CORBELED DOME DESIGN USING HCEB

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Bachelor thesis

By

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Reference for Title Page Figure: (Stigt, 2020)

Preface

This bachelor thesis is written as a completion of the Bachelor Civil Engineering at the faculty of Civil engineering and Geo-sciences of Delft University of Technology. I would like to express my appreciation and thanks to Ir. S. Pasterkamp and Dr. Ir. P.C.J. Hoogenboom for supervising me during this bachelor thesis. And for proving me with the required. Also, I want to thank Jurriaan van Stigt for proving information about the scope of this thesis.

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Abstract

In the mid-west of Mali, buildings are being realized by Partners Pays-Dogon in collaboration with the architectural firm LEVS. These buildings are build using HCEB, these are Hydraulic Compressed Earth Blocks, made of local material. The importance of using these bricks is that they are low in costs and better for the environment. Only the constructional properties of this material are less known and therefore there are no design rules that can guarantee the strength and stability of buildings with this material.

This bachelor thesis deals with creating design rules for a corbelled dome. A corbelled dome is a dome build by stacking the bricks horizontally on top of each other, unlike an ordinary dome in which the bricks are stacked with an angle increment. A few geometric boundaries have been drawn up that must be met. By complying with these boundaries, the strength and stability of the dome are guaranteed. Also, the dome complies with the NEN.

The corbelled dome is designed for two stress boundaries; No tensile membrane stresses may occur, and the shear strength must be higher than the shear stresses.

A relationship has been found between the base angle ϕ and the base radius r. The design rules are shown below:

- Maximal base angle ϕ is 60°
- $\frac{r}{(sin\phi - sin15)} < 4.11$

A maximal base radius r can be found by combining these two design rules. The maximal base radius is 2.50 m, and the total span is 5.00m

List of symbols

Symbol	Meaning
$\boldsymbol{a}$	transverse radii of curvature
$\boldsymbol{M}$	Bending moment
$\boldsymbol{N}$	Norman force
$\boldsymbol{e}$	Eccentricity
$\boldsymbol{t}$	Thickness of cross-section
$\boldsymbol{R}$	Radius at a point on the curve (general)
$\boldsymbol{\phi}, \boldsymbol{\theta}$	Angle of curvature
$\boldsymbol{r_2}$	Distance between a point on curvature and the intersection between the curvature radius and the rotation axis.
$\boldsymbol{r}, \boldsymbol{r_1}$	Curvature radii of ϕ, θ
$\boldsymbol{l_{ac}}, \boldsymbol{l_{cd}}$	Lengths corresponding to r, r_1
$\boldsymbol{N\phi}, \boldsymbol{N\theta}$	Meridian and hoop membrane forces
$\boldsymbol{p}$	External forces
$\boldsymbol{p_\phi}, \boldsymbol{p_r}$	Components of external forces in ϕ, θ direction
$\boldsymbol{F(\phi)}$	Resultant of forces acting on the shell above angle ϕ
$\boldsymbol{x}$	Vertical coordinate
$\boldsymbol{b}$	Heights brick
$\boldsymbol{y}$	Horizontal coordinate
$\boldsymbol{c}$	Length brick
$\boldsymbol{\eta}$	Number brick
$\boldsymbol{G}$	Permanent load
$\boldsymbol{Q_{k,i}}$	Variable load
$\boldsymbol{\psi_{0,i}}$	Simultaneity factor
$\boldsymbol{f_k}$	characteristic compressive strength masonry
$\boldsymbol{f_b}$	Normalized mean compressive strength bricks in direction of the applied load
$\boldsymbol{f_m}$	Mean compressive strength mortar
K, $\boldsymbol{\alpha}, \boldsymbol{\beta}$	Constant variables
$\boldsymbol{f_{vk}}$	Shear strength
$\boldsymbol{f_{vko}}$	Initial shear strength
$\boldsymbol{\sigma_d}$	Compressive strength perpendicular to longitudinal joints
$\boldsymbol{\sigma_\varphi}, \boldsymbol{\sigma_\theta}$	Meridian and hoop stresses
$\boldsymbol{\tau_\varphi}$	Shear stresses
$\boldsymbol{W}$	Concentrated load
$\boldsymbol{w}$	Uniform load
$\boldsymbol{\tau_\varphi}$	Shear stresses
$\boldsymbol{f_d}$	The design value of compressive strength masonry
$\boldsymbol{\psi_m}$	Partial factor
$\boldsymbol{\sigma_{cr}}$	Critical buckling strength
$\boldsymbol{E}$	Modulus of elasticity
$\boldsymbol{\nu}$	Poisson’s ratio

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Introduction

Over the past decade Partners Pays-Dogon, in collaboration with the architectural firm LEVS, has realized several education buildings in the mid-west of Mali for the Dagon people. A number of these building projects use HCEB (Hydraulic compressed earth blocks) as a building material. Some roofs have already been realized using HCEB. These roofs have a corbelled dome construction.

These HCEB bricks can be made from local material, which results in lower construction costs due to less use of cement and steel, which is also better for the environment. A roof made of HCEB performs better in terms of thermal insulation. People are also less dependent on materials that must be supplied. Besides, no framework is required when making a corbelled dome, unlike normal domes. The advantage of a dome over a flat roof made of concrete slabs is that larger spans can be achieved. With large spans, the concrete slabs become thicker resulting in a high own weight. This creates a higher load on the underlying construction.

1.1 Problem

In the design of the dome construction for the roof, little attention has been paid to the load-bearing system. The physics underlying the corbelled dome is not well known yet, especially in combination with the HCEB. The corbelled domes that have been made during these building projects are designed to be safe, using small spans. The problem is that there is no standard design that can be used for all spans, so every time a new design must be developed.

1.2 Goal

The object of this report is to set up several calculation rules for the design of a load-bearing system using a corbelled dome. It must be possible to easily make a roof construction with these calculation rules, without having to make a new design. The strength and stability of the dome must be guaranteed by using these calculation rules.
To achieve this goal, the physics background of corbelled domes must be studied. Analytical of the geometry. To achieve this, a literature study will be conducted first, after which the theory will be applied to find an optimal design.
The design of the domes must be suitable for buildings made of HCEB in Mali with a maximum of two building layers and where the domes span a maximum of 10 m. These domes must comply with the Dutch Eurocode about masonry. Hardly any earthquakes occur in Mali, so they are not included in the design.

1.3 Research questions

Mean question:
What is the geometry of a corbelled dome made of HCEB with varying span where strength requirements are met and that satisfies the NEN. How can this be presented in the form of calculation rules or a design table?

