate capacity and do a check, the model has to be tested and the corresponding literature for combining axial-, shear- and bending stresses has to be used.

The last question is a conclusion of the research. Recommendations and problems have to be derived from the previously discussed chapters. Also, the conclusion gives a description of the usefulness of the simple equilibrium model.

1

Boundary conditions and assumptions for creating a simple model

To create a working model, a few assumptions have to be made in order to make it feasible. Another point to discuss, are the boundary conditions for which the model is valid. This is an important step in the research to clarify and converge the problem. For that, it will be a more specific problem and it produces a less common solution.
For one thing, the different types of bearing will be discussed and they are narrowed down to one of them. Secondly, the equilibrium conditions (also its derivations) and assumptions for the calculation model are described. To support and clarify these statements, some figures have been added. All of the calculations or statements will be made in a two- dimensional plane, which means that for all quantities, it is calculated per 1 mm depth.
Another assumption is that an end- support is calculated. This also applies to a column which is supporting two separate beams at either side. This means there are no additional support bending stresses generated, because there is no bending moment present in the support.

Reinforcement within the column is not only counteracting bending moment of this complete element. It also has a positive effect on it's bearing capacity. The lack of reinforcement at the edges is a reason of failure while pure concrete cannot withstand a large number of tensile stresses. The influence of reinforcement is beyond the scope of this paper.

1.1. Bearing principles

Not all types of bearings will be discussed or calculated in this paper. First of all, a distinction between non- isolated- and isolated joints is made. The isolated joints do not include the so- called sandwiched bearing pad, whereas the non- isolated supports do. The main advantage of these non-isolated bearing type is the cleared cover zone. This bearing pad can be constructed in several ways. Four different types of non-isolated bearings are sketched and discussed below in figure 1.1 [1].

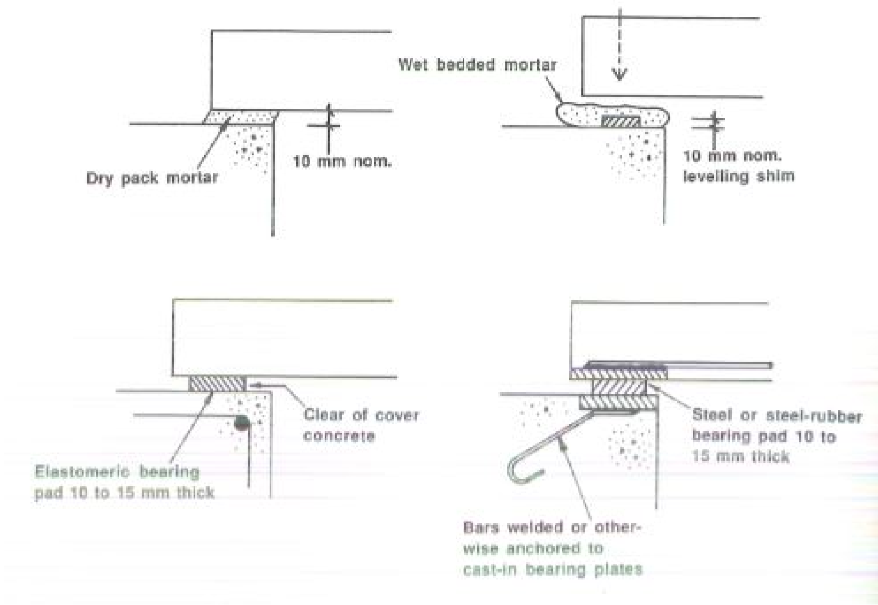

Figure 1.1: Non- isolated bearing types [1]

The upper two sketches represent either a dry- packed or wet- bedded mortar. While this cement grouting type of supporting is quite common, the focus of this paper is aimed at the third type of bearing. This type is often used as well, while elastomeric bearing pads are relatively flexible. Elastomeric bearing pad characteristics are not included in this research.

1.2. DIMENSIONS

The dimensions of the possible failure plane within the supporting concrete column, have to be set and defined. In figure 1.2 all the possible variables are shown. The indirect variables (this means they can be derived from the primary- or direct variables) are marked.

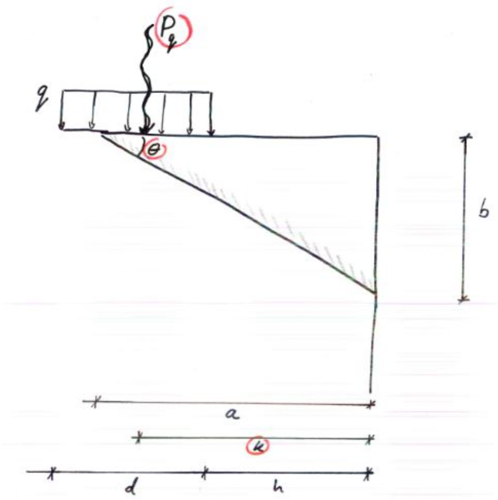

Figure 1.2: Direct- and indirect (red) variables

The (indirect) variables θ, k en P_q are composed of direct variables:

- $\theta = tan^{-1}(\frac{b}{a})$ [rad]

- $k = h + 0,5d_{\text{Eff}}$ [mm]
- $P_{\text{q}} = q \times d_{\text{Eff}}$ [kN]

Where, the effective length to which the support stress applies, 'd$_{\text{Eff}}$ = a-h' when 'd+h' is larger than 'a'. Otherwise, 'd$_{\text{Eff}}$' is equal to 'd'. Conclusively, if 'a=h', 'd$_{\text{Eff}}$' has a zero value.
The cross- sectional distance 'c' is an indirect variable as well and can be derived according to Pythagoras with the variables 'a' and 'b'.

1.3. EQUILIBRIUM CONDITIONS AND ASSUMPTIONS

To create a simple calculation model, some basic equilibrium conditions such as $\sum F = 0$ has to be met in all directions, which is in this case: $\sum F_{\text{H}} = 0$ and $\sum F_{\text{V}} = 0$ Another basic equilibrium condition is: $\sum M = 0$. This results in a force- and momentum scheme as in figure 1.3. The resulting force and bending moment act halfway of the cross- sectional distance 'c'.

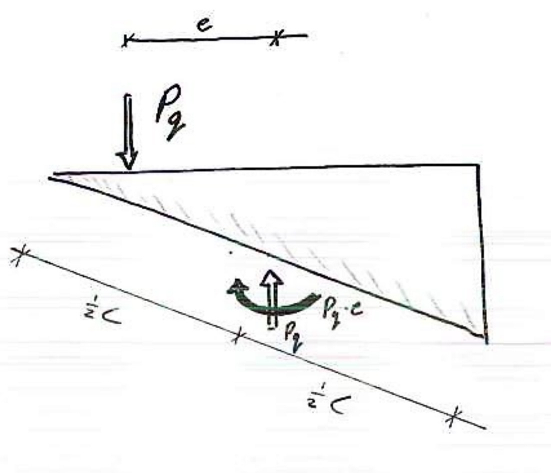

Figure 1.3: Force- and momentum equilibrium on a possible failure plane

The mentioned eccentricity 'e' is due to dimensional calculations: $e = k - (0,5 \times c \times cos(\theta))$ [mm]. If the value for eccentricity should be positive, the force quantity P$_{\text{q}}$ acts on the left side of the assigned midpoint and vice versa. This resulting internal moment and -force, will produce stresses on the cross section. Decomposition (figure 1.4) of these forces enables a calculation of the normal- and bending stresses (due to P$_{\text{qv}}$ and $P_{\text{q}} \times e$) as well as calculating the shear stress due to P$_{\text{qh}}$.

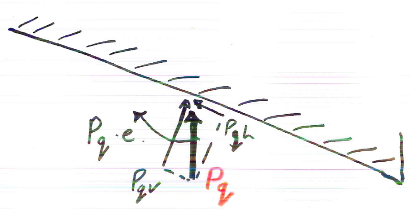

Figure 1.4: Decomposed forces P$_{\text{qh}}$ and P$_{\text{qv}}$

The normal-, bending- and shear stresses will be parametrically determined and plotted numerically in *Chapter 2: Parameter study and model*.

2

PARAMETER STUDY AND MODEL

In the previous chapter, the definitions of the variables were given and all assumptions that were made to create a model, were mentioned as well. Now, the method and calculations will be discussed, supported by some clarifying sketches. The model is computed in a python program of which the code is displayed in the *Appendix*.

First of all, the calculation of the axial- and bending stresses will be discussed. Then, a possible variation of parameters will be discussed. These involve plots which will give a lot of information about the relations between different variables. Some of the variables are of influence to the shear stress calculation as well, so the calculation of the shear stress is partially the same as the first one about axial- and bending stresses. The topic of shear stresses will be addressed in the third section. The last part is about the interpretation of the shear stress distributions, supported by some relevant graphs.

