Concentrated loads on anticlastic shells

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Preface

In this Bachelor's thesis, formulas have been derived for the deflection, normal forces and bending moments right underneath a perpendicular point load on an anticlastic shell.

This study covers the modelling of an anticlastic shell in Ansys and the derivation of the formulas mentioned above. Different shell types and their structural behaviour are presented in chapter 2. The process of finding a suitable mesh and optimizing it, is described in chapter 3. The derivation of the formulas and their limitations are described in chapters 5 to 7. Finally, the conclusions and recommendations are presented in chapter 8.

Hereby, I would like to thank my supervisors dr.ir. P.C.J. Hoogenboom and dr.ir. F.P. van der Meer for their valuable insights and willingness to help.

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Nathalie Ramos

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List of symbols

P	Concentrated load	[N]
E	Young's modulus	[N/mm^2]
k_x	Curvature in x-direction	[mm^{-1}]
k_y	Curvature in y-direction	[mm^{-1}]
l_x	Length shell in x-direction	[mm]
l_y	Length shell in y-direction	[mm]
n_x	Number of elements in x-direction	[-]
n_y	Number of elements in y-direction	[-]
h	Element size in middle strips	[mm]
v (nu)	Poisson's ratio	[-]
d	Diameter circular surface of loading	[mm]
t	Thickness shell	[mm]
w	Deformation/displacement in z-direction	[mm]
n_{xx}	Normal force in x-direction	[N/mm]
n_{yy}	Normal force in y-direction	[N/mm]
m_{xx}	Bending moment about the x-axis	[N]
m_{yy}	Bending moment about the y-axis	[N]

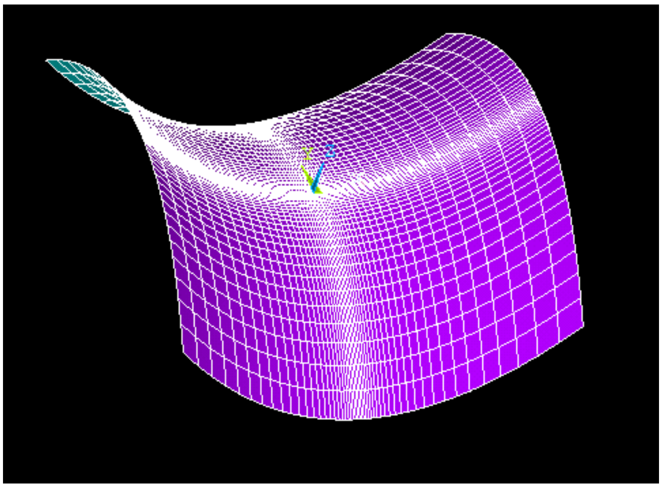


Figure I Saddle with fine middle strips

Abstract

In this study the influence of a perpendicular working point load on the deformation, normal forces and bending moments in an anticlastic shell is investigated. To do so, the finite element package Ansys is used.

The modelling in Ansys starts with defining a model. As the geometry of the anticlastic shell is already chosen, it is important to find an optimal mesh which yields accurate results. A rectangular mesh is applied since it turned out that a radial mesh is not suited to model saddle-shaped shells [2]. In the middle of the shell, strips with finer elements are used. The point load is distributed over a circular surface to prevent bending moments and shear forces approaching infinity.

When the mesh is optimized, the influence of a perpendicular working point load on the deformation, normal forces and bending moments can be expressed in the formulas depicted below.

Deformation (chapter 5)

$$w \approx 0.92 \frac{P}{Et^2} \sqrt{-\frac{1}{k_x k_y}} \sqrt{1 - 0.5v^2} \; for \; l_x = l_y \geq 20 \; m \; (\mathbf{5.14})$$

This formula has an accuracy of 4.97% when the following limitations are satisfied:

$$\left|\frac{k_{min}}{k_{max}}\right| < 4.5 \text{ and } t|k_{max}| < 0.06$$

Normal forces (chapter 6)

$$n_{xx} \approx -0.13 \frac{P}{t} k_x \sqrt{-\frac{1}{k_x k_y}} (1 - 0.45v^2) \; for \; l_x = l_y \geq 3m \quad (\mathbf{6.13})$$

$$n_{yy} \approx 0.13 \frac{P}{t} k_y \sqrt{-\frac{1}{k_x k_y}} (1 - 0.45v^2) \; for \; l_x = l_y \geq 3m \quad (\mathbf{6.14})$$

The formula for the normal force in the x-direction has an accuracy of 9.87% when the following limitations are respected:

$$-\frac{k_x}{k_y} \geq 0.5 \text{ and } t|k_{min}| < 0.02$$

The formula for the normal force in the y-direction has an accuracy of 14.59% provided that:

$$-\frac{k_y}{k_x} < 6 \text{ and } t|k_{min}| < 0.02$$

Bending moments (chapter 7)

$$m_{xx} = 0.163\frac{P}{\sqrt[4]{d}}t^{0.15}\left(\frac{1}{k_y}\right)^{0.1}(1+1.05v)\ for\ l_x = l_y \geq 4\,m \quad (\mathbf{7.22})$$

$$m_{yy} = 0.163\frac{P}{\sqrt[4]{d}}t^{0.15}\left(\frac{1}{k_x}\right)^{0.1}(1+1.05v)\ for\ l_x = l_y \geq 4\,m \quad (\mathbf{7.23})$$

The formula for m_{xx} has an accuracy of 4.7% provided that $tk_{min} > 0.0025$.

The formula for m_{yy} has an accuracy of 5.27% when the following requirements are met:

$$k_y < \frac{1}{500}mm^{-1}\ and\ tk_x > 0.002$$

It is conspicious that the formulas for the deformation and the bending moments are at least twice as accurate than the formulas for the normal forces. This might be due to the influence of the curvatures which has been not been taken into account accurately.

An important conclusion that can be drawn is that the stresses in anticlastic shells are not carried in the directions of zero curvature, as A. Semiary presumed [2]. Tensile stresses are carried in the direction of negative curvature (positive z-axis pointed downwards) and compressive stresses are carried in the direction of positive curvature (figure 4-3).

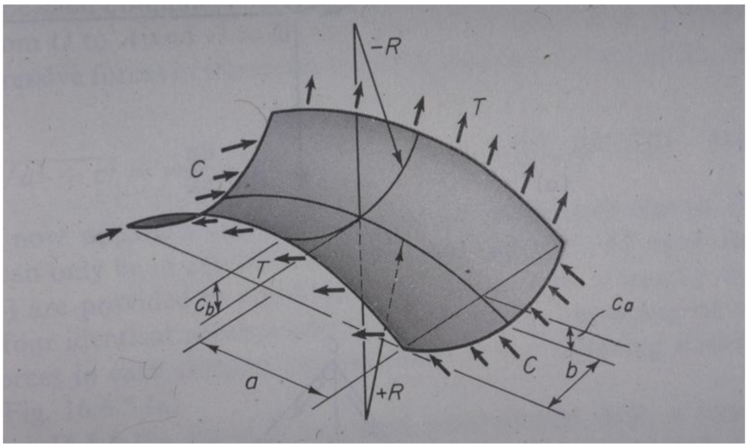


Figure 4-3 Tension and compression in an anticlastic shell

As the results generated in Ansys are compared to the solutions of the Sanders-Koiter equations [16], the observation is made that the deformations and normal forces generated in Ansys correspond to the results of the differential equations. However, when comparing the bending moments, large deviations are observed. The reason for these differences is yet unknown.

Chapter 1 Introduction

1.1 Background

Over the past years shell structures have been taking up a more prominent role in modern architecture. The range of use of shells is broad and it does not limit itself to civil engineering. Mechanical engineering, aerospace engineering are all fields of application. Even though man –made shell structures have been in existence for many years, one can think of the many domes built during the Renaissance or even during the 'Roman era', we can see a revival of interest in shell structures. One could wonder what it is that makes shell structures appealing. Cost factors, materials availability and labour supply all play a part [1]. The generally high strength-to-weight ratio of the shell form, combined with its inherent stiffness, has formed the basis of modern applications of shell structures [1].

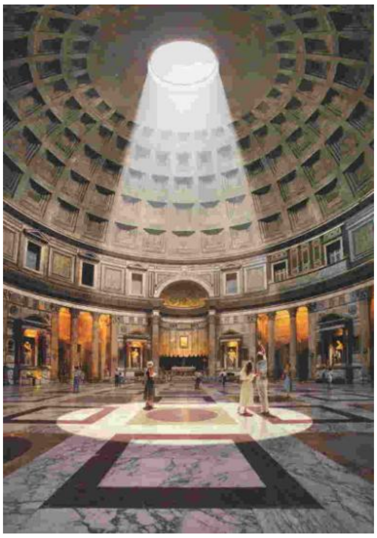


Figure 1-1 Left: L'hemisferic by Satiago Calatrava, right: The Pantheon in Rome (http://tx.english-ch.com/teacher/albert/others/tourist-attractions-in-spain/ http://theproegers.com/2012/02/do-you-have-a-pantheon/)

1.2 Purpose and scope of the thesis

When making a structural model of shell structures, finite element methods ought to be applied. While it is a very accurate numerical method, disadvantages are that the results often don't give good insight in the behaviour of the structure. Furthermore, when modelling a structure with many elements, much calculation time can be required to display results. In early stages of the design process, it can be preferable to make use of formulae to calculate stresses and displacements in shell structures.

Formulae for the displacements of domes and cylindrical shell structures have already been derived by A. Semiary [2] and respectively G. Van Bolderen [3]. Semiari also took a look at hyperbolic paraboloids, also known as saddles. He made an important observation. When saddles are loaded by a concentrated load, the biggest stresses are carried in the directions of zero curvature. This is remarkable since these are the least stiff directions. The phenomenon gave rise to the presumption that stresses in anticlastic shells are carried along the characteristics of the hyperbolic partial differential equation. The wave equation is an example of such an equation.

In this Bachelor's thesis, the goal is to express the influence of a concentrated load on the deflection of an anticlastic shell, the normal force and the moment in an anticlastic shell in analytical formulae. At the same time, the so far undissolved phenomenon of the characteristics will be analysed.

1.3 Research questions

Main question

How does a perpendicular working concentrated load influence the deformation, normal forces and bending moments in an anticlastic shell and how can this influence be expressed in analytical formulae?

Sub-questions

1. *Which type of shell structures are there and what mechanical properties do they have.*
2. *What is the relation between a perpendicular working concentrated load and the displacement, the curvatures, the thickness and the Poisson's ratio of a shell.*
3. *What is the relation between a perpendicular working concentrated load and the normal force, the curvatures, the thickness and the Poisson's ratio of a shell.*
4. *What is the relation between a perpendicular working concentrated load and the bending moment, the curvatures, the thickness and the Poisson's ratio of a shell.*
5. *Does a relation exist between the propagation of the stresses in an anticlastic shell and the characteristics of the hyperbolic partial differential equations.*

Chapter 2 Shell structures

2.1 Classification of shells

There are several ways to classify shell structures. Shells can be divided into different categories by making a distinction between the materials applied, for instance reinforced concrete, steel or wood. Each material has different properties which can determine the shape of the structure. The second criterion is the shell thickness. Shells can be divided into thick and thin shells. A shell is classified as thin when the thickness of the shell is negligible compared to the radius of curvature [4]. Last, the Gaussian curvature can be used to classify the shape of the shell surface. The Gaussian curvature is defined as the product of the two principal curvatures at a given point of the curved surface. In figure 2-1 the various types of shell surfaces are depicted.

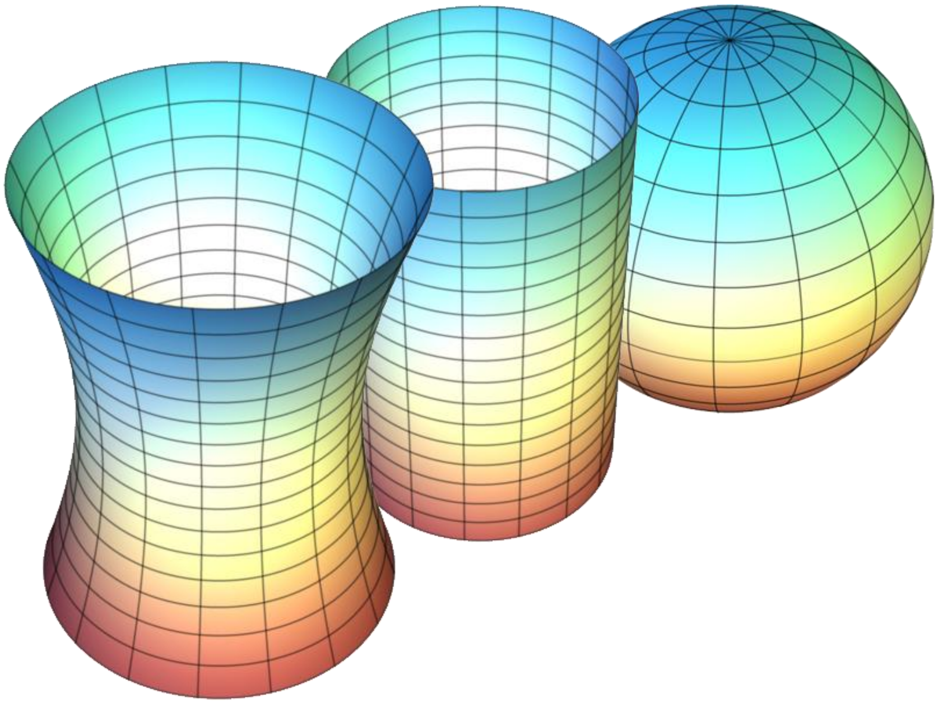


Figure 2-1 Shell surfaces. From left to right: Anticlastic surface, Non-developable surface, Synclastic surface [http://www.peacham.com/italy/capri.htm]

Synclastic shells (concave or convex curves) have positive Gaussian curvature. Anticlastic surfaces have negative Gaussian curvature . The surface is often saddle-shaped. Shell structures can also have developable surfaces. These are characterized by zero Gaussian curvature. Two examples of such surfaces are the cone and the cylinder.

2.2 Structural properties of shells

The main difference between shells compared to plates and beams is that shells possess both surface and curvature providing both strength and stiffness. The beam resists any applied transverse loading by bending, unlike the arch which resists such a loading primarily by internal thrust action. The plate, which can be idealized as a flat surface, has to bend to carry applied transverse loading. Shells, which can be idealized as curved surfaces, are able to resist transverse loading through the action of internal forces in the tangential plane of the shell's midsurface, i.e. in-plane actions. From this, it is evident that the property that makes both shells and arches capable of resisting external transverse loading by extensional action is curvature [5]

Thin shells are designed with the purpose to resist loads through membrane forces, i.e. tensile and compressive forces [6]. In practice usually a mixture of both membrane forces and flexural action (i.e. bending and shear forces) occurs. Good design consists of minimizing the flexural actions.
For shells, the relative proportions of extensional and flexural effects at a given point depend on several factors, including the type of shell surface [7].

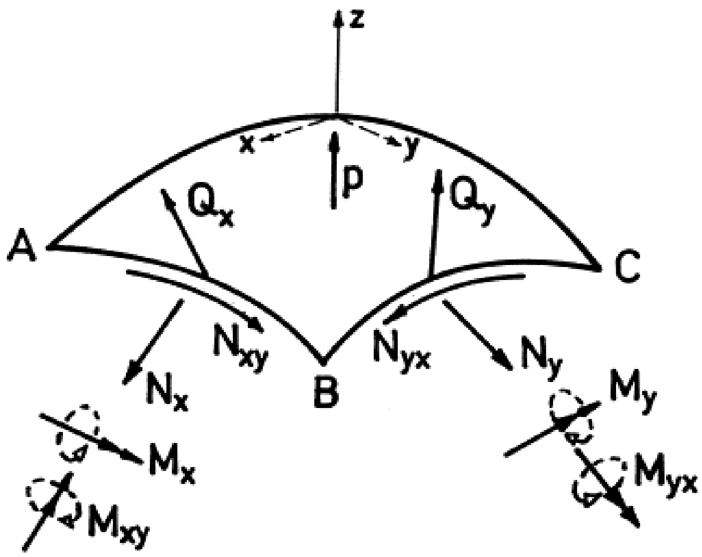


Figure 2-2 Infinitesimal shell element under in-plane and bending actions [http://www.sciencedirect.com/science/article/pii/S0141029697002277]

Shell surfaces can be divided into edges and interior regions. At the edges transverse shearing forces, bending moments and twisting moments are, in general, required in order to satisfy the boundary conditions. These locations include external supports, discontinuities in loading or shell geometry, and concentrated line loadings. The interior regions are essentially free from the so-called edge effects associated with localized bending, although, in general, there may still be subject to some relatively small bending actions [8].

Distinction has been made between developable en non-developable surfaces by considering their geometric characteristics solely. Structural capabilities must be taken into consideration too when selecting forms [9]. Synclastic and anticlastic surfaces that are supported at all edges have an inherent quality that stiffens them, since they cannot change shape freely. Not being developable , they have high resistance against deformations produced by loads.

Since the goal is to find the relation between a concentrated load and in-plane and bending actions in the middle of the shell surface of anticlastic shells, the main focus will be solely on the modelling of anticlastic shells. Formulae of the relation between a concentrated load and the deflection on the shell surface of synclastic shell surfaces and developable shell surfaces have already been derived [2].

2.3 Geometry of anticlastic shells

The hyperbolic paraboloid or hypar, which will be the centre of attention, is just one out of many quadric surfaces. The hypar has the distinctive property that it envelops a point at its center which appears to be a maximum point in one plane, a minimum point in another plane, and actually is neither. This point is called a saddle-point [10]. Mathematically a saddle can be depicted by the following formula:

$$f(x, y) = x^2 - y^2 \quad \mathbf{(2.1)}$$

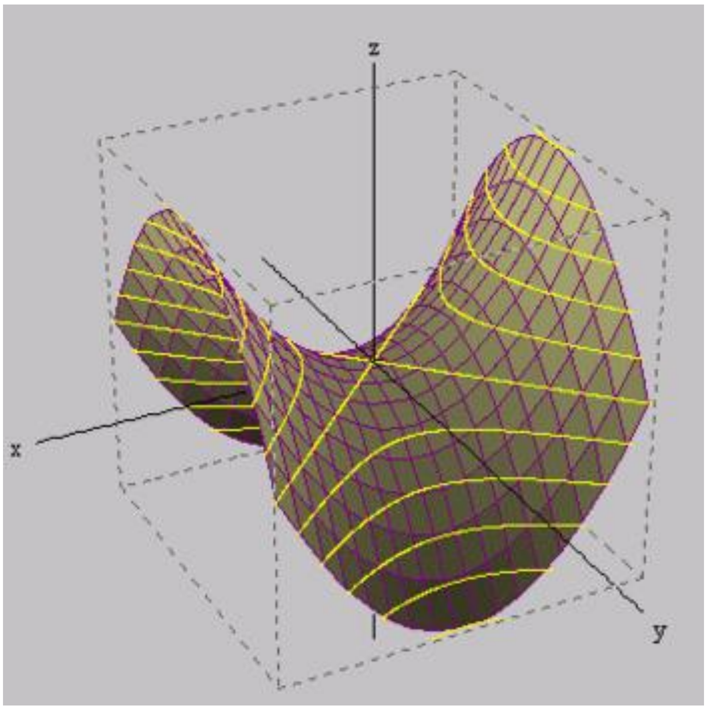


Figure 2-3 Saddle (http://www.peacham.com/italy/capri.htm)

In this study a shell with the geometry from equation (2.1) will be analysed with the finite element package Ansys. The geometry will be generated by using a script as depicted in appendix A. The script is written in APDL (Ansys Parametric Design Language) code which makes it possible to parametrize the model and automate common tasks [11].

Chapter 3 Mesh refinement

3.1 Mesh type and elements

The modelling in Ansys starts with defining the model. When defining a model, it is critical to define an optimal mesh. In finite elements analysis, small elements are used due to their accuracy. But in advance it is not certain if the elements are sufficiently small. Therefore, the element size must be optimized. When reducing the element size one must take into account more computation time and even more computer memory to perform the calculation [12]. Consensus must be found between the computation time and sufficient accuracy.

Semiari had already tried to model saddles by applying a radial mesh consisting of rings with triangles and rings with four node quadrilaterals. This mesh did not turn out to be as accurate as predicted. While a radial mesh is perfectly suited to predict the behaviour of a dome loaded by a concentrated load, it is not suited to model a saddle. The idea is to apply a mesh as depicted in figure 3-1.

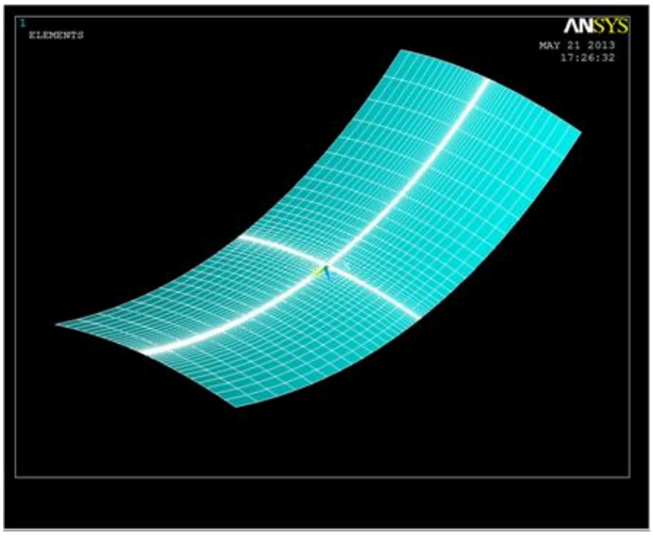


Figure 3-1 Mesh Ansys

The elements used are shell181 elements as depicted in figure 3-2. These elements are suitable for analysing thin to moderately-thick shell structures [13]. The elements have eight nodes and each node has six degrees of freedom: translations in the x, y and z direction, and rotations about the x, y, and z axes. After having determined the optimal number of elements and the distribution of these elements in the mesh (the middle strips contain a finer mesh), the model is used to generate results, i.e. deformations and stresses due to the concentrated load. The calculations performed are linear-elastic.

It is preferable to eliminate the influences of the edges. Therefore the shell size is chosen in such a way that the edge effects are not present in the middle of the shell. Therefore the influence of the supports will be eliminated too. The supports applied are hinged supports.

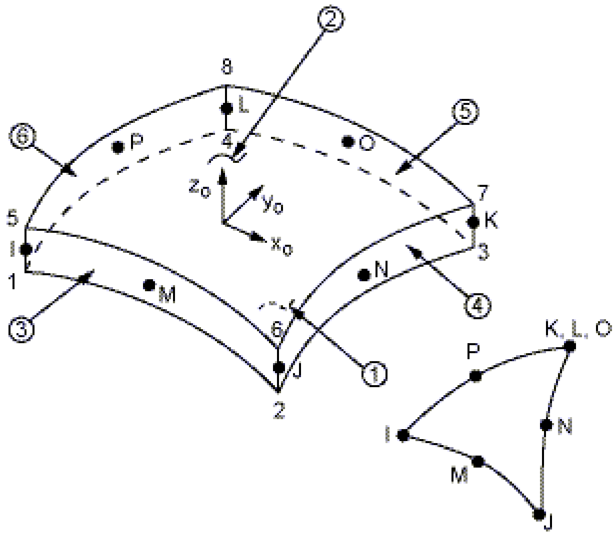


Figure 3-2 Shell281 elements [http://ans2.vm.stuba.sk/html/elem_55/chapter4/ES4-181.htm]

3.2 Mesh optimization: Concentrated load

Goal is to generate a mesh with very fine elements in the middle strips. This is done in order to find accurate values of the stresses in the shell. If the entire mesh would contain very fine elements, this would require too much calculation time. Since the deformation and stresses will problably be concentrated around the location of loading, it is more efficient to apply the finer elements in those middle strips of the shell.

The concentrated load must work upon a small surface on the shell. When the pointload is working on one point, this will lead to infinite moments and shear forces right underneath the concentrated load. Therefore the load must be distributed over a small surface and the mesh around the concentrated load ought to be very fine.

When defining a mesh, it is important to generate a mesh which is accurate enough and does not require too much calculation time. In order to find out the optimal number of elements in the mesh, the geometry of the shell and the material constants are kept fixed while the number of elements is varied (the element size in the middle strips will be kept constant too). While nx is varied, its influence on the deformation is monitored. As the number of elements increases, the deformation should converge.

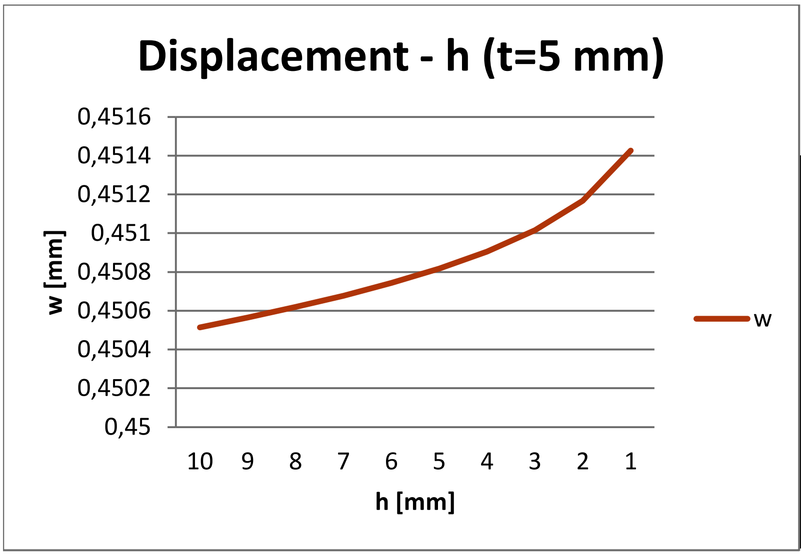
In table 3-1, the number of elements is depicted against the calculation time in Ansys. An important conclusion that can be drawn from these results, is that the elements outside the middle strips have a very small influence on the deformation. Therefore, it is convenient to choose that number of elements which generates the least calculation time considering the available time span.

kx=1/4000
ky=-1/4000
kxy=0
E=2,1e5
P=1000
v=0
h=5
lx=1000
ly=2000

nx	*ny*	*w*	*Δw [%]*	*t [min]*
50	150	0,450817		1
100	150	0,450815	0,000444	6
150	150	0,450816	0,000222	20
200	150	0,450816	0	28
250	150	0,450816	0	40

Table 3-1 Influence number of elements on deformation

Until now, the element size in the middle strips has been kept constant. When varying the fine element size h, a unforeseen observation is made.



kx=1/4000
ky=-1/4000
kxy=0
E=2,1e5
P=1000
v=0
h=5
lx=1000
ly=2000

Figure 3-3 Influence of element size h on deflection w of shell with thickness 5 mm

The deformation does not converge when decreasing the element size of h. This is remarkable since a more accurate mesh is generated when the element size is decreased. This should cause the deformation to converge.

The same trend as seen in figure 3-3 is detected when the thickness of the shell is varied or the size of the shell is modified. In figures 3-3 and 3-4 the influence of the element size h on the deformation of two shells with different thickness is displayed.