Sub-questions:
To answer the main question, it will be supplemented with a few sub-questions. The sub-questions can be divided into two groups. The first group of sub-questions can be answered through a literature study and the second group of sub-questions will be elaborated using a design procedure.

Literature study:

1. What are the mechanical principles of a shell structure? How can these principles be applied to a masonry dome?
2. What are the differences between a corbelled dome and an ordinary dome?
3. What are the properties of the HCEB, and which standards and safety margins must be met?
4. How is the shape of a corbelled dome mathematically described?

Design:

5. What is the optimal shape for a corbelled dome that can be easily scaled?
6. What forces act on the structure besides its weight? How are these forces transferred and where do the biggest stresses occur?
7. What is the largest span that can be realized?
8. How are the geometrical parameters, such as the height and the distance between the bricks, related to the span?

To find an answer to the mean question, the sub-questions are divides into chapters. First, in Chapter 2 some information is given about corbelled dome and their history. Thereafter, the mechanical behavior of shell structures is studied in Chapter 3 and in Chapter 4 the corbelling theory is investigated. In Chapter 5, the loads acting on the dome are defined, followed by Chapter 6 giving the masonry properties. A mathematical shape for the dome is described in Chapter 7 and in Chapter 8 the dimension boundaries are found for the dome. Chapter 9 gives the resulting maximal span for a corbelled dome. Chapter 10 describes some solutions for the trust forces and in Chapter 11 a conclusion is given.

2. Corbelled dome

2.1 Definition

A corbelled dome is made of horizontal stone elements that are cantilevering towards the center of the dome. The corbelled dome is a corbelled arch extended in three dimensions. Making a corbelled dome requires simple techniques and does not need a temporary supporting structure like building an ordinary dome. The corbelled dome has not the same structural behavior as an ordinary dome, which is based on a tension line. The corbelled dome is therefore also called 'false vault'. (Foti P, 2017) (Fraddosio A, 2019)

2.2 History

Early on, people in many different countries realized that corbelled domes and corbelled arches were good constructions to easily build all kinds of buildings and bridges. The use of the corbelled principle in contrast to the original arches was widely used by all kinds of peoples. Following is a timeline of places that used corbelled domes. A few well-known corbelled domes from history are further explained below.

Timeline

- The oldest corbelled domes have been found in Ireland and date from the period 3200 - 2500 BC. But also, later, around the early Middle Ages, corbelled domes were built. The so-called beehive huts were used by monks as homes. In Ireland, the building can have different shapes. For example, the more rectangular shaped buildings were used as an oratory. (Stalley, n.d.)
- The Egyptians used corbelled arches for separating rooms in the construction of the pyramids around 2600 BC. They preferred rectangular shapes for which the normal arch was less suitable and therefore they used corbelled vaults. Egyptians used large courses for building the domes, as they did when building the Pyramids. The builders, therefore, had to understand the force distribution because it was accompanied by great forces. In addition to corbelled arches, they also used very large angled stones that were placed together to support the load and form an opening. (Moyer, n.d.)
- In today's Syria and Israel, corbelled domes and arches were used as tombs and as homes. Nomads made these domes because they followed their grazing flock. In these places, wood is scarce, which is the reason for building domes. What makes these domes special is their smooth outer surface, due to mortar application. What makes the domes special is the thin shell of the domes. (Rovero, 2012)
- The ancient Greeks built corbelled domes for a long time. From the Mycenean period starting in 1600 BC until the Hellenistic period in 31 BC. One of the most famous Greek domes is the Treasure of Atreus.
- The Mayan civilization mainly builds corbelled arches, which made them unique. This started around 2500 BC, which is the beginning of the Classic period. Many buildings were made with this technique, such as gates, temples, and houses. Corbelled vaults were built on pyramids, so they represented the surrounding mountains. These were mainly used as tombs. The Maya architecture is known for its elongated low constructions, the opposite of a corbelled arch which is most stable when it is high and narrow. Yet this construction technique was often used because the dark, narrow spaces gave the feeling of a traditional Maya hut. (Seldom Scene Photography, n.d.) (carolinarh, 2017) (FLAAR, 2009)

- In India, corbelled arches were first built before switching to real arches. The arches in India are unique in that the stones are sawn exactly in such a way that the opening has a special shape. In India, people started using this masonry technique around 950 AD.

Famous corbelled domes

Corbelled Passage of Newgrange 3200 BC, Ireland

The corbelled vault that is part of the Newgrange, a historical monument in Ireland, is built around 3200 BC. The Newgrange site consists of an entrance, a 19-meter-long passage, the corbelled vault, and three adjacent chambers. The chambers were used as tombs and preserved human bones and grave goods. In Newgrange, many stones are engraved with megalithic art. This is a type of art that includes the art that has been made by engraving stones during the prehistoric period in Europe(wiki). To this day the hole construction is still intact, which is rare compared to other constructions of this age. The Newgrange is one of the oldest remaining constructions as it is older than Stonehenge and the Egyptian pyramids. (Wikipedia, n.d.)

Sic meters above the ground floor, the corbelled vault is closed on top by a capstone. The reason why the dome is still in a good shape is partly that almost no water can enter the vault. The stones used at the outside of the vault are tilted downwards and the outside of the stone vault is surrounded by smaller stones, called the cairn. The rainwater will be transported to de cairn, so the vault remains dry. In figure 1 a cross-section and a plan of Newgrange are shown. (O'Kelly, n.d.) (Morgan, n.d.)

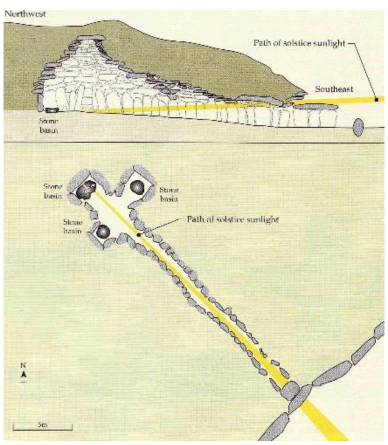

Figure 2.1: Cross-section and plan Newgrange (NEWGRANGE, n.d.)

