# Resistance of the columns of the CEG building against blast loading

by


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# Preface

This report is the end project for the bachelor of civil engineering at the TU Delft. It is intended to display the things the author has learned over the course of his studies and should be read by fellow students who are interested in the structural side of civil engineering. This report should not be read as actual structural analysis report.

I then want to thank my supervisors, Dr. ir. P.C.J. Hoogenboom and Prof. dr. H.M. Jonkers. Without their guidance and feedback the project would not have been the way it is now.

*Ocker Ledeboer*
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# Summary

In a response to the Russian invasion of Ukraine, the question of national defense has become a more relevant topic. Is the Netherlands properly prepared for war? One of these preparations are enough bomb shelters for the civilian population to take cover during air raids and bombings. The faculty of civil engineering and geo sciences is a large building with many concrete elements and a big concrete cellar. Therefore the ability of the building to resist a bombing is modeled and from these models estimations are made.

The main structural elements of the building bearing the vertical loads into the ground are the columns in the building. This report will therefore look at these columns and see if they can resist the loading of different blasts. The main question of the report is: **What is the explosive power which is necessary to blow away the supoorting columns of the CITG building?**

Firstly these columns are looked at more closely, what are their dimensions. What properties do they have, and what is the mostly likely mode of failure. These columns consist of reinforced concrete. First we look at how such a column behaves in a linear elastic loading range, then there is a more detailed into what happens when the plastic limits get reached and parts of the column can not take any more loads. Here we found that the first area's to fail are the edges, when about $1700kN$ gets applied, then the middle also fails at a load of $2300kN$. This creates a mechanism, meaning the column can freely move and therefore collapses. This happens when the maximum displacement is $7.37mm$ and $19.68mm$ respectively.

Then the blast forces are investigated, for different types of explosives. From literature the relation between the blast weight, distance between blast and important point and incoming pressure are found. This gets further elaborated upon in subsection 4.2 and the resulting forces can be found in tables 4.2, 4.3 and 4.4.

All these forces are and properties of the column are then used in a model, as a mass spring-system. From which a ordinary differential equation can be set up. This differential equation is then approached by a numerical program using the Euler forward method. Here it uses a linearisation of the local derivative with a finitely small time step to find the next point. This method is $O(h)$ meaning error linearly decreases with the time step. The blast forces get modeled as a force that acts for a small time in the beginning and then stops.

Finally from all these values it is found that the blast has to occur very close to have any kind of impact on the system. And only really collapses when it has a blast load greater than $50kg$ equivalent of TNT. So a lot of mid-sized bombs will not be enough to cause the column to collapse.

# Contents

# 1 Introduction

In febuari 2022 Russia invaded Ukraine. In Europe the fear that the war might escalate started spreading. With this fear came the question: are we prepared for a war should it find us?
One of the ways to be prepared for a war is having a safe place for the civilian population to hide away during the conflict, for example a bomb shelter. This report will therefore focus on seeing if the celar of the faculty civil engineering and geosciences of the TU Delft is adequately dimensioned to withstand different blasts and finally if it can be used as a bomb shelter.

Another report [7], by F. Saab modeled what would happen should the explosion blow away one of the columns and the effect that has on the cellar with the falling rubble. However these columns do not simply disappear, and the main question therefore of this report will center around: **What is the explosive power which is necessary to blow away the supoorting columns of the CITG building?**

The chapter 2 will focus on the column and the dimensions it has. It will then see what kind of properties is has and translate that into numbers for the mathematical model. Similarly with the forces coming from the blast. Then in chapter 3 the nunmerical method used to make the model will be discussed. Which equations are critical and how we go from the data to our results. Finally in chapter 4 all the different data is collected and shown which is then interpreted in chapter 5.

# 2

# From reality to model

To be able to actually calculate the damage done to the building, it first has to be simplified into a model. This chapter will therefore analyse the structure and translate it into a or multiple equations. To get there some assumptions and idealisations have to be made, these will also be justified in this chapter.

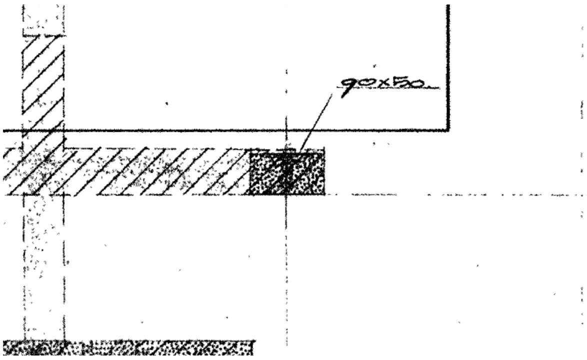


**Figure 2.1:** Top down view of columns and

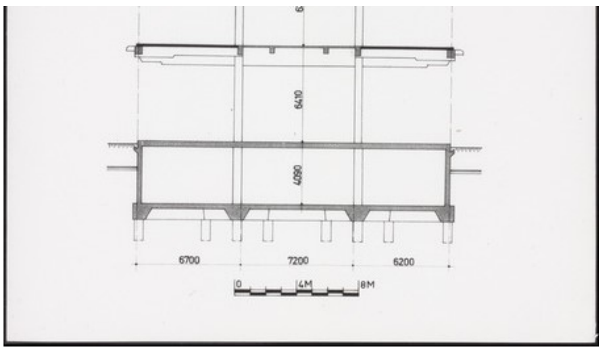


**Figure 2.2:** lengthwise viewing of the building, lower floors

## 2.1. The building and the columns

The building rests on multiple lines of columns that are lined up. They are 900x500 concrete columns with reinforcement as seen in figure 2.1. The exact reinforcement is however not online available. Important for the model is that the column already has to bear the load of the floors on top, which means that the column might fail first on crushing or the yielding of the reinforcement. from this we model the entire column as a bar under compression that is fully fixed from end to end (no rotation near the ends). This full length is 6410 mm which can be seen in figure 2.2. However this height includes the horizontal concrete beam, which is about 1.7 meters in height. The full lenght is then 4750 mm (measured).

The column is under compression. Because of this the resistance against moments becomes different. Where normally the first things to fail should be the reinforcement because that generally is the most ductile mode of failure, so columns get designed to fail that way. In columns with compression in them however an extra compressive stress has to be taken into account. In the first area cases the tensile reinforcement will yield before the concrete crushes. In the second area the compressive reinforcement will also yield, peaking the moment that gets taken. Then in the third area, the concrete crushes before the tensile reinforcement yields. After this the capacity drops significantly until the compressive stresses get so great that no moment can be taken anymore at all because the full cross-section is at it’s limit.

These resistances against the moment can be found using a few different formula’s each for another cases with how much normal force is present. Beginning with no normal force present.[3]

$$M_{Rd} = A_s f_{yd}(h/2 - d_2) + A_{s2}\sigma_{s2}(h/s - d_2) + \alpha x b f_{cd}(h/2 - \beta x) \tag{2.1}$$

where:

- $M_{RD}(kNm) =$ the moment capacity of the cross section
- $A_{s2}(mm^2) =$ the reinforcement area of the compressed area
- $A_s(mm^2) =$ the reinforcement area of the tensioned area
- $d_2(mm) =$ the distance from the edge of the column to the working reinforcement
- $d(mm) =$ the useful height, so the height minus the distance between the reinforcement and the edge (such that $d_2 + d = h$
- $x(mm) =$ the height concrete in the cross section in compression
- $\alpha(-) =$ a reduction factor due to some of the concrete already yielding (in lower strenght 0.75)
- $\beta(-) =$ a reducion factor having to do with the working line of the concrete force with respect to the normal centre (0.39)
- $f_{cd}(N/mm^2) =$ the concrete design yield strength (in this case 19)
- $f_{yd}(N/mm^2) =$ the reinforcement steel yield strength (in this case 435)
- $\sigma_{s2}(N/mm^2) =$ the stress in the compressed part of the reinforcement