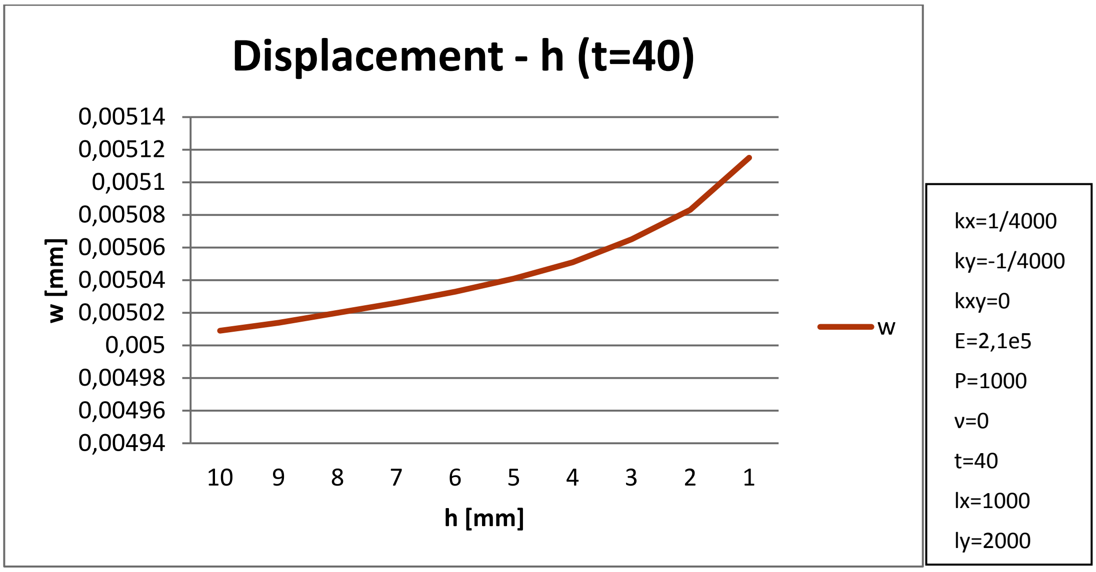


Figure 3-4 Influence of element size h on deflection w of shell with thickness 40 mm

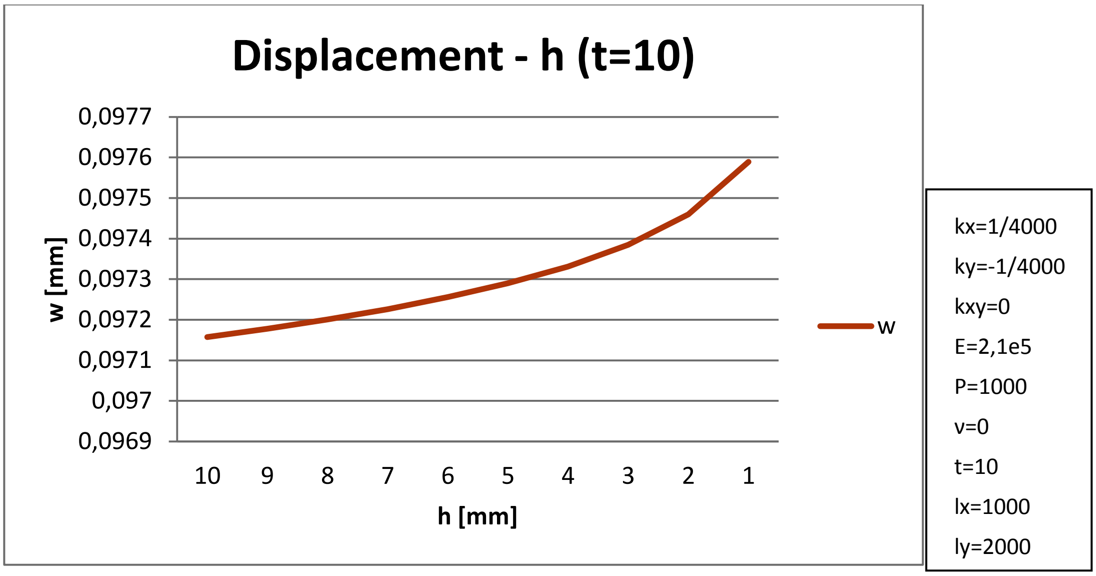


Figure 3-5 Influence of element size h on deflection w of shell with thickness 10 mm

From these figures it can be concluded that the lack of convergence of the deformation is not due to the thickness nor size of the shell.

When applying a thicker shell another observation is made. The deformations might be smaller, but the relative deviations of the deformation are bigger. It is probable that this phenomenon occurs due to the Poisson's ratio which has been equated to zero. The elements which have been used allow for shear-deformation (figure 3-6). Usually when thin shells are modelled, the Kirchhoff-Love theory is valid [14]. In this theory shear strains and thus shear stresses are negligible. When the thickness of the shell increases, the Mindlin-Reissner theory is applied [15]. This theory deals with thick plates or shells, in which shear stresses across the thickness of the shell are not negligible.

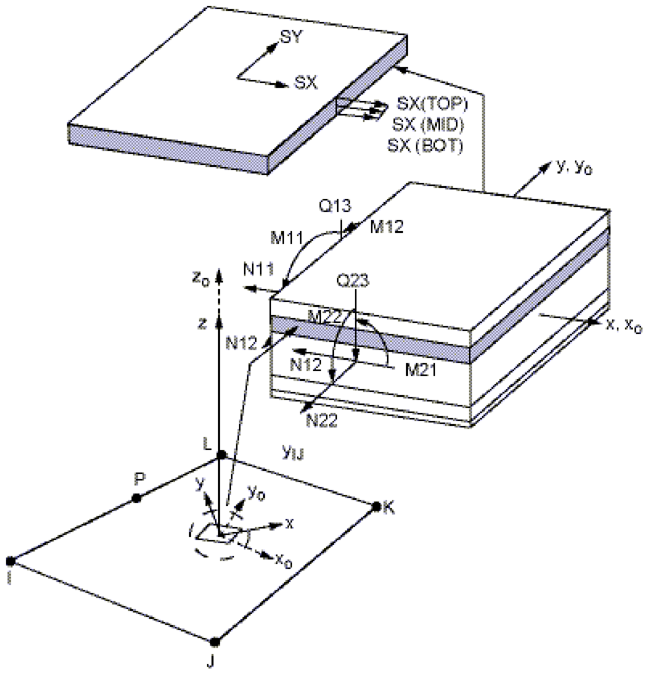


Figure 3-6 Shear stresses in Shell281 elements [http://www.sharcnet.ca/Software/Fluent13/help/ans_elem/Hlp_E_SHELL281.html]

It is important to point out that until now, it has been assumed that loading the shell with a concentrated pointload in one point would lead to finite deformations. This is true, since the deformation measured at an element size h of 0.1 mm is still finite. However, not distributing the concentrated load over a small area could lead to the deformation not converging.

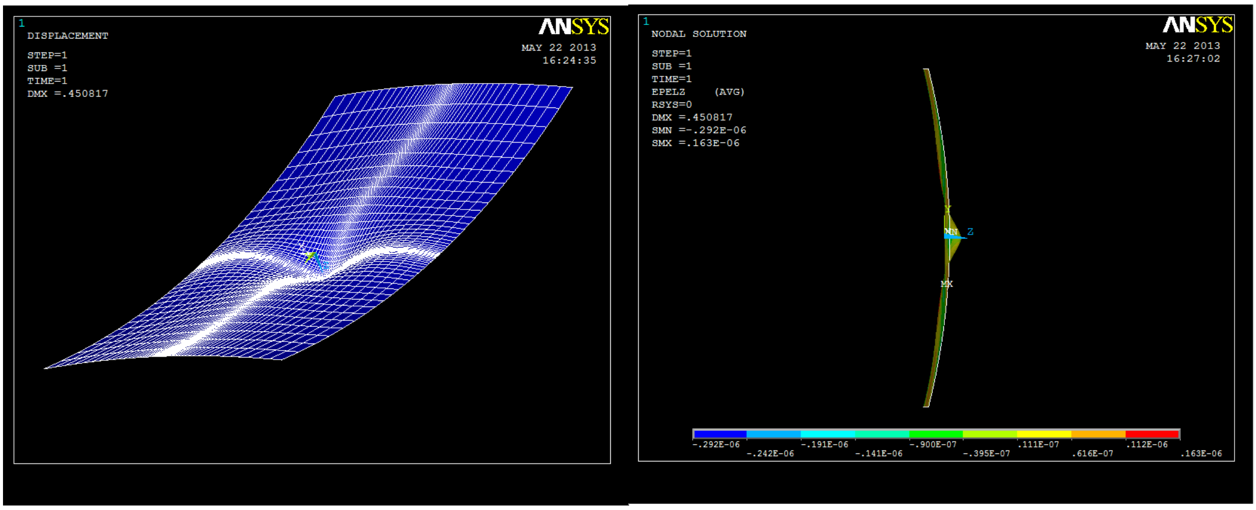


Figure 3-7 Left: deformed shell, right: side view of deformed shell

3.3 Optimization mesh: Distributed pointload

The concentrated load is distributed over a circular surface with diameter d. The diameter of the surface is coupled to the element size h in the middle strips. This means that the diameter of the surface can only be chosen as a multitude of h. This is illustrated in figure 3-8.

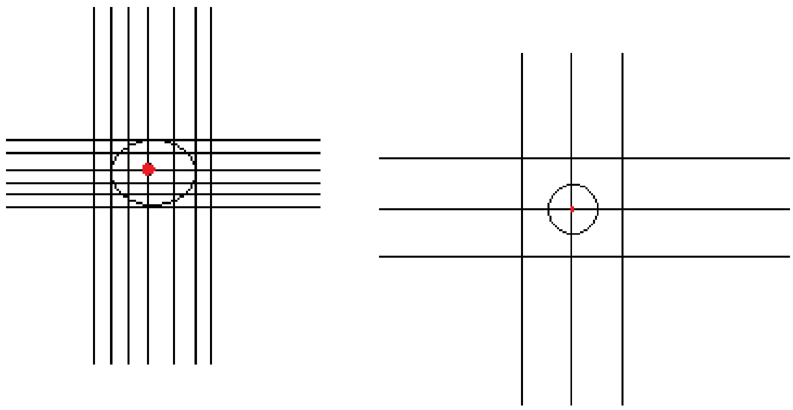


Figure 3-8 Left: d=4h, right: d=h

The shell is loaded in the centre of the circular surface. Again, the value of the Poisson's ratio is equal to zero. The measurements of the shell will be kept constant. Since it was seen that the deviations in the deformation were quite different when using thin or thick shells, calculations will be performed at different shell thicknesses.

At first the diameter d is kept constant. The ratio d/h increases which means that the element size h decreases. This is done to see what the influence is of the ratio d/h on the deformation.

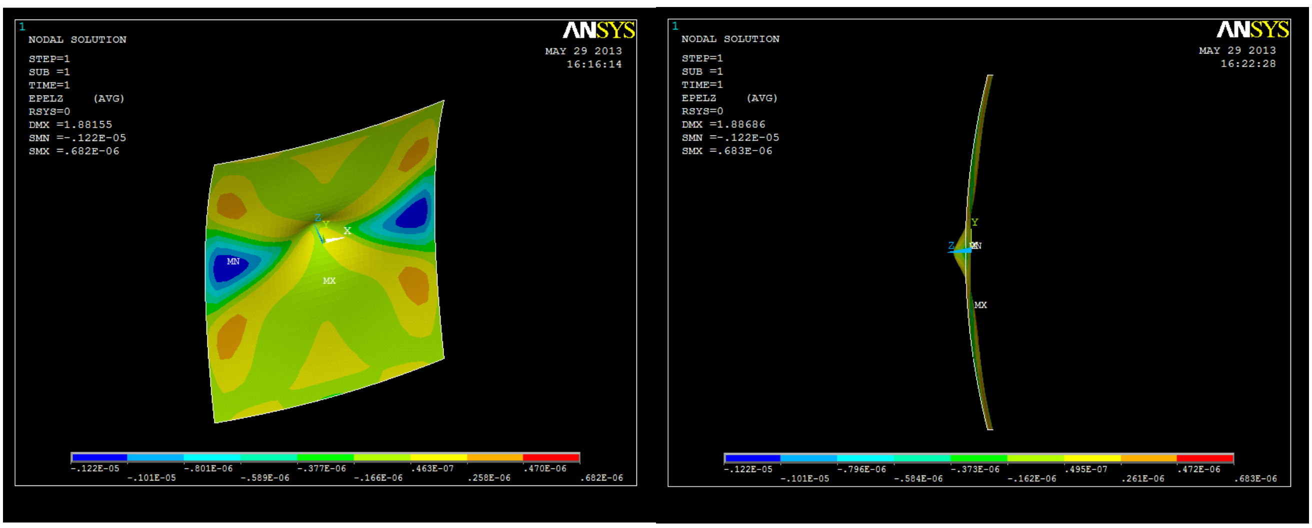


Figure 3-9 Deformation; the colours represent the intensity of the strain in the z-direction

d	*h*	*d/h*	*w*
10	10	1	0,44946268
10	5	2	0,44915086
10	2,5	4	0,4490332
10	1,25	8	0,4490105
9	9	1	0,44960691
9	4,5	2	0,44933643
9	2,25	4	0,44923272
9	1,125	8	0,44921437
8	8	1	0,44974702
8	4	2	0,44951477
8	2	4	0,44942505
8	1	8	0,44940842
6	6	1	0,45001918
6	3	2	0,44985342
6	1,5	4	0,44978819
6	0,75	8	0,44977391

lx=1000

ly=2000

kx=1/4000

ky=-1/4000

t=5

Table 3-2 Influence of d on deformation

In table 3-2 the influence of d on the deformation is depicted. It's obvious that the deformation converges to one value. To amplify this, some results of table are plotted in figure 3-10. Other results can be found in appendix C.

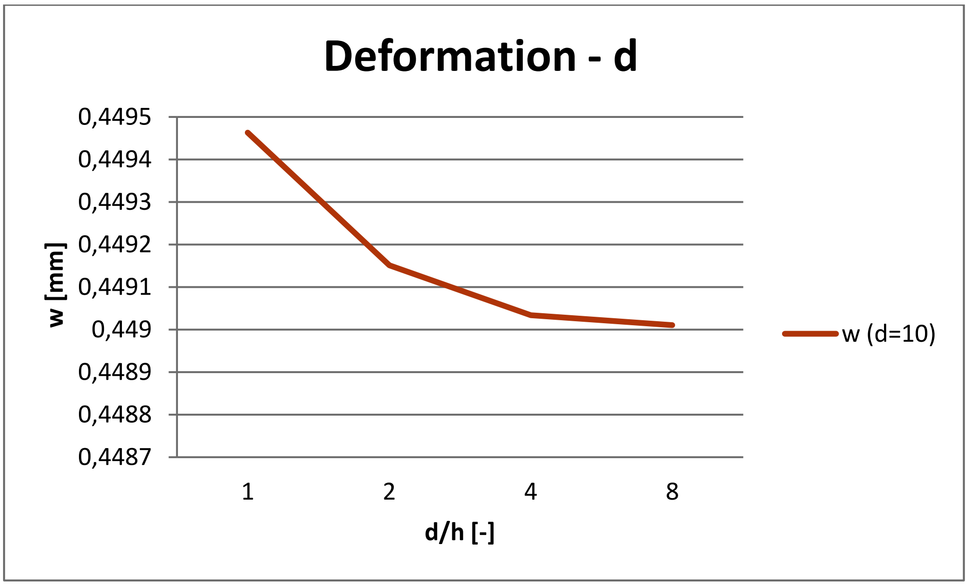


Figure 3-10 Influence of d on deformation

With a model which is converging, it is possible to choose a mesh which will be used to derive the formulas for the deformation, normal force and bending moments. The following mesh will be applied:

- 50 elements in both the x-direction and y-direction

- The ratio d/h is equal to 4. At this ratio the calculation time is acceptable (< 1 min).
- The diameter of the circular area is equal to 10 mm. This means that elements in the middle strips have a size of 2.5 mm.

3.4 Differential equation synclastic shells

There are no analytical solutions for the deflection, normal forces and bending moments under a point load working on an anticlastic shell. The Sanders-Koiter equations for a spherical cap loaded by a perpendicular working point load have been solved by Eric Reissner in 1946 [16]. The solutions of the differential equations are:

- Deflection $$u_z = \frac{\sqrt{3}}{4}\frac{Pa}{Et^2}\sqrt{1-\nu^2} \quad \textbf{(3.1)}$$
- Membrane forces $$n_1 = n_2 = -\frac{\sqrt{3}}{8}\frac{P}{t}\sqrt{1-\nu^2} \quad \textbf{(3.2)}$$
- Bending moments $$m_1 = m_2 = \frac{1}{4\sqrt[4]{12}}\frac{P\sqrt{at}}{d}\frac{(1+\nu)}{\sqrt[4]{1-\nu^2}} \quad \textbf{(3.3)}$$

Even though these results have been derived for synclastic shells, they can be used to verify the results generated by the Ansys script. Right underneath the point load, the magnitude of the deflection, membrane forces and bending moments for both synclastic and anticlastic shells are comparable.

Ansys								
t	**kx**	**ky**	**d**	**w**	**nxx**	**nyy**	**mxx**	**myy**
5	1/4000	1/4000	10	0,3310039	-43,1821	-43,1821	270,1442	268,7718
30	1/500	1/500	10	0,0019115	-7,23725	-7,24273	256,0279	254,1010
45	1/6000	1/6000	10	0,0063697	-4,81866	-4,81910	373,9243	371,9759

Table 3-3 Results generated with Ansys

Solutions differential equations						
t	**kx**	**ky**	**d**	**uz**	**n1=n2**	**m1=m2**
5	1/4000	-1/4000	10	0,329914	-43,3013	1899,589
30	1/500	-1/500	10	0,001146	-7,21688	1645,093
45	1/6000	-1/6000	10	0,00611	-4,81125	6979,536

Table 3-4 Results differential equations

When comparing the bending moments from tables 3-3 and 3-4, it is striking to see the major differences between the results from Ansys and the results of equation 3.3. Since the deformations and normal forces from Ansys do correspond to those generated by the formulas, it could mean that either the solutions of the differential equations for the bending moments are not correct or that the script is not modelling the behaviour of anticlastic shells properly. Unfortunately, investigation of the nature of these large deviations exceeds the scope of this thesis.

Chapter 4 Stresses in anticlastic shells

Semiari observed that the stresses in anticlastic shells were carried in the directions of zero curvature of the shells. This is quite remarkable since these directions are the least stiff directions of the shell. This load transfer could not be explained. It did lead to the presumption that the stresses in anticlastic shells propagate along the characteristics of the hyperbolic partial differential equation. This behaviour can be either confirmed or dismissed by looking at the stresses in the shell.

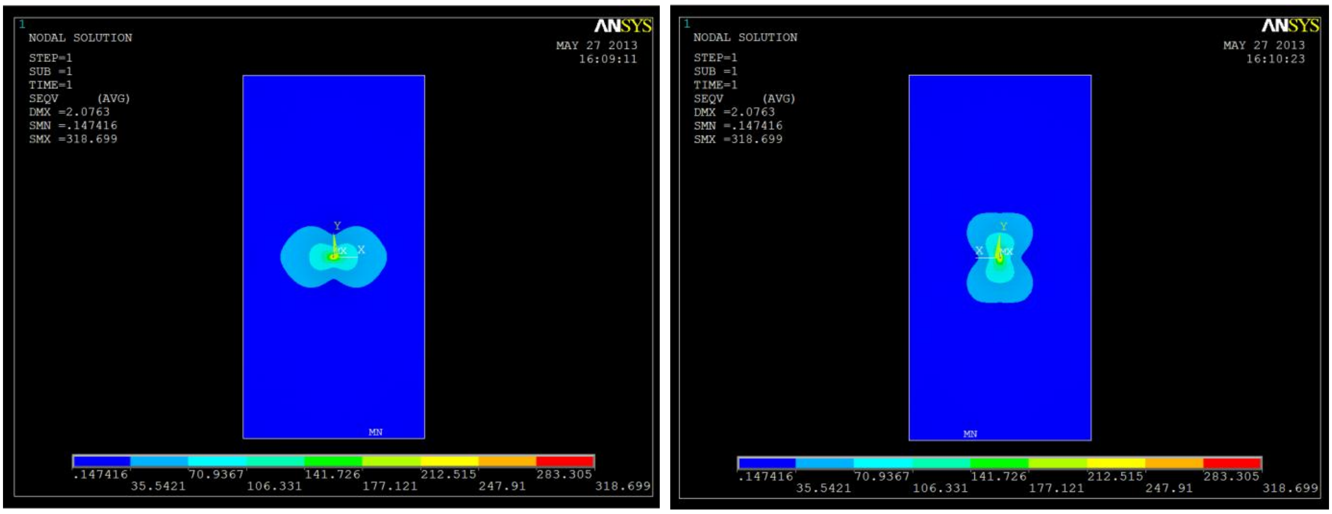


Figure 4-1 Von Mises stresses in shell with a thickness of 5 mm. Left: front view, right: back view

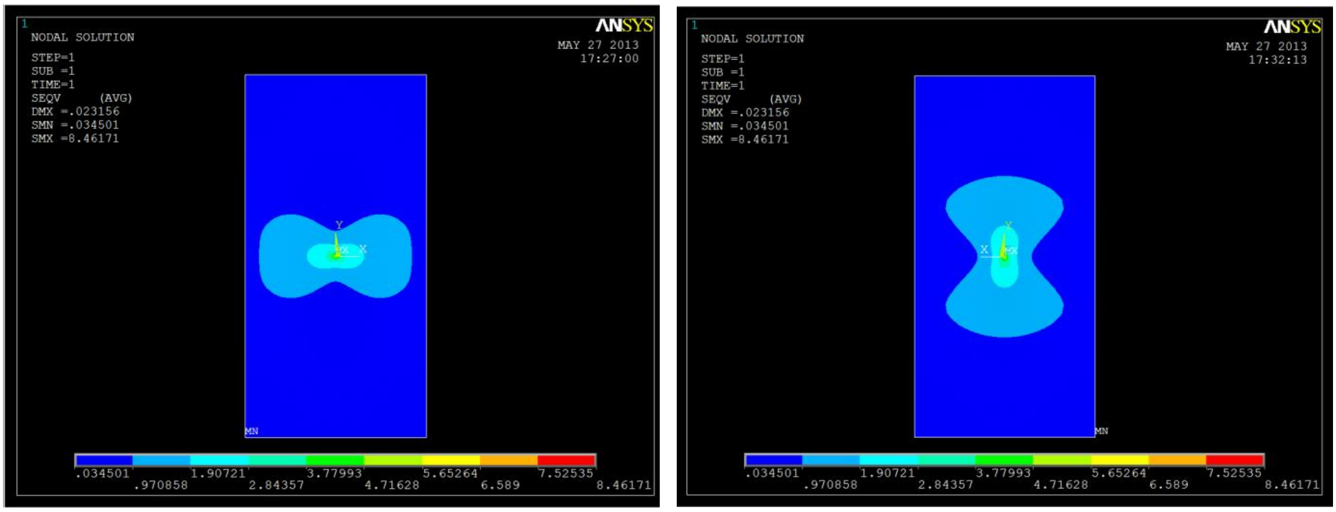


Figure 4-2 Von Mises stresses in shell with a thickness of 40 mm. Left: front view, right: back view

As can be concluded from the figures above, the stresses are not carried in the directions of zero curvature. The Von Mises stresses are concentrated in an eight-shaped surface around the surface of loading. This shape is not surprising. Due to the concentrated load, the shell is subjected to tension in one direction and compression in the other as can be seen in figure 4-3. In the top layers of the shell, tensile stresses are dominant. This causes the compressive stresses to be compensated for and the tensile stresses to be amplified. In the bottom layers the exact opposite occurs, this leads to the stress profile having the same shape but with a 90 degree turn.

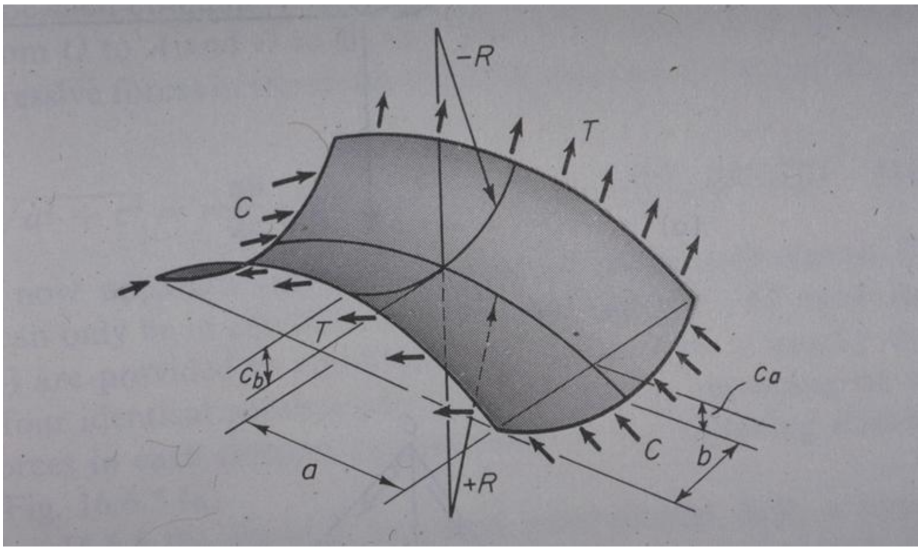


Figure 4-3 Compression and tension in an anticlastic shell [http://www.columbia.edu/cu/gsapp/BT/BSI/SHELL_MS/shell_ms.html]

In both figures 4-1 and 4-2, the Poisson's ratio has not been taken into account by equating it to zero. Since v has a small influence on the deformations and stresses, one might expect the stress profile to remain the same (with only the stress values slightly changing). However, when v is given a value, the stress profile does change a bit as can be seen in figure 4-4.

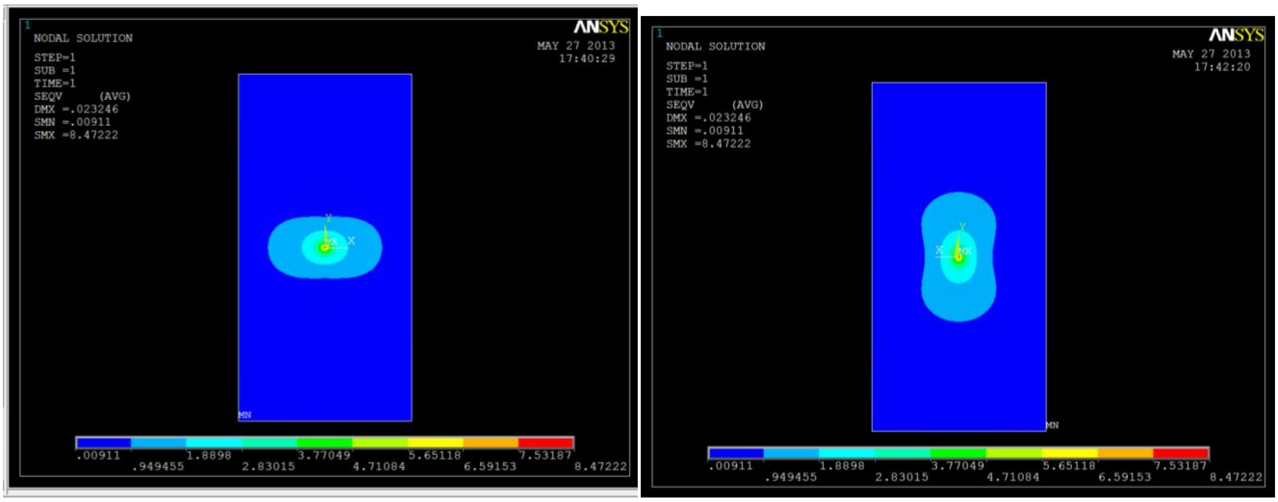


Figure 4-4 Stress profile. Left: front view, right: back view

The stress profile is not eight-shaped, but elliptically shaped. All other parameters except for v in figures 4-4 and 4-1 are identical. The exact influence of the Poisson's ratio on the stresses in anticlastic shells is yet to be determined. When the formulas for the normal force and bending moments are derived, it might be possible to clarify this observation.