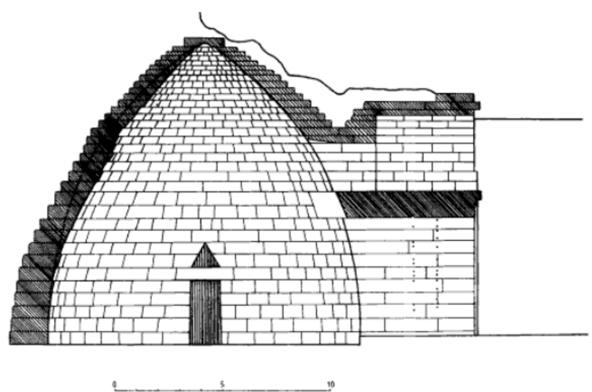

Figure 2.2: Cross-section Treasure of Atreus (Donaldson)

Treasury of Atreus 1250 BC

The Treasury of Atreus has been built around 1250 BC in Mycenae, Greece. It is a pointed dome excavated into the side of a hill, served as a tomb. One thing that makes this dome special are the dimensions. The diameter and the height of the tomb are slightly less than 15 and 14 meters respectively (Encyclopaedia Britannica, n.d.). The dome can be entered by an enormous doorway. The dome was constructed by digging a vertical hole in the hill and then making the dome from inside. When the dome was finished the vertical hole was refilled. The stones used for the dome were shaped so the internal surface of the dome would be smooth, like a true dome. This was done to improve the stability of the vault. It is no true dome, as no keystone is used, and the shape of the dome is ogival. To facilitate the force distribution at the position of the entrance a triangular-shaped lintel was used, transporting the weight above the entrance to the ground. (Wikipedia, n.d.)

After a further investigation is found that the stones used for the vault do not lay horizontal. If this is done on purpose or that this is due to settlements is unknown. The way the stones are now positioned they can follow the trust line better because they are perpendicular to this line.

It is unclear whether horizontal hoop forces are used in the vault, as 2/3 of the ring is opened to facilitate the entrance. The corbelling technique is the main mechanical principle that is used in this dome. The soil of the hill surrounding the dome ensures the resisting moment. Figure 2 shows a cross-section of the Treasure of Atreus. (Cavanagh, 1981)

3. Mechanical behavior shell structure

A dome is a thin shell construction. A dome transfers external forces through a pressure line to the supports, like an arch. A particular feature of a pressure line is that there is always a horizontal and vertical support reaction depending on the angle at which the arch arrives and the magnitude of the external load. Extra attention should be paid to the horizontal part of the support reaction, as it is much larger compared to other constructions that span a ditto distance. This force can be absorbed by a buttress or a tension tie.

The pressure line follows from the balance of the internal forces in the structure and the external forces acting on the structure. The shape of the trust line is like an inverted chain subject to gravity or additional external forces. The chain adapts to an external load and because a chain can only absorb tension, the reverse chain will only have compressive forces. When the external force only consists of the dead load, the top of the arch is in the middle. This results in an optimal shape of the arch, known as a funicular arch.

If the pressure line goes exactly through the center of the cross-section, only pressure forces will occur. If this is not the case and the pressure line goes outside the center of the cross-section, besides a pressure force also a moment with the magnitude: $M = N * e$ occurs. When e is smaller than $\frac{1}{6}t$ with t the thickness of the cross-section, no tensile forces occur. One does not want tensile forces occurring in the cross-section because it cannot be absorbed by the masonry and it causes cracks. Besides, the pressure force increases in the remaining cross-section. It is assumed that the masonry can withstand this pressure force and that any cracks that may occur are acceptable. Tensile forces are thus allowed in the cross-section and the assumption that the pressure line must lie in the cross-section.

In addition to the forces resulting from the trust lines, domes are subjected to hoop forces or parallel forces. These forces ensure that the pressure lines in the individual arch segments remain within the cross-section of the arch if the arch does not have the same shape as the pressure line. These forces arise from the rotation of an arch around a vertical axis and can be transferred to the trust by gravity. A dome can be thought of as a series of arch segments that rotated around a center. The forces that arise in the trust line in each arch and the combined hoop forces together form a network of compressive and tensile forces across the surface of the dome.

The transfer of forces in a network can be represented with the membrane theory. This theory is based on tensile and compressive forces only assuming the dome has no rigidity against bending and torsion. Therefore, the membrane theory is simpler than the bending theory. The membrane theory is usually applied to thin shells. A shell is a thin shell if the requirement R/t> 20 is met, with R the local radius of curvature and t the thickness (Heyman, 1967). In advance of the design of the domes, it is unknown if this rule applies to the domes. Bending and torsional moments can arise due to variation in thickness, abrupt changes in curvature, or the presence of concentrated loads and near the boundaries.

In using the membrane theory for domes, assumptions are made:

- As mentioned earlier, the shell thickness t is negligibly small compared to the radius R. The ratio R/t >20 must be satisfied.
- Strains and displacements arising in the shell are small.
- The normal stresses σ transverse to the middle surface are small and can be neglected.
- The shell has a constant thickness t.

A shell is described by a surface obtained by rotating a curve around an axis that lies in the plane of the curve. The two coordinated associated with the lines of the curve are ϕ (meridian coordinates) and θ (longitude coordinate). A point on the shell is located by these coordinates and a surface element, ABCD which is identified by the meridian and parallel increment dϕ and dθ. These are all shown in Figure 3.1 The curvature radii denoted by respectively r and r_1 are related to parallel and meridian curves and r_2 is the distance between the point and the intersection between the curvature radius and the rotation axis. The derivation of the meridian and hoop stresses can be found in Appendix A.

The formulas for σ_ϕ and σ_θ are shown in Formula 3.1 and 3.2.

$$\sigma_\phi = -\frac{F(\phi)}{2\pi r \sin\phi\, t} \quad [3.1]$$

$$\sigma_\theta = \left(-p\cos\phi\frac{r}{\sin\phi} + \frac{F(\phi)}{2\pi r(\sin\phi)^2} * \frac{r}{r_1}\right) * \frac{1}{t} \quad [3.2]$$

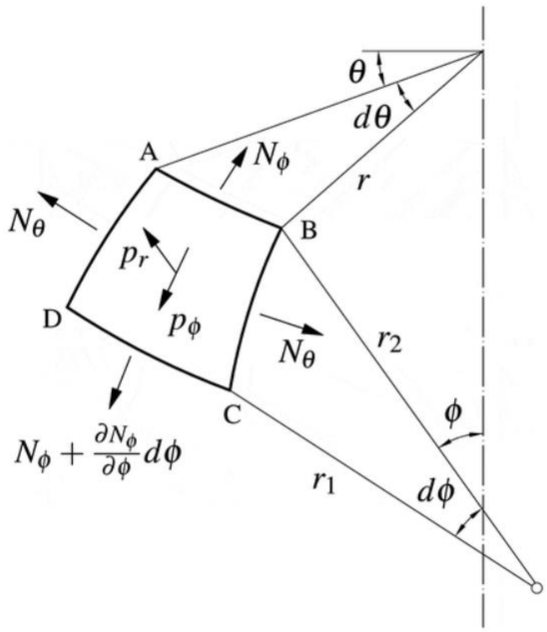

Figure 3.1 Surface element ABCD (Vittone, 2014)

4. Corbelling theory

In this chapter some information is given about the corbelling theory and if this theory can be used for designing the domes. The corbelling theory is used to understand the structural behavior of a corbelled dome. This theory assumes that the external forces are only transferred vertically between the horizontally lying rings of stones. This means that a trust line with a horizontal component is not assumed, like in ordinary domes. The equilibrium analyzes are done by choosing a tipping point and determining the resisting and overturning moment for this point.