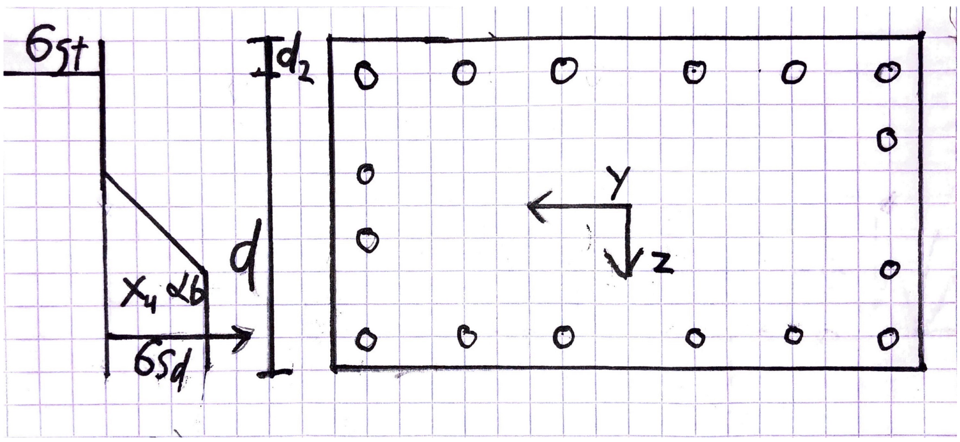


**Figure 2.3:** A sketch of the column and the important properties

In the equation 2.1, since we are looking at the weakest axis around which to bend, the height is 500, while the width is 900. Because of this we also have 6 active reinforcement bars on either side of the column, making it 12 total. The a sketch of the cross section of the column can be seen in figure 2.3

The only unknowns in this formula right now are the stress in the compression reinforcement and the $x$ as height of concrete under compression.[3]

$$x_1 = \left(\frac{\epsilon_{cu3}}{\epsilon_{cu3} - \epsilon_{yd}}\right) d_2 \tag{2.2}$$

$$\sigma_{s2} = E_s\left(1 - \frac{d_2}{x}\right)\epsilon_{cu3} \tag{2.3}$$

Here:

- $\epsilon_{cu3}(\%_0) =$ the ultimate strain for concrete (which is 3.5)
- $\epsilon_{c3}(\%_0) =$ the yield strain for concrete (which is 1.75)
- $\epsilon_{yd}(\%_0) =$ the yield strain for reinforcement concrete ( 2.175)
- $E_s(N/mm^2)$ = the elasticity modulus of steel (which is 210000)

This gives the moment capacity in the first case. Then for the next cases we use a similar basic formula for the moment itself, however mainly the area under compression and the stresses in the reinforcement will mostly differ.

$$x_2 = (\frac{\epsilon_{cu3}}{\epsilon_{cu3} + \epsilon_{yd}})d \tag{2.4}$$

In this case both reinforcements are yielding, meaning $\sigma_t = \sigma_d = f_{yd}$, which makes:

$$M_{Rd} = A_s f_{yd}(d - d_2) + \alpha x_2 b f_{cd}(h/2 - \beta x_2) \tag{2.5}$$

In this case, the moment is fully taken by the reinforcement, so the compressive force is fully taken by the concrete:

$$N_{ED} = N_c = \alpha x b f_{cd} \tag{2.6}$$

The next important point is when there is no more tension in the entire cross section, Here $x = h$

$$\epsilon_s = (d/h - 1)\epsilon_{cu3} \tag{2.7}$$

$$N_s = A_s * E_s * \epsilon_s \tag{2.8}$$

$$N_{s2} = A_{s2} f_{yd} \tag{2.9}$$

$$N_c = \alpha b h f_{cd} \tag{2.10}$$

$$N_{Ed} = N_{s2} + N_c + N_s \tag{2.11}$$

$$M_{Rd} = N_{s2}(h/2 - d_2) + N_s(h/s - d_2) + N_c(h/2 - \beta h) \tag{2.12}$$

This is the last cross section with any sort of moment Resistance. Finally we have the case of pure compression. All the elements are yielding at their full strength.

$$N_{ED} = A_s f_{yd} + A_{s2} f_{yd} + b h f_{cd} \tag{2.13}$$

These equations can then be used to get a diagram plotting the moment capacity versus the acting normal force.

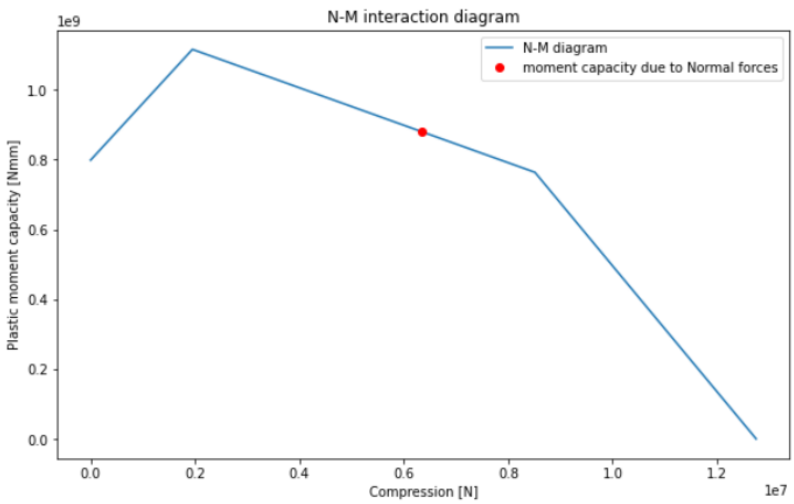


**Figure 2.4:** A interaction diagram between the normal forces and moment capacity

When trying to find the actual moment capacity we first try to estimate the force acting through the column. We assume that the actual force nears the design strength.

$$N_{Ed} = bhf_{cd}/\gamma_{concrete} \tag{2.14}$$

Here we use the concrete's design strength combined with a safety factor of 1.5 for concrete. After that we use linear interpolation to gain the strength from the graph that is shown in figure 2.4.

| | situation 1 | situation 2 | situation 3 | situation 4 | actual situation |
|---|---|---|---|---|---|
| xu (mm) | 164 | 274 | 500 | 500 | n. d. |
| N (kN) | 0 | 1951 | 8511 | 12748 | 5700 |
| MRD (kNm) | 798 | 1115 | 763 | 0 | 914 |

**Table 2.1**

This 914 kNm from table 2.1 will be the plastic moment capacity of the column going forward. The other values are the moment capacities and normal Forces that are used in figure 2.4

### 2.1.1. column in the model

In the model the column will be modeled as a mass spring system. Here a few aspects of the column are very important, firstly the moving mass $[m]$, the spring stiffness $[k]$ and the damping coefficient $[c]$. Firstly the mass that gets moving in the system is simply the mass of the column. However due to the fixed ends, the column is restricted in moving. So a effective length of the moving column is taken only looking at about 0.7 of the total length.

$$m = (A_c * \rho_c + A_s * \rho_s) * l_{eff} \tag{2.15}$$

The spring stiffness follows from a structural "forget-me-not". This is a standard case for how a doubly clamped bar reacts to a uniformly distributed load.

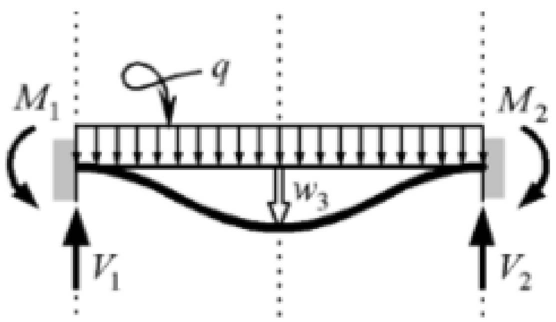


**Figure 2.5:** Forget met not [10]

For any elastic behaviour of a cross section, it is important to know how stresses distribute over those cross sections. Important here is the $EI$ or the Elasticity modulus multiplied with the surface area moment. In composite cross sections like reinforced concrete, the more stiff material pulls more stress to it's surface, which can make this $EI$ hard to calculate. Since however this is a relatively symmetrical cross section, a lot simplifies. First off, the $EI$ can be split into that of the steel part and the concrete.

$$EI = E_s I_s + E_c I_c \tag{2.16}$$

Most of the steel however does not centre around the normal centre. To find this moment of inertia we can then use Steiner's theorem or the parallel axis theorem, which states:

$$I_z = I_x + Ar^2 \tag{2.17}$$

Finally we then get

$$EI = E_s n_w(\frac{\pi}{4}(\phi_{lw}/2)^4 + \frac{\pi}{4}(\phi_{lw})^2(h/2 - d_2)^2) + E_c\frac{1}{12}Bh^3 \tag{2.18}$$

where:

- $n_w =$ the amount of active reinforcement bars in a cross section (in this case 6)
- $\phi_{lw} =$ the diameter of the reinforcement steel (here 32 mm)
- $E_c =$ the stiffness modulus of concrete (usually around 30000, but if we assume it cracks, then the stiffness gets reduced by a factor 3)

because $\phi_{lw} << h$, means that the contribution of $(\phi_{lw}/2)^4$ is probably negligible in comparison to the other addition, and will be assumed as such

From the standard case in figure 2.5 we find that the following relation between the displacement and the acting forces:

$$w_c = \frac{1}{384}\frac{ql^4}{EI} \tag{2.19}$$

Looking back at our standard spring model, we find that normally the forces from a spring can be found as the following:

$$F_{spring} = -k * u \tag{2.20}$$

We can rewrite equation 2.19 in the following way to make it look more like equation 2.20:

$$ql = \frac{384EI}{l^3} * w_c \tag{2.21}$$

Here the $ql$ is similar to the $F_{spring}$, only this is the load required to move the column, while the $F_{spring}$ is the reaction force resisting the movement. Then we can also see that our $k$ can be described as $\frac{384EI}{l^3}$, which will also be the k used in our model. Important is to note that this only applies to the elastic realm. Once plastic deformation happens, the moment and strain distribution will become different and a different stiffness will be used.