2.1. AXIAL- AND BENDING STRESSES: CALCULATION

Quantifying the axial- and bending stresses will be done by the following known expressions for cross- sectional stresses:

$$\sigma_{\mathrm{R}} = \frac{N}{A} + \frac{M \times z}{I_{zz}}$$

and

$$\sigma_{\mathrm{L}} = \frac{N}{A} - \frac{M \times z}{I_{zz}}$$

The cross- sectional area A_c is in this case: $c \times 1 = c$ [mm^2], because the in- depth distance was assumed at all places as 1 mm. The same way, $I_{zz;c}$ is defined as $\frac{1}{12}c^3$ [mm^4]. The normal force 'N' is equall to -P_{qv} and the bending moment 'M' is given as $P_q \times e$. The bending stress components will be the same (but positive/ negative), because the assumption is a small rectangular cross- section with a height of length 'c'. Thus, the value of 'z' is in both cases is $\frac{1}{2}c$.

The function that is eventually written, will return a stress value for each position along the cross- sectional distance 'c', which means that x_{min} = 0 [mm] and x_{max} = c [mm]. The positional stress value is obtained by defining a certain linear function depending on the values c, σ_{R} and σ_{L}. An example of a cross- sectional stress distribution is given in 2.1.

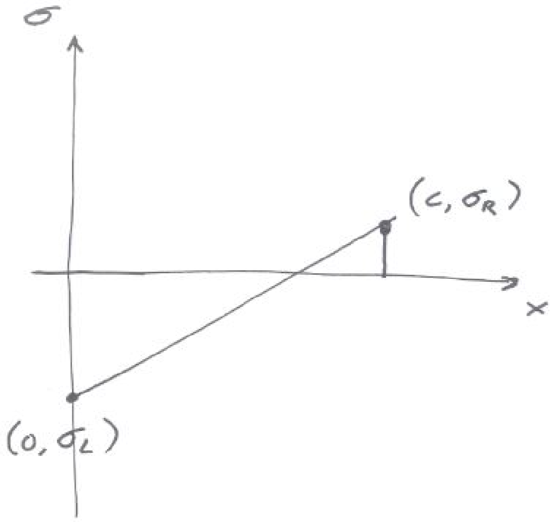

Figure 2.1: Stress distribution along the x- axis

The linear expression will be:

$$y(x) = \sigma_\mathrm{L} + \frac{\sigma_\mathrm{R} - \sigma_\mathrm{L}}{c} x \tag{2.1}$$

2.2. AXIAL- AND BENDING STRESSES: INTERPRETATION

The expression 2.1 is the eventual function and is dependant on 'x', 'a', 'b', 'd', 'h' and 'q'. Having a range of values for 'h' can give a lot of information when plotting 'h' versus stress y(x) in different scenarios. For example, what is the influence of an alternating distance 'a' on the 'h', stress- relation? And what happens to this relation if the value for 'q' changes? These questions can now be answered by plotting the expression 2.1, combined with assuming and altering certain variables.

Plotting location 'x' versus compressive stress while altering distance 'h' presents a graph that shows the actual stress distribution within the possible concrete crack plane (figure 2.2). For 'h=50 mm' (not plotted in the figure) a horizontal line would emerge, because of the absence of a bending moment. The resulting force $\mathrm{P_q}$ is then acting without any eccentricity, so only axial (and no bending) stresses will be present. The fact that 'h=200 mm' only produces zero- values is due to the fact that all support stress is fully taken by the column when '$h \geq a$' (i.e. '$\mathrm{d_{Eff}}$=0').

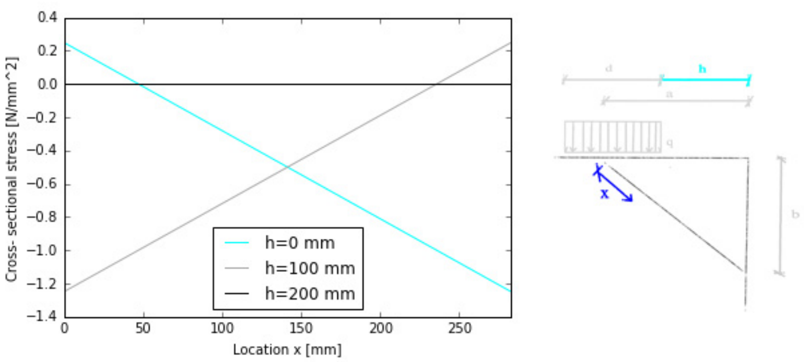

Figure 2.2: The influence of distance 'h' on the location 'x', sigma- relation
Constants are: a=200 mm, b=200 mm, d=100 mm, q=2 N/mm^2

For 'h=0' a descending line was found, whereas 'h=100' shows an ascending line. The reason for this is the differing eccentricity. While the resulting force acts left of the cross- sectional midpoint for 'h=100', the 'h=0'- configuration produces a different bending moment, because P_q is now on the right side of the midpoint. When a location of the support would be fixed, plotting the stress distribution could be quite useful to determine a concrete strength class for example. The maximum stresses do not always occur at the exact same place. Changing distance 'h', alters the stress distribution, but also it's magnitude.

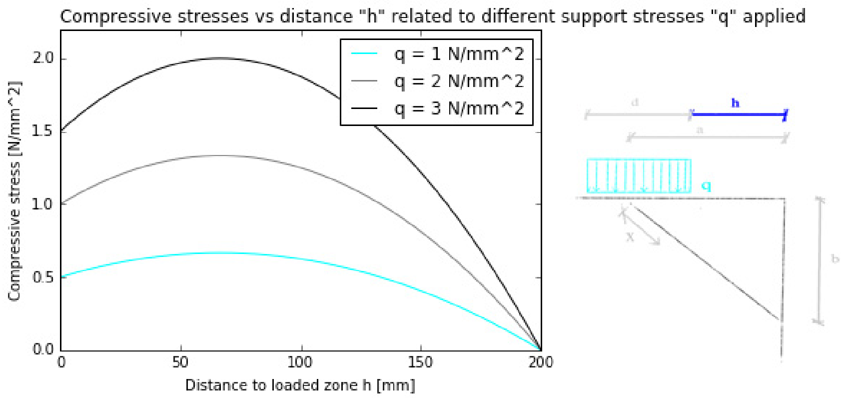

Figure 2.3: The influence of the support stress 'q' on the distance 'h', sigma- relation. Constants are: a=200 mm, b=200 mm, x=0 mm, d=200 mm

The second relation to be discussed, is the influence of the support stress on the distance 'h', sigma- relation. Figure 2.3 shows three different values for the support stress and as expected, a larger support stress produces a larger cross- sectional stress (at position 'x=0 mm'). The interesting thing here, is the maximum showing for each value of q. The value h for which the maximum stress occurs is in all cases the same, so it could be stated that there is one value of 'h' for which the cross-sectional stress is maximal. The value of 'q' does not influence this one value of 'h' and is only of importance to the stress quantity. The zero- value at 'h=200' mm is not surprising. With a constant value of 200 mm for 'a' as well, it means that 'h=a' and there is thus no support stress acting on the failure part. All these support stresses are further taken by the column.

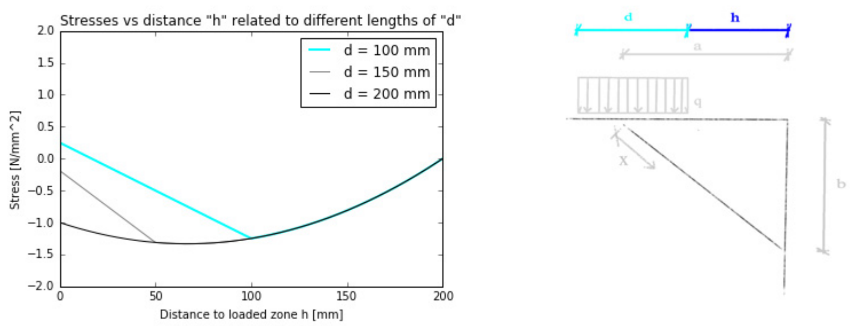

Figure 2.4: The influence of length 'd' on the distance 'h', sigma- relation. Constants: a=200 mm, b=200 mm, x=0 mm, q=2 N/mm^2

As a third, the influence of the length at which the support stress acts (also: the effective bearing pad length) will be presented in figure 2.4. This plot shows the effect of altering 'd' to the axial- and bending stress value at position 'x=0 mm'. Only in some cases, this is the maximum value (see figure 2.2). The stress graph is a smooth curve when 'd' is equal to 200 mm. Having a value of 'd' which is smaller than dimension 'a', causes for a transition. In this case, a transition from linear to a curved graph. The maximum stress value at 'x=0 mm' is not totally dependant on the value of 'd', while for both 'd=150 mm' and 'd=200 mm' the curve is at some point the same. The corresponding 'h' value is the same as the 'h' value for maximum stress in the 'q'- influence plot (figure 2.2).