Chapter 5 Formula deformation

5.1 Influence of P, E and l on the deformation

In this chapter a formula for the deformation will be derived. At first the Poisson's ratio will be equated to zero, since its contribution to the deformation is small. But at the same time, its influence is rather complicated. The influence of k_{xy} is not taken into account for it has a seemingly complicated relation with the deformation. In the available time span, it is not possible to accurately investigate its influence.

Since all calculations performed are linear-elastic, the deformation is proportional to the concentrated load. When the load is doubled, the deformation will be twice as big. The Young's modulus and the concentrated load are inversely proportional to each other. When a material is twice as stiff, the deformation will decrease by half. Therefore the formula for the deformation will have the following form:

$$w \approx \frac{P}{E} * f(t, k_x, k_y, l_x, l_y, d) \quad \mathbf{(5.1)}$$

Formula 5.1 has many parameters, which means that the formula for the deformation will be a complicated one when taking all parameters into acoount. Therefore it is preferable to eliminate some parameters, for instance the size of the shell which is expressed in l_x and l_y. The influence of the size of the shell on the deformation will be negligible when the measurements of the shell are big enough. The influence of the shell size on the deformation is depicted in figure 5-1.

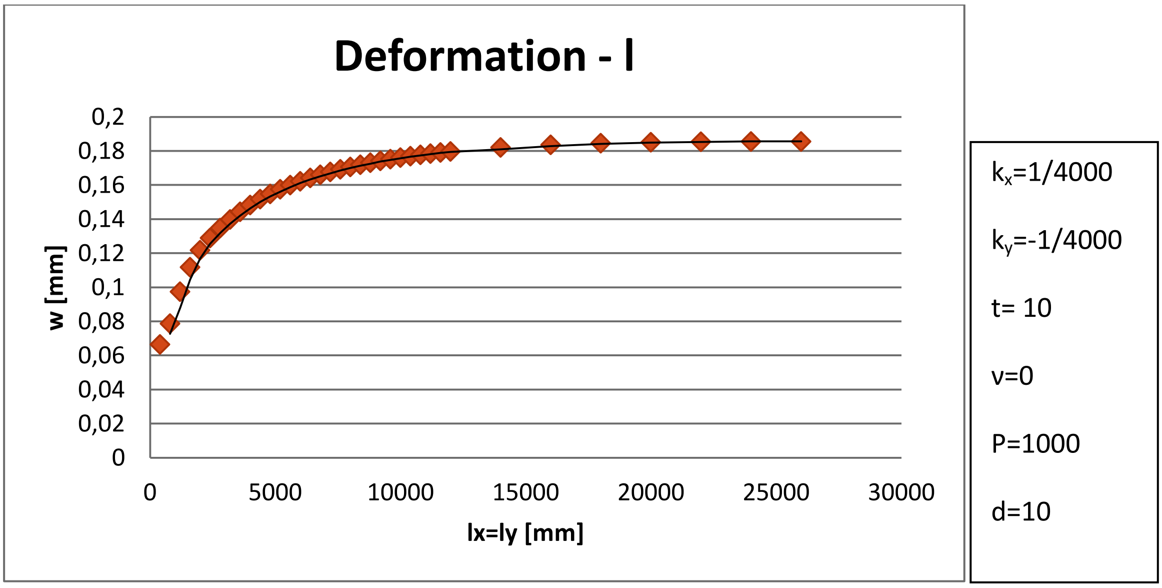


Figure 5-1 Influence of length on deformation

The data which belongs to figure 5-1 can be found in appendix D. The deformation starts to converge at a shell size of approximately 20.000x20.000 mm^2. This means that shells with a size equal to or larger than 20x20 m^2 will have a deformation which is not influenced by the shell size. The formula for the deformation will have the following form:

$$w \approx \frac{P}{E} * f(t, k_x, k_y, d)\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.2})$$

5.2 Influence of d on deformation

In chapter 3 the observation was made that the deformation did not change much when d was varied. This leads to the presumption that d does not influence the deformation much. This can be checked easily by varying d to look at its influence on the deformation. At the same time, the geometry and the other parameters are kept fixed.

d [mm]	*d/h [-]*	*h [mm]*	*t [mm]*	*kx [1/mm]*	*ky [1/mm]*	*w [mm]*	*Δw [%]*
50	4	12,5	30	1/4000	-1/4000	0,019269548	
45	4	11,25	30	1/4000	-1/4000	0,019290479	0,108622
40	4	10	30	1/4000	-1/4000	0,019311086	0,106825
35	4	8,75	30	1/4000	-1/4000	0,019331303	0,104692
30	4	7,5	30	1/4000	-1/4000	0,019351306	0,103476
25	4	6,25	30	1/4000	-1/4000	0,019371344	0,103546
20	4	5	30	1/4000	-1/4000	0,019392586	0,109658
15	4	3,75	30	1/4000	-1/4000	0,019416809	0,124912
10	4	2,5	30	1/4000	-1/4000	0,01944689	0,154919
5	4	1,25	30	1/4000	-1/4000	0,019493254	0,238413

Tabel 5-1 Influence of d on deformation

Again, Δw has been calculated by using formula 5.3. From table 5-1 it can be concluded that d has a very small influence on the deformation. In order to be sure that this behaviour does not depend on the thickness or curvature of the shell, the same variation of d has been applied at different thicknesses and curvatures. The results can be found in appendix D. The same conclusion can be drawn. The diameter d does not influence the deformation enough to involve it in the formula for w, which will have the same form as formula 5.4.

$$\Delta w = \left(\frac{w_{j+1} - w_j}{w_j}\right) * 100\ (\mathbf{5.3})$$

$$w \approx \frac{P}{E} * f(t, k_x, k_y)\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.4})$$

5.3 Influence of t on the deformation

Until now we have seen that thicker shells have smaller deformations than shells with a smaller thickness. This means that the deformation and the thickness are inversely proportional to each other. It is unknown what exact relation these two parameters have. This can be studied by varying the thickness while keeping the other parameters fixed (i.e.

the geometry, curvatures and material constants). In table 5-2 and figure 5-2 the influence of the thickness on the deformation is depicted.

t [mm]	*kx [1/mm]*	*ky [1/mm]*	*w [mm]*
5	1/4000	-1/4000	0,753880669
10	1/4000	-1/4000	0,185185325
15	1/4000	-1/4000	0,080913535
20	1/4000	-1/4000	0,044832455
25	1/4000	-1/4000	0,028320231
30	1/4000	-1/4000	0,01944689
35	1/4000	-1/4000	0,014149961
40	1/4000	-1/4000	0,010743579
45	1/4000	-1/4000	0,008427743
50	1/4000	-1/4000	0,006783931

Table 5-2 Influence of t on deformation

When looking at the values of the deformation in table 5-2, a certain pattern becomes evident. By reducing the thickness to half of its original value, the deformation becomes approximately four times as small. The thickness and the deformation are inversely proportional to each other, the exact power of this proportionality has yet to be determined. This can be done by applying the curve fitting function in Excel (figure 5-2).

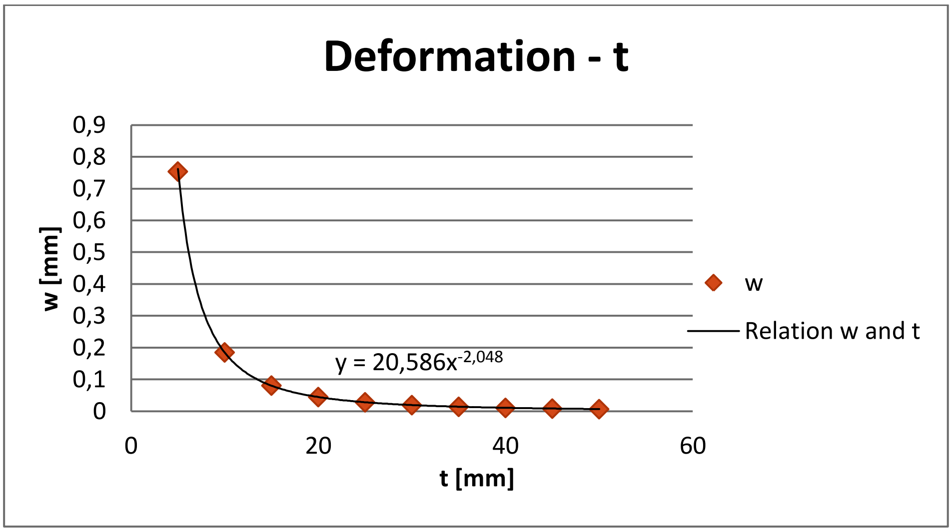


Figure 5-2 Relation between deformation and thickness

Since the deformation is proportional to $\frac{1}{t^2}$, the formula for the deformation has the same form as eq. 5.5.

$$w \approx \frac{P}{Et^2} * f(k_x, k_y)\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.5})$$

5.4 Influence of the curvatures on the deformation

In order to complete the formule for the deformation, the influence of the curvatures in both x-direction and y-direction has to be taken into account. Since the shell is assumed to be homogeneous and isotropic, the curvature in the x-direction will have the same influence on the deformation as the curvature in the y-direction. Therefore it is sufficient to look at the influence of k_x, this will also give us the relation between k_y and the deformation.

The relation between k_x and the deformation is determined by varying the value of k_x, while keeping all other parameters (including the curvature in the y-direction) constant. This yields the results given in table 5-3.

kx	*ky*	*kxy*	*t*	*w*
1/400	-1/1000	0	5	0,108025741
1/800	-1/1000	0	5	0,157969288
1/1200	-1/1000	0	5	0,195360249
1/1600	-1/1000	0	5	0,226335828
1/2000	-1/1000	0	5	0,253066538
1/2400	-1/1000	0	5	0,276808179
1/2800	-1/1000	0	5	0,298055657
1/3200	-1/1000	0	5	0,317709965
1/3600	-1/1000	0	5	0,335690541
1/4000	-1/1000	0	5	0,352508152
1/4400	-1/1000	0	5	0,368285295
1/4800	-1/1000	0	5	0,383034493
1/5200	-1/1000	0	5	0,396968262
1/5600	-1/1000	0	5	0,410220964
1/6000	-1/1000	0	5	0,422820432

Table 5-3 Influence of kx on deformation

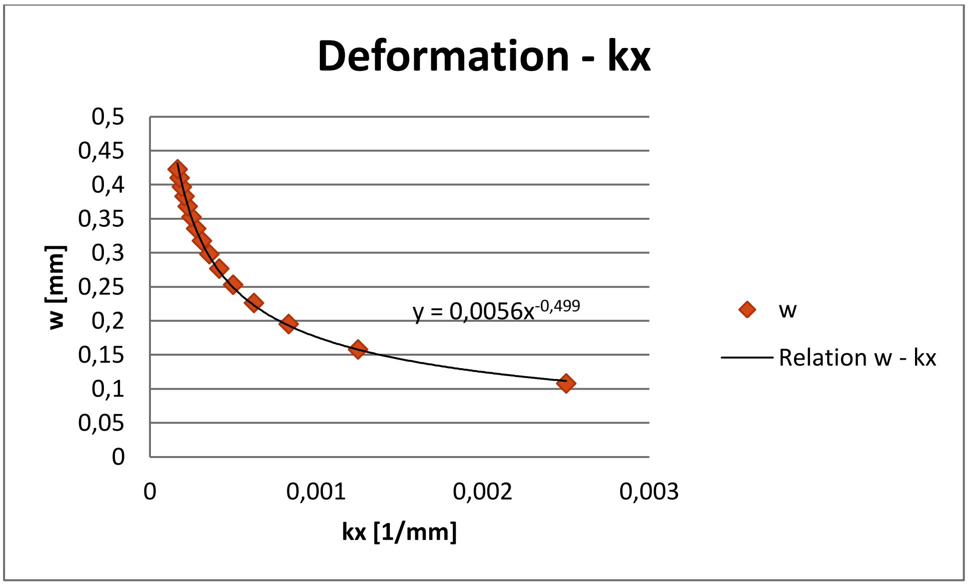


Figure 5-3 Relation between kx and deformation

In figure 5-3 the relation between the curvature in the x-direction and the deformation is depicted. The relation can be interpreted as follows: as the curvature increases, the deformation decreases. Thus, when the radius in x-direction decreases, the deformation decreases too. Since the radius and the curvature have the relation $k_x = \frac{1}{r_x}$, this makes sense. By including the influence of k_x on the deformation, the formula for w will have the same form as eq. 5.6.

$$w \approx \frac{P}{Et^2}\sqrt{r_x} * f(k_y)\; for\; l_x = l_y \geq 20\, m \;(\mathbf{5.6})$$

Since the curvature in y-direction has the same relation with the deformation as the curvature in x-direction, the formula for the deformation can be written as eq. 5.7.

$$w \approx \frac{P}{Et^2}\sqrt{r_x r_y}\; for\; l_x = l_y \geq 20\, m \;(\mathbf{5.7})$$

$$w \approx \frac{P}{Et^2}\sqrt{\frac{1}{k_x k_y}}\; for\; l_x = l_y \geq 20\, m \quad (\mathbf{5.8})$$

All parameters except v have been included in the formula for the deformation. Since the Poisson's ratio does not have a unit, it is possible to perform dimension analysis to check if the formula for the deformation is physically valid. This means that the formula should have the same unit as the deformation, which is mm.

$$\frac{[\mathrm{N}]}{\left[\frac{\mathrm{N}}{\mathrm{mm}^2}\right] * [\mathrm{mm}^2]}([\mathrm{mm}][\mathrm{mm}])^{\frac{1}{2}} = [\mathrm{mm}^2]^{\frac{1}{2}} = [\mathrm{mm}] \quad (\mathbf{5.9})$$

From eq. 5.9 it can be concluded that the formula for the deformation is valid. However, one small adjustment has to be made. Since the shell is anticlastic, the product of k_x and k_y will be negative. Since the square root has bo be taken, this product must be positive. After all, the deformation has to be real. Therefore, a minus sign has to be placed in front of the Gaussian curvature.

$$w \approx \frac{P}{Et^2}\sqrt{-\frac{1}{k_x k_y}}\; for\; l_x = l_y \geq 20\, m \quad (\mathbf{5.10})$$

5.5 Influence of v on the deformation

The formula for the deformation is almost complete. Until now the value of v has been equal to zero. In order to find out the influence of the Poisson's ratio on the deformation, v will be varied while all other parameters are kept constant. Van Bolderen found that the influence of v on the deformation of cylinders can be expressed by the following term: $(1 - 0.75\nu^2)$ [3]. The influence of v on the deformation of anticlastic shell might be expressed in a similar form. This is investigated by comparing the deformation at a certain value of v with the deformation at v=0. The factor C1 is defined by dividing the deformation at a certain value of v by the deformation at v=0. This factor will be compared to influence term of v having the following form: $(1 - av^2)$. From these comparisons, the parameter a can be determined.

v	t	w
0,05	5,00	0,1798904
0,10	5,00	0,1792681
0,15	5,00	0,1781593
0,20	5,00	0,1765546
0,25	5,00	0,1744406
0,30	5,00	0,1717995
0,35	5,00	0,1686077
0,40	5,00	0,1648356
0,45	5,00	0,1604451

Table 5-4 Deformations for different values of v

Reference value		
v	**t**	**w_ref**
0,00	5,00	0,1800318

Table 5-5 Deformation for v=0

v	w/w_ref (C1)	$1 - 0.75v^2$	$1 - 0.5v^2$	$1 - 0.4v^2$	$1 - 0.45v^2$
0,05	0,999215	0,998125	0,99875	0,999	0,998875
0,10	0,995758	0,9925	0,995	0,996	0,9955
0,15	0,989599	0,983125	0,98875	0,991	0,989875
0,20	0,980685	0,97	0,98	0,984	0,982
0,25	0,968943	0,953125	0,96875	0,975	0,971875
0,30	0,954273	0,9325	0,955	0,964	0,9595
0,35	0,936544	0,908125	0,93875	0,951	0,944875
0,40	0,915591	-11	0,92	0,936	0,928
0,45	0,891204	0,848125	0,89875	0,919	0,908875

Table 5-6 Determination of influence term v

From table 5-6 it becomes clear that the influence of v on the deformation can be expressed best by the term $(1 - 0.5v^2)$. Even though this term slightly overestimates the deformations as v increases, it can be compensated for by the constant A which has to determined by comparing the results generated with Ansys and the results given by formula 5.11.

$$w \approx A\frac{P}{Et^2}\sqrt{-\frac{1}{k_x k_y}}(1 - 0.5v^2)\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.11})$$

Instead of using the term $(1 - 0.5v^2)$ to describe the influence of v, the term $\sqrt{1 - 0.5v^2}$ could be used also. This is the term Reissner used when he solved the Koiters-Sander equations [16]. When expanding $\sqrt{1 - 0.5v^2}$ in a Taylor series, the following result is obtained:

$$\sqrt{1 - 0.5v^2} = 1 - \frac{1}{2}v^2 - \frac{1}{8}v^4 + O(v^6)\quad (\mathbf{5.12})$$

The term $\frac{1}{8}v^4$ will have a very small influence since v is small too. Therefore it is preferable to use $\sqrt{1 - 0.5v^2}$. This small adjustment yields the following equation:

$$w \approx A\frac{P}{Et^2}\sqrt{-\frac{1}{k_x k_y}}\sqrt{1-0.5v^2}\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.13})$$

5.6 Completing the formula: determination of constant A

The formula for the deformation is almost finished. In order to complete it, the constant A has to be determined. First values for the deformation will be generated by using the formula. The same parameters will be used to generate results in Ansys. By dividing these two values, the constant A will be determined. The parameters can be chosen randomly. The results are displayed in table 5-7.

t	*kx*	*ky*	*kxy*	*v*	*w_Ansys*	*w_formule*	*A*
5	1/1000	-0,001	0	0,25	0,17324883	0,18452381	0,938896874
10	1/500	-0,001	0	0,25	0,02984049	0,03261951	0,914804863
15	1/2000	-0,002	0	0,25	0,01813343	0,02050265	0,884443454
15	1/1000	-0,0005	0	0,25	0,02690029	0,02899512	0,927752343
25	1/1000	-0,002	0	0,25	0,00488554	0,00521912	0,936085304

Table 5-7 Determining constant A

The average of the constant is approximately 0.92. Hereby the formula for the deformation can be completed. The final form of the formula is depicted in eq. 5.14.

$$w \approx 0.92\frac{P}{Et^2}\sqrt{-\frac{1}{k_x k_y}}\sqrt{1-0.5v^2}\ for\ l_x = l_y \geq 20\ m\ (\mathbf{5.14})$$

5.7 Accuracy of the formula

The accuracy of the formula can be determined by comparing results generated in Ansys with results generated with formula 5.14. The curvatures and the thickness are varied within the following ranges:

- $\frac{1}{5000} \leq k_x \leq \frac{1}{500}\ mm^{-1}$
- $-\frac{1}{500} \leq k_y \leq -\frac{1}{5000} mm^{-1}$
- $5 \leq t \leq 50\ mm$

Within these ranges, the biggest deviation of 11.6% occurs when the shell thickness equals 50 mm, k_x=1/500 mm^{-1} and k_y=-1/500 mm^{-1}. It turns out that the errors do not depend on the curvatures or the thickness separately. Instead, there are two conditions which determine the error:

- The ratio of the curvatures: $\left|\frac{k_{max}}{k_{min}}\right|$
- The product of the thickness and the minimum curvature: $t|k_{max}|$

When $\left|\frac{k_{max}}{k_{min}}\right| < 4.5$ and $t|k_{max}| < 0.06$, the maximum error is limited to 4.97%.

Chapter 6 Formula normal force

6.1 From stress to normal force

Ansys can be used to determine deformations and stresses in constructions. It does not report normal forces and moments. If interested, these must be computed by the user. In this case, the stresses are converted into normal forces by implementing the required commando in the script (Appendix A). Equations 6.1 and 6.2 are used to convert stresses into normal forces. Distinction is made between the normal force in the x-direction and y-direction.

$$n_{xx} = \frac{(s_{xxb} + s_{xxt})t}{2} \quad \mathbf{(6.1)}$$

$$n_{yy} = \frac{(s_{yyb} + s_{yyt})t}{2} \quad \mathbf{(6.2)}$$

S_{xxb} = stress in bottom fiber shell [N/mm^2] (x-direction)
S_{xxt} = stress in top fiber shell [N/mm^2] (x-direction)

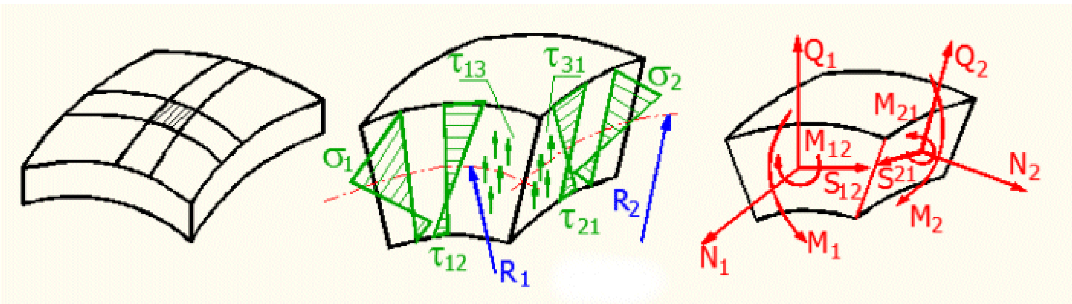


Figure 6-1 Stress distribution in a shell (http://www.mitcalc.com/doc/shells/help/en/shells.htm)

In figure 6-1 the stress distribution in the shell is displayed. Since all calculations performed are linear elastic, the normal force can be calculated easily by multiplying the stress working on the middle fiber (average of stress in top and bottom fiber) with the thickness of the shell.

6.2 Influence of P, E, l and d on the normal force

The first difference between the formula for the deformation and the formula for the normal force, is that the formula for the normal force does not contain the Young's modulus. This is confirmed when performing calculations in Ansys. When all parameters but the Young's modulus are kept constant, the normal force does not change. Therefore it is possible to formulate a relation between the normal force and other parameters as in eq. 6.3.

$$n_{xx} = -n_{yy} \approx P * f(l_x, l_y, k_x, k_y, d, t) \quad (\mathbf{6.3})$$

In this stage, the normal force in the x-direction equals the normal force in the y-direction. It is probable that both formulas will be symmetrical to one another. The main difference is that tension occurs in one direction and compression holds in the other. This is likely to cause both formulas to have the same form, but with different signs.

Of course it is preferable to eliminate the influence of the shell size on the normal forces. By investigating the influence of the shell size on the normal forces, a shell size can be chosen at which the values of the normal forces converge.

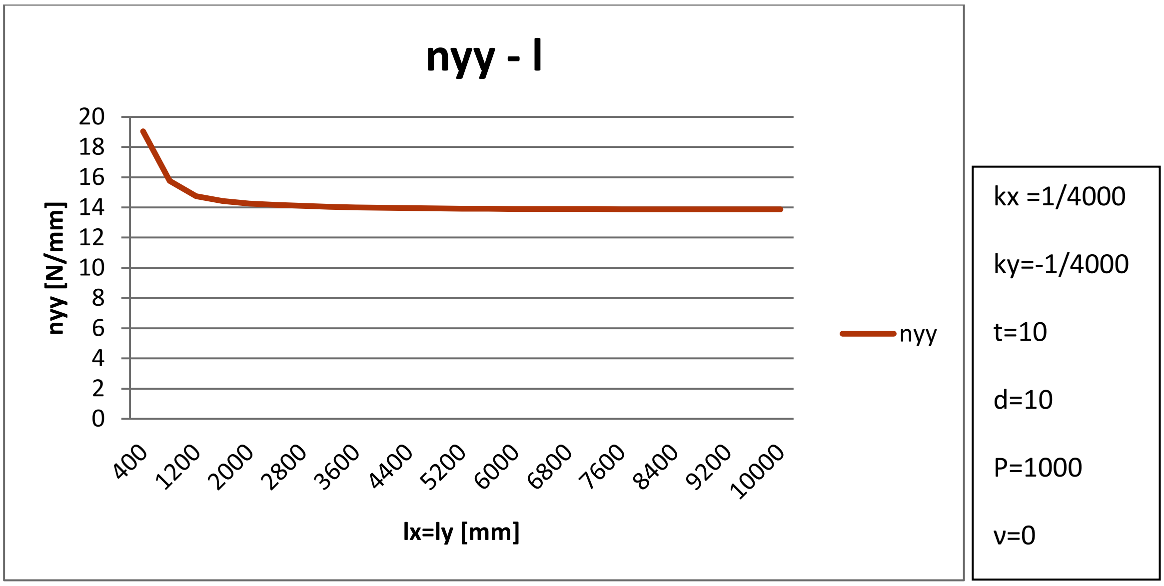


Figure 6-1 Influence of the length on the normal force in y-direction

From figure 6-1 it becomes obvious that the influence of the length on the normal force damps more quickly than its influence on the deformations. As long as the shell size is chosen to be bigger than 3000*3000 mm^2, length influences are negligible. In that case the formula for the normal force will have the same form as eq. 6.4.

$$n_{xx} = -n_{yy} \approx P * f(k_x, k_y, d, t) \; for \; l_x = l_y \geq 3m \quad (\mathbf{6.4})$$

As for the influence of the diameter of the surface of loading, it is practically negligible. Again the influence of d has been investigated at different shell thicknesses and different shell curvatures. These results can be found in appendix E. In table 6-1 some of the results are depicted. When varying the diameter from 50 to 5 mm at a shell thickness of 30 mm, the normal force (in both directions) changes approximately 1.8%. Therefore, the influence of d is considered to be negligible and it will not be taken into the formula for the normal force.

d	d/h	h	t	kx	ky	nxx	nyy
50	4	12,5	40	1/4000	-1/4000	-3,502546997	3,502315277
45	4	11,25	40	1/4000	-1/4000	-3,506287841	3,506073158
40	4	10	40	1/4000	-1/4000	-3,509934631	3,509795961
35	4	8,75	40	1/4000	-1/4000	-3,513571425	3,513429783
30	4	7,5	40	1/4000	-1/4000	-3,517193939	3,517043049
25	4	6,25	40	1/4000	-1/4000	-3,520966486	3,520967543
20	4	5	40	1/4000	-1/4000	-3,525318161	3,525260764
15	4	3,75	40	1/4000	-1/4000	-3,530725958	3,53058871
10	4	2,5	40	1/4000	-1/4000	-3,540350216	3,540411627
5	4	1,25	40	1/4000	-1/4000	-3,566371793	3,566264427

Table 6-1 Influence of d on normal forces

The formula for the normal force will have the following form:

$$n_{xx} = -n_{yy} \approx P * f(k_x, k_y, t)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.5})$$

6.3 Influence of t on the normal force

The influence of the thickness on the deformation is an important one. The relation between the normal force and the shell thickness can be determined by varying the thickness while all other parameters are kept constant. By plotting the thickness against the normal force, the right relation can be determined. The shell thickness is varied at different shell curvatures. This has been done to see whether the curvatures influence the relation between the shell thickness and the normal force. The results are depicted in figures 6-3 and 6-4. Other results can be found in appendix E.