### 2.1.2. plastic deformation

Going into plastic stiffness it gets more complicated as different parts of the column will start to behave non linearly after the plastic limit is reached [6]. The first parts of the beam to reach it's plastic limit will be the ends at the bottom and top, as the moment distribution is highest there in this standard case.

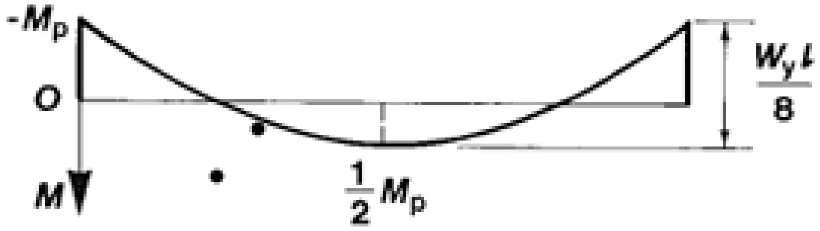


**Figure 2.6:** Moment distribution before moment failure [6]

In the figure 2.6 the Mp is the plastic moment capacity we found from figure 2.4. As seen in the forget me not's, the moment here will be given by $M_{end} = \frac{ql^2}{12}$, so $q_{collapse}l = 12M_p/l$. Plugging this into the forget me not for the deflection, the following can be stated:

$$w_y = \frac{12M_p l^2}{384EI} \tag{2.22}$$

This displacement still holds, since moment just before the failure is still wholly elastic. After the ends collapse the standard situation more closely resembles a hinged supported beam with two moments acting on both ends with the size of the plastic moment capacity. This does mean that from now on the angle at the end of the beam $\phi_1 > 0$

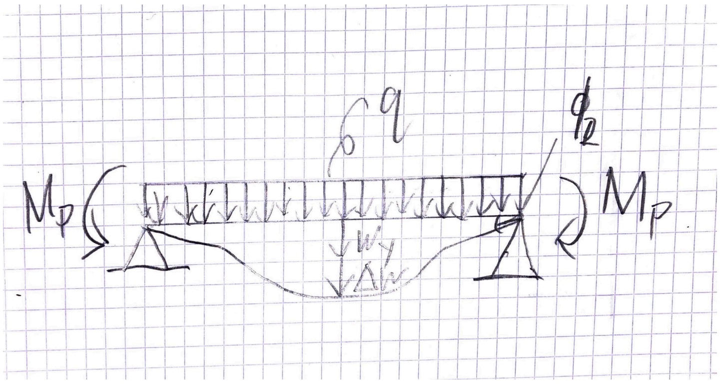


**Figure 2.7:** Displacement case after the ends fail

In figure 2.7 the $w_y$ is the displacement caused by the load until the ends fail, the subsequent $\Delta w$ is the displacement added onto it by the load that adds itself on after the ends have already yielded $\Delta q = q_{fullcollapse} - q_y$. The moment distribution will finally be the Mp's at the edges of the column and the Mp at the centre. Finally the full collapse displacement is given as $w_c = w_y + \Delta w$.

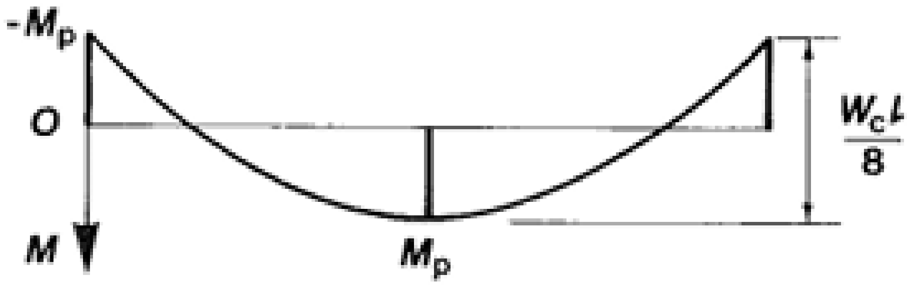


**Figure 2.8:** Moment distribution when the full beam collapses

Since in figure 2.8 the ends are now hinges, the moment distribution will also become like that of a hinged beam. So for the middle of the beam the moments will change as follows:

$$0.5Mp + \frac{\Delta q l^2}{8} = M_{middle} \tag{2.23}$$

When the middle collapses $M_{middle} = M_p$ from which we can find the following relations:

$$\Delta q l = 4M_p/l \tag{2.24}$$

$$q_y l = 12M_p/l \tag{2.25}$$

$$q_c l = 12Mp/l + 4Mp/l = 16Mp/l \tag{2.26}$$

$$w_c = \frac{q_y l^4}{384EI} + \frac{5\Delta q l^4}{384EI} = \frac{12M_p l^2}{384EI} + \frac{20M_p l^2}{384EI} = \frac{M_p l^2}{12EI} \tag{2.27}$$

From these equations, we find the plastic limits and the deflections that accompany it. After these limits are reached the column stops behaving elastically. It will then start to act plastically.

| | yield | collapse |
|---|---|---|
| w (mm) | 4.05 | 10.81 |
| ql(kN) | 2310 | 3080 |

**Table 2.2:** the displacements and accompanying loads of collapse

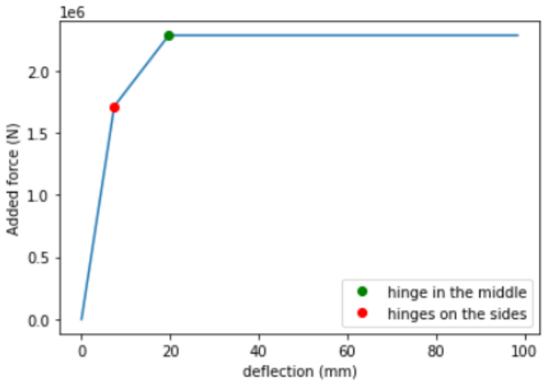


**Figure 2.9:** The plasticity graph and it's collapse load versus displacement

In table 2.2 we find the different displacements, first when the edges start to fail, causing the beam to act like a hinged beam. Then the deflection when the entire beam collapses. Then there are also the loads at which this happens, which are finally all graphed in figure 2.9. These values are important for the final models, which are discussed further in chapter 4.