By stacking identical stones on top of each other, the shape in Figure 4.1 can be obtained. One assumes that the joint center of gravity of the stones above does not lay outside the underlying stone. With this method, one finds a curve with the corresponding Formula 4.1. The variables are shown in Figure 4.1

$$x = nb; \quad y = \sum_{\eta=1}^{n} \frac{c}{2\eta} \qquad [4.1]$$

To obtain a larger span, one can move the center of gravity of the above stones by putting more weight on the outside of the arch. The dome at Newgrange and Treasury of Atreus are examples where the resisting moment is increased by placing soil on the outside of the dome. (Post, 2020)

With this method force transfer using shear forces between stones is not considered. It is also important to mention that no mortar is used between the stones. The corbelling theory can only be used with corbelled arches. Because of the three-dimensional nature of the dome, the internal action between the forces in the adjacent arches and the hoop forces resulting in extra strength and stability.

the modified corbelling theory is an improvement of the original corbelling theory. This theory is made specifically for domes, whereby the horizontal interaction between adjacent arches has been included in the equilibrium equations. For the equations that could be made, some assumptions are done. Frictionless blocks are used, and no mortar was used, so an infinitesimal space between the adjacent layers. The equilibrium equations for the resisting and overturning moments are set up for a spherical wedge as shown in Figure 4.2 and an extra equation for the interaction between the wedges. The results show that adjacent wedges can support each other, so a couple of adjacent wedges can withstand the overturning moment. This means that the force interaction between the wedges is essential for corbelled arch wedges to be stable (Foti P, 2017). Unfortunately, the corbelling theory cannot be used for designing the domes, as these use a mortar and therefore can transfer load also in the horizontal direction, like an ordinary dome.

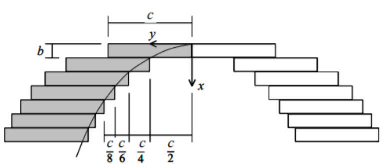

Figure 4.1 Pile of shifted blocks (Hoogenboom P.)

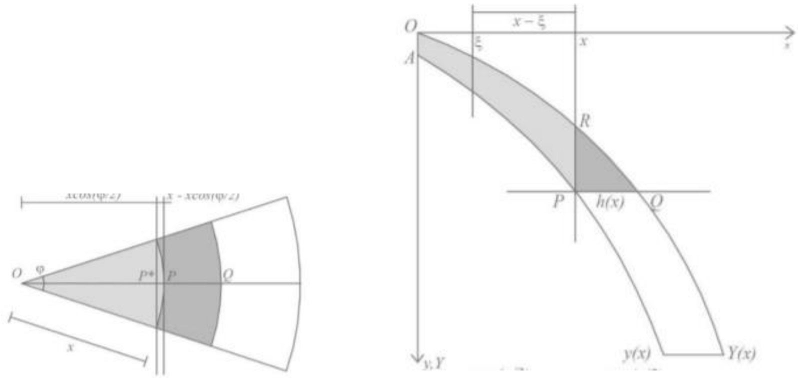

Figure 4.2 Infinitesimal meridian wedge of a dome which equilibrium is evaluated by the corbelling theory (Pilade Foti)

5. Defining loads

In this chapter the loads acting on the dome are determined. The load combinations are determined based on Eurocode 0 and 1. Different load combinations will be drawn up and compared. A distinction is made here between permanent and variable load.

The domes will be part of a residential building and possibly public buildings. The corresponding consequence class is CC2 according to the national appendix NEN-EN1990. This consequence class results in a factor k_{fi} of 1.0. The domes will only be tested for the ultimate limit state (ULS) and not for the serviceability limit state, as the deformations of a dome are negligible due to its shape and absence of frequent load combinations.

The load combinations belonging to the ULS are:

FC1: $1.35 * G + \sum\ 1.5 * \psi_{0,i} * Q_{k,i}$

FC2: $1.2 * G + 1.5 * Q_{k,1} + \sum\ 1.5 * \psi_{0,i} * Q_{k,i}$

First, the permanent load will be determined. This consists of the self-weight load of the dome and the water-resistant layer.

5.1 Uniform load

Self-weight masonry

The unit weight of the masonry is made up of the unit weight of the HCEB bricks, which is 22 kN/m3 and the mortar, with a unit weight of 21.25 kN/m$^{A.}$ The dimensions of the HCEB bricks are 295x140x70 mm. The horizontal joints are 15mm and the vertical joints are 12 mm. Rounded, this results in a joint unit weight of 22 kN/m$^{A.}$ For a first estimate of the thickness of the dome, the length of a brick, 295 mm, is assumed. The length of the bricks is used because with a smaller width it is difficult to span a large distance without resulting in a large height. This results in a distributed load of G_masonry = 6.5 kN/m^2.

Water-resistant layer

The water-resistant mortar layer on top of the dome can be made of various materials. The layer that has been used for the domes that have been built are made of soil, sand, and lime. The density of such a mortar is around 18 kN/mA. The thickness of the mortar is estimated at 70 mm. This results in a distributed load of G_mortar= 1.3 kN/m^2.

The total permanent load is G= 7.8 kN/m^2

5.2 Concentrated load

Second, the variable load will be determined. The only variable load that will be considered is a concentrated load due to a worker standing on the roof. One position will be investigated, namely a concentrated load at the crown of the dome. The magnitude of the concentrated load is Q_k = 1.5 kN. This force is applied on a surface of 0.1 – 0.1 m^2. The simultaneity factor applicable to loads on roofs is $\psi_{0,i}$= 0

For the variable load, no wind load is considered because it is expected to have a negligible effect on the stresses in the dome. This is because the loads due to the dome's weight are large, compared to the loads due to wind load.