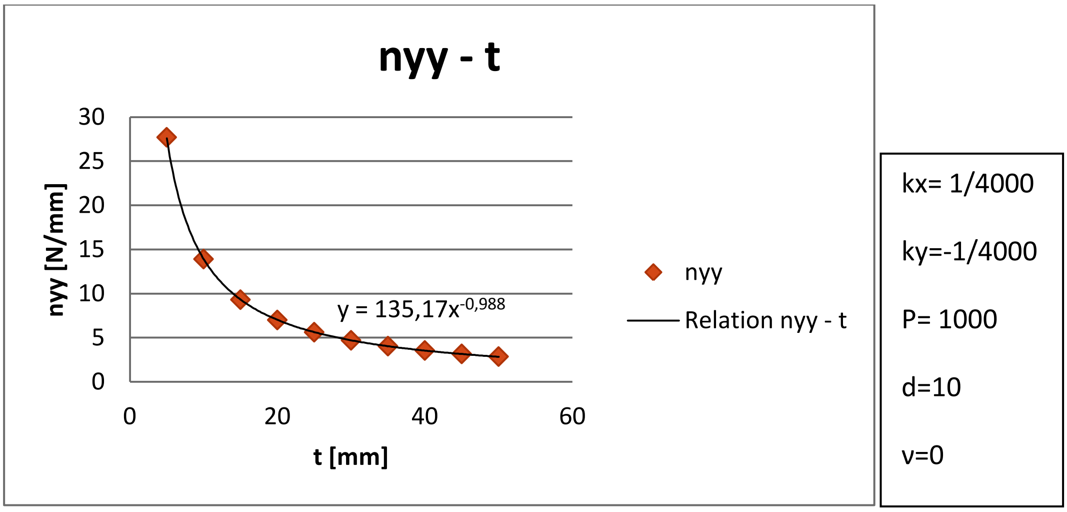


Figure 6-3 Influence of thickness on normal force

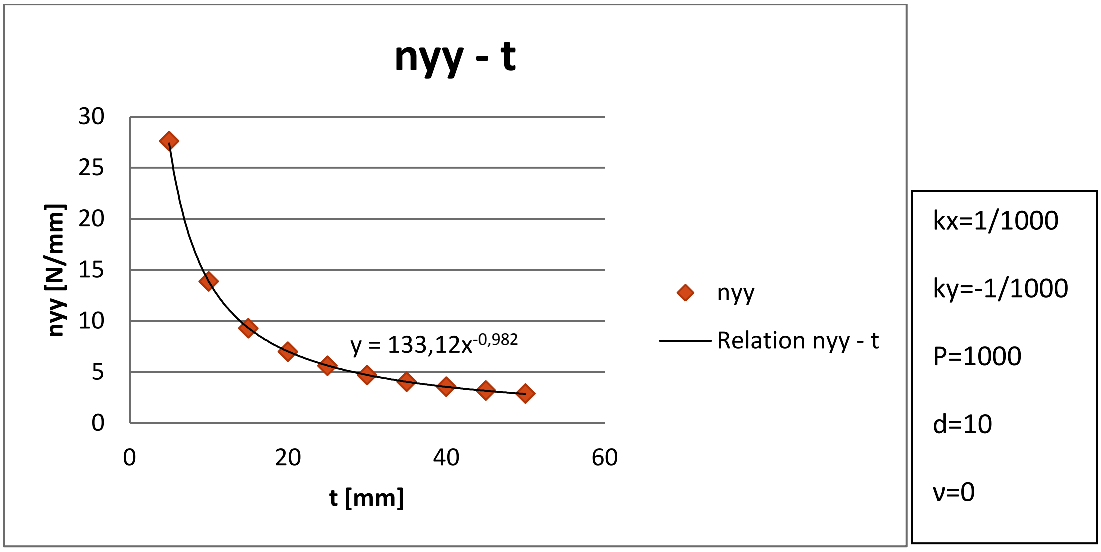


Figure 6-4 Influence of thickness on normal force

From both figures the same relation between the thickness and the normal force can be derived. As the thickness increases, the normal force decreases. The shell thickness and normal force are inversely proportional to each other. When this relation is taken into account, the formula for the normal force will have the same form as eq. 6.6.

$$n_{xx} = -n_{yy} \approx \frac{P}{t} * f(k_x, k_y)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.6})$$

6.4 The influence of the curvatures on the normal force

The unit of n_{xx} or n_{yy} is N/mm. As the dimensions in eq. 6.6 are checked, the term P/t has the unit N/mm. This means that the remaining parameters should be dimensionless. Of course separately, the curvatures have the dimension [1/mm]. To make them dimensionless, the influence of the ratio of the curvatures on the normal force can be checked.

First the influence of the ratio $\frac{k_x}{k_y}$ on the normal forces in both x-direction and y-direction will be checked. The results are displayed in figures 6-5 and 6-6.

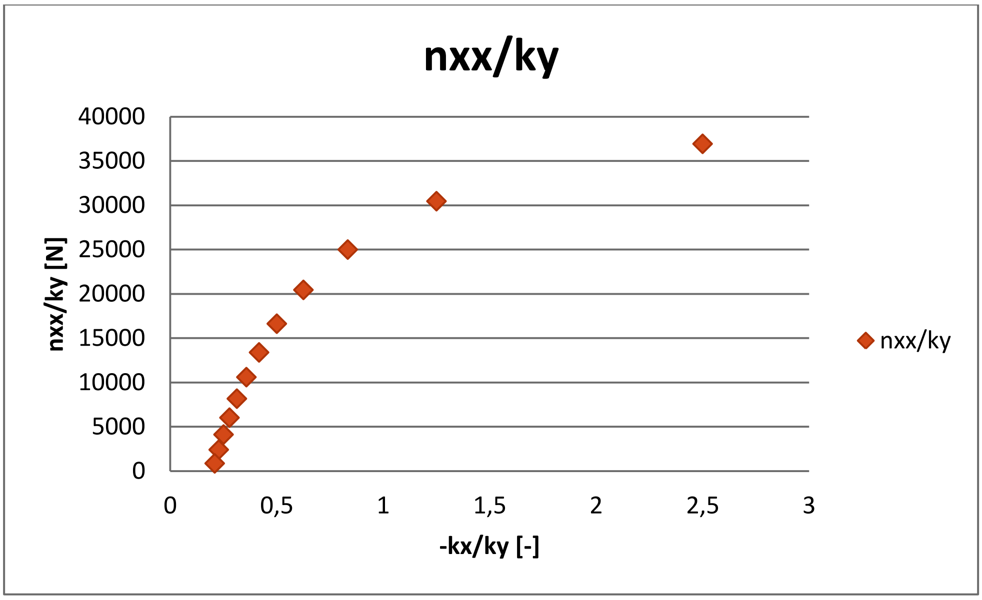


Figure 6-5 Influence of -kx/ky on normal force in x direction

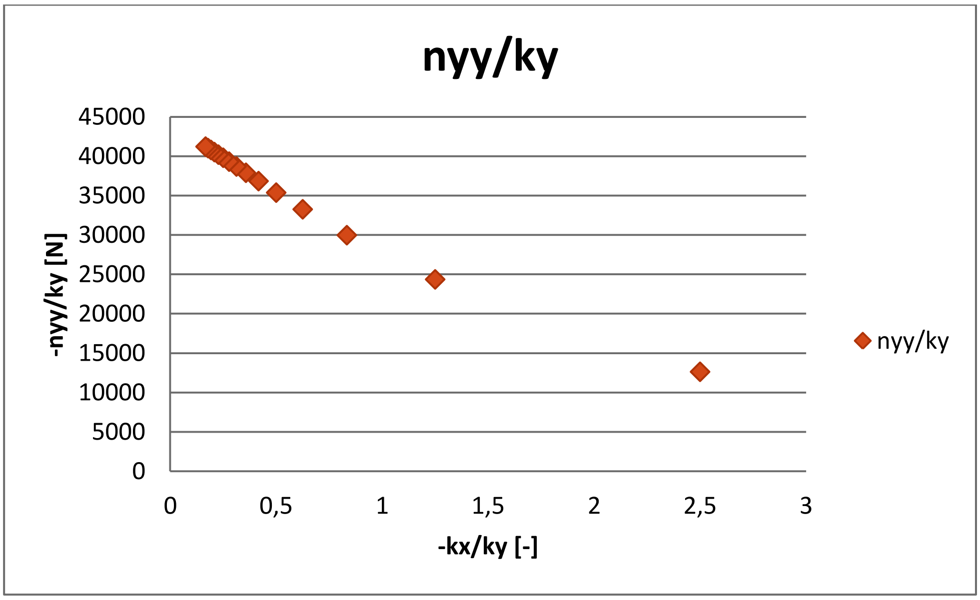


Figure 6-6 Influence of -kx/ky on normal force in y-direction

It is striking to see that the ratio $-\frac{k_x}{k_y}$ has a different influence on the normal forces in both directions. Since the formulas for the normal forces are expected to be symmetrical, it is convenient to look at the influence of $-\frac{k_y}{k_x}$ on the normal forces. These relations are depicted in figures 6-7 and 6-8.

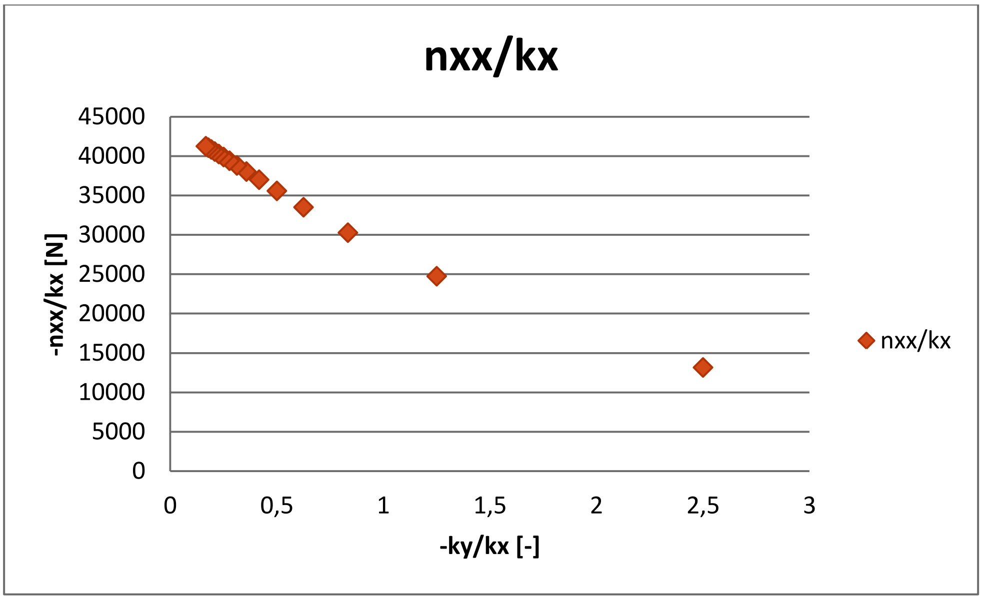


Figure 6-7 Influence of -ky/kx on the normal force in the x-direction

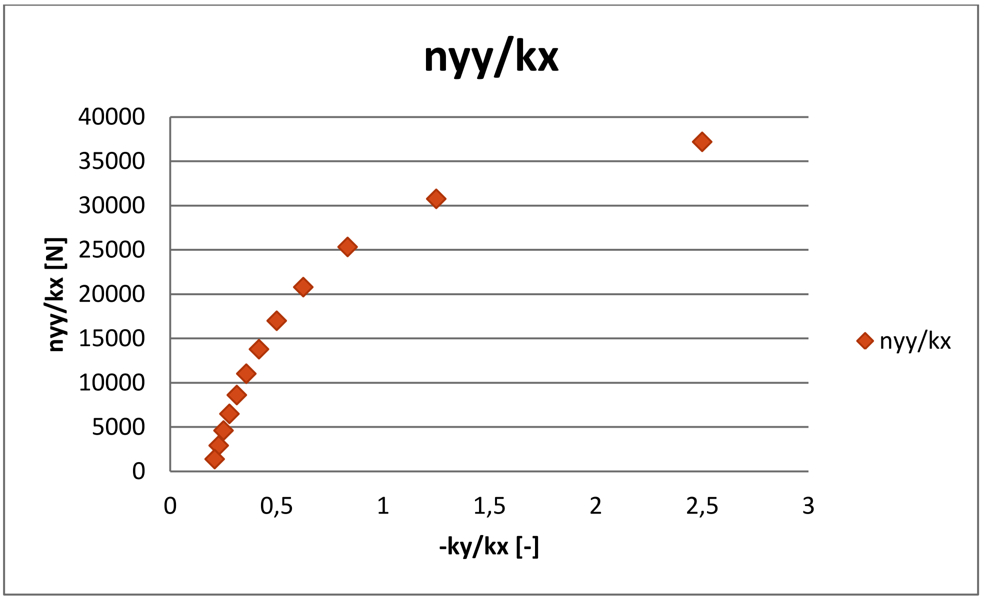


Figure 6-8 Influence of –ky/kx on the normal force in the y-direction

When comparing figures 6-7 and 6-8 with figures 6-5 and 6-6, the following conclusions can be drawn:

- The ratio $-\frac{k_x}{k_y}$ has the opposite effect of the ratio $-\frac{k_y}{k_x}$ on the normal forces.
- The relation between the normal force in the x-direction and the ratio $-\frac{k_x}{k_y}$ seems to be described by a square function.
- Similarly, the relation between the normal force in the y-direction and the ratio $-\frac{k_y}{k_x}$ can be described by a square function.

There is one little problem when a square function is chosen to describe the relation between the normal forces and the ratio of k_x and k_y. For small ratios of k_x and k_y, a square function can be used best to describe the relation. But as the ratio of the curvature increases, the normal forces seem to converge. This means that a square function will overestimate the value for the normal force as the ratio of the curvatures increases. Two things can be done to compensate for this over-estimiation:

- Intervals can be distinguished when determining the constants for the formulas. The areas in which the normal force is over-estimated will have smaller constants.
- Definition of limitations to constrain the errors when the results of the formulas deviate too much from results generated in Ansys.

For now the formulas for the normal forces will have the following form:

$$n_{xx} \approx -\frac{P}{t}\sqrt{-\frac{k_x}{k_y}}\ for\ l_x = l_y \geq 3m \quad \mathbf{(6.7)}$$

$$n_{yy} \approx \frac{P}{t}\sqrt{-\frac{k_y}{k_x}}\ for\ l_x = l_y \geq 3m \quad \mathbf{(6.8)}$$

The equations above can be rewritten in a more convenient form. The validity of the formulas should not depend on the choice of the coordinate system. Therefore, in both x-direction and y-direction tension or compression could occur. By using formulas 6.9 and 6.10, dependence of the coordinate system is ruled out.

$$n_{xx} \approx -\frac{P}{t}k_x\sqrt{-\frac{1}{k_x k_y}}\ for\ l_x = l_y \geq 3m \quad \mathbf{(6.9)}$$

$$n_{yy} \approx \frac{P}{t}k_y\sqrt{-\frac{1}{k_x k_y}}\ for\ l_x = l_y \geq 3m \quad \mathbf{(6.10)}$$

6.5 Influence of ν on the normal force

The formula for the normal force will be completed by taking the influence of the Poisson's ratio into account. The influence of the Poisson's ratio is depicted in figure 6-9.

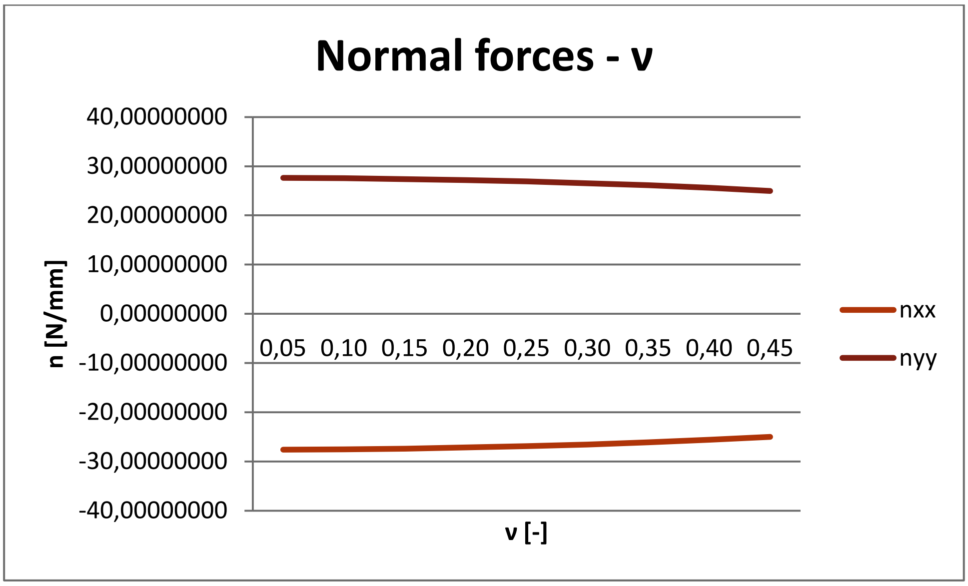


Figure 6-9 Influence of v on normal forces

The Poisson's ratio does not influence the normal forces much. It is not a linear relation, so again the influence of v might be expressed in the form $(1 - av^2)$. The exact term can be found by dividing the normal force at v=0 by the normal force at another value for v. This factor, say C1, will be compared to possible influence terms of v.

nu	*t*	*nxx*	*nyy*
0,05	5,00	-27,60020022	27,600613330
0,10	5,00	-27,53204639	27,532434790
0,15	5,00	-27,39422805	27,394584870
0,20	5,00	-27,18544701	27,185764480
0,25	5,00	-26,90379263	26,904061830
0,30	5,00	-26,54666061	26,546871140
0,35	5,00	-26,11063465	26,110774180
0,40	5,00	-25,59131775	25,591371350
0,45	5,00	-24,98309138	24,983040590

Table 6-2 Influence of v on normal force (data belonging to figure 30)

Reference value:			
nu	t	nxx_ref	nyy_ref
0,00	5,00	-27,59942805	27,599859750

Table 6-3 Reference value v

nu	*nxx/nxx_ref*	*nyy/nyy_ref*	*1-0,75v²*	*1-0,5v²*	*1-0,4v²*	*1-0,45v²*
0,05	1,0000280	1,00002730	0,998125000	0,99875	0,999	0,998875
0,10	0.9975586	0,99755705	0,992500000	0,995	0,996	0,9955
0,15	0,9925651	0,99256247	0,983125000	0,98875	0,991	0,989875
0,20	0,9850004	0,98499647	0,970000000	0,98	0,984	0,982
0,25	0,9747953	0,97478980	0,953125000	0,96875	0,975	0,971875
0,30	0,9618555	0,96184804	0,932500000	0,955	0,964	0,9595
0,35	0,9460571	0,94604735	0,908125000	0,93875	0,951	0,944875

0,40	0,9272409	0,92722831	0,880000000	0,92	0,936	0,928
0,45	0,9052032	0,90518723	0,848125000	0,89875	0,919	0,908875

Table 6-4 Influence term v

From table 6-4 it can be seen that the influence of v can be expressed best by the term $(1 - 0.45v^2)$. The formulas for the normal forces are equal to equations 6.11 and 6.12.

$$n_{xx} \approx -A_x \frac{P}{t} k_x \sqrt{-\frac{1}{k_x k_y}} (1 - 0.45v^2)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.11})$$

$$n_{yy} \approx A_y \frac{P}{t} k_y \sqrt{-\frac{1}{k_x k_y}} (1 - 0.45v^2)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.12})$$

The formulas for the normal forces are almost complete. The constant A has yet to be determined. This will be done in the next paragraph.

6.6 Completing the formulas: determination of constant A

The constants A_x and A_y can be determined by generating results with both Ansys and the formulas for the normal forces. By dividing the values from Ansys by the results of the formulas, the constants will be obtained. In table 6-6 the values for A_x and A_y are displayed.

t	*kx*	*ky*	*v*	*nxx(ansys)*	*nyy(ansys)*	*nxx(formula)*	*nyy(formula)*
5	1/1000	-0,001	0,25	-27,269552	27,267613	-194,375	194,375
10	1/500	-0,001	0,25	-17,923871	8,50050924	-137,443881	68,72194
15	1/2000	-0,002	0,25	-1,5743856	13,6882218	-32,3958333	129,5833
15	1/1000	-0,0005	0,25	-11,912584	5,60677826	-91,6292537	45,81463
25	1/1000	-0,002	0,25	-3,6584558	7,75094298	-27,4887761	54,97755

Table 6-5 Results from Ansys and formulas

Ax	*Ay*
0,140294	0,140284
0,130409	0,123694
0,048598	0,105633
0,130009	0,12238
0,133089	0,140984

Table 6-6 Constants Ax and Ay

The constants A_y are not very deviant. However, when looking at the constants A_x, there is one value which does not match the other values at all: A_x=0.048598. This might be due to the ratio of $-\frac{k_x}{k_y}$ which is quite small, or due to the combination of the thickness and the curvatures. Since this value deviates too much from the other values for A_x, it is not taken into account when determining the average value for A_x. When determining the accuracy of both formulas, the reason for this deviation might be declared.

The formulas will have the same form as eq. 6.13 and 6.14.

$$n_{xx} \approx -0.13\frac{P}{t}k_x\sqrt{-\frac{1}{k_xk_y}}(1-0.45\nu^2)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.13})$$

$$n_{yy} \approx 0.13\frac{P}{t}k_y\sqrt{-\frac{1}{k_xk_y}}(1-0.45\nu^2)\ for\ l_x = l_y \geq 3m \quad (\mathbf{6.14})$$

6.7 Accuracy of the formulas

The accuracy of the formulas can be determined by comparing the results generated by the formulas with results generated by Ansys. The ranges in which the parameters will be varied are as follows:

- $\frac{1}{5000} \leq k_x \leq \frac{1}{500}\ mm^{-1}$
- $-\frac{1}{500} \leq k_y \leq -\frac{1}{5000} mm^{-1}$
- $5 \leq t \leq 50\ mm$

When comparing the results from Ansys with the results generated by the formula, it is immediately clear that the formulas do not represent the influence of the curvatures accurately, since the occuring errors are enormous (appendix E). Two observations are made:

- When $-\frac{k_x}{k_y} < 0.5$, the formula for n_{xx} is not accurate anymore. The errors increase enormously as this ratio becomes smaller.
- The formula for n_{yy} seems to behave differently, since the errors are much smaller. One would expect the formulas to behave identically, since they have the same form.

For n_{xx} the following limitations can be formulated:

1) $-\frac{k_x}{k_y} \geq 0.5$
2) $t|k_{min}| < 0.02$

If these two limitations are respected, the maximum error will be restricted to 9.87%

As for the formula for n_{yy}, the following restrictments hold:

1) $-\frac{k_y}{k_x} < 6$
2) $t|k_{min}| < 0.02$

If these restrictments are met, the maximum error will be constrained to 14.59%.

Chapter 7 Formula bending moments

7.1 From stress to bending moment

Before the derivation of the formula for the bending moments can begin, it is important to convert the stresses generated by Ansys to bending moments. As said earlier, Ansys does not display bending moments or membrane forces.

The formula for determining the bending stress due to a bending moment is:

$$\sigma = \frac{My}{I_z} \quad \textbf{(7.1)}$$

In this case the moment of inertia is measured per unit of width:

$$I_z = \frac{1}{12}t^3 \quad \left[\frac{mm^4}{mm}\right] \quad \textbf{(7.2)}$$

Rewriting eq. 7.1 yields the following equation:

$$m = \frac{\sigma I_z}{y} \quad \left[\frac{Nmm}{mm}\right] \quad \textbf{(7.3)}$$

σ can be defined as the difference between the stress in the top fiber and bottom fiber. These stresses are different for the x-direction and y-direction.

$$m_{xx} = \frac{(s_{xxb} - s_{xxt})t^2}{12} \quad \textbf{(7.4)}$$

$$m_{yy} = \frac{(s_{yyb} - s_{yyt})t^2}{12} \quad \textbf{(7.5)}$$

Equations 7.4 and 7.5 are used to convert the stresses in Ansys to bending moments.

7.2 Influence of P, E and I on the bending moments

It comes as no surprise that the bending moments are proportional to the applied point load. As for the Young's modulus, it does not influence the bending moments. This can be checked by varying the value for E while all other constants are kept constant. It will be clear that the value for the bending moments doesn't change. This means that the bending moments have the following relation with the parameters:

$$m_{xx} = P * f\left(t, d, l_x, l_{y,} k_x, k_y\right) \quad \textbf{(7.6)}$$

$$m_{\boldsymbol{yy}} = P * f\left(t, d, l_x, l_{y,} k_x, k_y\right) \quad \textbf{(7.7)}$$

Again, the influence of v is initially neglected by equating it to zero. In the end the influence of v on the bending moments will be determined.

It is preferable to have a formula in which the shell size doesn't play a role. When the size of the shell is chosen to be big enough, its influence on the bending moments will be negligible. Therefore the influence of the shell size has to be investigated. The results are depicted in figure 7-1.

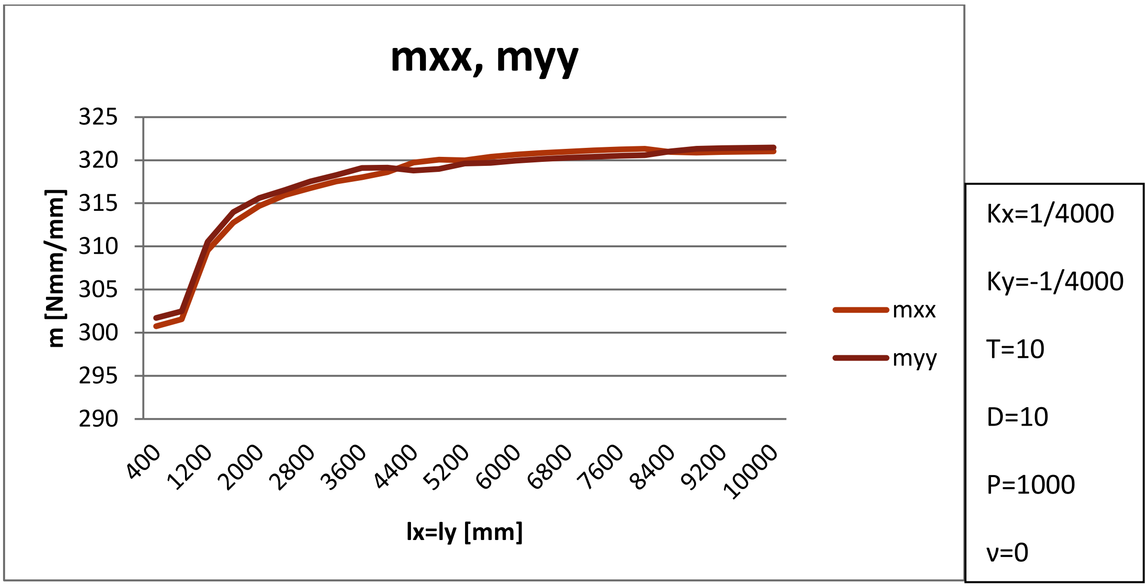


Figure 7-1 Influence of shell size on bending moments

The bending moments start to converge when the shell size is aprroximately 4000*4000 mm^2. The formula for the bending moments is as follows:

$$m_{xx} = P * f(t, d, k_x, k_y) \;\; for\ l_x = l_y \geq 4\,m \;(\mathbf{7.8})$$

$$m_{yy} = P * f(t, d, k_x, k_y) \;\; for\ l_x = l_y \geq 4\,m \;\;(\mathbf{7.9})$$

7.3 Influence of d on bending moments.

To derive a relation between d and the bending moments, the same approach will be used as has been done so far: d will be varied while other parameters are kept constant. This will be done while applying different shell thicknesses to make sure that the shell thickness does not influence the relation between the bending moments and the diameter of the surface in which the point load is applied. Some results are depicted below, an overview of all the results can be found in appendix F.