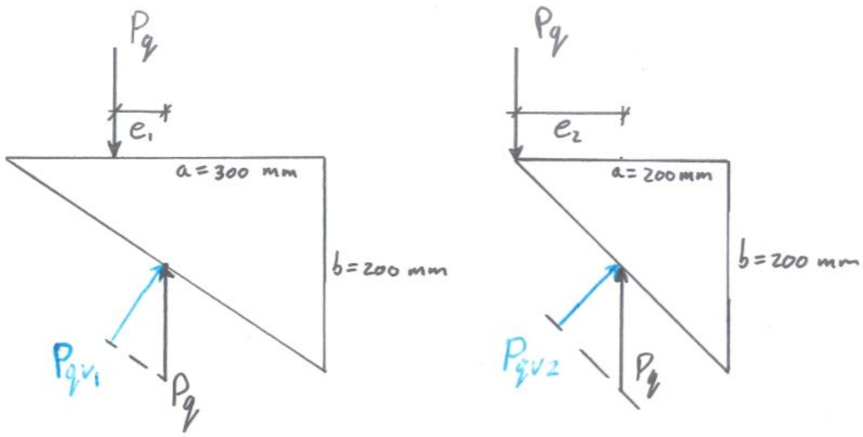

Figure 2.5: The effect of proportional crack dimensions on force and eccentricity.

The influence of crack dimension 'a' will be shown in figure 2.6. The crack dimension 'b' is kept as a constant, so this figure is merely showing the proportional crack dimension influence. The crack dimension size influence will be discussed later. The effect of proportional change to the force distribution is illustrated in figure 2.5.

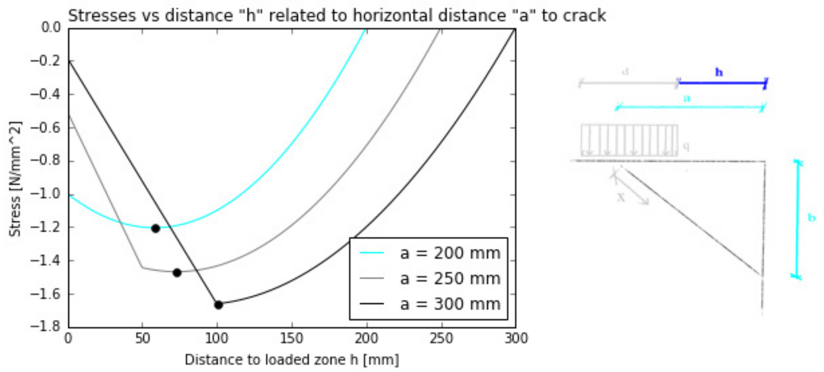

Figure 2.6: The influence of the crack dimension 'a' on the distance 'h', sigma- relation.
Constants: b=200 mm, x=0 mm, d=200 mm, q=2 N/mm^2

Clearly visible in figure 2.6, is the influence of 'a' on the value of 'h' for which the cross- sectional compression is maximal. The location of the maximum value is not necessarily the same as the location for which the function transforms from linear to parabolic. For 'a=200 mm' there is no transition present and the maximum value is not at 'h=0'. This transition seems to be present only for cases in which the support stress can be partially taken by the column. Of course, for every 'h>0' (with 'd ≥ a=200 mm') the graph should then be a curve.

Values for 'a' as 250 mm and 300 mm do result in a transitional curve with a transition for 'h=50 mm' and 'h=100 mm' respectively. This is a value which can thus be easily obtained by the expression 'h=a-d', where 'd=200 mm' (constant) in this particular case. The maximum compression values with variable 'a' are not linearly related to each other, as is visible in figure 2.6.

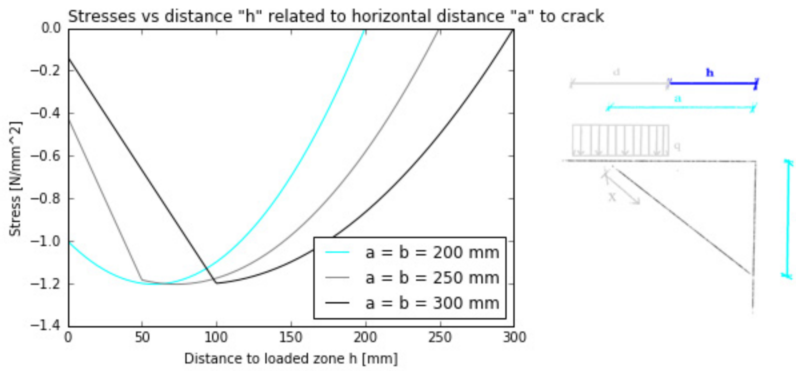

Figure 2.7: The influence of the crack dimension size on the distance 'h', sigma- relation.
Constants are: d=200 mm, q=2 N/mm^2 and x=0.1c

'a=b' [mm]	Distance 'h' [mm]	Max. Stress value [N/mm^2]
200	58.291457286432156	-1.2041665614504682
250	73.293172690763058	-1.2041612232060772
300	100.33444816053512	-1.1997740517443871

Table 2.1: Maximum cross- sectional compression for different values of 'h'.
Constants are: d=200 mm, q=2 N/mm^2 and x=0.1c

Now, the size of the crack dimension has been taken into account. In figure 2.7 the dimensions 'a' and 'b' are now variables, but their proportions are not: the angle θ is equal to $\frac{1}{4}\pi$ rad (constant), which means that 'a=b'. These graphs are very similar to the ones in figure 2.6, but in this case the magnitude of compression is not actually influenced by the variables 'a' and 'b'. The values for the maximum compression at 'x=0.1c' are about 1.2 N/mm^2. This value does barely fluctuate as can be seen in table 2.1. The corresponding values of distance 'h', belong to an optimum.
This optimum considers two separate phenomena. Shifting the distributed load to the left, decreases the magnitude of force P_{qv} (so it decreases the axial stress), but it can increase the eccentricity substantially. Considering these two contradictions, a certain optimum can be found. Hence, the odd- looking numbers for a corresponding 'h'- value.

2.3. SHEAR STRESSES: CALCULATION

The internal concrete cross- section is also subjected to a shear force, which is equal to P_{qh}. The magnitude of P_{qh} is influenced by distance 'h' and possible crack dimension 'a'. These two variables influence the value of P_q and thus influence P_{qh}. But not only the magnitude of indirect variable P_q modifies P_{qh}. This shear force also changes when the proportions 'a' and 'b' (i.e. $theta$) change.

The general expression for shear stress τ can be rewritten like in expression 2.2. In this expression, the width of the rectangular cross- section was taken as 1 mm. Variable S_x is dependent on 'x', which eventually makes τ dependant on position 'x' as well (expression 2.3).

$$\tau = \frac{V * S_x}{b * I_{zz}} = \frac{P_{qh} * S_x}{\frac{1}{12} * c^3} \tag{2.2}$$

Where:

$$S_x = 0.5 * x * (c - x) \tag{2.3}$$

The maximum value of shear stress is being calculated by the general formula, rewritten in expression 2.4. For a rectangular cross- section, this maximum shear stress always occurs halfway it's length and width. So in this case: 'x=0.5c'.

$$\tau_{max} = \frac{V * S_x}{b * I_{zz}} = \frac{P_{qh} * \frac{1}{8} * c^2}{\frac{1}{12} * c^3} = \frac{3}{2} * \frac{P_{qh}}{\sqrt{a^2 + b^2}} \tag{2.4}$$

2.4. SHEAR STRESSES: INTERPRETATION

This last section is about the results of calculating the shear stress distribution. First, an example is shown in a plot and it's maximum value is checked by expression 2.4. Secondly, the effect of applying different support stresses will be discussed and while the effect of length 'd' is limited, it is still basically the same as altering 'q'. An 'h'- versus shear stress plot will be made as a third point of attention. Next, the influence of the possible crack dimension 'a' on the shear stress distribution, will be discussed. And the last subject will be the influence of the support stress position 'h' for different values of 'a'.

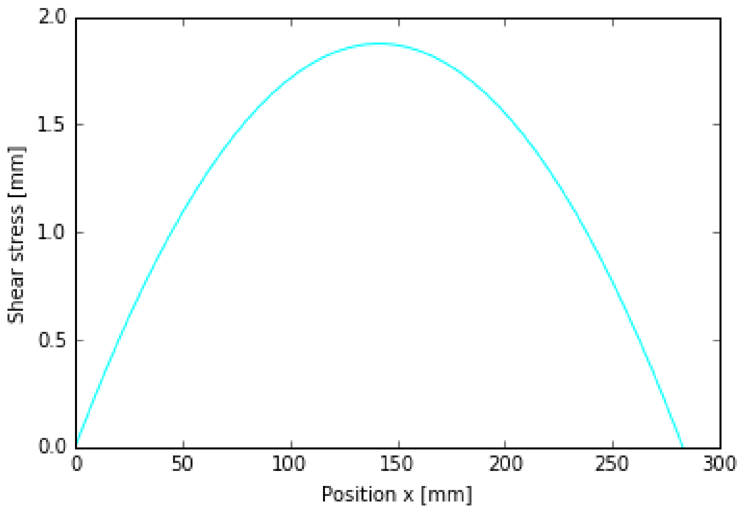

Figure 2.8: Shear stress distribution [N/mm^2] alongside the cross- section with position x. Constants are: a=200 mm, b=200 mm, d=200 mm, h=100 mm, q=5 N/mm^2

An example of a shear distribution is given in figure 2.8. The maximum value returned by the program is: 1.8749762541 N/mm^2. Calculating this by using the formula, gives:

$$\tau_{max} = \frac{3}{2} * \frac{q * d_{Eff}}{\sqrt{a^2 + b^2}} = \frac{3}{2} * \frac{5 * (200 - 100)}{\sqrt{200^2 + 200^2}} = 1.875[N/mm^2] \tag{2.5}$$

The direction of the shear force is not dependant on the location at which it acts (like when calculating bending stresses). The direction of the shear stress is the same each time, but the magnitude of it's maximum is the most important in terms of evaluating it's value with regards to the concrete material capacity.