## 2.2. Finding the blast forces

The main force the column will have to withstand will consist of the blast load from the explosion. These forces come from the shock wave that radiates out from the explosion. These shock waves are most often modeled as a wave of high pressure followed by an area of negative pressure. These waves travel with the speed of sound through the air, meaning they pass a very short time. In this report all the explosions happen at relatively short distance, so the blast will get modeled as a uniform, evenly distributed load working horizontally. Important is that the impulse absorbed from the explosion is the same as the initial momentum from the column.

$$I = \int_{t_0}^{t_{end}} F_{explosion}\, dt \tag{2.28}$$

The force will be modeled as constant so equation 2.28 easily simplifies into the force times the active time. The most important part of the function is that this energy release is constant, meaning that the impulse stays the same. For the purposes of this system therefore, the input will be given as a set impulse, which then gets divided by a active time. When the time is lower then the active time, the righthandside of the main equation 3.2 will have this force acting, after this time it will disappear. This has the benefit of being stable, while not requiring a extremely small time step in the forward Euler method. It does start to behave weirdly when the bomb time is too high, since the spring forces start to become significant before the bomb stops acting on the system. This is mainly due to the numerical method 3.1

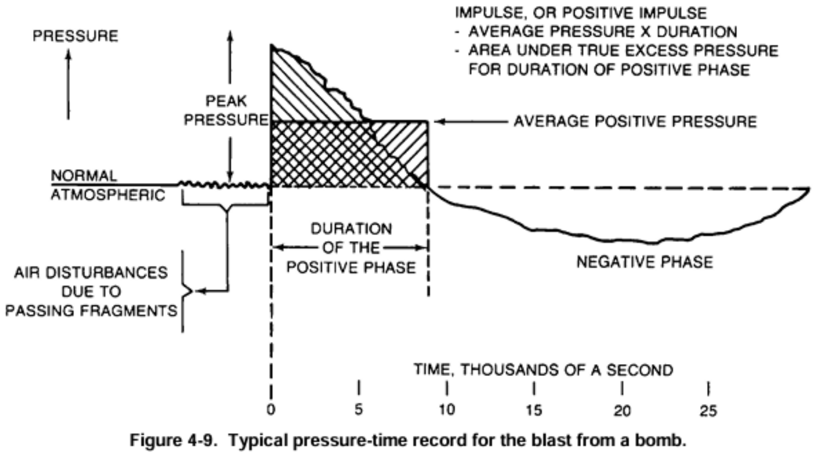


**Figure 2.10:** Graph showing the shockwave passing [5]

First that impulse that is created can be also be found as an integral of the shockwave [8]. In reality the wave takes about 0.01 second to pass as seen in figure 2.10.

$$i_s = \int_{t0}^{t_{end}} P_{s0}\, dt \tag{2.29}$$

Here the pressure is the incoming pressure wave. To model this we will make a very sweeping generalisation, since we are mostly interested in the structural reaction to the blast and not necessarily the blast itself. If we make a linearisation from the peak pressure to the 0 point we can integrate over a line. This makes:

$$i_s = \frac{1}{2} P_{s0} * t_{end} \tag{2.30}$$

This decreases over the distance it travels. Here we use a scaled distance Z. [4]

$$Z = R/W_{TNT}^{1/3} \tag{2.31}$$

here:

- $R(m) =$ the distance from the explosion to the point of interest
- $W_{TNT}(kg) =$ the weight of the explosive as it's tnt equivalent

This can then be expressed using bakers equation [1] [8], which use parameters from empirical results to describe the blast waves relation to the relative distance as such:

$$P_{s0} = 20.06Z^{-1} + 1.94Z^{-2} - 0.04Z^{-3}(0.05 \leq Z \leq 0.5) \tag{2.32}$$

$$P_{s0} = 0.67Z^{-1} + 3.01Z^{-2} - 4.31Z^{-3}(0.5 \leq Z \leq 70.9) \tag{2.33}$$

Where $P_{so}$ is the pressure in bar.

In the formula we use the TNT equivalent weight. This is because different kinds of explosives have different types of explosive power. To standardise this a bit, so different types of explosives can be better compared to each other. For this project we will limit it to a few different types of explosives: PETN charges, TNT and nitroglycerin charges.

| type of explosive | $W_{tnt}$ equivalent factor |
|---|---|
| TNT | 1.1 |
| Nitroglycerine | 1.2 |
| PETN | 1.7 |

**Table 2.3:** Factors for the explosiveness of substances [5]

We use the factors from table 2.3 to find the TNT equivalent as follows:

$$W_{pTNT} = (\frac{P}{P_{TNT}})W \tag{2.34}$$

Where the $\frac{P}{P_{TNT}}$ is the same as the equivalent factor [9].

With these equations we can find the final forces acting on the structure, simply by multiplying the incoming pressure with the area it acts over (900 and 6410), these can be found in chapter 4.

# 3

# Python program

Last chapter we discussed about how the real life column got transformed into a mass spring system, where stiffness and the mass got decided by the dimensions of the columns. In this chapter there will be an expansion upon the mass spring system, where different loads are unleashed on the system, which will then be expanded upon.

## 3.1. the forward Euler method

To approximate the solution to the system, a numerical method is used, the forward Euler method. This method uses a linearisation around a certain time step of the function. The main principle centres around the following equation

$$y_{n+1} = y_n + f(y_n, t_n)h \tag{3.1}$$

Here the function $f(y_n, t_n)$ is the rate of change of $y_n$ at that specif moment. In the specific case of the column, the full system is governed by the basic mass spring equation. [2]

$$m\ddot{w} + kw + c\dot{w} = F_{external} \tag{3.2}$$

The equation 3.2 can be rewritten to be an explicit way to calculate the acceleration at that specific moment as a function of the place and the speed. here the k is the spring coefficient, c is the damping factor.

$$a_n = (F_{external} - kw_n - cv_n)/m \tag{3.3}$$

When having found the acceleration at a certain point, this can be used to find the speed in a following time step $h$.

$$v_{n+1} = v_n + a_n h \tag{3.4}$$

This speed can then be used to find the next displacement of the column.

$$w_{n+1} = w_n + v_n h \tag{3.5}$$

It should be noted that there are some limiting factors when using this method, since it is only a linearisation around the specific point in time, it is not fully accurate. However it gets more and more accurate as the h decreases. This algorithm is O(h), so the error linearly decreases with the step size.A

To actually go through the time steps, it loops through the system, with the same amount of time steps as are given. Arrays are made for each value that is of interest in which the value gets saved.

## 3.2. Blast forces

The blast forces will be modeled as incoming shock waves. These shock waves are already transformed into a point pulse acting on the mass-spring system. This pulse is then divided into a force acting over a time. This is the time the bomb is active. Then this gets added as a constant to the system. After this time the force will be set to 0.

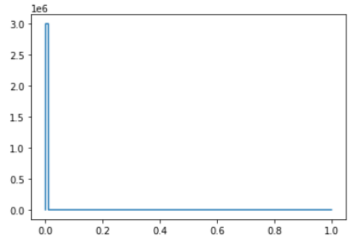


**Figure 3.1:** example of blast force

The numbers in figure 3.1 are arbitrary and it is more important to show how the force works.

## 3.3. stiffness and plasticity

From section 2.1.1 it follows that the initial stiffness of the system is found from the forget me not as a constant. Then in section 2.1.2 it is seen that this behaviour changes from the moment the plastic capacity gets reached. A few different models got made to take this into account. One where the stiffness $k$ stays constant. This is a best case scenario and failure here means the column probably does really collapse under the blast.

The second model investigates everything as linear, up until the moment the ultimate deflection gets reached, the column cannot take any more load. Meaning the spring force will stay constant as long as it is above that ultimate deflection load. The response this gives can be seen in figure 3.2

The third model Is similar to the second one, but then adds a third component. Here we also take into account that the edges can start to fail before creating a full mechanism. So the column will not fully stop being able to resist deformation, but it will become less stiff. This secondary stiffness can be expressed in the following equation.

$$k = \frac{384EI}{5l^3}$$

When the hinges get formed at the edges, these edges can rotate. The the other forget me not can then be used for the new stiffness. This can be seen in figure 3.3

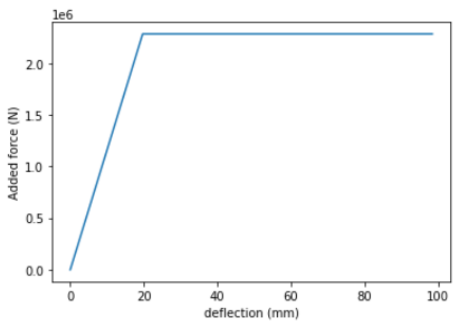


**Figure 3.2:** F-w diagram where only the ultimate collapse displacement is important

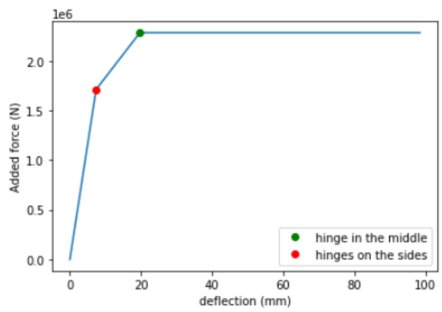


**Figure 3.3:** F-w diagram with the first hinges also displayed

# 4 Inputs and data

In this chapter the final inputs will be calculated and be used to graph the dynamic response of the system. Because there are multiple models some data will be repeatedly modeled, however if this is for some reason redundant, it will not be included.