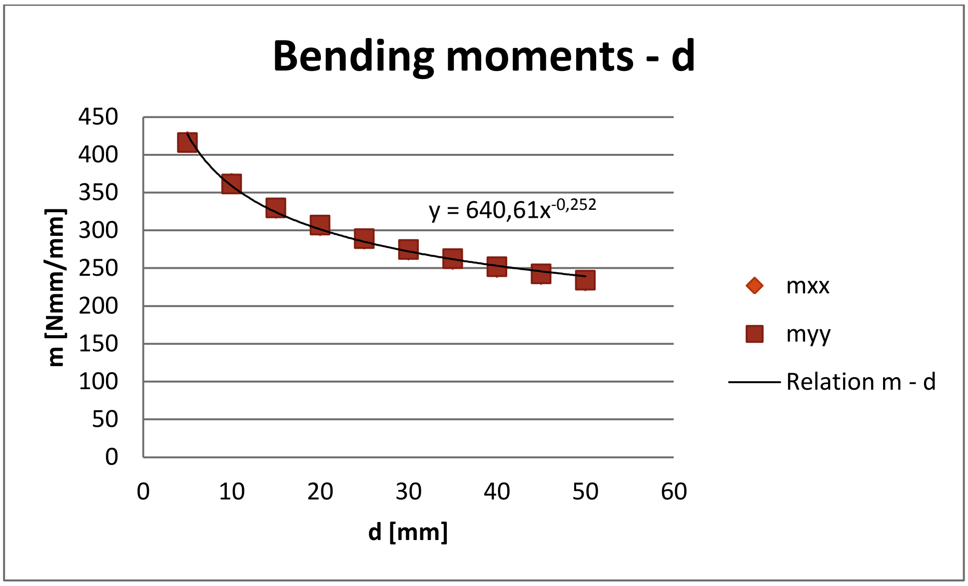


Figure 7-2 Influence of d on bending moments (t=30 mm)

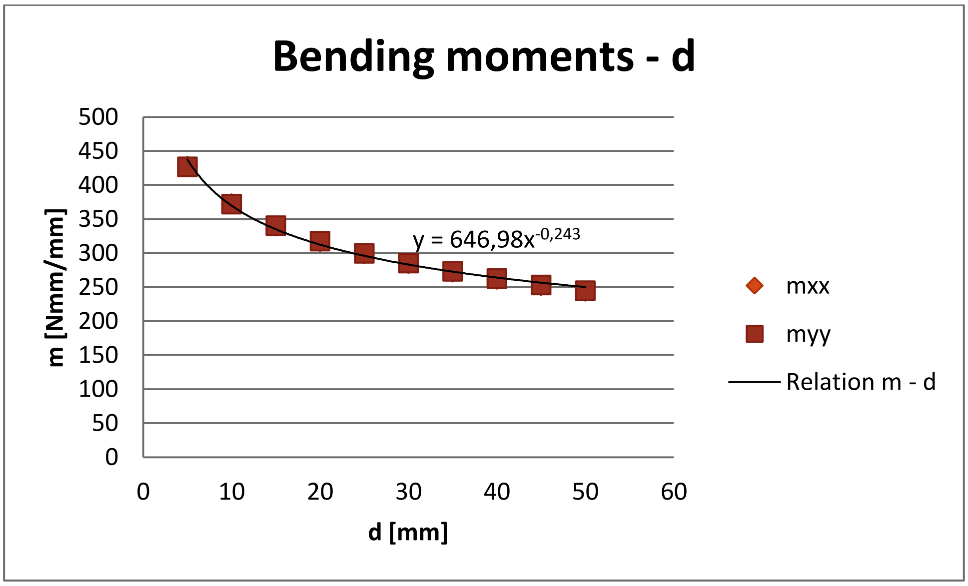


Figure 7-3 Influence of d on bending moments (t=40 mm)

The relation between the bending moments and d is depicted in figures 7-2 and 7-3. By changing the thickness of the shell, this relation does not change much. As the diameter of the circular surface increases , the bending moments in x- and y-direction increase. This is not surprising as it was already known that spreading the point load over a small surface would lead to finite bending moments. As the point load is spread over a bigger area, the bending moments underneath the point load will grow smaller. The formula for the bending moments can be expanded with this information as displayed in eq. 7.10 and eq. 7.11

$$m_{xx} = \frac{P}{\sqrt[4]{d}} * f(t, k_x, k_y) \;\; for\; l_x = l_y \geq 4\, m \; (\mathbf{7.10})$$

$$m_{yy} = \frac{P}{\sqrt[4]{d}} * f(t, k_x, k_y) \;\; for\; l_x = l_y \geq 4\, m \; (\mathbf{7.11})$$

7.4 Influence of t on the bending moments

The influence of the thickness on the bending moments is depicted in figures 7-4 and 7-5.

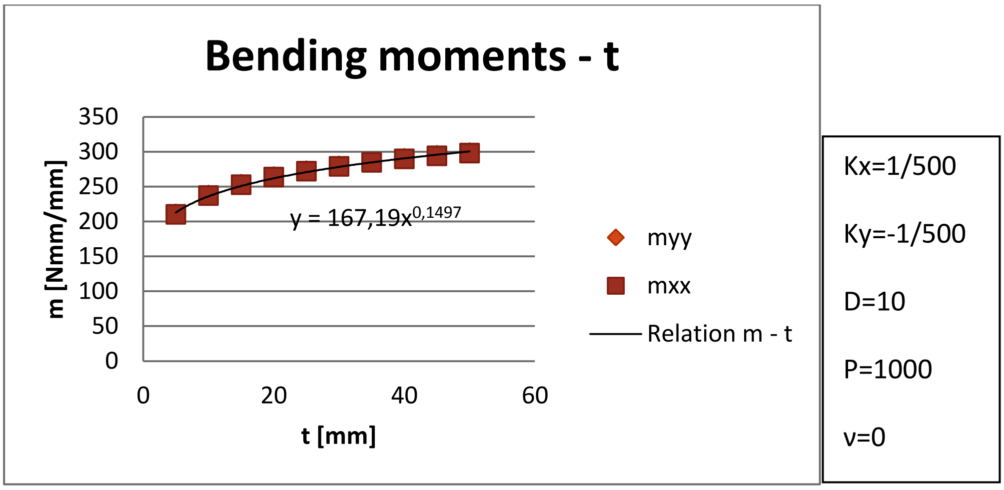


Figure 7-4 Influence of t on bending moments

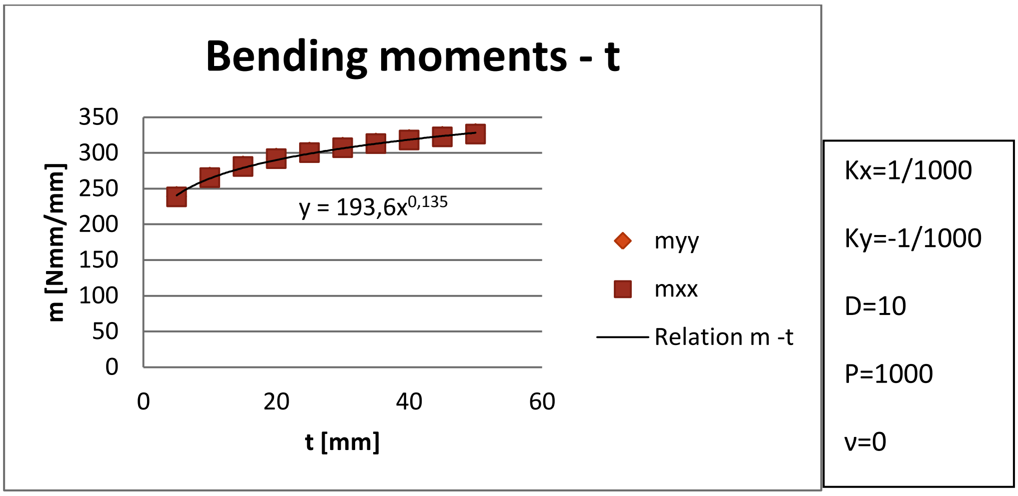


Figure 7-5 Influence of t on bending moments

All data that were used can be found in appendix F. When comparing figure 7-4 and figure 7-5, a minor difference can be seen in the relations between the thickness and the bending moments. This can be due to the influence of the curvature. In figure 7-5 a smaller curvature has been chosen than in figure 7-4. Since this difference is quite small, the bending moments in both x-direction and y-direction are assumed to be propotional to $t^{0.15}$. Small differences can be compensated for when comparing the formula with the results from Ansys. The formula for the bending moments are given in eq. 7.12 and eq. 7.13.

$$m_{xx} = \frac{P}{\sqrt[4]{d}} t^{0.15} * f(k_x, k_y) \ for \ l_x = l_y \geq 4\,m \ (\mathbf{7.12})$$

$$m_{yy} = \frac{P}{\sqrt[4]{d}} t^{0.15} * f(k_x, k_y) \ for \ l_x = l_y \geq 4\,m \ (\mathbf{7.13})$$

7.5 Influence of the curvatures on the bending moments

7.5.1 Influence of kx on the bending moments

Until now, the formulas for the bending moments in the x-direction and the y-direction have turned out to be completely identical, which is not surprising. When taking the influence of the curvatures into account, this might change. First the influence of the curvature in the x-direction will be looked into. By varying k_x with all other parameters fixed, the influence of k_x can be displayed. K_x will be vaired at several values for k_y to see if k_y influences the relation of the curvature in the x-direction with the bending moments.

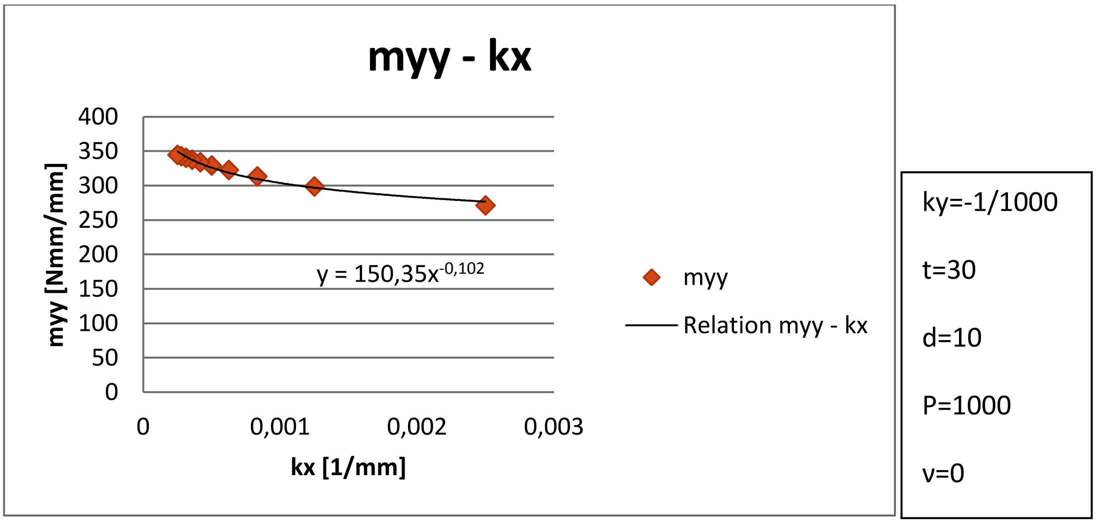


Figure 7-6 Influence of kx on myy

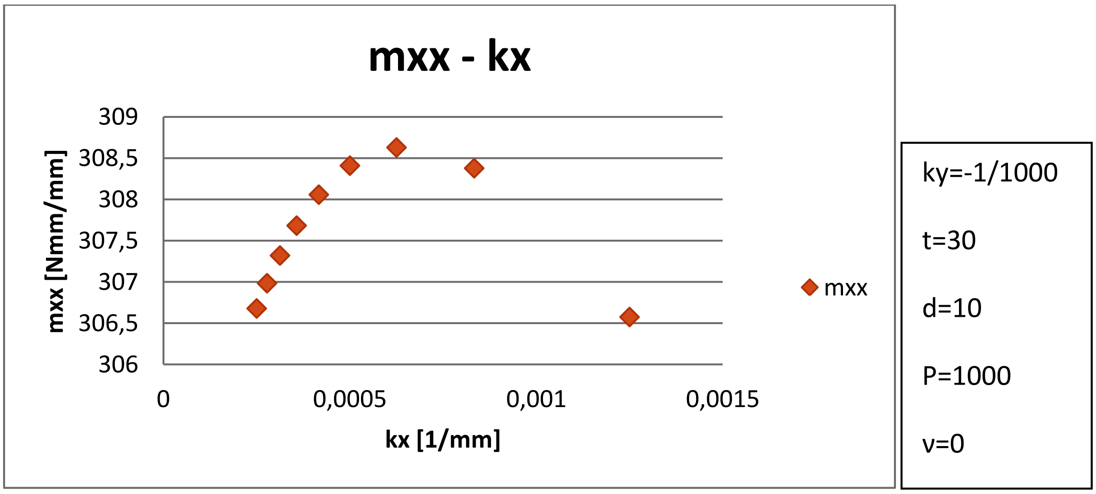


Figure 7-7 Influence of kx on mxx

The first thing that stands out when taking a close look at figures 7-6 and 7-7, is that kx has a relation with myy which does not match the relation between kx and mxx. Another observation is made when looking at the exact values of the bending moments when the curvature in the x-direction is varied. The results, coinciding with the figures above, are displayed in table 7-1.

kx	ky	t	mxx	myy
1/800	-1/1000	30	306,5764	298,8115
1/1200	-1/1000	30	308,3784	313,4707
1/1600	-1/1000	30	308,6294	322,7675
1/2000	-1/1000	30	308,4095	329,2142
1/2400	-1/1000	30	308,0585	333,9673
1/2800	-1/1000	30	307,6835	337,6135
1/3200	-1/1000	30	307,3215	340,4935
1/3600	-1/1000	30	306,9858	342,8228
1/4000	-1/1000	30	306,68	344,7453

Table 7-1 Influence of kx on the bending moments

The bending moments m_{xx} change very little when k_x is varied. Especially when the change in m_{xx} is compared to the change in m_{yy}, the influence of k_x on m_{xx} is negligible. Therefore the choice is made not to include the parameter k_x in the formula for m_{xx}. The formulas for the bending moments become as follows:

$$m_{xx} = \frac{P}{\sqrt[4]{d}} t^{0.15} * f(\,k_y)\ \ for\ l_x = l_y \geq 4\,m\ (\mathbf{7.14})$$

$$m_{yy} = \frac{P}{\sqrt[4]{d}} t^{0.15} r_x^{0.1} * f(\ k_y)\ \ for\ l_x = l_y \geq 4\,m\ \ (\mathbf{7.15})$$

In eq. 7.15 r_x (radius in x-direction) has been included instead of k_x. When going back to table 7-1 and figure 7-7, it can be seen that m_{xx} increases as k_x decreases. This is equivalent to saying that m_{xx} increases as r_x increases. Eq. 7.15 can be written with the parameter k_x instead of r_x. In that case the formula will have the same form as eq. 7.16.

$$m_{yy} = \frac{P}{\sqrt[4]{d}} t^{0.15} \left(\frac{1}{k_x}\right)^{0.1} * f(\ k_y)\ \ for\ l_x = l_y \geq 4\,m\ \ (\mathbf{7.16})$$

7.5.2 Influence of ky on the bending moments

The formulas for m_{xx} and m_{yy} are expected to be symmetrical. Therefore it is probable that the curvature in y-direction will influence the bending moments in the exact opposite way as k_x did. The influence of the curvature in y-direction is displayed in figures 7-8 and 7-9.

As predicted the curvature in the y-direction has the opposite effect of the curvature in x-direction. The influence of k_y on m_{yy} will be neglected. Its influence will be compensated for by the constant A which has yet to be determined. The formulas for the bending moments are as follows:

$$m_{xx} = \frac{P}{\sqrt[4]{d}} t^{0.15} \left(\frac{1}{k_y}\right)^{0.1}\ \ for\ l_x = l_y \geq 4\,m\ (\mathbf{7.17})$$

$$m_{yy} = \frac{P}{\sqrt[4]{d}} t^{0.15} \left(\frac{1}{k_x}\right)^{0.1} \quad for\ l_x = l_y \geq 4\,m \quad \mathbf{(7.18)}$$

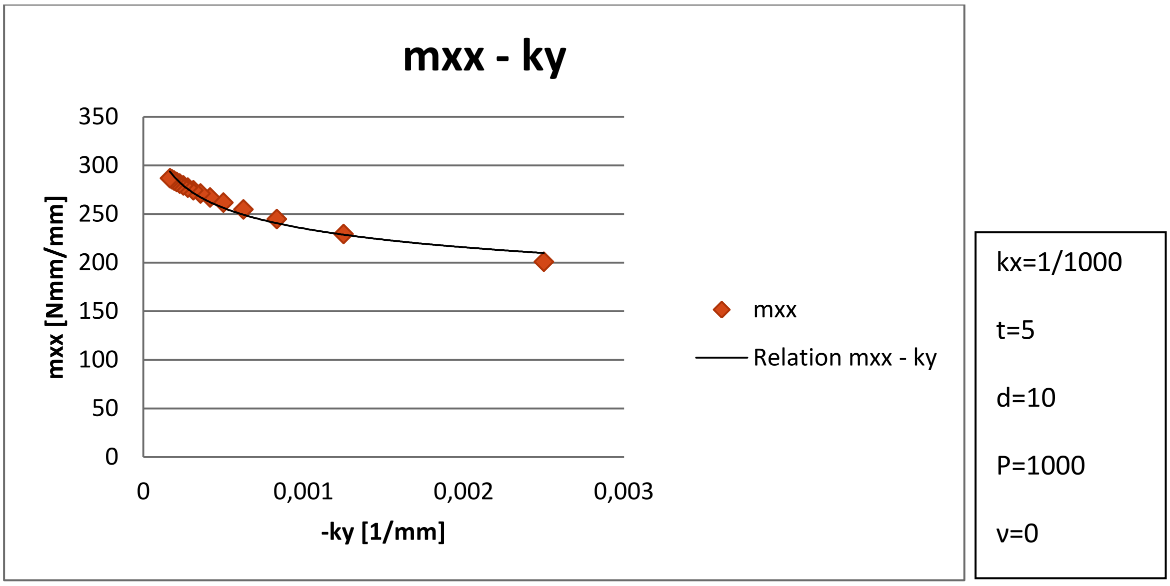


Figure 7-8 Influence of ky on mxx

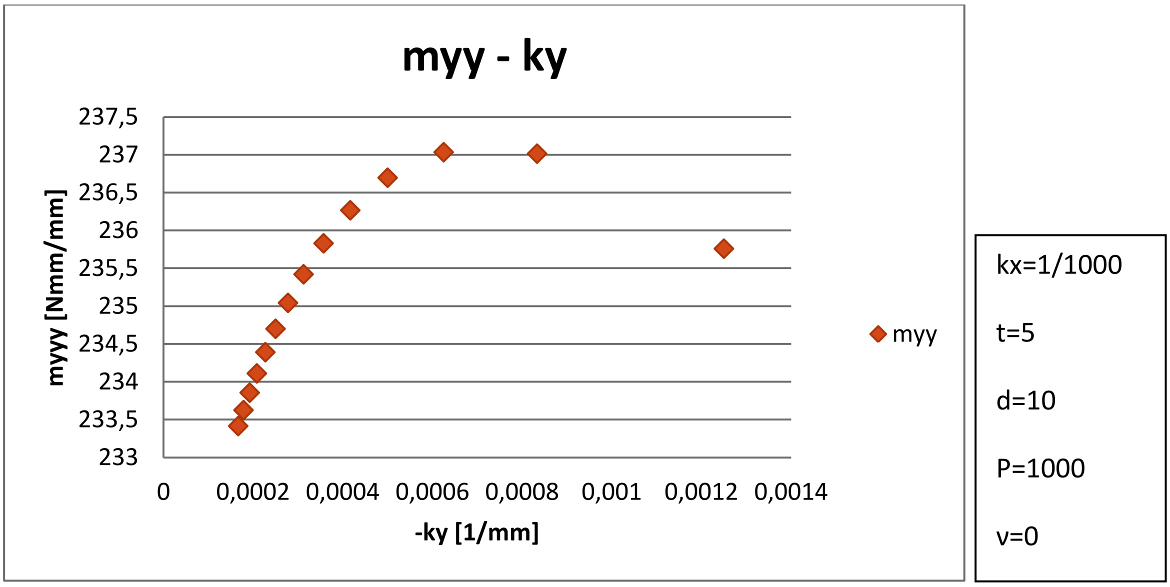


Figure 7-9 Influence of ky on myy

As all parameters but the Poisson's ratio have been taken into account, the dimensions should be correct since v does not have a dimension.

$$[m_{xx}] = \frac{[N]}{[mm]^{0.25}}[mm]^{0.15}\left(\frac{1}{[mm^{-1}]^{0.1}}\right) = \frac{[N]}{[mm]^{0.25}}[mm]^{0.25} = [N] \quad \mathbf{(7.19)}$$

The dimensions of the formula for the bending moments are indeed correct. Since the formula for m_{yy} has the same parameters, all dimensions are equal to those in equation 7.19.

7.6 Influence of ν on the bending moments

The influence of the Poisson's ratio on the bending moments can be determined using the same method that was used for determining the influence of ν on the deformation and normal forces:

- The bending moments are calculated at different values for ν, say m_{nu}.
- The bending moments are calculated for ν=0 ($m_{reference}$).
- By dividing m_{nu} by $m_{reference}$ a ratio C is obtained.
- Analysis of this ratio C at different values of ν will provide us with the influence term for ν.

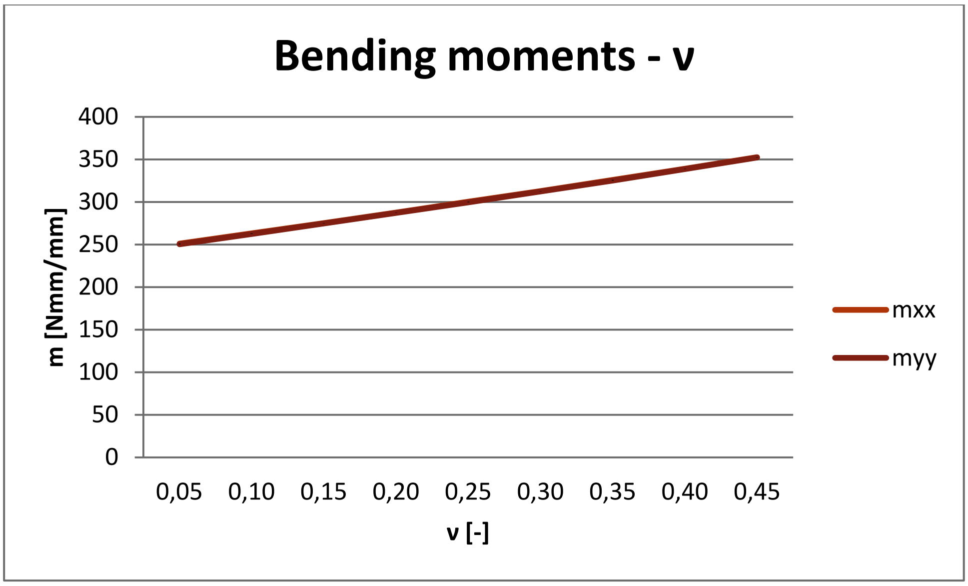


Figure 7-10 Influence ν on bending moments

nu	***t***	***mxx***	***myy***
0,05	5,0000000	250,8691	250,2407
0,10	5,0000000	262,9572	262,3373
0,15	5,0000000	275,1755	274,5642
0,20	5,0000000	287,5436	286,9411
0,25	5,0000000	300,0841	299,4907
0,30	5,0000000	312,8234	312,2393
0,35	5,0000000	325,7932	325,2187
0,40	5,0000000	339,0316	338,467
0,45	5,0000000	352,5859	352,0314

Table 7-2 influence of ν on the bending moments

Reference value:			
nu	t	mxx_ref	myy_ref
0,00	5,0000000	238,8938	238,2572

Table 7-3 Reference value bending moments

nu	***mxx/mxx_ref***	***myy/myy_ref***
0,05	1,050128	1,050296
0,10	1,100728	1,101067
0,15	1,151874	1,152386
0,20	1,203646	1,204333
0,25	1,25614	1,257006
0,30	1,309466	1,310514
0,35	1,363757	1,36499
0,40	1,419173	1,420595
0,45	1,475911	1,477527

Table 7-4 factors C1

The relation between ν and the bending moments is a linear one as can be seen in figure 7-10 and table 7-4. The influence of ν can be expressed by the following term: $(1 + 1.05\nu)$. By including this term in the formula for the bending moments, the formulas are almost complete.

$$m_{xx} = A_x \frac{P}{\sqrt[4]{d}} t^{0.15} \left(\frac{1}{k_y}\right)^{0.1} (1 + 1.05\nu) \;\; for\; l_x = l_y \geq 4\, m \;(\mathbf{7.20})$$

$$m_{yy} = A_y \frac{P}{\sqrt[4]{d}} t^{0.15} \left(\frac{1}{k_x}\right)^{0.1} (1 + 1.05\nu) \;\; for\; l_x = l_y \geq 4\, m \;\; (\mathbf{7.21})$$

7.7 Completing the formulas: determining Ax and Ay

In order to complete the formulas, the factors A_x and A_y have to be determined. If these factors are not applied, the results generated with the formulas with deviate significantly from the results generated in Ansys. A_x and A_y can be determined by dividing the results from Ansys by the results generated with the formula. For instance, if the formula for m_{xx} produces bending moments which are 2.4 times as big as the bending moments in Ansys, the factor A_x will be equal to $\frac{1}{2.4}$ which compensates for the over-estimation. The factors are calculated and depicted in tables 7-5 and 7-6.

t	kx	ky	v	mxx_ansys	myy_formula	mxx_formula	myy_formula
5	1/1000	-0,001	0,25	299,112352	297,630799	1803,336975	1803,337
10	1/500	-0,001	0,25	321,49478	302,852291	2000,927655	1866,931
15	1/2000	-0,002	0,25	327,554082	355,888117	1984,001865	2279,02
15	1/1000	-0,0005	0,25	377,206287	357,995464	2279,019679	2126,4
25	1/1000	-0,002	0,25	349,762228	362,902265	2141,999615	2295,738

Table 7-5 Determining Ax and Ay

Ax	Ay
0,165044	0,165044
0,151356	0,162219
0,179379	0,156158
0,157083	0,168357
0,169422	0,158076

Table 7-6 Ax and Ay

Both factors A_x and A_y linger around 0.16. With this value, the formulas for the bending moments can be completed. The final formulas are given in eq. 7.22 and eq. 7.23.

$$m_{xx} = 0.163\frac{P}{\sqrt[4]{d}}t^{0.15}\left(\frac{1}{k_y}\right)^{0.1}(1+1.05v) \;\; for\; l_x = l_y \geq 4\,m \;(\mathbf{7.22})$$

$$m_{yy} = 0.163\frac{P}{\sqrt[4]{d}}t^{0.15}\left(\frac{1}{k_x}\right)^{0.1}(1+1.05v) \;\; for\; l_x = l_y \geq 4\,m \;(\mathbf{7.23})$$

7.8 Accuracy of the formulas

Even though the formulas for m_{xx} and m_{yy} are symmetrical, the errors between the both formulas and Ansys are quite different. The curvatures and thickness have been varied within the following ranges:

- Thickness: $5 \leq t \leq 50\, mm$
- Curvature in x-direction: $\frac{1}{5000} \leq k_x \leq \frac{1}{500}\, mm^{-1}$
- Curvature in y-direction: $-\frac{1}{500} \leq k_x \leq -\frac{1}{5000}\, mm^{-1}$

The formula for m_{xx} turns out to be quite stable. The errors remain small within the ranges mentioned above. When the limitation $t\left[k_y\right] > \frac{5}{2000}$ is applied, the maximum error will be limited to 4.7%. This error is much higher than the average error.