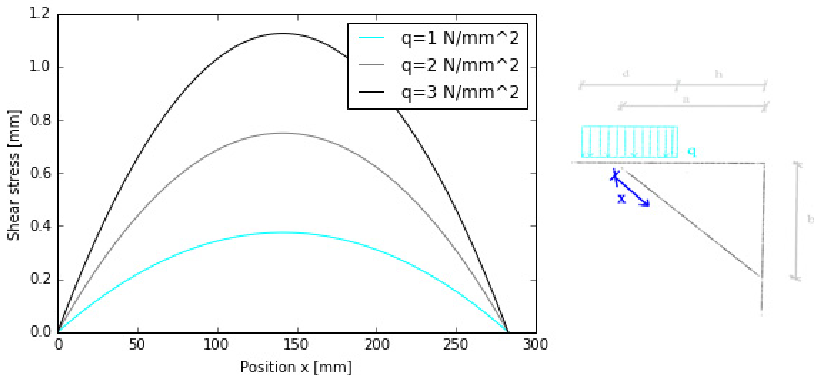

Figure 2.9: Shear stress distribution [N/mm^2] for different values of 'q' alongside the cross- section with position x. Constants are: a=200 mm, b=200 mm, d=200 mm, h=100 mm

Figure 2.9 illustrates the effect of altering the support load magnitude. The maximum value of shear will be at midpoint, no matter the location of the distributed support load. The increase in 'q' and the increase in (maximum) shear stress are one- to- one related.

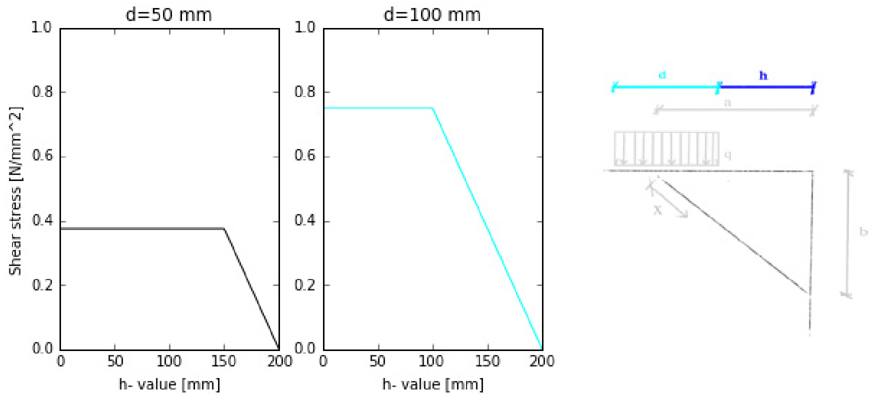

Figure 2.10: Shear stress distribution [N/mm^2] for different values of 'd' while varying 'h'. Constants are: a=200 mm, b=200 mm, q=2 N/mm^2 and x=0.5c

Plotting the shear stress distribution to position 'x' for the influence of 'd', is basically the same as for support load 'q' (figure 2.9), only when '$d + h \geq a$', maximum value of τ is reached. The stress distribution will remain the same (with a constant 'h'), no matter the increase of value 'd'. So a plot with regards to a changing 'h' (figure 2.10), will show a constant value of τ_{max} up until a certain value of 'h' is reached. τ_{max} will then decrease linearly.

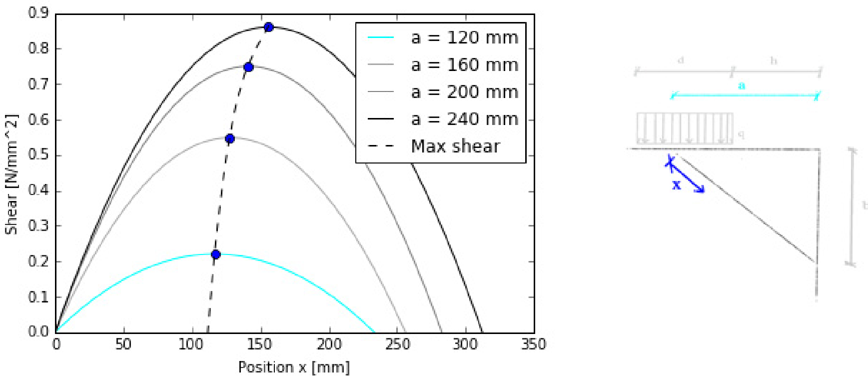

Figure 2.11: Shear stress distribution [N/mm^2] for different values of 'a' alongside the cross- section with position x. Constants are: b=200 mm, d=200 mm, h=100 mm, q=2 N/mm^2

In figure 2.11 the shear stress distribution was plotted for some different values of variable 'a'. The 'x'- values for which the maximum values occur change when 'a' increases. This makes sense when realizing that 'c' increases when 'a' increases. However, this is not a one- on- one relation (Pythagoras), so when the 'x'- value for τ_{max} is always half the value of c, then this line on which each maximum point must lie, will represent a certain square root function. An approximation of this line was plotted in figure 2.11.

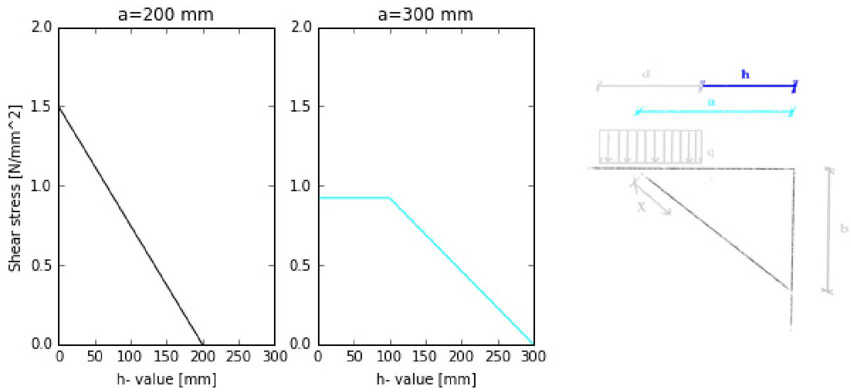

Figure 2.12: Support stress location 'h' plotted versus shear stress. Constants are: b=200 mm, d=200 mm, q=2 N/mm^2 and x=0.5c

Figure 2.12 shows the shear stress for every value of 'h' from 0 to 'a' for two different values of 'a'. When 'a=200 mm', the maximum shear stress occurs for a value of 'h=0 mm', but for 'a=300 mm', this will no longer be the case. A value of 300 mm for 'a' presents a constant value of shear stress until the point of 'h=100 mm' is reached. Of course, when 'h=a-d', the support stress magnitude starts to be taken by the column itself. So when 'a' or 'd' changes, the value of 'h' for which the transition will take place, will change. The interesting thing here, is the fact that the maximum shear stress is also different. This change is due to the altering angle of θ. When θ decreases, P_{qv} increases and P_{qh} decreases. The result is a reduction of the maximum shear force value, which will always occur when h=0, but when $d \neq a$, this maximum shear force will present itself up until 'h=a-d'.

3

Current standards

Current dutch standards do not fully, theoretically apply to the case of bearing.

The British Standard (BS8110) mostly prescribes empirical formula's and is not quite accurate. This standard generates a minimum value for the effective and nominal bearing length. The ultimate bearing stress is also processed and tested as a unity check against the cube compressive strength [2]. In case of a non- isolated support, this check holds for all three materials and their corresponding f_{cu} (ultimate compressive strength of the material).

First of all, the (effective) bearing length and the consequences of its value will be thoroughly discussed. Like the simplified model, this theory applies to a bearing- plate type of support. All the other possible types of support were quickly addressed to in *Chapter 1: Boundary conditions and assumptions for creating a simple model.*

The ultimate bearing stress and the unity check is the next subject described. This is also limited to padded bearing and especially elastomeric bearing.

To conclude this chapter, the dutch standards will be discussed and compared to the British Standard serving the purpose of showing the differences and similarities in procedure of calculation.