## 4.1. confirmation of the model

In numerical models it is hard to be sure that your model is accurate as some methods can be unstable and are prone to errors. However in mass-spring systems some properties should be visible in a graphing of the problem. Mainly the natural frequency of the oscillator. This is found as:[9]

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{4.1}$$

This is a frequency at which such a system will naturally want to oscillate. To find the period from this we simply use:

$$T = \frac{2\pi}{\omega_0} \tag{4.2}$$

From this we find a natural frequency and the period, we can do this with both the regular stiffness and the secondary stiffness (the stiffness when the hinges at both ends form. From this we get:

| type | stiffness ($N/mm$) | natural frequency $\omega_0$ (Hz) | Period T (s) |
|---|---|---|---|
| k1 | 570055 | 12.04 | 0.52 |
| k2 | 114011 | 5.38 | 1.16 |

**Table 4.1:** analytical properties of the different ossicilations

From table 4.1 we can confirm that the model is fairly accurate, since the model with only k1 has 39 peaks in 20 seconds (figure 4.1), which results in a period of 0.51 seconds, while the model with the secondary stiffness has 21 peaks in 20 seconds (figure 4.9). This gives a period of 0.95 seconds. But this model uses both stiffnesses so it should indeed lie between 0.52 and 1.16.

## 4.2. Blasts

First a table of close blasts. This happens at 0.2 meters from the column.

| type explosive | weight explosive (kg) | weight eq factor | weight tnt eq (kg) | distance r (m) | Relative distance Z | pressure bakers eq (Pa) | relative impulse (Nm/s/m2) | impulse (Nm/s) |
|---|---|---|---|---|---|---|---|---|
| TNT | 5 | 1.1 | 5.5 | 0.2 | 0.113 | 30066520 | 30067 | 128534 |
| TNT | 50 | 1.1 | 55 | 0.2 | 0.053 | 80786483 | 80786 | 345362 |
| nitroglycerine | 5 | 1.2 | 6 | 0.2 | 0.110 | 31240067 | 31240 | 133551 |
| nitroglycerine | 50 | 1.2 | 60 | 0.2 | 0.051 | 83598137 | 83598 | 357382 |
| PETN | 5 | 1.7 | 8.5 | 0.2 | 0.098 | 36419639 | 36420 | 155694 |
| PETN | 50 | 1.7 | 85 | 0.2 | 0.045 | 95360941 | 95361 | 407668 |

**Table 4.2:** Different blasts happening at 0.2 meters distance from the column

The blasts from table 4.2 give dynamic responses visible in figures 4.1, 4.2, 4.3, 4.4, 4.5 and 4.6 all figures were using the smallest possible time step of 0.000005. This was the smallest because the time steps were halved until python ran out of data to store:

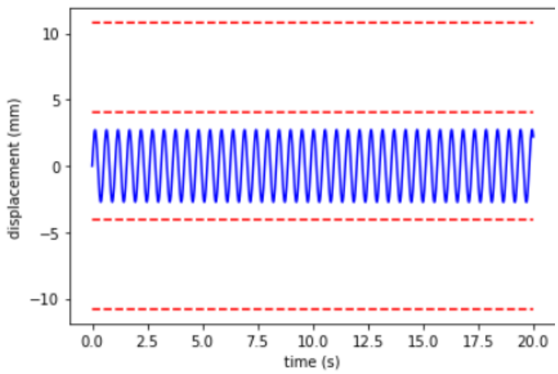


**Figure 4.1:** 5kg of TNT

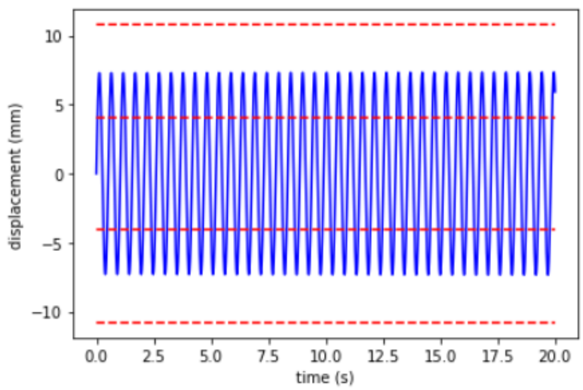


**Figure 4.2:** 50 kg of TNT

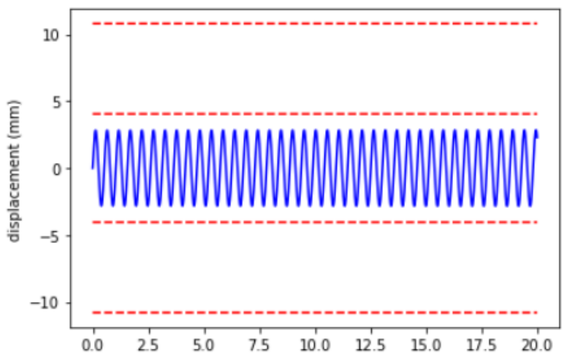


**Figure 4.3:** 5kg of nitroglycerine

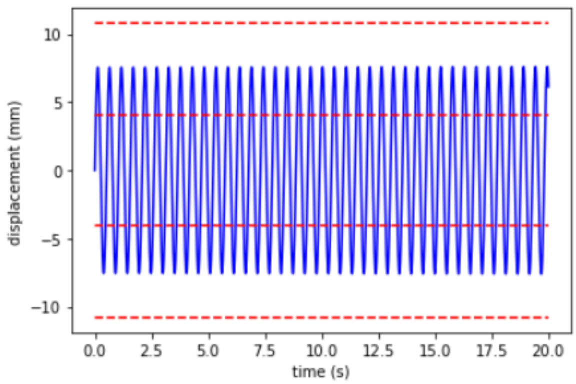


**Figure 4.4:** 50kg of nitroglycerine

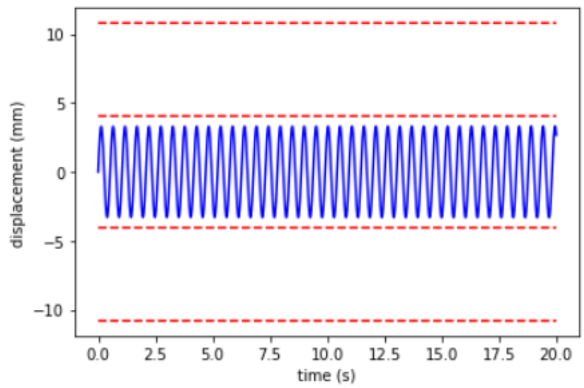


**Figure 4.5:** 5kg of PETN

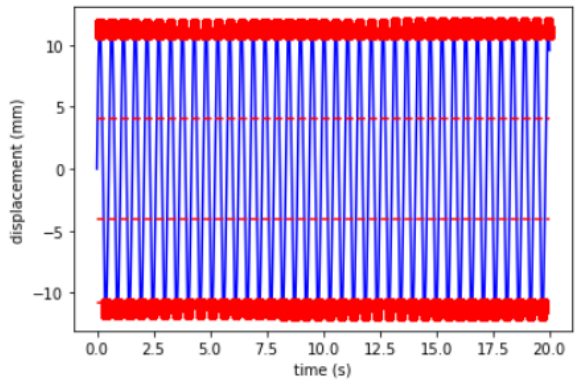


**Figure 4.6:** 50kg of PETN

In these models the stiffness k stays constant. Meaning a mechanism only forms when 50 kg PETN gets detonated. Further away blasts scale down quickly.

| type explosive | weight explosive (kg) | weight eq factor | weight tnt eq (kg) | distance r (m) | Relative distance Z | pressure bakers eq (Pa) | relative impulse (Nm/s/m2) | impulse (Nm/s) |
|---|---|---|---|---|---|---|---|---|
| TNT | 5 | 1.1 | 5.5 | 0.5 | 0.283 | 9323770 | 9324 | 39859 |
| TNT | 50 | 1.1 | 55 | 0.5 | 0.131 | 24720304 | 24720 | 105679 |
| nitroglycerine | 5 | 1.2 | 6 | 0.5 | 0.275 | 9660583 | 9661 | 41299 |
| nitroglycerine | 50 | 1.2 | 60 | 0.5 | 0.128 | 25679571 | 25680 | 109780 |
| PETN | 5 | 1.7 | 8.5 | 0.5 | 0.245 | 11147822 | 11148 | 47657 |
| PETN | 50 | 1.7 | 85 | 0.5 | 0.114 | 29921799 | 29922 | 127916 |