The formula for m_{yy} is a bit more tedious. The maximum error of 10.8% occurs when the curvature in the y-direction is equal to -1/500 mm^{-1}. Even though k_y has not been included in the formula for m_{yy}, its influence for is quite noticable when curvatures in the y-direction are big. These errors are the result of neglecting k_y in the formula for m_{yy}. The maximum error can be constrained to 5.24% when the following limitations are imposed for m_{yy}:

$$k_y < \frac{1}{500} mm^{-1} \; and \; tk_x > 0.002$$

Chapter 8 Conclusions and recommendations

Formulas have been derived for the deformation, normal forces and bending moments in an anticlastic shell loaded by a perpendicular working point load.

The formulas for the deformation and the bending moments are at least twice as accurate than the formulas for the normal forces. It is recommended to investigate why the formulas for the normal forces are less accurate. Also, the formulas for the normal forces do not represent the influence of the curvatures well. It is unknown how this influence could be modelled more accurately.

The influence of k_{xy} has not been taken into account in the derived formulas. At first sight, its influence on the deformation, normal forces and bending moments seemed quite complicated. Also, the available time span has proven too short to determine the relation between k_{xy} and the deformation, normal forces and bending moment. Further research is recommended.

A. Semiary [2] concluded that the stresses in anticlastic shells were transferred in the directions of zero curvature. It was presumed that the stresses in anticlastic shells propagated along the characteristics of the hyperbolic differential equations. By applying a rectangular mesh instead of a radial mesh, it can be concluded that the stresses are not carried in the directions of zero curvature. Tensile stresses are carried in the direction of negative curvature and compressive stresses are carried in the direction of positive curvature.

When deformations, normal forces and bending moments generated in Ansys are compared to those generated with the solutions of the Sanders-Koiters equations [17], the deformations and normal forces correspond nicely. However, considerable deviations are observed when the bending moments are compared. It is recommended to investigate why these results differ this much.

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Appendix A : Ansys script

```
! anticlastische schaal met scharnierend ondersteunde randen en een puntlast in het midden
! achtknoopselementen, rechthoekige schaal, kleine elementen in het midden, momenten
en normaalkrachten berekend,
! verdeelde puntlast
! P.C.J. Hoogenboom, 24 mei 2013
!
! define variables
kx  = 1/2000          ! 1/mm  kromming in de x-richting
ky  =-1/4000          ! 1/mm  kromming in de y-richting
kxy = 0               ! 1/mm  twist
t   = 5               ! mm    dikte
lx  = 1000            ! mm    lengte in de x-richting
ly  = 2000            ! mm    lengte in de y-richting
E   = 2.1e5           ! N/mm2 elasticiteitsmodulus
nu  = 0.35            ! -     dwarscontractiecoefficient
P   = 1000            ! N  puntlast
d   = 10              ! mm    diameter van het oppervlak waarover de puntlast is verdeeld
nx  = 70              ! -     aantal elementen in de y-richting (moet even zijn)
h   = 1               ! mm    elementgrootte in het midden

/PREP7
MPTEMP,,,,,,,,        ! materiaal: isotroop
MPTEMP,1,0
MPDATA,EX,1,,E
MPDATA,PRXY,1,,nu
ET,1,SHELL281         ! element type: 8 node quadrilateral
R,1,t,t,t,t, , ,      ! element dikte

gx=-2*nx*(nx-2)*(nx+2)      ! knopen
gy=-2*ny*(ny-2)*(ny+2)
mx=(4-nx*nx)*lx
my=(4-ny*ny)*ly
qx=2*nx*(lx-nx*h)
qy=2*ny*(ly-ny*h)
*DO,j,0,ny-1
 *DO,i,0,2*nx
  x=(i-nx)*((nx*h-lx)*i*i+qx*i+mx)/gx
  k=2*j
  y=(k-ny)*((ny*h-ly)*k*k+qy*k+my)/gy
  z1=0.5*kx*x*x
  z2=kxy*x*y
  z3=0.5*ky*y*y
  z=z1+z2+z3
```

```
  N,,x,y,z,,,
 *ENDDO
 *DO,i,0,nx
  k=2*i
  x=(k-nx)*((nx*h-lx)*k*k+qx*k+mx)/gx
  k=2*j+1
  y=(k-ny)*((ny*h-ly)*k*k+qy*k+my)/gy
  z1=0.5*kx*x*x
  z2=kxy*x*y
  z3=0.5*ky*y*y
  z=z1+z2+z3
  N,,x,y,z,,,
 *ENDDO
*ENDDO
*DO,i,0,2*nx
 x=(i-nx)*((nx*h-lx)*i*i+qx*i+mx)/gx
 y=ly/2
 z1=0.5*kx*x*x
 z2=kxy*x*y
 z3=0.5*ky*y*y
 z=z1+z2+z3
 N,,x,y,z,,,
*ENDDO

SHPP,OFF                    ! geen waarschuwingen aspect ratio
*DO,j,1,ny                  ! elementen
 *DO,i,1,nx
  k1=1+(i-1)*2+(j-1)*(3*nx+2)
  k2=1+i+(2+(j-1)*3)*nx+(j-1)*2
  k3=1+(i-1)*2+j*(3*nx+2)
  E,k3,k3+2,k1+2,k1,k3+1,k2+1,k1+1,k2
 *ENDDO
*ENDDO

*DO,i,1,2*nx+1              ! scharnierende randen
 D,i                 ,,0,,,,UX,UY,UZ,,,
 D,(2*nx+1)*ny+(nx+1)*ny+i      ,,0,,,,UX,UY,UZ,,,
*ENDDO
*DO,j,1,ny
 D,(2+(j-1)*3)*nx+2*(j-1)+2  ,,0,,,,UX,UY,UZ,,,
 D,(3+(j-1)*3)*nx+2*(j-1)+2  ,,0,,,,UX,UY,UZ,,,
*ENDDO
*DO,j,2,ny
 D,1+(j-1)*(3*nx+2)        ,,0,,,,UX,UY,UZ,,,
 D,1+(j-1)*(3*nx+2)+2*nx     ,,0,,,,UX,UY,UZ,,,
*ENDDO
```

```
FCUM,ADD                   ! puntlast
n=4*d/h
*DO,k,1,n
 dr=d/2/n
 ro=(k-(3*k-2)/(6*k-3))*dr
 df=2*3.1415/(6*k)
 *DO,l,1,6*k
  x=ro*cos(l*df)
  y=ro*sin(l*df)
  dP=P/(1/4*3.1415*d*d)*dr*ro*df
  r1=d
  m=d/2/h+1
  *DO,j,(ny/2-m),(ny/2+m)
   *DO,i,(nx/2-m),(nx/2+m)
    p=(2*nx+1)*j+(nx+1)*(j-1)+i+1
    *GET,xp,NODE,p,LOC,X
    *GET,yp,NODE,p,LOC,Y
    rp=SQRT((xp-x)**2+(yp-y)**2)
    *IF,rp,LT,r1,THEN
     r1=rp
     n1=p
    *ENDIF
    p=(2*nx+1)*j+(nx+1)*j+2*i
    *GET,xp,NODE,p,LOC,X
    *GET,yp,NODE,p,LOC,Y
    rp=SQRT((xp-x)**2+(yp-y)**2)
    *IF,rp,LT,r1,THEN
     r1=rp
     n1=p
    *ENDIF
    p=(2*nx+1)*j+(nx+1)*j+2*i+1
    *GET,xp,NODE,p,LOC,X
    *GET,yp,NODE,p,LOC,Y
    rp=SQRT((xp-x)**2+(yp-y)**2)
    *IF,rp,LT,r1,THEN
     r1=rp
     n1=p
    *ENDIF
   *ENDDO
  *ENDDO
  F,n1,FZ,dP
 *ENDDO
*ENDDO
FINISH
```

```
/SOLU                     ! bereken
SOLVE
FINISH

/POST1                    ! momenten en normaalkrachten
k=(2*nx+1)*ny/2+(nx+1)*ny/2+nx+1
*GET,uz,NODE,k,u,z        ! doorbuiging uz onder de puntlast
SHELL,TOP
*GET,sxxt,NODE,k,S,X      ! spanning sxx onder de puntlast in het top-oppervlak van de
schaal (z<0)
*GET,syyt,NODE,k,S,Y      ! spanning syy
*GET,sxyt,NODE,k,S,XY     ! spanning sxy
SHELL,BOT
*GET,sxxb,NODE,k,S,X      ! spanning sxx onder de puntlast in het bottom-oppervlak van de
schaal (z>0)
*GET,syyb,NODE,k,S,Y      ! spanning syy
*GET,sxyb,NODE,k,S,XY     ! spanning sxy
nxx=(sxxb+sxxt)*t/2
nyy=(syyb+syyt)*t/2
nxy=(sxyb+sxyt)*t/2
mxx=(sxxb-sxxt)*t*t/12
myy=(syyb-syyt)*t*t/12
mxy=(sxyb-sxyt)*t*t/12
FINISH
```

Appendix B : Optimization mesh (concentrated load)

Step 1: Variation of m; n=50; t=5			
m	w	Δw [%]	t [min]
50	0,412894		<1
100	0,413502	0,147253	<1
150	0,413769	0,06457	<1
200	0,413938	0,040844	2
250	0,414061	0,029715	8
300	0,414158	0,023426	9

Variation of m; n=100; t=5			
m	w	Δw [%]	t [min]
50	0,413237		<1
100	0,413608	0,089779	2
150	0,413824	0,052223	7
200	0,413971	0,035522	9
250	0,414086	0,02778	24

Variation of m; n=150; t=5			
m	w	Δw [%]	t [min]
50	0,413452		1
100	0,413686	0,056597	20

Step 2: Variation of m; n=50; t=40			
m	w	Δw [%]	t [min]
50	0,005073		<1
100	0,005111	0,749064	<1
150	0,005135	0,469575	1
200	0,005152	0,331061	1
250	0,005165	0,252329	3
300	0,005176	0,212972	3

Variation of m; n=100; t=40			
m	w	Δw [%]	t [min]
50	0,005088		<1
100	0,005116	0,550314	<1
150	0,005137	0,410477	6
200	0,005154	0,330932	10

Optimization number of elements outside middle strips

thickness=5 mm				
nx	ny	w	Δw [%]	t [min]
50	50	0,450817		<1
100	50	0,450815	0,000444	<1
150	50	0,450815	0	1
200	50	0,450814	0,000222	2
250	50	0,450814	0	7
50	100	0,450817		<1
100	100	0,450815	0,000444	1
150	100	0,450815	0	8
200	100	0,450815	0	12
250	100	0,450815	0	15
50	150	0,450817		1
100	150	0,450815	0,000444	6
150	150	0,450816	0,000222	20
200	150	0,450816	0	28
250	150	0,450816		40

thickness=40 mm				
nx	ny	w	Δw [%]	t [min]
50	50	0,005041		<1
100	50	0,005041	0	<1
150	50	0,005041	0	1
200	50	0,005041	0	4
250	50	0,005041	0	10

Optimization number of elements middle strips

nx=50; ny=50; t=40				
nx	ny	h	w	Δw [%]
50	50	10	0,005009	
50	50	9	0,005014	0,09982
50	50	8	0,00502	0,119665
50	50	7	0,005026	0,119522

50	50	6	0,005033	0,139276
50	50	5	0,005041	0,158951
50	50	4	0,005051	0,198373
50	50	3	0,005065	0,277173
50	50	2	0,005083	0,35538
50	50	1	0,005115	0,629549

nx=100; ny=100; t=40				
nx	ny	h	w	Δw [%]
100	100	10	0,005009	
100	100	9	0,005014	0,09982
100	100	8	0,00502	0,119665
100	100	7	0,005026	0,119522
100	100	6	0,005033	0,139276
100	100	5	0,005041	0,158951
100	100	4	0,005051	0,198373
100	100	3	0,005064	0,257375
100	100	2	0,005083	0,375197
100	100	1	0,005114	0,609876

nx=50; ny=50; t=5					
t	nx	ny	h	w	Δw [%]
5	50	50	10	0,450514	
5	50	50	9	0,450565	0,01132
5	50	50	8	0,45062	0,012207
5	50	50	7	0,450678	0,012871
5	50	50	6	0,450743	0,014423
5	50	50	5	0,450817	0,016417
5	50	50	4	0,450905	0,01952
5	50	50	3	0,451015	0,024395
5	50	50	2	0,451167	0,033702
5	50	50	1	0,451427	0,057628

nx=100; ny=100;t=5					
t	nx	ny	h	w	Δw [%]
5	100	100	10	0,450513	
5	100	100	9	0,450564	0,01132
5	100	100	8	0,450618	0,011985
5	100	100	7	0,450677	0,013093
5	100	100	6	0,450742	0,014423
5	100	100	5	0,450815	0,016196
5	100	100	4	0,450902	0,019298
5	100	100	3	0,451012	0,024396

5	100	100	2	0,451163	0,03348
5	100	100	1	0,451417	0,056299

t	***nx***	***ny***	***h***	***w***	***Δw [%]***
10	50	50	10	0,097157	
10	50	50	9	0,097178	0,021615
10	50	50	8	0,097201	0,023668
10	50	50	7	0,097226	0,02572
10	50	50	6	0,097256	0,030856
10	50	50	5	0,09729	0,034959
10	50	50	4	0,097331	0,042142
10	50	50	3	0,097385	0,055481
10	50	50	2	0,09746	0,077014
10	50	50	1	0,097589	0,132362

t	***nx***	***ny***	***h***	***w***	***Δw [%]***
20	50	50	10	0,021468	
20	50	50	9	0,021478	0,046581
20	50	50	8	0,021489	0,051215
20	50	50	7	0,021501	0,055843
20	50	50	6	0,021515	0,065113
20	50	50	5	0,021532	0,079015
20	50	50	4	0,021552	0,092885
20	50	50	3	0,021579	0,125278
20	50	50	2	0,021616	0,171463
20	50	50	1	0,021681	0,300703

t	***nx***	***ny***	***h***	***w***	***Δw [%]***
30	50	50	10	0,009112	
30	50	50	9	0,009118	0,065847
30	50	50	8	0,009125	0,076771
30	50	50	7	0,009133	0,087671
30	50	50	6	0,009143	0,109493
30	50	50	5	0,009154	0,120311
30	50	50	4	0,009168	0,152939
30	50	50	3	0,009185	0,185428
30	50	50	2	0,00921	0,272183
30	50	50	1	0,009253	0,466884

nx=50; ny=50; t=5; lx=500; ly=500					
lx; ly	nx	ny	h	w	Δw [%]
500	50	50	10	0,300726	
500	50	50	9	0,300777	0,016959
500	50	50	8	0,300832	0,018286
500	50	50	7	0,30089	0,01928
500	50	50	6	0,300955	0,021603
500	50	50	5	0,301028	0,024256
500	50	50	4	0,301115	0,028901
500	50	50	3	0,301225	0,036531
500	50	50	2	0,301376	0,050129
500	50	50	1	0,301631	0,084612

nx=50; ny=50; t=5; lx=750; ly=750					
lx; ly	nx	ny	h	w	Δw [%]
750	50	50	10	0,363825	
750	50	50	9	0,363876	0,014018
750	50	50	8	0,363931	0,015115
750	50	50	7	0,363989	0,015937
750	50	50	6	0,364054	0,017858
750	50	50	5	0,364128	0,020327
750	50	50	4	0,364215	0,023893
750	50	50	3	0,364325	0,030202
750	50	50	2	0,364477	0,041721
750	50	50	1	0,364734	0,070512

nx=50; ny=50; t=5; lx=1000; ly=1000					
lx; ly	nx	ny	h	w	Δw [%]
1000	50	50	10	0,422362	
1000	50	50	9	0,422413	0,012075
1000	50	50	8	0,422468	0,01302
1000	50	50	7	0,422527	0,013966
1000	50	50	6	0,422592	0,015384
1000	50	50	5	0,422666	0,017511
1000	50	50	4	0,422754	0,02082
1000	50	50	3	0,422864	0,02602
1000	50	50	2	0,423016	0,035945
1000	50	50	1	0,423274	0,060991

nx=50; ny=50; t=5; lx=1500; ly=1500					
lx; ly	nx	ny	h	w	Δw [%]
1500	50	50	10	0,496283	
1500	50	50	9	0,496334	0,010276
1500	50	50	8	0,496389	0,011081

1500	50	50	7	0,496448	0,011886
1500	50	50	6	0,496513	0,013093
1500	50	50	5	0,496586	0,014703
1500	50	50	4	0,496674	0,017721
1500	50	50	3	0,496784	0,022147
1500	50	50	2	0,496936	0,030597
1500	50	50	1	0,497196	0,052321
1500	50	50	0,5	0,497459	0,052897
1500	50	50	0,25	0,497734	0,055281
1500	50	50	0,1	0,498142	0,081971

Appendix C: Distributed pointload

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
10	10	1	1,87702		307,892	
10	5	2	1,88155	0,24134	300,21	-2,49503
10	2,5	4	1,94111	3,165475	299,696	-0,17121
10	1,25	8	2,0763	6,964572	318,699	6,340759
10	1	10	2,14499	3,308289	328,894	3,198943
10	0,5	20	2,49012	16,09005	381,882	16,11097

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
9	h	d/h	w	Δw [%]	σ	Δσ
9	9	1	1,87805		316,267	
9	4,5	2	1,88253	0,238545	308,608	-2,42169
9	3	3	1,91058	1,490016	307,363	-0,40342
9	1,5	6	2,00897	5,149745	317,802	3,39631
9	1	9	2,11166	5,111575	333,758	5,020736
9	0,75	12	2,21493	4,890465	349,401	4,686929
9	0,5	18	2,42206	9,351537	381,716	9,248686

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ [%]
8	8	1	1,87903		325,641	
8	4	2	1,88347	0,236292	318,011	-2,34307
8	2	4	1,94288	3,154284	318,051	0,012578
8	1	8	2,07816	6,962859	338,36	6,385454
8	0,5	16	2,35399	13,2728	382,659	13,09227

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ [%]
6	6	1	1,88089		348,591	
6	3	2	1,88522	0,023021	341,044	-2,165
6	1,5	4	1,94454	0,314658	342,096	0,308465
6	1	6	2,01152	0,344452	352,5	3,041252
6	0,5	12	2,21767	1,024847	387,247	9,857305

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ [%]
5	5	1	1,88172		363,182	
5	2,5	2	1,88605	0,230109	355,696	-2,06123
5	1,25	4	1,94533	3,143077	357,255	0,438296
5	1	5	1,97849	1,704595	362,173	1,376608
5	0,5	10	2,14954	8,645482	392,288	8,315087

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ [%]
4	4	1	1,88255		381,1	
4	2	2	1,88686	0,228945	373,693	-1,94358
4	1	4	1,9461	3,139608	376,013	0,62083
4	0,5	8	2,08152	6,958532	400,071	6,398183

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ [%]
2	2	1	1,88434		437,322	
2	1	2	1,88861	0,226605	430,435	-1,57481
2	0,5	4	1,9478	3,134051	433,813	0,784787
2	0,25	8	2,0833	6,956566	461,115	6,293495

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
10	10	1	0,020678		5,94792	
10	5	2	0,020692	0,067704807	5,83004	-1,98187
10	2,5	4	0,021324	3,05432051	5,85763	0,473239
10	1,25	8	0,022806	6,949915588	6,23618	6,462511

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
9	9	1	0,020701		6,07882	
9	4,5	2	0,020714	0,062798899	5,96146	-1,93064
9	2,25	4	0,021347	3,055904219	5,99214	0,514639
9	1,125	8	0,02283	6,947112006	6,38355	6,532057

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
8	8	1	0,020725		6,22535	
8	4	2	0,020738	0,062726176	6,10858	-1,87572
8	2	4	0,021372	3,0571897	6,14481	0,5931
8	1	8	0,022857	6,948343627	6,54373	6,491983

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
7	7	1	0,020752		6,39174	
7	3,5	2	0,020766	0,067463377	6,27564	-1,81641
7	1,75	4	0,0214	3,053067514	6,31961	0,700646
7	0,875	8	0,022886	6,943925234	6,7275	6,454354

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
6	6	1	0,020783		6,5842	
6	3	2	0,020797	0,067362748	6,4689	-1,75116
6	1,5	4	0,021433	3,058133385	6,52091	0,804001
6	0,75	8	0,022919	6,933233798	6,93407	6,335926

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
5	5	1	0,02082		6,81239	
5	2,5	2	0,020833	0,062439962	6,69806	-1,67827
5	1,25	4	0,02147	3,057648922	6,75797	0,894438
5	0,625	8	0,02296	6,939916162	7,19131	6,412281

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
4	4	1	0,020864		7,09263	
4	2	2	0,020877	0,062308282	6,97954	-1,59447
4	1	4	0,021516	3,060784595	7,0512	1,026715
4	0,5	8	0,023009	6,939022123	7,50843	6,484428

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
2	2	1	0,020999		7,97195	
2	1	2	0,021014	0,071431973	7,86668	-1,32051
2	0,5	4	0,021656	3,05510612	7,95451	1,116481
2	0,25	8	0,023156	6,926486886	8,46171	6,376257

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
1	10	1,87702	
1	9	1,87805	0,054874216
1	8	1,87903	0,052181784
1	6	1,88089	0,098987243
1	5	1,88172	0,044128046
1	4	1,88255	0,044108582
1	2	1,88434	0,095083796

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
2	10	1,88155	
2	9	1,88253	0,052084717
2	8	1,88347	0,049932803
2	6	1,88522	0,092913612
2	5	1,88605	0,044026692
2	4	1,88686	0,0429469
2	2	1,88861	0,09274668

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
4	10	1,94111	
4	8	1,94288	0,091184941
4	6	1,94454	0,085440171
4	5	1,94533	0,040626575
4	4	1,9461	0,039581973
4	2	1,9478	0,087354196

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
1	10	0,020678	
1	9	0,020701	0,111229326
1	8	0,020725	0,115936428
1	7	0,020752	0,130277443
1	6	0,020783	0,149383192
1	5	0,02082	0,178030121
1	4	0,020864	0,211335255
1	2	0,020999	0,647047546

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
2	10	0,020692	
2	9	0,020714	0,106321284
2	8	0,020738	0,115863667

2	7	0,020766	0,135017842
2	6	0,020797	0,149282481
2	5	0,020833	0,17310189
2	4	0,020877	0,211203379
2	2	0,021014	0,656224553

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
4	10	0,021324	
4	9	0,021347	0,107859689
4	8	0,021372	0,117112475
4	7	0,0214	0,13101254
4	6	0,021433	0,154205607
4	5	0,02147	0,17263099
4	4	0,021516	0,214252445
4	2	0,021656	0,650678565

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;			
d/h	d	w	Δw [%]
8	10	0,022806	
8	9	0,02283	0,105235464
8	8	0,022857	0,11826544
8	7	0,022886	0,126875793
8	6	0,022919	0,144192956
8	5	0,02296	0,178890877
8	4	0,023009	0,213414634
8	2	0,023156	0,638880438

As from now, a nonzero value is assumed for nu (nu=0.35).