3.1. British Standard: Bearing length

To calculate the ultimate bearing stress, the effective surface of the bearing plate has to be determined and in order to do so, the effective bearing length and the net bearing width have to be obtained. The application of the effective surface will be further discussed in *Section 3.2: British Standard: Ultimate bearing stress.*
First, the effective bearing length ($l'_{bearing}$) is defined as the minimum of three values:

- $l_{bearing}$ (The actual bearing length)
- $0.5 \times l_{bearing} + 100mm$
- 600 mm

So, this effective length is always less or equal to the actual length of the bearing pad.
The net bearing width is just like $l'_{bearing}$ a length quantity [m] and is defined as in figure 3.1 [1].

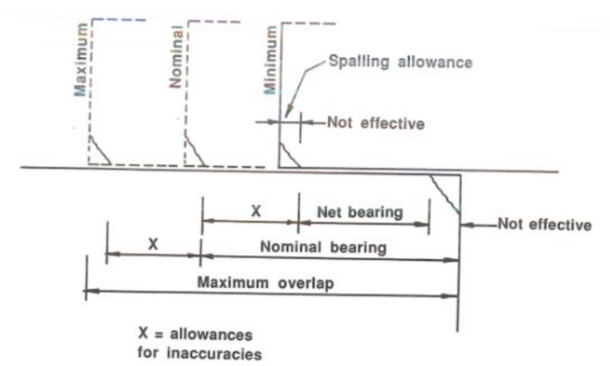

Figure 3.1: Definitions of bearing lengths [1]

From the figure, it can be shown that:
$Net\, bearing\, width + 2 \times Spalling\, allowance = Actual\, bearing\, width$
The minimum value for the net bearing width (b'_{net}) for non- isolated joints is 40 mm.

3.2. BRITISH STANDARD: ULTIMATE BEARING STRESS

According to the formula for stresses ($\sigma = \frac{F}{A}$) and taking only the effective plane loaded into account, the ultimate bearing stress will be:

$$f_{\mathrm{b}} = \frac{V_{\mathrm{support;ultimate}}}{l'_{\mathrm{bearing}} \times b'_{\mathrm{net}}}$$

So, the equation basically presents the effective surface as the loaded plane. The assumption of stress variation throughout the bearing pad was made in the British Standard. This varying stress distribution is more explicitly mentioned in the early Dutch Standard (Section 3.3). Eventually, this ultimate bearing stress must not exceed the compressive strength conditions of the material applied. For elastomeric bearing pads, the value which cannot be exceeded is between 0,4f_{cu} and 0,6f_{cu} (in which f_{cu} is the ultimate cube compressive strength of concrete):

$$f_{\mathrm{b}} \leq 0,5 f_{\mathrm{cu}}$$

3.3. DUTCH STANDARD BASED ON VARIABLE SUPPORT STRESS- DISTRIBUTION

The earlier dutch Standard VBC 1990 ("Voorschriften Beton Constructieve eisen") does not have a precise calculation method for the bearing length, based on all the parameters that were mentioned in *Chapter 1: Boundary conditions and assumptions for creating a simple model.*

The second thing to notice about the dutch standard, is the influence of using a bearing pad, on the support stress distribution [3]. The stress distribution curve alters with either the bearing type or the limit state of support (ULS or SLS).

The dutch standard defines a few variables:

a =	minimum support length
a_b =	support width
F_d =	design value for the support reaction
f_b' =	design value for the concrete compressive strength
∅ =	diameter of the reinforcement in the lower part of the beam

The minimum support length for an isolated support, will depend on the full span (L), the possible length to the crack (a_2) and the concrete cover in the beam (c). Eventually; '$a \geq a_1 + a_2 + c$', where '$a_1 = max(50 + 0,004L; \frac{F_\text{d}}{\frac{2}{3}f'_\text{b} \times a_\text{b}}; 6\varnothing \geq 70mm)$'. The fact that '$a_1$' could be equal to '$\frac{F_\text{d}}{\frac{2}{3}f'_\text{b} \times a_\text{b}}$', is due to a parabolic support stress distribution.

In the VBC 1990, 'a_2' is set to be equal to '$\frac{F_\text{rep}}{\frac{1}{2}f'_\text{b} \times a_\text{b}}$', where '$F_\text{rep}$' is defined as the representative value of the support reaction. Should 'a_2' be larger than 25 mm, a bearing pad should be used.

When the bearing pad is applied, the support length 'a' should be larger than- or equal to: '$a_1 + a_\text{r} + c$' and 'a_r' (the distance from the bearing pad to the edge of the support) can't be larger than 25 mm.

3.4. COMPARING THE STANDARDS

Both standards use the support stress distribution and simplify it to have a more fast and easy calculation method. The British standard clearly distinguishes isolated- and non- isolated bearings, which are basically the same as supporting with- or without bearing pad, discussed in the Dutch Standard.

The current Dutch standard NEN-EN 13670:2009 nl [4] only suggests certain failure allowances, but does not specify a bearing length ('h') or support stress magnitude ('q') for that matter.

Obvious about these existing standards is the amount of assumptions and rules of thumb. Most of them are more or less empirically determined. There is no precise in- depth calculation for the required support length present in either standard.

4

ULTIMATE CAPACITY

Plotting several varying parameters seems to be insufficient, when a piece of column needs to be evaluated about it's capacity. This capacity involves shear- and axial stress capacity and has to have a combined limit. When the support edge is loaded with both shear- and axial stress, there obviously must be a lower limit than when the cross- section is solely loaded with shear stress for example.

The Mohr- Coulomb curve describes the capacity limit for a combination of all stresses. This curve and it's origin are the first thing to be discussed. After that, some parameter estimations have to be made in order to do a capacity check. Varying crack size by altering the value for 'a=b', results in a certain σ-, τ curve, which is bound by the capacity line. A capacity check can then be executed.

4.1. MOHR- COULOMB FAILURE CURVE

The principle of the capacity line is dependant on the material properties of a certain concrete type (e.g. C45/55). Particularly the tensile- and compressive strength are of importance. The first step in the Mohr- Coulomb failure hypothesis, is to plot two failure circles for both compressive strength (f_c) and tensile strength (f_t). The tangent line of both circles presents the failure line, which is the capacity limit [5]. The corresponding expression for this curve is:

$$\tau = c + \sigma * tan(\phi)$$

Where:

$$c = \frac{1}{2} f_c f_t \qquad f_c = \frac{2c*cos(\phi)}{1-sin(\phi)}$$

$$sin(\phi) = \frac{f_c - f_t}{f_c + f_t} \qquad f_t = \frac{2c*cos(\phi)}{1+sin(\phi)}$$

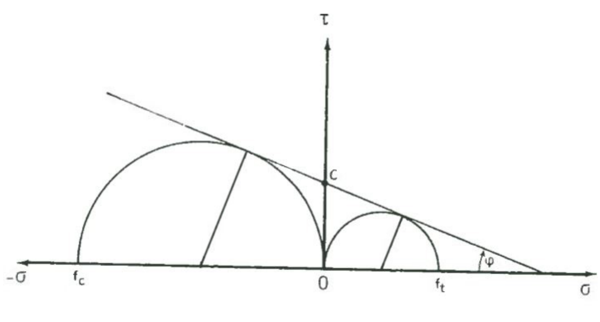

Figure 4.1: Mohr- Coulomb failure curve [5]

4.2. MAXIMUM STRESSES COMBINED

Combining shear stress τ and axial-/ bending stress σ is not a simple matter of summing two values. The maximum values of these stresses have to be obtained first, but the problem for this would be, that τ_{max} occurs at 'x=0.5c' and σ_{max} presents itself at either 'x=0' or 'x=c' (see figure 4.2).

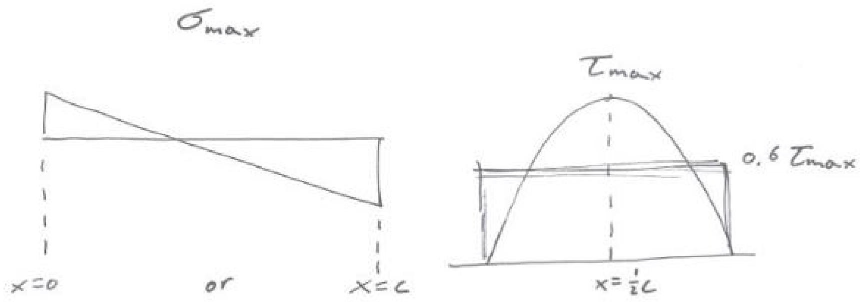

Figure 4.2: Locations of maximum stress

The assumption of having a constant maximum shear stress of 60% of the actual maximum value, gives a maximum combination at either 'x=0' or 'x=c'. However, when 'k=0.5a or 'h≥a', the x-location of the combined maximum stresses is not important, because both stress distributions will then be constant (or equal to zero).