**Table 4.3:** Different blasts happening at 0.5 meters distance from the column

| type explosive | weight explosive (kg) | weight eq factor | weight tnt eq (kg) | distance r (m) | Relative distance Z | pressure bakers eq (Pa) | relative impulse (Nm/s/m2) | impulse (Nm/s) |
|---|---|---|---|---|---|---|---|---|
| TNT | 5 | 1.1 | 5.5 | 2 | 1.133 | 296315 | 296 | 1267 |
| TNT | 50 | 1.1 | 55 | 2 | 0.526 | 2963137 | 2963 | 12667 |
| nitroglycerine | 5 | 1.2 | 6 | 2 | 1.101 | 323253 | 323 | 1382 |
| nitroglycerine | 50 | 1.2 | 60 | 2 | 0.511 | 3232513 | 3233 | 13819 |
| PETN | 5 | 1.7 | 8.5 | 2 | 0.980 | 457941 | 458 | 1958 |
| PETN | 50 | 1.7 | 85 | 2 | 0.455 | 4579391 | 4579 | 19577 |

**Table 4.4:** Different blasts happening at 2 meters distance from the column

From table 4.4 and table 4.3 we can see that even a small distance causes the explosions to have immensely less power compared to the blasts happening next to the column. These blasts certainly will not reach the moment capacity and are therefore irrelevant to show.

What will be shown again however is the differences between a linear k and a changing k.

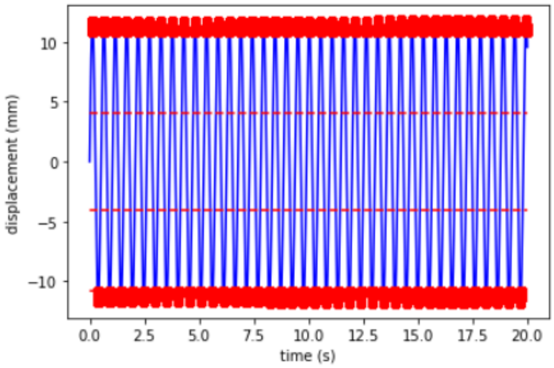


**Figure 4.7:** Blast with constant k

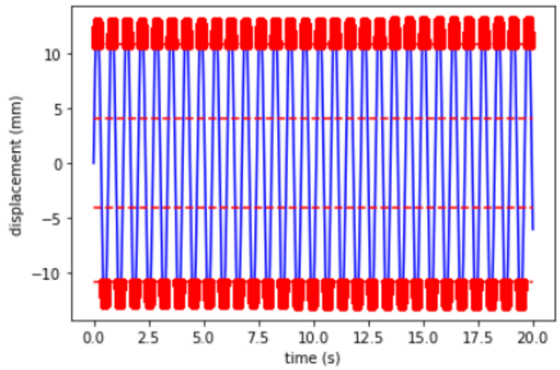


**Figure 4.8:** Blast with hinge at collapse load

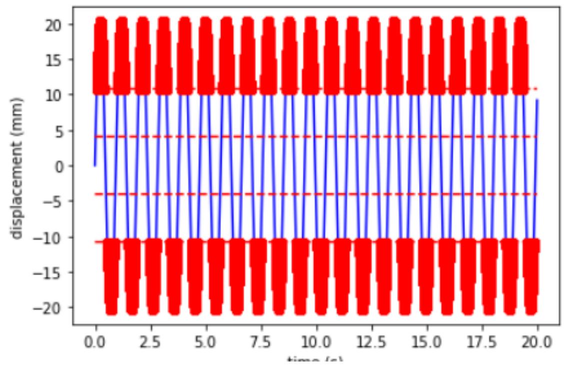


**Figure 4.9:** Blast with plastic hinges

You can see in figures 4.7 and 4.8 the blast hangs a bit more in the mechanism range when the k changes, versus the constant elastic version. However this difference is not very extreme and is barely visible. The biggest change is visible when we also change the hinges at the ends in figure 4.9. A max deflection which is 1.5 times as high as the other ones.

We can also remodel this with the blasts where the hinges do form, namely the 50 kg TNT and the 50 kg nitroglycerin.

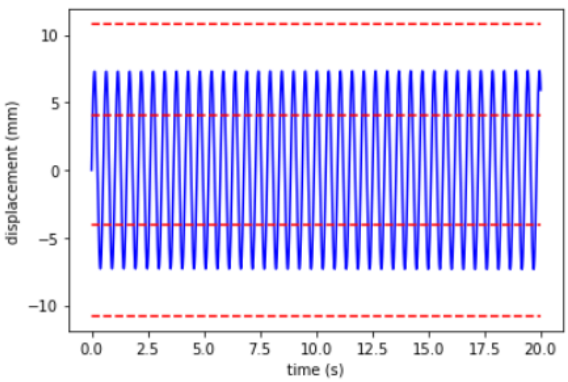


**Figure 4.10:** 50 kg TNT

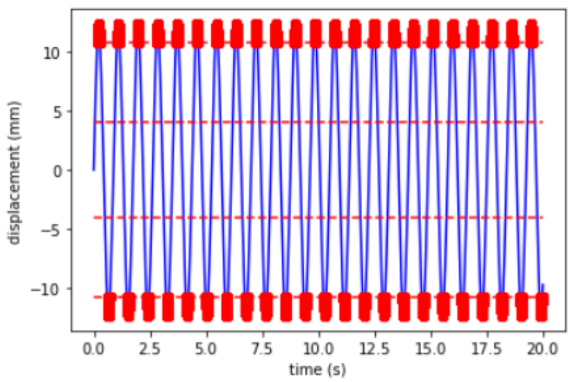


**Figure 4.11:** 50 kg TNT with plastic hinges

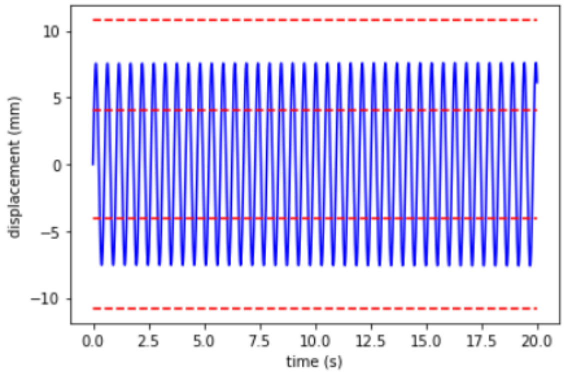


**Figure 4.12:** 50 kg nitroglycerine

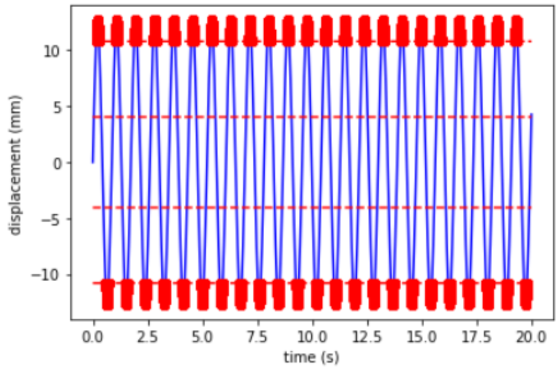


**Figure 4.13:** 50 kg nitroglycerine with deformed hinges

Between figure 4.10 and figure 4.11 is a very visible difference between the two models, as one forms a mechanism while the other does not. Meaning the difference between plastic deformed ends vs stiff ends does make a significant contribution to the system. The exact same holds for figure 4.12 and figure 4.13.

## 4.3. damping

In the previous section we examined the dynamic response of the column to the blast forces. Those oscilations are idealised models and don't take losses from friction into equation. This effect is called damping. The damping coëfficient $c$ can be found in equation 3.2, but is up until now taken as 0. This damping ratio is found by equation 4.3:

$$c = \zeta * 2 * m * \omega_0 \tag{4.3}$$

or

$$c = \zeta * 2 * \sqrt{m * k} \tag{4.4}$$

Here the $\zeta$ is the damping ratio, a percentage of kinetic energy that fades per oscillation. When the damping ratio is 1 or greater than 1, the system is over damped and will asymptotically approach the equilibrium instead of oscillating around it. The damping in the case for a concrete column is around 2.5 % The effect the dampening has on the oscilation will be showcased on figures 4.14 and 4.15.