Now the load combinations can be determined:

FC1: 1.35*7.8[kN/m^2] + 1.5*0*1.5[kN] = 10.5 kN/m^2

FC2: 1.2*7.8 [kN/m^2] + 1.5*1.5[kN] = 9.4 kN/m^2 + 2.3 kN

6. Masonry properties

Masonry is usually made up of elements joined with mortar. It is an old construction technique that was used as far back as 6000 AD. Masonry used to be used in many structures such as bridges, aqueducts, and houses. An advantage to masonry is that it can be easily done by hand, hence the name masonry. Also, masonry is good for thermal and sound insulation. Due to the high compressive forces that masonry can absorb, it is extremely suitable as a load-bearing construction. The brick gives strength to the masonry and the mortar creates a bond between the bricks.

The HCEB bricks used for the dome are the modern version of the ancient adobe bricks, that are molded sun-dried earth bricks. By using a compressing machine, the quality and performance of the bricks are improved. Many types of compressing machines are designed powered by human or an engine. The main difference between the different pressing machines is production speed, dimensions, and compression capacity. The machine the HCEB bricks are made with is developed by the Dutch company OSKAM v-f. It is a mobile hydraulic press on wheels, which also includes all the machinery for making the blocks. With this machine, it is possible to have full production directly on-site, which also can quickly be moved.

The bricks that are produced using the above-mentioned technique, have varying mechanical properties because they strongly depend on the raw earth mixture and the soil type that is used. For load-bearing structures, the bricks can be stabilized to ensure the waterproofing properties of the HCEB. This can be done by adding lime and cement to the mixture. By testing it is proved that the compressive strength of stabilized blocks increased. The dry compressive strength of stabilized HCEB is f_b = 9 N/mm2 after curing for 28 days.

A mortar mixture is used to join the blocks together and it gives resistance to lateral forces. It is used in horizontal joints to distribute the vertical loads homogenously and making precise horizontal courses. The joints prevent air and water from passing through the construction. The mortar used is different from pure cement mortar, as this is too brittle. When the mortar used is stronger than the HCEB bricks, the bricks can crack due to stresses produced by small displacements of the masonry. This must always be avoided, especially in load-bearing constructions. Lime or cement is added to the mortar mix to make it waterproof. The compressive strength of the mortar is f_m =1 N/mm2.

6.1 Compressive strength

The compressive strength of the masonry can best be determined by testing several test samples. This is often not possible on a construction site, and it also takes time before the samples can be tested. Alternatively, the compressive strength can be determined using Formula 6.2 given by Eurocode. (EN1996-1-1: Formula 9.4.1)

$$f_k = K * f_b^{\alpha} * f_m^{\beta} \quad [6.1]$$

The constant variables depend on the type of brick and type of mortar and are determined in National Annex NB-A of NEN-EN1996-1-1. HCEB blocks are not included in Eurocode, therefore clay bricks with a total volume of perforations <25% are assumed. The HCEB bricks are pressed and will therefore contain virtually no perforations. This choice results in the following values for the constant variables. K = 0.6, α = 0.65, β = 0.25.

With Formula 6.2 and the above-mentioned constant variables, compressive strength can be found.
f_k = 2.5 N/mm^2.

For the design of the masonry, the design compressive strength can be calculated with Formula 3 (EN1990: formula 9.4.2)

$$f_d = \frac{f_k}{\psi_m} \quad [6.2]$$

For the ultimate limit state, the value of ψ_m is given in the Dutch National Annex EN1996-1-1. For consequence class CC2 and bricks in category 1, the value of ψ_m is 1.7. The bricks are in category 1 if the perforation<25%. The design compressive strength of the masonry is f_d = 1.47 N/mm^2.

For masonry, the tensile strength is assumed to be zero, because bricks and mortar have a minimum tensile strength.

6.2 Shear strength

The shear strength of masonry depends on the applied vertical load and the normal stress that occurs. Four failure mechanisms can be distinguished, according to Mann und Muller. First, gaping of the bed joints can lead to failure(a). Secondly, the shear strength increases linearly depending on the compressive stress. At certain shear stress, the horizontal joints fail(b). Next, diagonal cracks in the bricks can occur due to tensile stresses(c). Lastly, the masonry can fail under compression(d). The four mechanisms are shown in Figure 6.1.

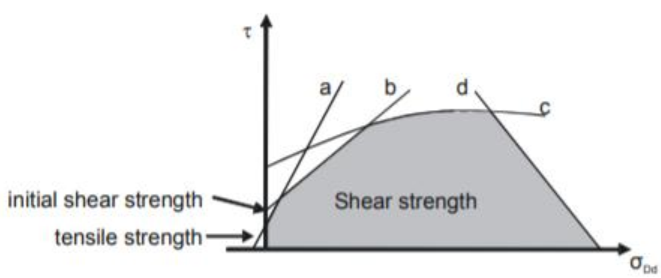

Figure 6.1: Failure mechanisms (Jäger, 2009)

The shear strength of the masonry is unknown but can be calculated by Formula 6.3 (EN1996-1-1 formula 9.4.3). In this formula, the left part represents the failure mechanism a, and the right part presents the failure mechanism c.

$$f_{vk} = f_{vko} + 0.4\,\sigma_d < 0.065 * f_b \quad [6.3]$$

f_{vko} can be found by defining the environmental class and brick category. The environmental class for this type of masonry is MX2, which corresponds to masonry exposed to moisture or water. The f_{vko} is 0.1 N/mm^2 for $f_m$between 1 and 2 N/mm^2 from table 5.4 of EN 1996-1-1:2019 (E). The compressive stress σ_d depends on the vertical force applied to the masonry. This is equal to the vertical component of the meridian stress at a certain ϕ.

The formula for σ_d is shown in Formula 7.5.

is unknown in advance. The right part of the formula results in and f_{vk} less than 0.59 N/mm^2. It is assumed that f_{vk} is smaller than 0.59 N/mm^2.