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
10	10	1	0,021097		8,06716	
10	5	2	0,021099	0,00948	7,91563	-1,87836
10	2,5	4	0,021737	3,02384	7,95768	0,531227
10	1,25	8	0,023246	6,94208	8,47222	6,465955

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
9	9	1	0,021126		8,2449	
9	4,5	2	0,021128	0,009467	8,09394	-1,83095
9	2,25	4	0,021767	3,024423	8,13979	0,566473
9	1,125	8	0,023279	6,946295	8,67203	6,538744

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
8	8	1	0,021159		8,44381	
8	4	2	0,02116	0,004726	8,2935	-1,78012
8	2	4	0,0218	3,024575	8,3469	0,643878
8	1	8	0,023314	6,944954	8,88904	6,495106

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
7	7	1	0,021195		8,6696	
7	3,5	2	0,021196	0,004718	8,52005	-1,72499
7	1,75	4	0,021838	3,028873	8,58405	0,751169
7	0,875	8	0,023353	6,937448	9,13798	6,453015

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
6	6	1	0,021236		8,93068	
6	3	2	0,021237	0,004709	8,78204	-1,66437
6	1,5	4	0,021881	3,032443	8,85701	0,853674
6	0,75	8	0,023397	6,928385	9,41824	6,336563

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
5	5	1	0,021284		9,24012	
5	2,5	2	0,021286	0,009397	9,09261	-1,59641
5	1,25	4	0,021931	3,030161	9,17847	0,944283
5	0,625	8	0,023451	6,930829	9,7665	6,406623

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
4	4	1	0,021343		9,61998	
4	2	2	0,021345	0,009371	9,47395	-1,51799
4	1	4	0,021993	3,03584	9,5749	1,065553
4	0,5	8	0,023518	6,934024	10,1961	6,487796

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
2	2	1	0,021525		10,811	
2	1	2	0,021529	0,018583	10,6735	-1,27185
2	0,5	4	0,02218	3,023828	10,7981	1,167377
2	0,25	8	0,023715	6,920649	11,4831	6,343709

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
10	10	1	1,72319		415,301	
10	5	2	1,72721	0,233288	405,409	-2,38189
10	2,5	4	1,78179	3,160009	404,948	-0,11371
10	1,25	8	1,90588	6,964345	430,643	6,345259

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
9	9	1	1,72417		426,681	
9	4,5	2	1,72815	0,230836	416,818	-2,31156
9	2,25	4	1,78266	3,15424	416,6	-0,0523
9	1,125	8	1,90682	6,964873	443,432	6,440711

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
8	8	1	1,72512		439,415	
8	4	2	1,72906	0,22839	429,588	-2,23638
8	2	4	1,78352	3,149688	429,854	0,06192
8	1	8	1,9077	6,962636	457,325	6,390775

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
7	7	1	1,72604		453,869	
7	3,5	2	1,72995	0,22653	444,085	-2,15569
7	1,75	4	1,78438	3,146334	445,03	0,212797
7	0,875	8	1,90857	6,95984	473,265	6,344516

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
6	6	1	1,72694		470,579	
6	3	2	1,73082	0,224675	460,851	-2,06724
6	1,5	4	1,78521	3,142441	462,504	0,358684
6	0,75	8	1,90941	6,957165	491,216	6,207946

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
5	5	1	1,72783		490,384	
5	2,5	2	1,73168	0,222823	480,728	-1,96907
5	1,25	4	1,78604	3,139148	483,086	0,490506
5	0,625	8	1,91029	6,956731	513,527	6,301362

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
4	4	1	1,72873		514,695	
4	2	2	1,73255	0,220971	505,136	-1,85722
4	1	4	1,78689	3,136417	508,47	0,66002
4	0,5	8	1,91122	6,957899	541,062	6,409818

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000;						
d	h	d/h	w	Δw [%]	σ	Δσ
2	2	1	1,73081		590,926	
2	1	2	1,73461	0,21955	581,935	-1,52151
2	0,5	4	1,78891	3,130387	586,854	0,845283
2	0,25	8	1,91333	6,955073	623,723	6,282483

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
1	10	1,72319	
1	9	1,72417	0,056871268
1	8	1,72512	0,055098975
1	7	1,72604	0,053329623
1	6	1,72694	0,052142476
1	5	1,72783	0,051536243
1	4	1,72873	0,052088458
1	2	1,73081	0,120319541

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35

d/h	d	w	Δw [%]
2	10	1,72721	
2	9	1,72815	0,054423029
2	8	1,72906	0,052657466
2	7	1,72995	0,051473055
2	6	1,73082	0,050290471
2	5	1,73168	0,049687431
2	4	1,73255	0,050240229
2	2	1,73461	0,118899887

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
4	10	1,78179	
4	9	1,78266	0,048827303
4	8	1,78352	0,048242514
4	7	1,78438	0,048219252
4	6	1,78521	0,046514756
4	5	1,78604	0,04649313
4	4	1,78689	0,047591319
4	2	1,78891	0,113045571

t=5; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
8	10	1,90588	
8	9	1,90682	0,049321049
8	8	1,9077	0,046150135
8	7	1,90857	0,045604655
8	6	1,90941	0,044012009
8	5	1,91029	0,046087535
8	4	1,91122	0,048683708
8	2	1,91333	0,110400686

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
1	10	0,021097	
1	9	0,021126	0,137460302
1	8	0,021159	0,156205623
1	7	0,021195	0,170140366
1	6	0,021236	0,193441849
1	5	0,021284	0,226031268
1	4	0,021343	0,277203533
1	2	0,021525	0,852738603

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
2	10	0,021099	
2	9	0,021128	0,137447272
2	8	0,02116	0,151457781
2	7	0,021196	0,170132325
2	6	0,021237	0,193432723
2	5	0,021286	0,230729387
2	4	0,021345	0,277177488
2	2	0,021529	0,862028578

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
4	10	0,021737	
4	9	0,021767	0,138013525
4	8	0,0218	0,151605642
4	7	0,021838	0,174311927
4	6	0,021881	0,196904478
4	5	0,021931	0,228508752
4	4	0,021993	0,282704847
4	2	0,02218	0,850270541

t=40; lx=1000; ly=2000; kx=1/4000; ky=-1/4000; nu=0,35			
d/h	d	w	Δw [%]
8	10	0,023246	
8	9	0,023279	0,141959907
8	8	0,023314	0,150350101
8	7	0,023353	0,167281462
8	6	0,023397	0,188412624
8	5	0,023451	0,23079882
8	4	0,023518	0,285702102
8	2	0,023715	0,837656263

Appendix D Data formula deformation

1) **Influence of length on deformation**

lx	*lx*	*t*	*kx*	*ky*	*w*
200	200	5	1/4000	-1/4000	0,185158876
400	400	5	1/4000	-1/4000	0,288312494
600	600	5	1/4000	-1/4000	0,321368001
800	800	5	1/4000	-1/4000	0,375473982
1000	1000	5	1/4000	-1/4000	0,420884465
1200	1200	5	1/4000	-1/4000	0,45619033
1400	1400	5	1/4000	-1/4000	0,483462273
1600	1600	5	1/4000	-1/4000	0,505002183
1800	1800	5	1/4000	-1/4000	0,523039648
2000	2000	5	1/4000	-1/4000	0,539062388
2200	2200	5	1/4000	-1/4000	0,553782887
2400	2400	5	1/4000	-1/4000	0,567418161
2600	2600	5	1/4000	-1/4000	0,579984341
2800	2800	5	1/4000	-1/4000	0,591492666
3000	3000	5	1/4000	-1/4000	0,602007021
3200	3200	5	1/4000	-1/4000	0,611638054
3400	3400	5	1/4000	-1/4000	0,620504189
3600	3600	5	1/4000	-1/4000	0,628718917
3800	3800	5	1/4000	-1/4000	0,636365299
4000	4000	5	1/4000	-1/4000	0,643512103
4200	4200	5	1/4000	-1/4000	0,650205598
4400	4400	5	1/4000	-1/4000	0,656485245
4600	4600	5	1/4000	-1/4000	0,662377615
4800	4800	5	1/4000	-1/4000	0,667911981
5000	5000	5	1/4000	-1/4000	0,673114824
5200	5200	5	1/4000	-1/4000	0,678004827
5400	5400	5	1/4000	-1/4000	0,682607172
5600	5600	5	1/4000	-1/4000	0,686939298
5800	5800	5	1/4000	-1/4000	0,691021829
6000	6000	5	1/4000	-1/4000	0,69486961
400	400	10	1/4000	-1/4000	0,066514769
800	800	10	1/4000	-1/4000	0,078740064
1200	1200	10	1/4000	-1/4000	0,09747334
1600	1600	10	1/4000	-1/4000	0,111840426
2000	2000	10	1/4000	-1/4000	0,121809038
2400	2400	10	1/4000	-1/4000	0,128994342
2800	2800	10	1/4000	-1/4000	0,134793109
3200	3200	10	1/4000	-1/4000	0,139867827
3600	3600	10	1/4000	-1/4000	0,144375033
4000	4000	10	1/4000	-1/4000	0,148336299
4400	4400	10	1/4000	-1/4000	0,151803916

4800	4800	10	1/4000	-1/4000	0,154858168
5200	5200	10	1/4000	-1/4000	0,157580313
5600	5600	10	1/4000	-1/4000	0,160028918
6000	6000	10	1/4000	-1/4000	0,162246143
6400	6400	10	1/4000	-1/4000	0,164260166
6800	6800	10	1/4000	-1/4000	0,166094303
7200	7200	10	1/4000	-1/4000	0,167768103
7600	7600	10	1/4000	-1/4000	0,169298404
8000	8000	10	1/4000	-1/4000	0,170699912
8400	8400	10	1/4000	-1/4000	0,17198627
8800	8800	10	1/4000	-1/4000	0,173166689
9200	9200	10	1/4000	-1/4000	0,174251737
9600	9600	10	1/4000	-1/4000	0,17524965
10000	10000	10	1/4000	-1/4000	0,176167751
10400	10400	10	1/4000	-1/4000	0,177013464
10800	10800	10	1/4000	-1/4000	0,17779132
11200	11200	10	1/4000	-1/4000	0,178506515
11600	11600	10	1/4000	-1/4000	0,179164679
12000	12000	10	1/4000	-1/4000	0,179769888
14000	14000	10	1/4000	-1/4000	0,182131808
16000	16000	10	1/4000	-1/4000	0,183658309
18000	18000	10	1/4000	-1/4000	0,18461537
20000	20000	10	1/4000	-1/4000	0,185185325
22000	22000	10	1/4000	-1/4000	0,185490257
24000	24000	10	1/4000	-1/4000	0,185614529
26000	26000	10	1/4000	-1/4000	0,185606828

2) Influence of d on deformation

d	***d/h***	***h***	***t***	***kx***	***ky***	***w***
50	4	12,5	5	1/4000	-1/4000	0,742926487
45	4	11,25	5	1/4000	-1/4000	0,744953589
40	4	10	5	1/4000	-1/4000	0,746823444
35	4	8,75	5	1/4000	-1/4000	0,748521162
30	4	7,5	5	1/4000	-1/4000	0,750024901
25	4	6,25	5	1/4000	-1/4000	0,751326219
20	4	5	5	1/4000	-1/4000	0,752412598
15	4	3,75	5	1/4000	-1/4000	0,753269911
10	4	2,5	5	1/4000	-1/4000	0,753880669
5	4	1,25	5	1/4000	-1/4000	0,754256771
50	4	12,5	30	1/4000	-1/4000	0,019269548
45	4	11,25	30	1/4000	-1/4000	0,019290479
40	4	10	30	1/4000	-1/4000	0,019311086
35	4	8,75	30	1/4000	-1/4000	0,019331303
30	4	7,5	30	1/4000	-1/4000	0,019351306
25	4	6,25	30	1/4000	-1/4000	0,019371344
20	4	5	30	1/4000	-1/4000	0,019392586

15	4	3,75	30	1/4000	-1/4000	0,019416809
10	4	2,5	30	1/4000	-1/4000	0,01944689
5	4	1,25	30	1/4000	-1/4000	0,019493254
50	4	12,5	40	1/4000	-1/4000	0,010633174
45	4	11,25	40	1/4000	-1/4000	0,010644619
40	4	10	40	1/4000	-1/4000	0,010656176
35	4	8,75	40	1/4000	-1/4000	0,010667818
30	4	7,5	40	1/4000	-1/4000	0,010679711
25	4	6,25	40	1/4000	-1/4000	0,010692059
20	4	5	40	1/4000	-1/4000	0,010705765
15	4	3,75	40	1/4000	-1/4000	0,010722207
10	4	2,5	40	1/4000	-1/4000	0,010743579
5	4	1,25	40	1/4000	-1/4000	0,010777787
50	4	12,5	40	1/2000	-1/2000	5,38E-03
45	4	11,25	40	1/2000	-1/2000	0,005395003
40	4	10	40	1/2000	-1/2000	0,005405866
35	4	8,75	40	1/2000	-1/2000	0,005416888
30	4	7,5	40	1/2000	-1/2000	0,005428233
25	4	6,25	40	1/2000	-1/2000	0,005440118
20	4	5	40	1/2000	-1/2000	0,005453432
15	4	3,75	40	1/2000	-1/2000	0,005469565
10	4	2,5	40	1/2000	-1/2000	0,005490699
5	4	1,25	40	1/2000	-1/2000	0,005524755

3) Influence of t on deformation

t	*kx*	*ky*	*w*
5	1/4000	-1/4000	0,753880669
10	1/4000	-1/4000	0,185185325
15	1/4000	-1/4000	0,080913535
20	1/4000	-1/4000	0,044832455
25	1/4000	-1/4000	0,028320231
30	1/4000	-1/4000	0,01944689
35	1/4000	-1/4000	0,014149961
40	1/4000	-1/4000	0,010743579
45	1/4000	-1/4000	0,008427743
50	1/4000	-1/4000	0,006783931
5	1/1000	-1/1000	0,17830635
10	1/1000	-1/1000	0,044561225
15	1/1000	-1/1000	0,019797547
20	1/1000	-1/1000	0,011144778
25	1/1000	-1/1000	0,007146719
30	1/1000	-1/1000	0,00497823
35	1/1000	-1/1000	0,003672268
40	1/1000	-1/1000	0,002825302
45	1/1000	-1/1000	0,002244789
50	1/1000	-1/1000	0,001829473

5	1/500	-1/500	0,086534698
10	1/500	-1/500	0,021914117
15	1/500	-1/500	0,009859107
20	1/500	-1/500	0,005621669
25	1/500	-1/500	0,003651623
30	1/500	-1/500	0,002575745
35	1/500	-1/500	0,001923026
40	1/500	-1/500	0,001496582
45	1/500	-1/500	0,001202237
50	1/500	-1/500	0,000990246

4) Influence of curvatures in deformation

kx	*ky*	*kxy*	*t*	*w*
1/400	-1E-10	0	5	0,486773152
1/800	-1E-10	0	5	0,814341827
1/1200	-1E-10	0	5	1,10286802
1/1600	-1E-10	0	5	1,36887073
1/2000	-1E-10	0	5	1,61922042
1/2400	-1E-10	0	5	1,85769672
1/2800	-1E-10	0	5	2,08669431
1/3200	-1E-10	0	5	2,30785919
1/3600	-1E-10	0	5	2,52238738
1/4000	-1E-10	0	5	2,73118559
1/4400	-1E-10	0	5	2,93496262
1/4800	-1E-10	0	5	3,13428357
1/5200	-1E-10	0	5	3,32960675
1/5600	-1E-10	0	5	3,52131096
1/6000	-1E-10	0	5	3,70971578
1/400	-1/1000	0	5	0,108025741
1/800	-1/1000	0	5	0,157969288
1/1200	-1/1000	0	5	0,195360249
1/1600	-1/1000	0	5	0,226335828
1/2000	-1/1000	0	5	0,253066538
1/2400	-1/1000	0	5	0,276808179
1/2800	-1/1000	0	5	0,298055657
1/3200	-1/1000	0	5	0,317709965
1/3600	-1/1000	0	5	0,335690541
1/4000	-1/1000	0	5	0,352508152
1/4400	-1/1000	0	5	0,368285295
1/4800	-1/1000	0	5	0,383034493
1/5200	-1/1000	0	5	0,396968262
1/5600	-1/1000	0	5	0,410220964
1/6000	-1/1000	0	5	0,422820432
1/400	-1/2000	0	5	0,149559663
1/800	-1/2000	0	5	0,22350574
1/1200	-1/2000	0	5	0,27972531

1/1600	-1/2000	0	5	0,32670956
1/2000	-1/2000	0	5	0,367698189
1/2400	-1/2000	0	5	0,403746038
1/2800	-1/2000	0	5	0,436837504
1/3200	-1/2000	0	5	0,466901077
1/3600	-1/2000	0	5	0,495045526
1/4000	-1/2000	0	5	0,521133657
1/4400	-1/2000	0	5	0,545745513
1/4800	-1/2000	0	5	0,56901125
1/5200	-1/2000	0	5	0,590980631
1/5600	-1/2000	0	5	0,611870099
1/6000	-1/2000	0	5	0,631827619
1e-10	1/400	0	5	0,486773133
1e-10	1/800	0	5	0,814341808
1e-10	1/1200	0	5	1,102868
1e-10	1/1600	0	5	1,36887071
1e-10	1/2000	0	5	1,61922041
1e-10	1/2400	0	5	1,8576967
1e-10	1/2800	0	5	2,08669429
1e-10	1/3200	0	5	2,30785918
1e-10	1/3600	0	5	2,52238736
1e-10	1/4000	0	5	2,73118558
1e-10	1/4400	0	5	2,9349626
1e-10	1/4800	0	5	3,13428356
1e-10	1/5200	0	5	3,32960673
1e-10	1/5600	0	5	3,52131094
1e-10	1/6000	0	5	3,70971576

5) Constant A

t	*kx*	*ky*	*v*	*w_Ansys*	*w_formule*	*A*
5	1/1000	-0,001	0,25	0,17324883	0,18452381	0,938896874
10	1/500	-0,001	0,25	0,02984049	0,03261951	0,914804863
15	1/2000	-0,002	0,25	0,01813343	0,02050265	0,884443454
15	1/1000	-0,0005	0,25	0,02690029	0,02899512	0,927752343
25	1/1000	-0,002	0,25	0,00488554	0,00521912	0,936085304

6) Accuracy formula deformation

t	*kx*	*ky*	*w(ansys)*	*w(formula)*	*error [%]*
5	1/2000	-1/2000	0,357312	0,339524	4,978274
10	1/2000	-1/2000	0,088408	0,084881	3,989697
15	1/2000	-1/2000	0,038937	0,037725	3,114069
20	1/2000	-1/2000	0,021742	0,02122	2,397949
25	1/2000	-1/2000	0,013835	0,013581	1,839292
30	1/2000	-1/2000	0,009567	0,009431	1,422786

35	1/2000	-1/2000	0,007008	0,006929	1,130308
40	1/2000	-1/2000	0,005356	0,005305	0,945359
45	1/2000	-1/2000	0,004228	0,004192	0,853953
50	1/2000	-1/2000	0,003424	0,003395	0,844846
5	1/500	-1/500	0,086535	0,087619	1,253081
10	1/500	-1/500	0,021914	0,021905	-0,04269
15	1/500	-1/500	0,009859	0,009735	-1,25425
20	1/500	-1/500	0,005622	0,005476	-2,58781
25	1/500	-1/500	0,003652	0,003505	-4,02182
30	1/500	-1/500	0,002576	0,002434	-5,50841
35	1/500	-1/500	0,001923	0,001788	-7,01407
40	1/500	-1/500	0,001497	0,001369	-8,52173
45	1/500	-1/500	0,001202	0,001082	-10,0247
50	1/500	-1/500	0,00099	0,000849	-14,2829
20	1/500	-1/1000	0,007559	0,007502	0,752925
20	1/1000	-1/1000	0,01086	0,01061	2,2966
20	1/1500	-1/1000	0,01319	0,012995	1,482905
20	1/2000	-1/1000	0,015065	0,015005	0,401529
20	1/2500	-1/1000	0,01664	0,016776	-0,81956
20	1/3000	-1/1000	0,017994	0,018377	-2,13027
20	1/3500	-1/1000	0,019191	0,01985	-3,43096
20	1/4000	-1/1000	0,020263	0,02122	-4,72325
20	1/4500	-1/1000	0,021237	0,022507	-5,98296
20	1/5000	-1/1000	0,022127	0,023725	-7,21992
40	1/3000	-1/500	0,002991	0,003249	8,611331
40	1/3000	-1/1000	0,004432	0,004594	3,673362
40	1/3000	-1/1500	0,005537	0,005627	1,622294
40	1/3000	-1/2000	0,006454	0,006497	0,674148
40	1/3000	-1/2500	0,007234	0,007264	0,41814
40	1/3000	-1/3000	0,007909	0,007958	0,608732
40	1/3000	-1/3500	0,008509	0,008595	1,013288
40	1/3000	-1/4000	0,009046	0,009189	1,572077
40	1/3000	-1/4500	0,00953	0,009746	2,262304
40	1/3000	-1/5000	0,009971	0,010273	3,027932
5	1/400	-1/1000	0,108026	0,107367	0,609934
5	1/800	-1/1000	0,157969	0,15184	3,880263
5	1/1200	-1/1000	0,19536	0,185965	4,809269
5	1/1600	-1/1000	0,226336	0,214734	5,126063
5	1/2000	-1/1000	0,253067	0,24008	5,131832
5	1/2400	-1/1000	0,276808	0,262994	4,99052
5	1/2800	-1/1000	0,298056	0,284066	4,693639
5	1/3200	-1/1000	0,31771	0,303679	4,416178
5	1/3600	-1/1000	0,335691	0,322101	4,048364
5	1/4000	-1/1000	0,352508	0,339524	3,683416
5	1/4400	-1/1000	0,368285	0,356096	3,309858
5	1/4800	-1/1000	0,383034	0,37193	2,899163
5	1/5200	-1/1000	0,396968	0,387117	2,481699

5	1/5600	-1/1000	0,410221	0,40173	2,069854
5	1/6000	-1/1000	0,42282	0,41583	1,653276

Appendix E Data formulas normal force

1) Influence of l on normal force

lx	ly	t	nxx	nyy
400	400	10	-19,04735473	19,0470791
800	800	10	-15,75046645	15,75020032
1200	1200	10	-14,74139609	14,74115575
1600	1600	10	-14,41950289	14,41937033
2000	2000	10	-14,26227662	14,26214382
2400	2400	10	-14,16799005	14,16801325
2800	2800	10	-14,09447524	14,0943897
3200	3200	10	-14,03807787	14,03801531
3600	3600	10	-13,99820281	13,99813332
4000	4000	10	-13,96993639	13,96982751
4400	4400	10	-13,94891163	13,9488891
4800	4800	10	-13,93228144	13,93237922
5200	5200	10	-13,91910531	13,9191887
5600	5600	10	-13,90839153	13,90847328
6000	6000	10	-13,89967099	13,89974739
6400	6400	10	-13,89256541	13,89274951
6800	6800	10	-13,88707136	13,88725106
7200	7200	10	-13,88254049	13,88271578
7600	7600	10	-13,8789149	13,87908578
8000	8000	10	-13,87620033	13,87636678
8400	8400	10	-13,87418351	13,8742867
8800	8800	10	-13,87266341	13,8725742
9200	9200	10	-13,87151406	13,87142136
9600	9600	10	-13,87077886	13,87068277
10000	10000	10	-13,87042405	13,87032467

2) Influence of d on normal forces

d/h	h	t	kx	ky	nxx	nyy
4	12,5	5	1/4000	-1/4000	-26,89572088	26,89739971
4	11,25	5	1/4000	-1/4000	-27,02934065	27,0307669
4	10	5	1/4000	-1/4000	-27,1551034	27,15592493
4	8,75	5	1/4000	-1/4000	-27,27220818	27,27284047
4	7,5	5	1/4000	-1/4000	-27,37995313	27,38062829
4	6,25	5	1/4000	-1/4000	-27,47574442	27,47636389
4	5	5	1/4000	-1/4000	-27,56315385	27,56357266
4	3,75	5	1/4000	-1/4000	-27,63676739	27,63687853
4	2,5	5	1/4000	-1/4000	-27,69838326	27,69850089
4	1,25	5	1/4000	-1/4000	-27,75027011	27,75019335
4	12,5	30	1/4000	-1/4000	-4,644118394	4,643923332

4	11,25	30	1/4000	-1/4000	-4,649915438	4,649731809
4	10	30	1/4000	-1/4000	-4,655471081	4,655349799
4	8,75	30	1/4000	-1/4000	-4,66085987	4,660731993
4	7,5	30	1/4000	-1/4000	-4,666015463	4,665877588
4	6,25	30	1/4000	-1/4000	-4,671056749	4,671066536
4	5	30	1/4000	-1/4000	-4,676374049	4,676323944
4	3,75	30	1/4000	-1/4000	-4,682216508	4,682082049
4	2,5	30	1/4000	-1/4000	-4,69086611	4,690928479
4	1,25	30	1/4000	-1/4000	-4,71301745	4,712915806
4	12,5	40	1/4000	-1/4000	-3,502546997	3,502315277
4	11,25	40	1/4000	-1/4000	-3,506287841	3,506073158
4	10	40	1/4000	-1/4000	-3,509934631	3,509795961
4	8,75	40	1/4000	-1/4000	-3,513571425	3,513429783
4	7,5	40	1/4000	-1/4000	-3,517193939	3,517043049
4	6,25	40	1/4000	-1/4000	-3,520966486	3,520967543
4	5	40	1/4000	-1/4000	-3,525318161	3,525260764
4	3,75	40	1/4000	-1/4000	-3,530725958	3,53058871
4	2,5	40	1/4000	-1/4000	-3,540350216	3,540411627
4	1,25	40	1/4000	-1/4000	-3,566371793	3,566264427
4	12,5	40	1/2000	-1/2000	-3,463014131	3,462538388
4	11,25	40	1/2000	-1/2000	-3,469820947	3,469380826
4	10	40	1/2000	-1/2000	-3,476506095	3,476223059
4	8,75	40	1/2000	-1/2000	-3,48324079	3,482953266
4	7,5	40	1/2000	-1/2000	-3,489989703	3,489684245
4	6,25	40	1/2000	-1/2000	-3,497105413	3,497104949
4	5	40	1/2000	-1/2000	-3,505439692	3,505322847
4	3,75	40	1/2000	-1/2000	-3,515919838	3,515644345
4	2,5	40	1/2000	-1/2000	-3,534873741	3,534996417
4	1,25	40	1/2000	-1/2000	-3,586655266	3,586440541

3) Influence of t on normal forces

t	*kx*	*ky*	*nxx*	*nyy*
5	1/4000	-1/4000	-27,6983833	27,69850089
10	1/4000	-1/4000	-13,899671	13,89974739
15	1/4000	-1/4000	-9,29731462	9,297382648
20	1/4000	-1/4000	-6,99429539	6,994360251
25	1/4000	-1/4000	-5,61206248	5,612125762
30	1/4000	-1/4000	-4,69086611	4,690928479
35	1/4000	-1/4000	-4,03324276	4,033304549
40	1/4000	-1/4000	-3,54035022	3,540411627
45	1/4000	-1/4000	-3,15724558	3,157306728
50	1/4000	-1/4000	-2,85101605	2,851016049
5	1/1000	-1/1000	-27,5994281	27,59985975
10	1/1000	-1/1000	-13,8600854	13,86038129
15	1/1000	-1/1000	-9,27305515	9,273322986
20	1/1000	-1/1000	-6,97966897	6,979926027

25	1/1000	-1/1000	-5,60537253	5,605624145
30	1/1000	-1/1000	-4,69144739	4,691695822
35	1/1000	-1/1000	-4,04091176	4,041158179
40	1/1000	-1/1000	-3,5552859	3,55553098
45	1/1000	-1/1000	-3,17969794	3,179942116
50	1/1000	-1/1000	-2,88119691	2,881440465
5	1/500	-1/500	-27,5059367	27,5067144
10	1/500	-1/500	-13,8499992	13,85058151
15	1/500	-1/500	-9,28078633	9,281317832
20	1/500	-1/500	-6,99708881	6,997600629
25	1/500	-1/500	-5,63105046	5,631552276
30	1/500	-1/500	-4,72556458	4,726060523
35	1/500	-1/500	-4,08413617	4,084628391
40	1/500	-1/500	-3,6082257	3,608715468
45	1/500	-1/500	-3,24287812	3,24336624
50	1/500	-1/500	-2,95494668	2,955433701
5	1/6000	-1/6000	-27,7381396	27,73821988
10	1/6000	-1/6000	-13,9292389	13,92929034
15	1/6000	-1/6000	-9,32168918	9,32173474
20	1/6000	-1/6000	-7,01664182	7,016685182
25	1/6000	-1/6000	-5,63350664	5,633548906
30	1/6000	-1/6000	-4,71099145	4,711033079
35	1/6000	-1/6000	-4,05145219	4,051493421
40	1/6000	-1/6000	-3,55626836	3,556309326
45	1/6000	-1/6000	-3,17081348	3,170854274
50	1/6000	-1/6000	-2,86230807	2,862348738

4) Influence of curvatures of normal forces

kx	***ky***	***kxy***	***t***	***nxx***	***nyy***
1/400	-1E-10	0	5	-42,8781453	-37,88692676
1/800	-1E-10	0	5	-43,1269235	-36,8649387
1/1200	-1E-10	0	5	-43,2303977	-36,17434773
1/1600	-1E-10	0	5	-43,2941937	-35,64016329
1/2000	-1E-10	0	5	-43,3409736	-35,20112636
1/2400	-1E-10	0	5	-43,3784829	-34,82189804
1/2800	-1E-10	0	5	-43,4101743	-34,48539463
1/3200	-1E-10	0	5	-43,4380171	-34,17994446
1/3600	-1E-10	0	5	-43,4632638	-33,90032275
1/4000	-1E-10	0	5	-43,4867051	-33,64348753
1/4400	-1E-10	0	5	-43,5088133	-33,40695362
1/4800	-1E-10	0	5	-43,5298521	-33,1882629
1/5200	-1E-10	0	5	-43,5499581	-32,9849423
1/5600	-1E-10	0	5	-43,5691976	-32,79460818
1/6000	-1E-10	0	5	-43,5876046	-32,61507465
1/400	-1/1000	0	5	-36,935913	12,6399239
1/800	-1/1000	0	5	-30,4595599	24,36993168