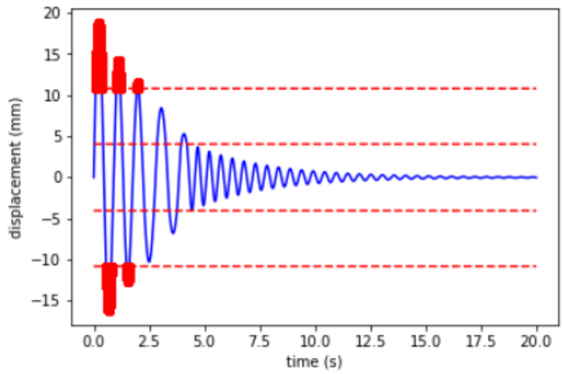


**Figure 4.14:** 50 kg PETN model 3 with damping

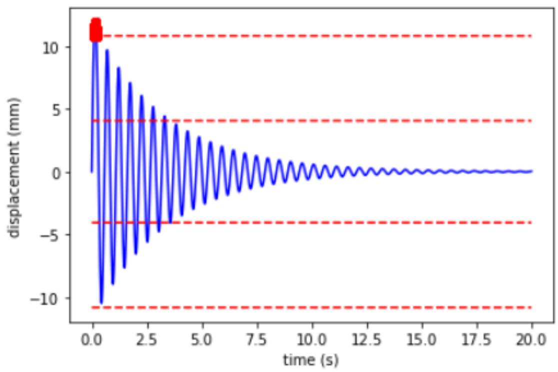


**Figure 4.15:** 50 kg PETN model 2 with damping

Here we can see the damping makes the oscillation die out quite quickly, after only 10 seconds it becomes insignificant in comparison to the maximum displacement.

# 5 Interpretations

In this chapter the data from chapter 4 will be quickly discussed.

Firstly important is to note the big difference that the distance makes to the power of the explosive. This is because the equations that were used were for open air explosions. This will mean short wave reflections against the roof and floor, or reflections inside the building itself were neglected. And instead the pressure had enough room to escape into all sides. Even at 0.5 meters distance the biggest explosive has a similar incoming pressure as that as seen in figure 4.1. There the dynamic response is very little.

Besides this the explosives while packing quite a punch were often not strong enough to blow away the column, the explosions themselves went up to a maximum of 50 kg. This was mainly done so the relative distance would not drop below 0.05, after this the baker equations 2.32 and 2.33 breaks down and will not be relevant anymore.

In the initial model the only explosion to lead where a full mechanism forms is the 50kg PETN charge. The other 50kg charges do cause the edges to fail and become hinges. In the final model however, when we take the effect of the hinged edges on the stiffness of the mass-spring system, the weakened column makes it easier for the other hinge to form and create a mechanism. This does accentuate the big impact of the different models.

To find out if the column actually collapses under the loads presented by the blast we have to know if the forces present will continue the collapse mechanism. These are the normal forces present in the column times the displacement from the second order effects.

$$M_{buc} = N * w_{max} \tag{5.1}$$

The moment from equation 5.1 is the moment keeping the failure mechanism going after the blast forces fade away. We take the maximum displacement in the worst case scenario. Which is the 50 kg in model 3 from figure 4.9. This is $20.2mm$, together with the normal force of $5700kN$ from table 2.1 we get $107kNm$, which is much less than the collapse moment of $914kNm$. Which leads to the conclusion that the column won't collapse at this load, but it does come very close to this point. Since the column stops behaving linearly, the pressure does not need to be 9 times as large to cause full failure, but only the displacement has to become 9 times as large.

# 6 Conclusion and recommendations

From the model of the column as a mass spring system, a few things can be noted. Firstly the explosion has to be significant and close to the column for it to fail. Otherwise the column is stable and does not near failure. This effect can already be seen when the distance increases to half a meter. A relatively big bomb has to be used for multiple columns to fail, so the entire building is fairly resistant to mid-sized bombings. Only direct hits with 50 kg bombs can really cause the columns to form a mechanism, which is not even a full collapse.

It is also necessary to say that the plastic analysis is very important to the behaviour of the system. When looking at the differences between the models that did take changes of behaviour due to local failure and the models that did not, it can be seen that these fail earlier, from which can be concluded that local failures easily compound onto each other into complete failure.

## 6.1. future research

There are some glaring missing pieces from this research that should be investigated when it comes to future research into this topic.

Firstly the how the blast itself was modeled. The leading equation was one that only holds in open air. Nothing about it says anything about the interaction between the dynamic response of the ground, the rest of the building or how it might reflect inside of the building. It was all pretty straight forward. But since this report was mainly focused on the structural part and not really the blast dynamics, this part was beyond the scope of this project.

Another aspect that did not get a lot of cover in this project is the impact of second order effects. Because this is a column under pressure the entire system is prone to buckling and extra stresses from eccentric loading. Especially because the explosion causes the column to oscillate, those displacements cause extra moments in the column when looking at the compression in the column, which in turn cause extra displacement. A future model where the effects of these extra forces cause extra displacement can be very interesting.

It is also interesting to look at the moment that the column collapses and see what the rest of the building does in response. Will the structure in the ceiling be enough to still carry the weight of the building or will the absence of the column cause a chain reaction. It can also be that the collapse stops midway, because load gets distributed through the roof into other columns.

Then there finally there should also be a more detailed analysis on the column, maybe in a finite element analysis where the column gets divided into multiple smaller elements which interact with each other. This can be used to say a lot more about the local responses to the load, as the individual elements can only take so much stress. A lot more can be said about how the pressure wave clashes with the column and a much more detailed analysis can be made about for local failures like spalling of the concrete and failure on the interaction between the reinforcement steel and the concrete itself.