The design shear strength can be found using Formula 6.4.

$$f_{vd} = \frac{f_{vk}}{\psi_m} = \frac{0.1+0.4*\sigma_d}{1.7} \quad [6.4]$$

6.3 Critical buckling membrane stresses

In a thin shell structure, the membrane stress for buckling can be lower than the compressive strength of the masonry and therefore be critical. buckling can occur because of irregularities in the masonry. It can also arise from deformation, whether initially or over time. Irregular settlement of the support can also result in deformations of the support, the eccentricity of loading, temperature stresses, or creep. (Hoogenboom D. P., 2020)

For a dome, the critical membrane stress σ_{cr} for a base radius $r > 3.8\sqrt{at}$ is shown in Formula 6.5. It is assumed that this condition is met.

$$\sigma_{cr} = \frac{-1}{\sqrt{3(1-\nu^2)}} \frac{Et^2}{a} \quad [6.5]$$

No value for the elasticity modulus is given, so an estimation is made. The elasticity modulus E of masonry is $f_k * K_E$, with K_E is 1000. With f_k found in Formula 6.1, E is 2500 N/mm^2. The Poisson's ratio for masonry is assumed to be 0.25. With Formula 6.5 critical membrane stress σ_{cr} is $-\frac{440*10^3}{a}$ N/mm^3.

Only for large values of a critical buckling strength is smaller than the design compressive strength. Buckling will therefore never be normative.

7. Modeling dome design

In this chapter, the various shapes for the dome will be investigated. This dome will all be represented by one mathematical shape. It is investigated which shape has the most efficient power distribution and which is most suitable for a corbelled dome. For these shapes, formulas are determined to calculate the stresses in the domes. The same starting points and boundaries apply to all domes and these are formulated below:

- There are no tensile forces in the dome because they cannot be absorbed by the brickwork. These must therefore be circumvented.
- In the design, no restrictions were placed on the size of the support reactions.
- A constant thickness t = 0.295 m is assumed
- The dome is completely closed and has no lantern at the crown.

7.1 Mathematical shape

The mathematical shape that will be used to present the cross-section of the domes is shown in Figure 7.1. Variable a represents the transverse radii of curvature, r is the radius at the base of the dome, ϕ is the angle belonging to a point on the shell and ϕ_0 is the angle at which the meridian arch intersects with the axis of revolution. The symbol t represents the constant thickness of the shell. The relation between variables r, a, and ϕ is shown in Formula 7.1.

This mathematical shape can produce different kinds of dome shapes. For example, with a ϕ_0 is 0° a segmental dome is represented, and with a bigger ϕ_0, this results in a pointed dome. The angle ϕ at the base can variate, resulting in the variable height of the dome. Also, the radius r can variate resulting in a change in the span.

$$r = a * (\sin\phi - \sin\phi_0) \qquad [7.1]$$

7.2 Stress analysis

In Chapter 3 the stresses σ_ϕ and σ_θ are determined for a point in a shell structure using the membrane theory. Formulas for the stresses in the mathematical dome shape introduced in section 7.1 are denoted in Formula 7.2 and 7.A. The derivation of the formulas is put in Appendix B. In these formulas two parts are distinguished. One part belongs to the uniform load corresponding to the self-weight of the dome and the second part belongs to a concentrated load at the top of the crown. The values of these loads are determined in Chapter 5.

For a corbelled dome, a third stress is relevant, the shear stress in the horizontal bed joints. In ordinary domes, this stress is neglectable, because the meridian stress acts almost perpendicular to the bed joints resulting in a small shear force. While the horizontal component of the meridian stress results in shear stress acting on a bed joint in a corbelled dome as shown in Figure 7.2. Formula 7.4 gives the shear stress due to dead load and a concentrated load. For a reference system, the outward acting stress is positive. The shear strength depends on the stress perpendicular to the bed joint. This stress is equal to the vertical component of the meridian stress and is shown in Formula 7.5. There is a relation between the shear stress and shear strength as these both depend on the meridian stress acting on the cross-section and the angle ϕ of the cross-section.

$$\sigma_\varphi = \left(-wa\left(\frac{(\cos\phi - cos\phi 0) - (\phi - \phi 0)\sin\ \phi)}{(\sin\phi - \sin\phi 0)\sin\phi}\right) - \frac{W}{2\pi\,\mathrm{a}(\sin\phi - \sin\phi 0)}\right) * \frac{1}{t} \quad [7.2]$$

$$\sigma_\theta = (\frac{-wa((\phi - \phi 0)s\ in\,\phi - (\cos\varphi - \cos\phi 0)\sin\phi + (\sin\phi - \sin\phi 0)\cos\phi\sin\phi)}{(\sin\phi)^2} + \frac{W}{2\pi\,\mathrm{a}(\sin\phi - \sin\phi 0)}) * \frac{1}{t} \quad [7.3]$$

$$\tau_\varphi = \sigma_\varphi * \cos(\varphi) \quad [7.4]$$

$$\sigma_d = \sigma_\varphi * \sin(\varphi) \quad [7.5]$$

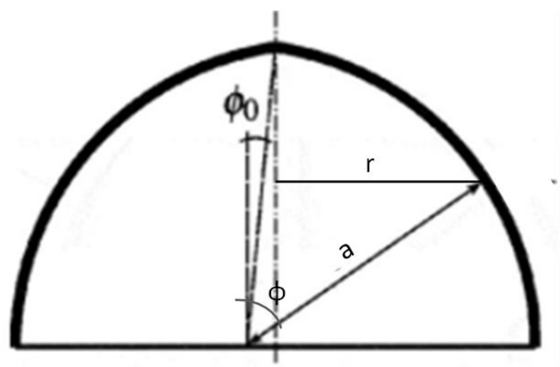

Figure 7.1 Cross-section mathematical shape dome

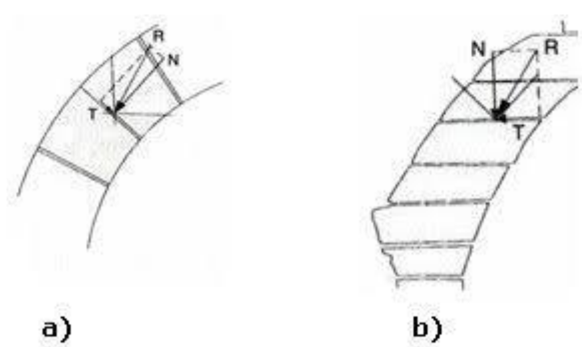

Figure 7.2 Shear forces a) ordinary dome, b) corbelled dome (Tempesta, 2018)

8. Stress analysis

For a better understanding of the relationship between the stresses and the variables, in Appendix D, the stresses in the dome are analyzed. Two load combinations found in Chapter 5 will be tested. This will be done for various angles ϕ_0 and ϕ at the base of the dome. The masonry strength parameters are calculated in Chapter 6 and are used to define if the external stresses exceed the strength of the masonry. For both load combination the meridian stresses, hoop stress, and shear stresses in the dome are calculated. The strength parameters result in boundaries for angle ϕ for different spans. The chosen angles for ϕ_0 are 0° and 5°. For the calculation of the meridian and hoop stresses a base radius of r = 1m is chosen. The calculations are done using the software Python and are added in Appendix C.