The Mohr failure envelope is a linear function $\tau(\sigma)$. So, when the modelled results are plotted they have to be below this curve in order to prevent failure of the concrete segment. The curve from the model is a σ versus τ plot, with a parameter 'a'. The first step is to generate a distance 'a'-, σ plot. This way, the crack size is increasing each time with a value of 'Δa', but has a constant value of 'θ'. For a constant value of 'h' (in this case 'h=100 mm'), there is a maximum tensile stress with a corresponding value of 'a' (and thus 'b') as can be seen in figure 4.3. The maximum stress is obviously dependent on the crack dimensions 'a' and 'b' (or for that matter the value of θ).

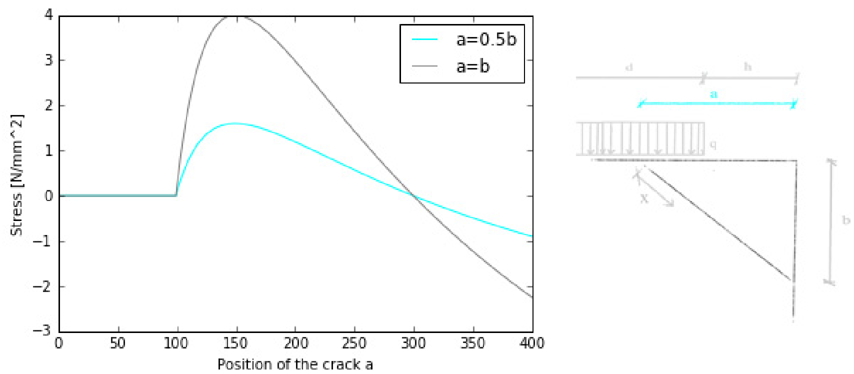

Figure 4.3: Determining possible crack locations for axial-/ bending stresses

The 60% maximum shear stress can also be plotted in terms of variable 'a', which results in a graph like figure 4.4. The shear stress in terms of 'a' does not really have a maximum value when a small part of the column is evaluated. However, when 'a=d+h', a transition occurs and the maximum shear stress value is present at 'a=d+h'. Considering a larger range of 'a' might be interesting to find this shear value, but for this check, a large value of length 'd' was considered to specify the problem solely to the changing crack size.

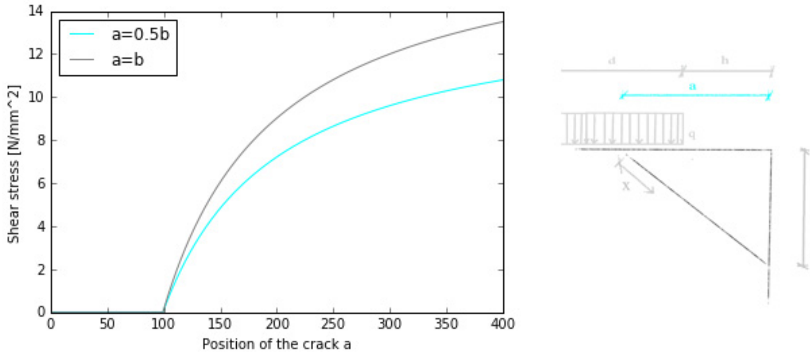

Figure 4.4: Determining possible crack locations for shear stress

Also for this figure 4.4, the stresses at all distances 'a' are lower for 'a=0.5b'. The conclusion can now be that in this case these proportions will not be the most probable crack dimensions. Another important thing to notice here is the shift of the graph caused by the value of 'h'. Only zero values are produced for the stress level when '$0 \leq a \leq h$' in both figures 4.3 and 4.4. Distance 'h' does not change the stress levels, it only changes the starting point of the graph. The all- zero values for τ and σ are due to the fact that there is no loading on the segment as long as distance 'h' is still larger than distance 'a'. The amount of zero- values is equal to 'h/Δa'.

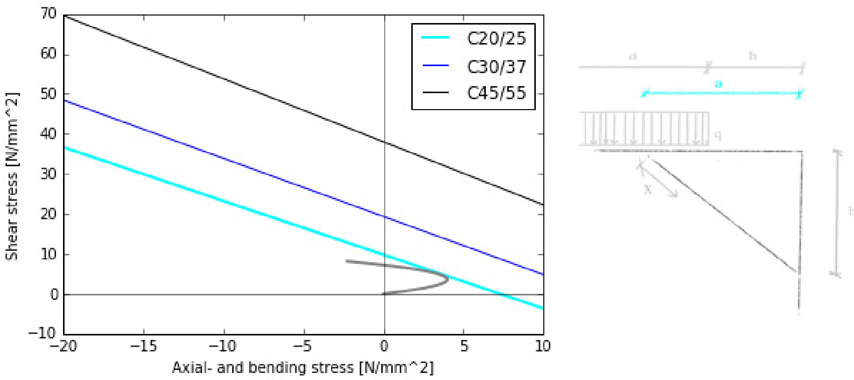

Figure 4.5: Capacity check for different values of 'a=b'

In figure 4.5 some failure envelopes for different concrete strength classes are plotted. Apparently, the value for 'ϕ' remains unaltered when a different concrete strength class is applied. The 'c'- value however is dependant on the strength class. In contrast to 'ϕ', variable 'c' is not a ratio of f_c and f_t, it is a multiplication, which explains the increase, while both tensile and compressive strength increase with a larger strength class.

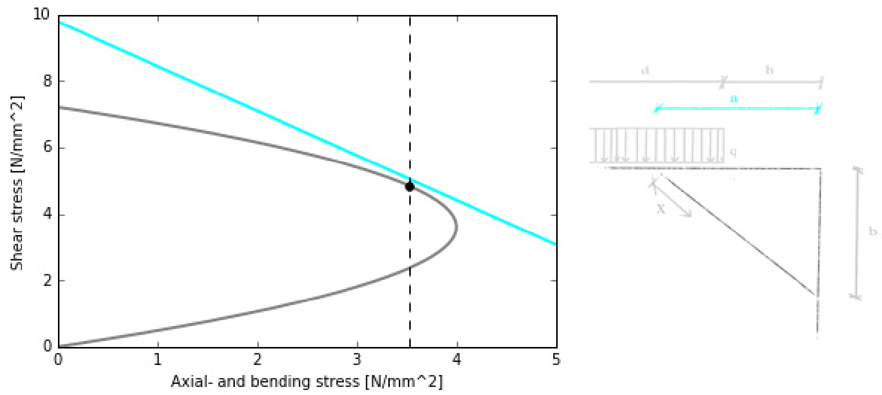

Figure 4.6: (Shear) stress function for different values of 'a=b'

The grey curve in figure 4.5, which is the σ versus τ plot, was composed of both the a-, σ- (figure 4.3) and the a-, τ plot (figure 4.4) for an 'a/b'- ratio of '1'. While considering the low concrete strength class (C20/25), this grey 'load curve' touches the capacity limit slightly. This means that a crack at a certain location 'a' will be formed. The intersection seems to appear at approximately 'σ=3.55 N/mm^2' (tensile stress). Returning this value, it appears that 'a=180 mm' is the most probable location, because 'σ(a=180)= 3.55 N/mm^2'. However, these statements are only valid for 'h=100 mm'.

Several likewise calculations show that the size of the crack is linearly related to distance 'h'. In table 4.1 some of the solutions were given and they are then plotted in figure 4.7.

Distance 'h' [mm]	Crack size 'a=b' [mm]
10	18
50	90
100	180
200	360

Table 4.1: Influence of distance 'h' to the crack size and it's location
Constants are: d= 600 mm and q=24 N/mm^2

Variable 'h' does not influence the magnitude of maximum (shear) stress, but it creates a shift in the stress graphs, because of the presence of 'h' times an 'a=0 mm' value.

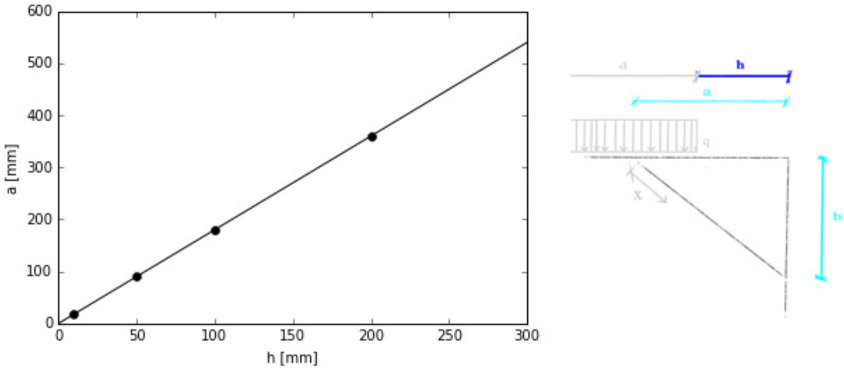

Figure 4.7: Influence of distance 'h' to the crack size and it's location
Constants are: d= 600 mm and q=24 N/mm^2

The reason for this crack size not to be increasing infinitely for an infinite increase of 'h' in actually occurring failure cases, is the applied reinforcement of the column, which will start to act when load 'q' is applied above this steel reinforcement. Because of the reinforcement, the bending moment can be taken more easily and also the forces P_{qv} and P_{qh} will be counteracted.