1/1200	-1/1000	0	5	-25,0025107	29,98169965
1/1600	-1/1000	0	5	-20,4506511	33,26589543
1/2000	-1/1000	0	5	-16,6350682	35,38483205
1/2400	-1/1000	0	5	-13,3952229	36,8465395
1/2800	-1/1000	0	5	-10,6060508	37,90570975
1/3200	-1/1000	0	5	-8,17496389	38,70235027
1/3600	-1/1000	0	5	-6,03271419	39,31944031
1/4000	-1/1000	0	5	-4,12693457	39,80897494
1/4400	-1/1000	0	5	-2,41737864	40,20504314
1/4800	-1/1000	0	5	-0,87258903	40,53083996
1/5200	-1/1000	0	5	0,532416627	40,80264533
1/5600	-1/1000	0	5	1,81774929	41,03219077
1/6000	-1/1000	0	5	2,999768229	41,22812314
1/400	-1/2000	0	5	-40,4186447	0,190990328
1/800	-1/2000	0	5	-37,1045762	12,65240925
1/1200	-1/2000	0	5	-33,6995589	19,7791511
1/1600	-1/2000	0	5	-30,5172931	24,42468576
1/2000	-1/2000	0	5	-27,6496479	27,64987341
1/2400	-1/2000	0	5	-25,0557521	30,02744674
1/2800	-1/2000	0	5	-22,6799985	31,86575975
1/3200	-1/2000	0	5	-20,5033716	33,32197582
1/3600	-1/2000	0	5	-18,5099773	34,49566767
1/4000	-1/2000	0	5	-16,6799208	35,4573635
1/4400	-1/2000	0	5	-14,9935863	36,25733861
1/4800	-1/2000	0	5	-13,4337699	36,93159308
1/5200	-1/2000	0	5	-11,9858386	37,50634959
1/5600	-1/2000	0	5	-10,6373457	38,00111966
1/6000	-1/2000	0	5	-9,37762606	38,43071612
ky	**kx**	**kxy**	**t**	**nxx**	**nyy**
-1/400	1/1000	0	5	-13,159733	37,18537631
-1/800	1/1000	0	5	-24,7424944	30,75248505
-1/1200	1/1000	0	5	-30,2713065	25,32566922
-1/1600	1/1000	0	5	-33,5027409	20,78862338
-1/2000	1/1000	0	5	-35,5772809	16,9948917
-1/2400	1/1000	0	5	-36,9998803	13,7864026
-1/2800	1/1000	0	5	-38,030431	11,02132035
-1/3200	1/1000	0	5	-38,8038868	8,613101231
-1/3600	1/1000	0	5	-39,4030753	6,489006985
-1/4000	1/1000	0	5	-39,8761021	4,606639056
-1/4400	1/1000	0	5	-40,2596453	2,913689906
-1/4800	1/1000	0	5	-40,5749372	1,384523577
-1/5200	1/1000	0	5	-40,8371423	-0,002268296
-1/5600	1/1000	0	5	-41,058417	-1,26980493
-1/6000	1/1000	0	5	-41,2474681	-2,436288952

kx	*ky*	*kxy*	*t*	*kx/ky*	*nxx/ky*	*nyy/ky*
1/400	-1/1000	0	5	2,5	36935,91298	12639,9239
1/800	-1/1000	0	5	1,25	30459,55992	24369,93168
1/1200	-1/1000	0	5	0,833333333	25002,51068	29981,69965
1/1600	-1/1000	0	5	0,625	20450,65114	33265,89543
1/2000	-1/1000	0	5	0,5	16635,06819	35384,83205
1/2400	-1/1000	0	5	0,416666667	13395,2229	36846,5395
1/2800	-1/1000	0	5	0,357142857	10606,05077	37905,70975
1/3200	-1/1000	0	5	0,3125	8174,963894	38702,35027
1/3600	-1/1000	0	5	0,277777778	6032,714186	39319,44031
1/4000	-1/1000	0	5	0,25	4126,934566	39808,97494
1/4400	-1/1000	0	5	0,227272727	2417,378637	40205,04314
1/4800	-1/1000	0	5	0,208333333	872,5890294	40530,83996
1/5200	-1/1000	0	5	0,192307692	-532,4166268	40802,64533
1/5600	-1/1000	0	5	0,178571429	-1817,74929	41032,19077
1/6000	-1/1000	0	5	0,166666667	-2999,768229	41228,12314
1/400	-1/2000	0	5	5	80837,2893	381,9806568
1/800	-1/2000	0	5	2,5	74209,15232	25304,8185
1/1200	-1/2000	0	5	1,666666667	67399,1177	39558,3022
1/1600	-1/2000	0	5	1,25	61034,58612	48849,37152
1/2000	-1/2000	0	5	1	55299,29572	55299,74682
1/2400	-1/2000	0	5	0,833333333	50111,50412	60054,89348
1/2800	-1/2000	0	5	0,714285714	45359,99702	63731,5195
1/3200	-1/2000	0	5	0,625	41006,74312	66643,95164
1/3600	-1/2000	0	5	0,555555556	37019,95456	68991,33534
1/4000	-1/2000	0	5	0,5	33359,84168	70914,727
1/4400	-1/2000	0	5	0,454545455	29987,17268	72514,67722
1/4800	-1/2000	0	5	0,416666667	26867,5398	73863,18616
1/5200	-1/2000	0	5	0,384615385	23971,67722	75012,69918
1/5600	-1/2000	0	5	0,357142857	21274,6913	76002,23932
1/6000	-1/2000	0	5	0,333333333	18755,25212	76861,43224
ky	**kx**	**kxy**	**t**	**ky/kx**	**nxx/kx**	**nyy/kx**
-1/400	1/1000	0	5	2,5	13159,73299	37185,37631
-1/800	1/1000	0	5	1,25	24742,49443	30752,48505
-1/1200	1/1000	0	5	0,833333333	30271,30652	25325,66922
-1/1600	1/1000	0	5	0,625	33502,74092	20788,62338
-1/2000	1/1000	0	5	0,5	35577,2809	16994,8917
-1/2400	1/1000	0	5	0,416666667	36999,88032	13786,4026
-1/2800	1/1000	0	5	0,357142857	38030,43102	11021,32035
-1/3200	1/1000	0	5	0,3125	38803,88677	8613,101231
-1/3600	1/1000	0	5	0,277777778	39403,07532	6489,006985
-1/4000	1/1000	0	5	0,25	39876,10213	4606,639056
-1/4400	1/1000	0	5	0,227272727	40259,64531	2913,689906
-1/4800	1/1000	0	5	0,208333333	40574,93721	1384,523577
-1/5200	1/1000	0	5	0,192307692	40837,14226	-2,26829562
-1/5600	1/1000	0	5	0,178571429	41058,41698	-1269,80493

-1/6000	1/1000	0	5	0,166666667	41247,46814	-2436,28895

5) Constants Ax and Ay

t	***kx***	***ky***	***v***	***nxx(ansys)***	***nyy(ansys)***	***nxx(formula)***	***nyy(formula)***	***Ax***	***Ay***
5	1/1000	-0,001	0,25	-27,269552	27,267613	-194,375	194,375	0,140294	0,140284
10	1/500	-0,001	0,25	-17,923871	8,50050924	-137,443881	68,72194	0,130409	0,123694
15	1/2000	-0,002	0,25	-1,5743856	13,6882218	-32,3958333	129,5833	0,048598	0,105633
15	1/1000	-0,0005	0,25	-11,912584	5,60677826	-91,6292537	45,81463	0,130009	0,12238
25	1/1000	-0,002	0,25	-3,6584558	7,75094298	-27,4887761	54,97755	0,133089	0,140984

6) Accuracy of the formulas

t	***kx***	***ky***	***nxx(ansys)***	***nxx(formula)***	***error***
5	1/2000	-1/2000	-27,12754087	-25,851875	4,70247516
10	1/2000	-1/2000	-13,65773454	-12,9259375	5,35811439
15	1/2000	-1/2000	-9,1849017	-8,617291667	6,17981609
20	1/2000	-1/2000	-6,954038056	-6,46296875	7,06164249
25	1/2000	-1/2000	-5,618447524	-5,170375	7,97502374
30	1/2000	-1/2000	-4,73034185	-4,308645833	8,9147049
35	1/2000	-1/2000	-4,09775467	-3,693125	9,87442399
40	1/2000	-1/2000	-3,624899036	-3,231484375	10,8531205
45	1/2000	-1/2000	-3,258379933	-2,872430556	11,8448243
50	1/2000	-1/2000	-2,966196288	-2,5851875	12,8450295
20	1/500	-1/1000	-9,439759916	-9,140018059	3,17531229
20	1/1000	-1/1000	-7,158128365	-6,46296875	9,71147176
20	1/1500	-1/1000	-5,522728215	-5,276991887	4,44954592
20	1/2000	-1/1000	-4,259975276	-4,57000903	7,27782988
20	1/2500	-1/1000	-3,24600257	-4,087540339	25,9253575
20	1/3000	-1/1000	-2,410355596	-3,731396748	54,8068988
20	1/3500	-1/1000	-1,704461599	-3,454602109	102,679961
20	1/4000	-1/1000	-1,100417559	-3,231484375	193,659834
20	1/4500	-1/1000	-0,574814535	-3,046672686	430,027078
20	1/5000	-1/1000	-0,112459519	-2,890327492	2470,1048
40	1/3000	-1/500	0,220324354	-1,319247972	698,775372
40	1/3000	-1/1000	-1,281006829	-1,865698374	45,6431247
40	1/3000	-1/1500	-2,181125835	-2,285004515	4,76261746
40	1/3000	-1/2000	-2,787127193	-2,638495944	5,33277598
40	1/3000	-1/2500	-3,221357495	-2,949928144	8,42593073
40	1/3000	-1/3000	-3,543495586	-3,231484375	8,80518131
40	1/3000	-1/3500	-3,793568049	-3,490402051	7,99157928
40	1/3000	-1/4000	-3,994690465	-3,731396748	6,59109184
40	1/3000	-1/4500	-4,159430198	-3,957743915	4,84889211
40	1/3000	-1/5000	-4,295853137	-4,171828389	2,88708072

t	*kx*	*ky*	*nyy*	*nyy (formula)*	*error*
5	1/2000	-1/2000	27,1265769	24,49125	9,7149261
10	1/2000	-1/2000	13,6565259	12,245625	10,3313308
15	1/2000	-1/2000	9,18361931	8,16375	11,1053091
20	1/2000	-1/2000	6,95272086	6,1228125	11,9364545
25	1/2000	-1/2000	5,61710837	4,89825	12,7976589
30	1/2000	-1/2000	4,72898633	4,081875	13,6839331
35	1/2000	-1/2000	4,09638578	3,49875	14,5893432
40	1/2000	-1/2000	3,62351872	3,06140625	15,51289
45	1/2000	-1/2000	3,25698964	2,72125	16,4489206
50	1/2000	-1/2000	2,9647972	2,449125	17,3931693
20	1/500	-1/1000	4,45192469	4,32948224	2,75032614
20	1/1000	-1/1000	7,15548537	6,1228125	14,4319053
20	1/1500	-1/1000	8,38240816	7,49888321	10,5402282
20	1/2000	-1/1000	9,06664666	8,65896448	4,49650456
20	1/2500	-1/1000	9,49549016	9,68101659	1,9538374
20	1/3000	-1/1000	9,78495694	10,6050223	8,38087899
20	1/3500	-1/1000	9,99174928	11,4547333	14,6419209
20	1/4000	-1/1000	10,1449109	12,245625	20,7070729
20	1/4500	-1/1000	10,2623158	12,9884467	26,5644809
20	1/5000	-1/1000	10,3545875	13,691025	32,2218287
40	1/3000	-1/500	5,98318222	7,49888321	25,3326898
40	1/3000	-1/1000	5,18943029	5,30251117	2,17906142
40	1/3000	-1/1500	4,66996693	4,32948224	7,29094446
40	1/3000	-1/2000	4,24189522	3,7494416	11,6092829
40	1/3000	-1/2500	3,86970633	3,35360252	13,3370277
40	1/3000	-1/3000	3,54257703	3,06140625	13,5825072
40	1/3000	-1/3500	3,24869711	2,83431144	12,7554418
40	1/3000	-1/4000	2,97981963	2,65125558	11,0263065
40	1/3000	-1/4500	2,73250664	2,49962774	8,52253754
40	1/3000	-1/5000	2,50483595	2,37135508	5,3289263

Appendix F Data formulas bending moments

1) Influence of d on bending moments

lx	ly	t	mxx	myy
400	400	10	300,7589	301,6993
800	800	10	301,5524	302,4898
1200	1200	10	309,548	310,5215
1600	1600	10	312,7649	313,9652
2000	2000	10	314,6904	315,6195
2400	2400	10	315,9291	316,5446
2800	2800	10	316,7515	317,5194
3200	3200	10	317,522	318,2914
3600	3600	10	318,0366	319,0923
4000	4000	10	318,6062	319,1385
4400	4400	10	319,7141	318,7857
4800	4800	10	320,0458	318,9726
5200	5200	10	319,9658	319,6248
5600	5600	10	320,3957	319,689
6000	6000	10	320,6469	319,9422
6400	6400	10	320,8494	320,1168
6800	6800	10	320,9932	320,2619
7200	7200	10	321,1176	320,3876
7600	7600	10	321,2255	320,4967
8000	8000	10	321,319	320,5913
8400	8400	10	320,9495	321,0064
8800	8800	10	320,8826	321,3089
9200	9200	10	320,9401	321,3751
9600	9600	10	320,9888	321,4325
10000	10000	10	321,0296	321,482

2) Influence of d on bending moments

d	d/h	h	t	kx	ky	mxx	myy
50	4	12,5	5	1/4000	-1/4000	165,8959	166,0433
45	4	11,25	5	1/4000	-1/4000	174,188	174,3046
40	4	10	5	1/4000	-1/4000	183,4103	183,6852
35	4	8,75	5	1/4000	-1/4000	193,9642	194,2871
30	4	7,5	5	1/4000	-1/4000	206,1177	206,5114
25	4	6,25	5	1/4000	-1/4000	220,5962	220,8793
20	4	5	5	1/4000	-1/4000	238,3453	238,6035
15	4	3,75	5	1/4000	-1/4000	261,256	261,3853
10	4	2,5	5	1/4000	-1/4000	293,948	293,3114
5	4	1,25	5	1/4000	-1/4000	349,1692	348,1477
50	4	12,5	30	1/4000	-1/4000	233,2123	234,1075

45	4	11,25	30	1/4000	-1/4000	241,5226	242,493
40	4	10	30	1/4000	-1/4000	250,8158	251,9219
35	4	8,75	30	1/4000	-1/4000	261,4414	262,5944
30	4	7,5	30	1/4000	-1/4000	273,8036	274,7328
25	4	6,25	30	1/4000	-1/4000	288,4288	289,058
20	4	5	30	1/4000	-1/4000	306,0394	307,0105
15	4	3,75	30	1/4000	-1/4000	329,0931	329,7069
10	4	2,5	30	1/4000	-1/4000	362,1067	361,3666
5	4	1,25	30	1/4000	-1/4000	417,3872	416,1715
50	4	12,5	40	1/4000	-1/4000	243,8235	244,7752
45	4	11,25	40	1/4000	-1/4000	252,1125	253,1883
40	4	10	40	1/4000	-1/4000	261,4142	262,6141
35	4	8,75	40	1/4000	-1/4000	272,0473	273,2847
30	4	7,5	40	1/4000	-1/4000	284,444	285,393
25	4	6,25	40	1/4000	-1/4000	299,0731	299,7175
20	4	5	40	1/4000	-1/4000	316,6758	317,68
15	4	3,75	40	1/4000	-1/4000	339,74	340,3653
10	4	2,5	40	1/4000	-1/4000	372,7587	372,0156
5	4	1,25	40	1/4000	-1/4000	428,0294	426,8089
50	4	12,5	40	1/2000	-1/2000	217,2559	218,208
45	4	11,25	40	1/2000	-1/2000	225,5337	226,6099
40	4	10	40	1/2000	-1/2000	234,8251	236,0254
35	4	8,75	40	1/2000	-1/2000	245,4489	246,6865
30	4	7,5	40	1/2000	-1/2000	257,8372	258,7865
25	4	6,25	40	1/2000	-1/2000	272,4584	273,1031
20	4	5	40	1/2000	-1/2000	290,0534	291,0577
15	4	3,75	40	1/2000	-1/2000	313,1089	313,7343
10	4	2,5	40	1/2000	-1/2000	346,1139	345,3708
5	4	1,25	40	1/2000	-1/2000	401,3433	400,1227

3) Influence of t on bending moments

t	***kx***	***ky***	***mxx***	***myy***
5	1/4000	-1/4000	293,948	293,3114
10	1/4000	-1/4000	320,6469	319,9422
15	1/4000	-1/4000	336,0761	335,3518
20	1/4000	-1/4000	346,9416	346,2088
25	1/4000	-1/4000	355,3111	354,5738
30	1/4000	-1/4000	362,1067	361,3666
35	1/4000	-1/4000	367,824	367,0821
40	1/4000	-1/4000	372,7587	372,0156
45	1/4000	-1/4000	377,0997	376,3557
50	1/4000	-1/4000	380,9747	380,2301
5	1/1000	-1/1000	238,8939	238,2572
10	1/1000	-1/1000	265,9158	265,211
15	1/1000	-1/1000	281,5089	280,7845
20	1/1000	-1/1000	292,4758	291,7428

25	1/1000	-1/1000	300,9303	300,1928
30	1/1000	-1/1000	307,8077	307,0674
35	1/1000	-1/1000	313,6024	312,8603
40	1/1000	-1/1000	318,6056	317,8622
45	1/1000	-1/1000	323,0045	322,2601
50	1/1000	-1/1000	326,9262	326,181
5	1/500	-1/500	210,8893	210,2525
10	1/500	-1/500	237,8733	237,1682
15	1/500	-1/500	253,4469	252,7222
20	1/500	-1/500	264,3955	263,6621
25	1/500	-1/500	272,8364	272,0984
30	1/500	-1/500	279,7007	278,9596
35	1/500	-1/500	285,4809	284,7378
40	1/500	-1/500	290,4702	289,7256
45	1/500	-1/500	294,8566	294,1109
50	1/500	-1/500	298,7707	298,0239
5	1/6000	-1/6000	309,7189	309,0824
10	1/6000	-1/6000	336,2506	335,5459
15	1/6000	-1/6000	351,5753	350,851
20	1/6000	-1/6000	362,335	361,6023
25	1/6000	-1/6000	370,6171	369,8799
30	1/6000	-1/6000	377,3561	376,6161
35	1/6000	-1/6000	383,0414	382,2995
40	1/6000	-1/6000	387,9574	387,2143
45	1/6000	-1/6000	392,2838	391,5398
50	1/6000	-1/6000	396,1423	395,3976

4) Influence of the curvatures on the bending moments

kx	***ky***	***t***	***mxx***	***myy***
1/400	-1/1000	30	298,0091	271,3382
1/800	-1/1000	30	306,5764	298,8115
1/1200	-1/1000	30	308,3784	313,4707
1/1600	-1/1000	30	308,6294	322,7675
1/2000	-1/1000	30	308,4095	329,2142
1/2400	-1/1000	30	308,0585	333,9673
1/2800	-1/1000	30	307,6835	337,6135
1/3200	-1/1000	30	307,3215	340,4935
1/3600	-1/1000	30	306,9858	342,8228
1/4000	-1/1000	30	306,68	344,7453
1/400	-1/1000	20	283,5706	255,2825
1/800	-1/1000	20	291,4122	283,3227
1/1200	-1/1000	20	292,9206	298,2666
1/1600	-1/1000	20	292,9949	307,7384
1/2000	-1/1000	20	292,6641	314,3157
1/2400	-1/1000	20	292,238	319,1725
1/2800	-1/1000	20	291,8084	322,9048

1/3200	-1/1000	20	291,405	325,8577
1/3600	-1/1000	20	291,0362	328,2473
1/4000	-1/1000	20	290,7028	330,2167
1/400	-1/1000	10	258,0275	227,9724
1/800	-1/1000	10	265,043	256,603
1/1200	-1/1000	10	266,2214	271,8834
1/1600	-1/1000	10	266,1075	281,5943
1/2000	-1/1000	10	265,6568	288,3552
1/2400	-1/1000	10	265,1414	293,3479
1/2800	-1/1000	10	264,6418	297,1856
1/3200	-1/1000	10	264,183	300,227
1/3600	-1/1000	10	263,7704	302,6961
1/4000	-1/1000	10	263,4017	304,7394
ky	kx	t	mxx	myy
-1/400	1/1000	5	200,8548	228,977
-1/800	1/1000	5	229,3975	235,7575
-1/1200	1/1000	5	244,7576	237,013
-1/1600	1/1000	5	254,682	237,0324
-1/2000	1/1000	5	261,7024	236,6952
-1/2400	1/1000	5	266,9635	236,265
-1/2800	1/1000	5	271,0652	235,8286
-1/3200	1/1000	5	274,367	235,4187
-1/3600	1/1000	5	277,0787	235,0412
-1/4000	1/1000	5	279,3533	234,6989
-1/4400	1/1000	5	281,2891	234,3895
-1/4800	1/1000	5	282,9538	234,109
-1/5200	1/1000	5	284,4035	233,8546
-1/5600	1/1000	5	285,679	233,6238
-1/6000	1/1000	5	286,8086	233,4135

5) Constants Ax and Ay

t	*kx*	*ky*	*v*	*mxx_ansys*	*myy_formula*	*mxx_formula*	*myy_formula*	*Ax*	*Ay*
5	1/1000	-0,001	0,25	299,112352	297,630799	1803,336975	1803,337	0,165044	0,165044
10	1/500	-0,001	0,25	321,49478	302,852291	2000,927655	1866,931	0,151356	0,162219
15	1/2000	-0,002	0,25	327,554082	355,888117	1984,001865	2279,02	0,179379	0,156158
15	1/1000	-0,0005	0,25	377,206287	357,995464	2279,019679	2126,4	0,157083	0,168357
25	1/1000	-0,002	0,25	349,762228	362,902265	2141,999615	2295,738	0,169422	0,158076

6) Accuracy of the formulas

t	*kx*	*ky*	*mxx(ansys)*	*mxx(formule)*	*error*
5	1/2000	-1/2000	334,2351734	316,9740549	5,164363263
10	1/2000	-1/2000	369,0961765	351,7047347	4,711899732
15	1/2000	-1/2000	389,1671855	373,7592106	3,959217402
20	1/2000	-1/2000	403,2671498	390,2408369	3,230194413

25	1/2000	-1/2000	414,1214237	403,5238572	2,559048116
30	1/2000	-1/2000	422,9344807	414,71181	1,944194934
35	1/2000	-1/2000	430,3459778	424,4127493	1,378711264
40	1/2000	-1/2000	436,735513	432,9993193	0,855481992
45	1/2000	-1/2000	442,3477671	440,7172932	0,368595487
50	1/2000	-1/2000	447,3503325	447,7377532	-0,08660342
20	1/500	-1/1000	354,5632372	364,1075754	2,691857822
20	1/1000	-1/1000	367,687117	364,1075754	0,973529232
20	1/1500	-1/1000	372,0806812	364,1075754	2,142843251
20	1/2000	-1/1000	374,1261274	364,1075754	2,677854133
20	1/2500	-1/1000	375,23333	364,1075754	2,965023008
20	1/3000	-1/1000	375,888124	364,1075754	3,134057134
20	1/3500	-1/1000	376,3056567	364,1075754	3,241535451
20	1/4000	-1/1000	376,5829046	364,1075754	3,312770977
20	1/4500	-1/1000	376,7750814	364,1075754	3,362086983
20	1/5000	-1/1000	376,9104965	364,1075754	3,396806718
40	1/3000	-1/500	376,7824782	376,9478014	0,043877611
40	1/3000	-1/1000	409,8716443	404,0026502	1,431910241
40	1/3000	-1/1500	428,4558783	420,7201765	1,805483879
40	1/3000	-1/2000	441,0114561	432,9993193	1,816763864
40	1/3000	-1/2500	450,2433215	442,770028	1,659834388
40	1/3000	-1/3000	457,3754378	450,9167203	1,412126002
40	1/3000	-1/3500	463,0802387	457,921483	1,114009046
40	1/3000	-1/4000	467,7572373	464,0771798	0,786745185
40	1/3000	-1/4500	471,6624591	469,5755388	0,442460556
40	1/3000	-1/5000	474,9785642	474,549166	0,090403692

t	*kx*	*ky*	*myy(ansys)*	*myy(formule)*	*error*
5	1/2000	-1/2000	332,7535542	311,175749	6,484620499
10	1/2000	-1/2000	367,2344315	345,2711116	5,980735483
15	1/2000	-1/2000	387,2006519	366,9221518	5,237207107
20	1/2000	-1/2000	401,2579808	383,102285	4,524694013
25	1/2000	-1/2000	412,0907787	396,1423232	3,870131604
30	1/2000	-1/2000	420,897469	407,1256183	3,272020315
35	1/2000	-1/2000	428,2951577	416,6491015	2,71916598
40	1/2000	-1/2000	434,6794156	425,0786001	2,208711795
45	1/2000	-1/2000	440,2879091	432,6553915	1,733528778
50	1/2000	-1/2000	445,2876769	439,5474285	1,289110106
20	1/500	-1/1000	336,8348091	333,50991	0,987100809
20	1/1000	-1/1000	365,677872	357,447071	2,25083376
20	1/1500	-1/1000	380,1218504	372,2381393	2,073995772
20	1/2000	-1/1000	389,0671001	383,102285	1,533107049
20	1/2500	-1/1000	395,1930288	391,747058	0,871971564
20	1/3000	-1/1000	399,6567078	398,9549594	0,175587785
20	1/3500	-1/1000	403,0631627	405,1525225	0,51837031
20	1/4000	-1/1000	405,7478432	410,5988625	1,195574873

20	1/4500	-1/1000	407,9202721	415,4636135	1,849219551
20	1/5000	-1/1000	409,7112384	419,8641008	2,478053175
40	1/3000	-1/500	406,9608732	450,9167203	10,80100079
40	1/3000	-1/1000	431,9448485	450,9167203	4,392197726
40	1/3000	-1/1500	442,8748604	450,9167203	1,815831205
40	1/3000	-1/2000	448,9720664	450,9167203	0,433134723
40	1/3000	-1/2500	452,7806662	450,9167203	0,411666404
40	1/3000	-1/3000	455,3195689	450,9167203	0,966979872
40	1/3000	-1/3500	457,1008048	450,9167203	1,352892933
40	1/3000	-1/4000	458,3963894	450,9167203	1,631703316
40	1/3000	-1/4500	459,3616523	450,9167203	1,838405958
40	1/3000	-1/5000	460,0992368	450,9167203	1,995768684