A relationship should be found between angle ϕ and the base radius r. For various angles ϕ_0 a relation can be found. The strength parameters result in boundaries for angle ϕ for different base radii. This results in a relation between angle ϕ and the base radius r for various angles ϕ_0.

The boundaries are given by the buckling strength for meridian and hoop stresses, the shear strength, and no tensile hoop stresses that may occur. These boundaries are found in the following paragraphs for a constant ϕ_0 = 15°. The value for ϕ_0 is obtained by analyzing existing domes and the practical building possibilities. Domes with a large span and high ϕ_0 value will result in large heights, this is not economical. Domes with small ϕ_0 are difficult to build, as the top of the dome is horizontal. When using a corbelled dome this is difficult to build and it results in less stability.

8.1 Tensile hoop stresses

In the lower region of a dome, hoop stresses can be tensile. This results in cracks and further degradation of the dome. The magnitude and the sign of the hoop stresses depend on the shape of the arch, that is, the angular difference between two membrane adjacent nodes, the corresponding stresses of these nodes, and the magnitude of the self-weight applied to the lower node. In the lower region, the self-weight dominates resulting in tensile hoop stresses. At which angle ϕ the hoop forces are zero largely depend on ϕ_0 and much less on the span and the magnitude of the self-weight and concentrated load. The hoop stresses are calculated using Formula 7.3. Figure 8.1 shows that the hoop stresses become tensile at ϕ is 60°. This angle is the maximal base angle of the dome.

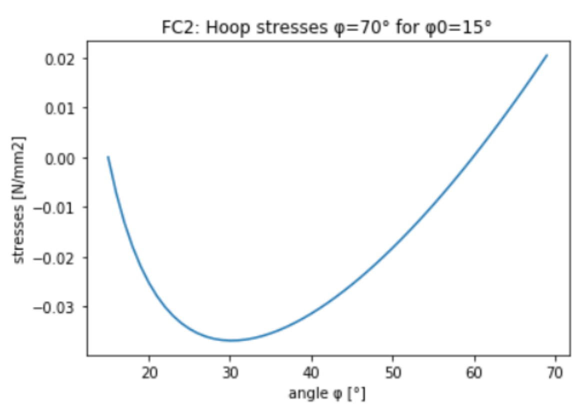

Figure 8.1 Hoop stresses for φ_0=15, r=1m, and φ=70

8.2 Shear stresses

The shear stresses may not exceed the shear strength of the masonry. The shear strength is shown in Formula 6.4 and dependent on the normal stress acting on the horizontal cross-section. A relation between base radius r and base angle ϕ can be found by setting the shear stress in Formula 7.4 equal to the shear strength. In Formula 6.4 and 6.5, instead of using thickness t, t/2 is used in the formula. This is done because in a corbelled dome the bricks at the top only overlap by half a brick. Failure due to shear firstly appears at the top. In Figure 8.2 for ϕ_0 from 0° until 40°, the maximal value for a can be found. The variable a is found for ϕ_0 = 15° and gives a lower boundary condition. For the two load combinations, the maximal value for variable a is 4.11 m resulting in Formula 8.1 obtained from Formula 7.1.

$$\frac{r}{(sin\phi - sin15)} < 4.11\ m \qquad [8.1]$$

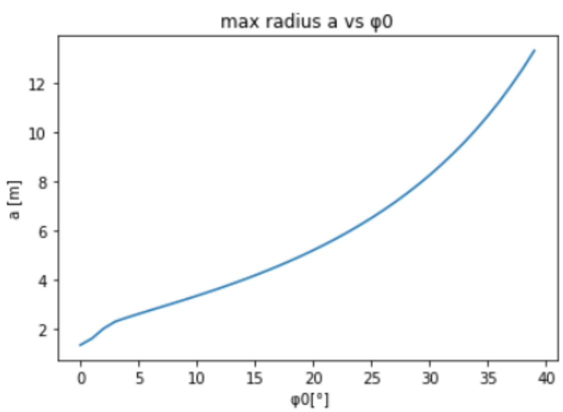

Figure 8.2 Radius a vs φ0 (thickness = t/2)

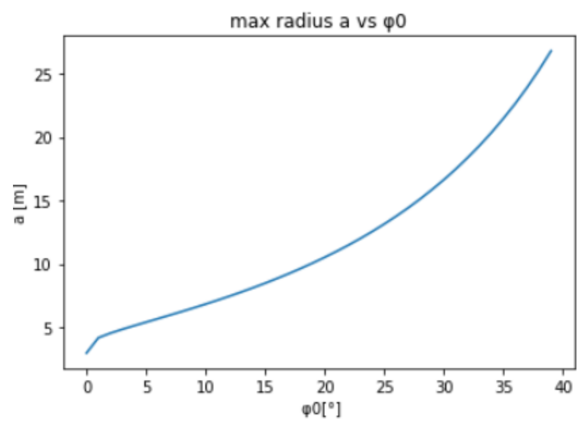

Figure 8.3 Radius a vs φ0 (thickness = t)

When using an ordinary dome, for Formulas 6.4 and 6.5 the thickness t has to be used. This results in higher shear resistance. In Figure 8.3 a is found for various ϕ_0 for thickness t. The value for a is 8.12 m for ϕ_0 is 15°. This means the dome can be larger when using an ordinary dome. Using Formula 7.1, for an ordinary dome, a radius r of 4.93 m is found.

9. Results

In this chapter, the results of Chapter 8 are combined to find a maximal radius r and a range for base angle ϕ. A design is made for the dome with the maximal span and the stresses in this dome are calculated.

In Section 8.1 and 8.2, several boundary conditions are found for the base radius and the base angle. Section 8.1 gives an upper boundary for the base angle, ϕ is 60°. This means that the base angle cannot be larger than 60° with a ϕ_0 is 15°. Section 8.2 gives a lower boundary for radius a, which is a relation between base radius r and base angle ϕ. The shear analysis for a corbelled dome gives a value for a which is 4.11 m. When combining the results of Section 8.1 and 8.2, one can obtain the largest radius. This value is 2.50 m for radius r and base angle ϕ is 60°. The design is safe for values of a lower than 4.11 m and values of ϕ lower than 60°. For a design with lower values for a and ϕ, the base radius is also smaller. The maximal span for a corbelled dome is 5 m and the height in the middle of this dome is 1.91 m. In Table 8.1, for different span, a range for the base angle and the height in the center of the dome is given.