It can be concluded that bearing distance 'h' does in fact influence the crack size , but not it's corresponding value of maximum (shear) stress.

Conclusion

As the described standards in chapter 3 have shown, current calculations are not as exact and explicit as desired.
In chapter 2, the influences of several variables on the possible crack size / -location were discussed, while chapter 4 was exactly the opposite way around.

In figures 2.3 and 2.4, an optimum for the stress was found with a corresponding 'h'- value (while having constant values for crack size and 'x'- position). Another remarkable thing was the linear to parabolic transition zone. Apparently, this happens when 'd_{Eff}' is no longer equal to 'd'.
From figures 2.10 and 2.12 in 2.4 it can be concluded that the maximum shear stress is always depending on the value of 'd_{Eff}'. If this value decreases (it becomes smaller than distance 'd'), τ_{max} will (linearly) decrease as well.

In the fourth chapter, a possible crack was enlarged each time to see which maximum stress values would be generated for each crack size. Doing this, means creating a more practical graph to determine a possible crack location. It was stated that support load location 'h' would have no influence on the value of stress, but it does have an apparent influence on when (at which value of 'a=b') the ultimate capacity is being reached. In the second chapter about the parameter variations, concluded at some point that distance 'h' alters both stress distribution and maximum values of stress. A few chapters later, this seems no longer to be the case. The reason for this is partially that only σ_{max} was used no matter the 'x'- position. Additionally, distance 'h' does alter stress values, but plotting 'a' versus 'σ' shows that this is only due to a caused shift of the graph. I.e. the stress propagation starts at a larger 'a'- value.

Observing actual column damages due to support capacity failure, the 'a/b'- ratio would often not be equal to '1', but would rather be '0.2 - 0.5'. But plotting the stresses for such a ratio does not reveal a larger maximum (shear) stress.
Another remark would be that the corresponding support load magnitude is equal to '24 N/mm^2', which is a ridiculously large value (e.g. 240 kN concentrated support load on a 100 mm x 100 mm bearing pad). These large support stresses would first cause for a failure of the bearing pad.

The simplified model does produce some problems.
First, some estimations of the crack dimensions *and* it's proportions have to be made in order for calculations to be made. These two assumptions already cause for enough speculation. After that, the influences of certain variables (e.g. 'q', 'd', 'h') have to be determined. They have to be varied one at a time to see it's influence on the stress distribution. The shape of the distributed load is assumed as well and is simplified to not complicate stress relations any further.
Creating a plot, gives more information and better insight, but cannot provide a waterproof statement about which distances have to be kept.

Too many assumptions have to be made, to make this model feasible. Some of the assumptions could be empirically obtained after a failure, but that will cause for a no longer, strictly theoretical model. Further research is needed in order to contemplate actual failure cases and parameter estimations.

Bibliography

[1] K. S. Elliot, *Multi-storey precast concrete framed structures* (Blackwell Science Ltd., 1996) pp. 320–325.

[2] B. S. Institution, *The structural use of concrete*, BS8110 (1985).

[3] ir.W.C. Vis and ing.R. Sagel, *Berekenen en tekenen van betonconstructies*, CEMENT EN BETON (VNC, 1991) pp. 70–73.

[4] *Nen-en 13670:2009 nl,* .

[5] H. Reinhardt, *Betonkunde*, pp. 138–140.