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# A
# The code

```
%matplotlib inline
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy import interpolate
```

```
#making a funciton to find the linear stiffness
def EI(H, B, Es, Ec, nw, phi_sw, phi_lw, c):
    EIzz = Ec * (1 / 12 * B * H ** 3) + Es * ((0.5 * H - c - phi_sw - 0.5 * phi_lw) ** 2 * (nw - 4) * (0.25 * phi_lw ** 2 * np.pi))
    EIyy = Ec * (1 / 12 * H * B ** 3) + Es * ((0.5 * B - c - phi_sw - 0.5 * phi_lw) ** 2 * nw * (0.25 * phi_lw ** 2 * np.pi))
    return EIzz, EIyy
```

```
    a, b = EI(900, 500, 200000, 10000, 12, 20, 32, 30)
print (a, b)
```

```
def forward_eu(k, m, zeta, P, t_end, dt, t0=0, tbomb=0.001, y_0=0, v0=0):
    n = (t_end - t0) / dt
    t = np.zeros(int(n+1))
    y_n = np.zeros(int(n+1))
    v = np.zeros(int(n+1))
    F = P / tbomb
    omega = np.sqrt(k/m)
    c = zeta * 2 * m * omega
    v[0] = v0
    y_n[0] = y_0
    t[0] = t0
    print(2 * np.pi / omega)
    print(omega)
    for i in range(int(n)):
        if t[i] <= tbomb:
            ft = F
        else:
            ft = 0
        v[i + 1] = v[i] + ((ft - k * y_n[i] - c * v[i]) / m) * dt
        y_n[i + 1] = y_n[i] + v[i] * dt
        t[i + 1] = t[i] + dt
    return y_n, t, n


#testing it out
x, t, n = forward_eu(100, 200, 0.02, -30000, 10, 0.00001)
print(x, t, n)
plt.plot(t, x)
```

```
#function to calculate design N force in column in kN
def descap(H, B, fck, gamma):
```

```
    N = fck * H * B / gamma
    return N
N = descap(900, 500, 19, 1.35)
print(N)

#function making a graph of the N-M interaction diagram

def NMdiag(H, B, philw, phisw, fyd, fck, nt, N, c=30, alpha=0.75, beta=0.38888, epsc3=1.75, epscu3=3.5, epsyd=2.175, Es=200):
    As = 0.25 * philw ** 2 * np.pi * nt
    a = c + phisw + 0.5 * philw
    xu0 = (As * fyd) / (alpha * H * fck)
    d = B - a
    Mp0 = As * fyd * (B/2 - a) + alpha * xu0 * H * fck * (B/2 - beta*xu0)
    N0 = 0
    xu1 = (epscu3/(epscu3 + epsyd)) * d
    Mp1 = As * fyd * (B / 2 - a) + alpha * xu0 * H * fck * (B / 2 - beta*xu1) + As * fyd * (B /2 - a)
    N1 = alpha * xu1 * B * fck
    xu2 = B
    epss = (d/B - 1) * epscu3
    Ns2 = epss * Es
    Mp2 =  As * fyd * (B / 2 - a) + Ns2 * (B / 2 - a) + alpha * B * H * fck * (B / 2 - beta * xu2)
    N2 = alpha * B * H * fck + As * fyd - Ns2
    Mp3 = 0
    N3 = B * H * fck + 2 * As * fyd
    plt.figure(figsize=(10,6))
    if N0 <= N < N1:
        mp = np.interp(N, [N0, N1], [Mp0, Mp1])
        #print(mp)
        #plt.plot(N, mp, 'ro')
    elif N1 <= N < N2:
        mp = np.interp(N, [N1, N2], [Mp1, Mp2])
        #print(mp)
        #plt.plot(N, mp, 'ro')
    elif N2 <= N < N3:
        mp = np.interp(N, [N2, N3], [Mp2, Mp3])
        #print(mp)
       # plt.plot(N, mp, 'ro')
    elif N > N3:
        mp = 0
        print('the column will crush before failing under the moment capacity')
    plt.plot([N0, N1, N2, N3], [Mp0, Mp1, Mp2, Mp3], label='N-M diagram')
    plt.plot(N, mp, 'ro', label='moment capacity due to Normal forces')
    #plt.plot(0, mp, 'go')
    plt.legend()
    plt.xlabel('Compression [N]')
    plt.ylabel('Plastic moment capacity [Nmm]')
    plt.title('N-M interaction diagram')
    return mp

mp = NMdiag(900, 500, 32, 10, 435, 19, 6, N)
print(mp)
```

```
def normcol(H, B, L, EI, ns, philw, rho_c=2400, rho_s=7850):
    leff = L * 0.7
    k = 384 * EI / (L ** 3)
    Ac = H * B
    As = 0.25 * philw ** 2 * np.pi * ns
    m = (rho_c * Ac * leff + rho_s * As * leff) / (10 ** 9)
    return k, m
k1, m1 = normcol(900, 500, 6410, b, 16, 32)
print(k1, m1)

w1, t1, n = forward_eu(k1, m1, 600, 100000, 20, 0.00005, tbomb=0.1)
print(w1, t1, n)
plt.plot(t1, w1)
```

```
def analytical(k1, k2, m):
    omega1 = np.sqrt(k1/m)
```

```
    T1 = np.pi * 2 / omega1
    omega2 = np.sqrt(k2/m)
    T2 = np.pi * 2 / omega2
    c1 = 0.025 * 2 * m * omega1
    c2 = 0.025 * 2 * m * omega2
    return omega1, T1, omega2, T2, c1, c2

o1, T1, o2, T2, c1, c2 = analytical(k1, k2, m1)
print(o1, o2, T1, T2, c1, c2)
print(k1, k2)
```

```
def plasticcol(Mp, l, EI):
    wy = Mp * l ** 2 / (32 * EI)
    wc = Mp * l ** 2 / (12 * EI)
    wend = wc * 5
    W1 = 12 * Mp / l
    W2 = 16 * Mp / l
    W3 = W2
    phi1 = - (4 * Mp * l) / (24 * EI)
    print(phi1)
    plt.plot([0, wy, wc, wend], [0, W1, W2, W3])
    return wy, wc, wend, W1, W2, W3

wy, wc, wend, Wy, Wc, Wend = plasticcol(mp, L, b)
print(wy, wc, Wy, Wc)
plt.plot(wc, Wc, 'go', label='hinge in the middle')
plt.plot(wy, Wy, 'ro', label='hinges on the sides')
plt.legend()
plt.xlabel('deflection (mm)')
plt.ylabel('Added force (N)')
```

```
def forward_eu2(k, m, zeta, P, t_end, dt, wlim, Wc, t0=0, tbomb=0.001, y_0=0, v0=0):
    n = (t_end - t0) / dt
    print(n)
    t = np.zeros(int(n+1))
    y_n = np.zeros(int(n+1))
    v = np.zeros(int(n+1))
    F = P / tbomb
    omega = np.sqrt(k/m)
    c = zeta * 2 * m * omega
    v[0] = v0
    y_n[0] = y_0
    t[0] = t0
    print(2 * np.pi / omega)
    for i in range(int(n)):
        if t[i] <= tbomb:
            ft = F
        else:
            ft = 0
        if y_n[i] > wlim:
            k1 = Wc / y_n[i]
        elif y_n[i] < -wlim:
            k1 = -Wc / y_n[i]
        else:
            k1 = k
        v[i + 1] = v[i] + ((ft - k1 * y_n[i] - c * v[i]) / m) * dt
        y_n[i + 1] = y_n[i] + v[i] * dt
        t[i + 1] = t[i] + dt
    return y_n, t, n
```

```
def forward_eu3(k, m, zeta, P, t_end, dt, wlim, k2, Wc, t0=0, tbomb=0.001, y_0=0, v0=0):
    n = (t_end - t0) / dt
    print(n)
    t = np.zeros(int(n+1))
    y_n = np.zeros(int(n+1))
    v = np.zeros(int(n+1))
    F = P / tbomb
    omega = np.sqrt(k/m)
    c = zeta * 2 * m * omega
    v[0] = v0
```

```
    y_n[0] = y_0
    t[0] = t0
    print(2 * np.pi / omega)
    for i in range(int(n)):
        if t[i] <= tbomb:
            ft = F
        else:
            ft = 0
        if y_n[i] > wlim:
            k1 = Wc / y_n[i]
        elif y_n[i] > wy:
            k1 = k2
        elif y_n[i] < -wlim:
            k1 = -Wc / y_n[i]
        elif y_n[i] < -wy:
            k1 = k2
        else:
            k1 = k
        v[i + 1] = v[i] + ((ft - k1 * y_n[i] - c * v[i]) / m) * dt
        y_n[i + 1] = y_n[i] + v[i] * dt
        t[i + 1] = t[i] + dt
    return y_n, t, n
```

```
#Define limits of the motion

wlim = np.ones(len(t1))
wlimcup = wlim * wc
wlimcdown = wlim * -wc
wlimyup = wlim * wy
wlimydown = wlim * -wy
```

```
w1, t1, n = forward_eu(k1, m1, 0, 710991, 20, 0.0001, tbomb=0.001)
#print(w1, t1, n)


plt.plot(t1[(w1 < wc) & (w1 > -wc)], w1[(w1 < wc) & (w1 > -wc)], 'b')
plt.plot(t1[(w1 > wc) | (w1 < -wc)], w1[(w1 > wc) | (w1 < -wc)], 'ro')
plt.plot(t1, wlimcup, 'r--')
plt.plot(t1, wlimcdown, 'r--')
plt.plot(t1, wlimyup, 'r--')
plt.plot(t1, wlimydown, 'r--')
plt.xlabel('time␣(s)')
plt.ylabel('displacement␣(mm)')
```

```
w1, t1, n = forward_eu2(k1, m1, 0, 710991, 20, 0.0001, wc, Wc, tbomb=0.001)
#print(w1, t1, n)

plt.plot(t1[(w1 < wc) & (w1 > -wc)], w1[(w1 < wc) & (w1 > -wc)], 'b')
plt.plot(t1[(w1 > wc) | (w1 < -wc)], w1[(w1 > wc) | (w1 < -wc)], 'ro')
plt.plot(t1, wlimcup, 'r--')
plt.plot(t1, wlimcdown, 'r--')
plt.plot(t1, wlimyup, 'r--')
plt.plot(t1, wlimydown, 'r--')
plt.xlabel('time␣(s)')
plt.ylabel('displacement␣(mm)')
```

```
w1, t1, n = forward_eu3(k1, m1, 0, 710991, 20, 0.0001, wc, k2, Wc, tbomb=0.001)
#print(w1, t1, n)

plt.plot(t1[(w1 < wc) & (w1 > -wc)], w1[(w1 < wc) & (w1 > -wc)], 'b')
plt.plot(t1[(w1 > wc) | (w1 < -wc)], w1[(w1 > wc) | (w1 < -wc)], 'ro')
plt.plot(t1, wlimcup, 'r--')
plt.plot(t1, wlimcdown, 'r--')
plt.plot(t1, wlimyup, 'r--')
plt.plot(t1, wlimydown, 'r--')
plt.xlabel('time␣(s)')
plt.ylabel('displacement␣(mm)')
```

Important to not is that these last few cells were used to make the graphs, they therefore have been run with other parameters, but putting all that in the appendix would not add anything.