Appendix

Appendix A: Python code

```
import numpy as np
import matplotlib.pyplot as plt
import numpy.random as rnd
from scipy.stats import norm
import scipy.optimize as sc
import ipywidgets as widgets
from ipywidgets import interact
%matplotlib inline

#General function for stress when 'h' is a list
def Function(x=5,a=200,b=200,d=200,q=20):
    c = np.sqrt((a**2)+(b**2))
    Theta = np.arctan(b/a)
    Phi = np.arctan(a/b)
    Ac = 1*c
    Ic = (1/12)*1*(c**3)
    h = np.linspace(0,a,a)
    d1 = np.zeros(a)
    for i in range(a):
        if d+h[i]>a:
            d1[i] = a-h[i]
        else:
            d1[i] = d
    k = h + (0.5*d1)
    e = k-(0.5*c*np.cos(Theta))
    Pq = q*d1
    Mq = e*Pq
    Pqh = np.cos(Phi)*Pq
    Pqv = np.cos(Theta)*Pq
    sigMr = ((Mq*0.5*c)/Ic)
    sigMl = -((Mq*0.5*c)/Ic)
    sigPqv = -(Pqv/Ac)
    sigR = sigMr + sigPqv
    sigL = sigMl + sigPqv
    y = sigL+((sigR-sigL)/c)*x
    Sx = 0.5*x*(c-x)
    tau = ((Pqh*Sx)/Ic)
    return h, y, tau, c
```

```
#General function for stress when 'x' is a list
def Yfunc(a=200,b=200,d=200,h=100,q=20):
    c = np.sqrt((a**2)+(b**2))
    Theta = np.arctan(b/a)
    Phi = np.arctan(a/b)
    Ac = 1*c
    Ic = (1/12)*1*(c**3)
    if d+h>a:
        d1 = a-h
    else:
        d1 = d
    k = h + (0.5*d1)
    e = k-(0.5*c*np.cos(Theta))
    Pq = q*d1
    Mq = e*Pq
    Pqh = np.cos(Phi)*Pq
    Pqv = np.cos(Theta)*Pq
    sigMr = ((Mq*0.5*c)/Ic)
    sigMl = -((Mq*0.5*c)/Ic)
    sigPqv = -(Pqv/Ac)
    sigR = sigMr + sigPqv
    sigL = sigMl + sigPqv
    x = np.linspace(0,c,c)
    y = sigL+((sigR-sigL)/c)*x
    Sx = 0.5*x*(c-x)
    tau = ((Pqh*Sx)/Ic)

    return x, y, tau, c

##Figure 2.2
plt.plot(Yfunc(200,200,100,0,2)[0],Yfunc(200,200,100,0,2)[1], 'cyan')
plt.plot(Yfunc(200,200,100,100,2)[0],Yfunc(200,200,100,100,2)[1], 'darkgrey')
plt.plot(Yfunc(200,200,100,200,2)[0],Yfunc(200,200,100,200,2)[1], 'k')
plt.xlim(0,c)

##Figure 2.3
plt.plot(Function(0,200,200,200,1)[0],np.abs(Function(0,200,200,200,1)[1]), 'aqua')
plt.plot(Function(0,200,200,200,2)[0],np.abs(Function(0,200,200,200,2)[1]), 'gray')
plt.plot(Function(0,200,200,200,3)[0],np.abs(Function(0,200,200,200,3)[1]), 'k')
plt.ylim(0,2.2)

##Figure 2.4
plt.plot(Function(0,200,200,100,2)[0],Function(0,200,200,100,2)[1], 'aqua')
plt.plot(Function(0,200,200,150,2)[0],Function(0,200,200,150,2)[1], 'gray')
plt.plot(Function(0,200,200,200,2)[0],Function(0,200,200,200,2)[1], 'k')
plt.ylim(-2,2)

##Figure 2.6
c4  = Function(0,200,200,200,2)[3]
```

```
c5 = Function(0,250,200,200,2)[3]
c6 = Function(0,300,200,200,2)[3]

X_1 = Function(0.1*c4,200,200,200,2)[0]
X_2 = Function(0.1*c5,250,200,200,2)[0]
X_3 = Function(0.1*c6,300,200,200,2)[0]

Y_1 = Function(0.1*c4,200,200,200,2)[1]
Y_2 = Function(0.1*c5,250,200,200,2)[1]
Y_3 = Function(0.1*c6,300,200,200,2)[1]

plt.plot(X_1,Y_1, 'aqua')
plt.plot(X_2,Y_2, 'gray')
plt.plot(X_3,Y_3, 'k')

Min_y_1 = min(Y_1)
Min_x_1 = X_1[Y_1.argmin()]
Min_y_2 = min(Y_2)
Min_x_2 = X_2[Y_2.argmin()]
Min_y_3 = min(Y_3)
Min_x_3 = X_3[Y_3.argmin()]

plt.plot([Min_x_1,Min_x_2,Min_x_3],[Min_y_1,Min_y_2,Min_y_3], 'ko')

##Figure 2.7
c4  = Function(0,200,200,200,2)[3]
c5 = Function(0,250,250,200,2)[3]
c6 = Function(0,300,300,200,2)[3]
plt.plot(Function(0.1*c4,200,200,200,2)[0],
    Function(0.1*c4,200,200,200,2)[1], 'aqua')
plt.plot(Function(0.1*c5,250,250,200,2)[0],
    Function(0.1*c5,250,250,200,2)[1], 'gray')
plt.plot(Function(0.1*c6,300,300,200,2)[0],
    Function(0.1*c6,300,300,200,2)[1], 'k')
plt.ylim(-1.4,0)

##Figure 2.8
plt.plot(Yfunc(200,200,200,100,5)[0],
    Yfunc(200,200,200,100,5)[2], 'cyan')

##Figure 2.9
plt.plot(Yfunc(200,200,200,100,1)[0],Yfunc(200,200,200,100,1)[2], 'cyan')
plt.plot(Yfunc(200,200,200,100,2)[0],Yfunc(200,200,200,100,2)[2], 'gray')
plt.plot(Yfunc(200,200,200,100,3)[0],Yfunc(200,200,200,100,3)[2], 'k')

#Figure 2.10
def plt_(d0=100):
    c0 = Function(0,200,200,d0,2)[3]
    X0 = np.linspace(0,c0,200)
```

```
    Y0 = Function(0.5*c0,200,200,d0,2)[2]
    Xb = Function(X0,200,200,d0,2)[0]
    return Xb, Y0

plt.subplot(1,2,1)
plt.plot(plt_(50)[0],plt_(50)[1],'k')

plt.subplot(1,2,2)
plt.plot(plt_(100)[0],plt_(100)[1],'cyan')

##Figure 2.11
a1 = 120
a2 = 160
a3 = 200
a4 = 240

c1 = Yfunc(a1,200,200,100,2)[3]
c2 = Yfunc(a2,200,200,100,2)[3]
c3 = Yfunc(a3,200,200,100,2)[3]
c4 = Yfunc(a4,200,200,100,2)[3]

X1 = Yfunc(a1,200,200,100,2)[0]
X2 = Yfunc(a2,200,200,100,2)[0]
X3 = Yfunc(a3,200,200,100,2)[0]
X4 = Yfunc(a4,200,200,100,2)[0]

Y1 = Yfunc(a1,200,200,100,2)[2]
Y2 = Yfunc(a2,200,200,100,2)[2]
Y3 = Yfunc(a3,200,200,100,2)[2]
Y4 = Yfunc(a4,200,200,100,2)[2]

plt.plot(X1,Y1, 'cyan')
plt.plot(X2,Y2, 'darkgrey')
plt.plot(X3,Y3, 'grey')
plt.plot(X4,Y4, 'k')


Max_y1 = max(Y1)
Max_x1 = X1[Y1.argmax()]
Max_y2 = max(Y2)
Max_x2 = X2[Y2.argmax()]
Max_y3 = max(Y3)
Max_x3 = X3[Y3.argmax()]
Max_y4 = max(Y4)
Max_x4 = X4[Y4.argmax()]

plt.plot([Max_x1,Max_x2,Max_x3,Max_x4],[Max_y1,Max_y2,Max_y3,Max_y4], 'ko')
```

```
p0,p1 = ([[Max_x1,Max_x2,Max_x3,Max_x4],[Max_y1,Max_y2,Max_y3,Max_y4]])

plt.xlim(0,350)
plt.ylim(0,0.9)
plt.plot(p0,p1, 'bo')
p2 = np.polyfit(p0,p1, 3)

ypol = np.zeros(len(p0))

for i in range(len(p0)):
    ypol[i] = p2[0]*(p0[i]**3) + p2[1]*(p0[i]**2) + p2[2]*p0[i] +p2[3]
#plt.plot(p0,ypol)

p3 = np.linspace(100, 157, 100)
ypol1 = np.zeros(100)
for i in range(len(p3)):
    ypol1[i] = p2[0]*(p3[i]**3) + p2[1]*(p3[i]**2) + p2[2]*p3[i] +p2[3]
plt.plot(p3,ypol1,linestyle = '--', color = 'k', label = 'Max shear');


##Figure 2.12
def plt_(a0=200):
    c0 = Function(0,a0,200,200,2)[3]
    X0 = np.linspace(0,c0,a0)
    Y0 = Function(0.5*c0,a0,200,200,2)[2]
    Xb = Function(X0,a0,200,200,2)[0]
    return Xb, Y0

plt.subplot(1,2,1)
plt.plot(plt_(200)[0],plt_(200)[1],'k')

plt.subplot(1,2,2)
plt.plot(plt_(300)[0],plt_(300)[1],'cyan')




#General function for determining stress maximum
def Maxs(a=200,b=200,d=200,h=100,q=20):
    c = np.sqrt((a**2)+(b**2))
    Theta = np.arctan(b/a)
    Phi = np.arctan(a/b)
    Ac = 1*c
    Ic = (1/12)*1*(c**3)
    if h>=a:
        d1 = 0
    elif d+h>a:
        d1 = a-h
```

```
    else:
        d1 = d
    k = h + (0.5*d1)
    e = k-(0.5*c*np.cos(Theta))
    Pq = q*d1
    Mq = e*Pq
    Pqh = np.cos(Phi)*Pq
    Pqv = np.cos(Theta)*Pq
    sigMr = ((Mq*0.5*c)/Ic)
    sigMl = -((Mq*0.5*c)/Ic)
    sigPqv = -(Pqv/Ac)
    sigR = sigMr + sigPqv
    sigL = sigMl + sigPqv
    x = np.linspace(0,c,c)
    y = sigL+((sigR-sigL)/c)*x
    z = np.max(y)
    return z

#General function for determining shear stress maximum
def Maxsh(a=200,b=200,d=200,h=100,q=24):
    c = np.sqrt((a**2)+(b**2))
    Theta = np.arctan(b/a)
    Phi = np.arctan(a/b)
    Ac = 1*c
    Ic = (1/12)*1*(c**3)
    if h>=a:
        d1 = 0
    elif d+h>a:
        d1 = a-h
    else:
        d1 = d
    k = h + (0.5*d1)
    e = k-(0.5*c*np.cos(Theta))
    Pq = q*d1
    Mq = e*Pq
    Pqh = np.cos(Phi)*Pq
    Pqv = np.cos(Theta)*Pq
    sigMr = ((Mq*0.5*c)/Ic)
    sigMl = -((Mq*0.5*c)/Ic)
    sigPqv = -(Pqv/Ac)
    sigR = sigMr + sigPqv
    sigL = sigMl + sigPqv
    x = np.linspace(0,c,c)
    y = sigL+((sigR-sigL)/c)*x
    Sx = 0.5*x*(c-x)
    tau = ((Pqh*Sx)/Ic)
    z = np.max(np.abs(tau))
    return z
```

```
##Figure 4.3
n2 = 200
z2 = np.zeros(n2)
ab2 = 1
for i in range(n2):
    z2[i] = Maxs((i+1)*ab2,(i+1)*2*ab2,800,10,24)
xab2 = np.linspace(0,n2*ab2,n2)
plt.plot(xab2,z2,'cyan')
plt.plot(xab1,z1,'grey')
Max_y2 = max(z2)
Max_x2 = xab2[z2.argmax()]
print('a=', Max_x2,'sig=',Max_y2)

##Figure 4.4
n4 = 200
z4 = np.zeros(n4)
ab4 = 1
for i in range(n4):
    z4[i] = Maxsh((i+1)*ab4,(i+1)*2*ab4,300,10,24)
xab4 = np.linspace(0,n4*ab4,n4)
plt.plot(xab4,z4,'cyan',label = 'a=0.5b')
plt.plot(xab3,z3,'grey',label = 'a=b')
plt.xlabel('Position of the crack a')
plt.ylabel('Shear stress [N/mm^2]')
Max_y4 = max(z4)
Max_x4 = xab4[z4.argmax()]
print(Max_x4,Max_y4)

##Figure 4.5
fck = 20 #N/mm^2   (For example C20/25)
fctk005 = 2.2 #N/mm^2

fcd = fck/1.5 #N/mm^2
fctd = fctk005/1.5 #N/mm^2

fck1 = 30
fctk1 = 2.9

fcd1 = fck1/1.5
fctd1 = fctk1/1.5

fck2 = 45
fctk2 = 3.8

fcd2 = fck2/1.5
fctd2 = fctk2/1.5

def cap(fcd,fctd):
    sigm = np.linspace(-20,10,200)
```

```
    phi = np.arcsin((fcd-fctd)/(fcd+fctd))
    c = 0.5*fcd*fctd
    tau = c-(sigm*np.tan(phi))
    return sigm, tau

plt.plot(cap(fcd,fctd)[0],cap(fcd,fctd)[1],'cyan')
plt.plot(z1,0.6*z3,'grey',)
plt.axhline(y=0,'k')
plt.axvline(x=0,'k')

plt.plot(cap(fcd1,fctd1)[0],cap(fcd1,fctd1)[1],'b')
plt.plot(cap(fcd2,fctd2)[0],cap(fcd2,fctd2)[1],'k')

##Figure 4.6
plt.plot(cap(fcd,fctd)[0],cap(fcd,fctd)[1],'cyan')
plt.plot(z1,0.6*z3,'gray')

X = z1
Y = cap(fcd,fctd)[1]
Z = z3

print(X[180])
plt.axvline(X[180],'k')
plt.xlim(0,5)
plt.ylim(0,10)

##Figure 4.7
plt.plot([10,50,100,200],[18,90,180,360],'ko')
plt.xlim(0,300)

def a_(h):
    return 1.8*h
h = np.linspace(0,300,300)
plt.plot(h,a_(h),'k')
```