Table 9.1 Dome span and corresponding height and base angle

Span [m]	**Base angle ϕ [°]**	**Height in the center [m]**
2	30 - 60	0.41 - 0.77
3	38.6 - 60	0.76 - 1.15
4	48.2 - 60	1.23 - 1.53
5	60	1.91

In Figure 9.1 a dome design is shown for a maximal span of 5 m. In Appendix E the meridian, hoop, and shear stresses are shown for this span.

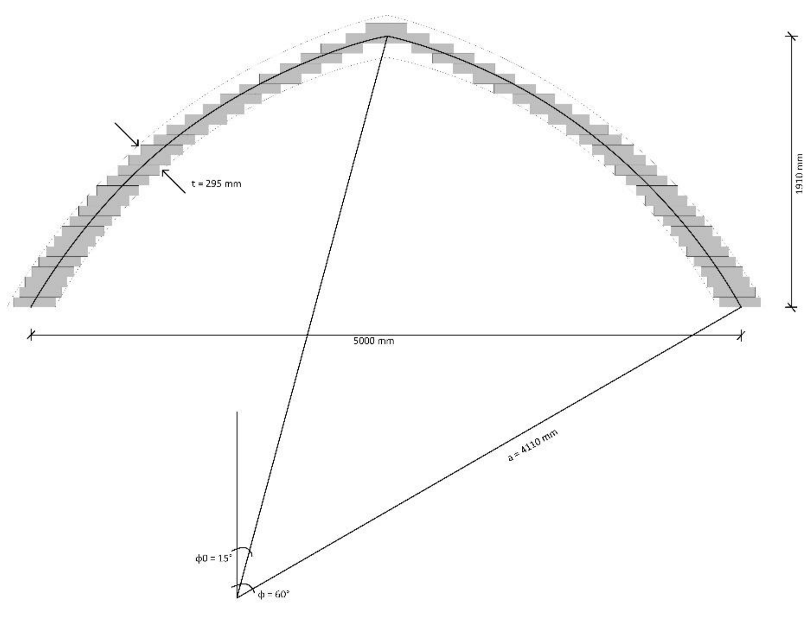

Figure 9.1 Dome design r = 2.5 m

10. Horizontal load bearing

In this chapter transfer of the horizontal forces at the base of the dome will be discussed. The magnitude and the direction of the thrust forces follow from the base angle, the loads, and the span of the dome. A small base angle results in large horizontal truss forces. The horizontal forces are given by Formula 10.1 and derived from Formula 7.2. $N\phi$ is the horizontal component of the meridian stresses multiplied by the thickness of the dome. If these forces are too high, these cannot be transferred by a single masonry structural wall. Several solutions can be found to accommodate the horizontal forces. Below some solutions are given.

$$N\phi = \left(\frac{-wa\left((\phi-\phi 0)s\,\mathrm{in}\,\phi-(\cos\varphi-\cos\phi 0)\sin\phi+(\sin\phi-\sin\phi 0)\cos\phi\sin\phi\right)}{(\sin\phi)^2}+\frac{W}{2\pi\,\mathrm{a}(\sin\phi-\sin\phi 0)}\right)*\cos\phi \qquad [10.1]$$

Thick walls

For domes with a large span and a large base angle, the height of the dome can partially be used as a living space. The walls will be shorter and thicker to accommodate the horizontal support reactions. The advantages of this design are that no extra material is used, as the walls are thicker but lower. Disadvantages are the arching walls, making it less efficient for large furniture. In Figure 10.1 an example is given of this design. This technique cannot be used for small base angles because the height will not suffice.

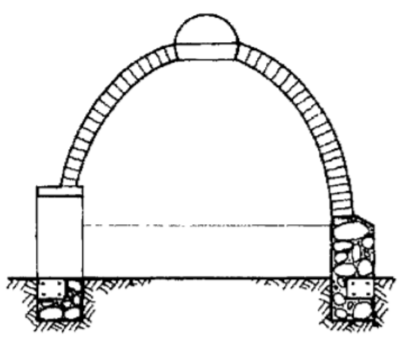

Figure 10.1 (Stabilisation of dome entrance)

Additional vertical forces

Another solution is using an additional vertical load at the position of the support. The additional force results in higher shear resistance of the wall, therefor it can bear more horizontal force. This solution will possible only be used with small domes, because in large domes

Ring beam

A ring beam can resist the horizontal support reactions by a tensile hoop force. This beam must be made from steel and must be rigidly attached to the dome so that the horizontal forces are carried by the ring beam. When this beam is made of reinforcement bars, it must be protected from the environment. This solution can only be used for small spans, as large ring beams are hard to fabricate. In the country where these domes are used, it is not desired to use steel, so this is not the most optimal solution.

Buttresses

For large domes, buttresses can be used to facilitate horizontal support reactions. An investigation must turn out what size and shape these must be and how the walls between the buttresses will react. Buttresses are used in large buildings, like churches.

11. Conclusions and recommendations

11.1 Conclusions

This thesis aims to make design rules for the building corbelled domes made of HCEB. This material is not standardly used, so there are no regulations regarding a safe dome design. This thesis answers the main question:

What is the geometry of a corbelled dome made of HCEB with varying span where strength and stability requirements are met and that satisfies the NEN. How can this be presented in the form of calculation rules or a design table?

To answer this problem, first, the mechanical behavior of a shell structure and the corbelling theory is studied. After this, the material properties and the loads acting on the dome are determined. Then a model for the dome is described using a constant thickness t and a constant ϕ_0 of 15°. Also, the membrane stresses for variating ϕ_0, and ϕ are determined. The design rules for the dome arise from equating the design strength and the stresses due to the load combinations. Doing this for the tensile stresses a maximal value for ϕ is found and for shear, a maximum value a is found. Variable a gives a relation between base angle ϕ and base radius r.

It is assumed that no tensile stresses may occur, resulting in a maximal base angle ϕ of 60° shown in Figure 8.1. In Figure 8.2 is the relation between a and ϕ_0 shown for the shear analysis. In this figure at ϕ_0 is 15°, the variable a is 4.11 m. The restrictions give a minimum and maximum relation between the base radius r and the base angle ϕ. Below the design rules are shown:

- Maximal base angle ϕ is 60°
- $a = \frac{r}{(sin\phi - sin15)} < 4.11\ m$

A maximal base radius r can be found by combining these two design rules. The maximal base radius is 2.50 m, and the total span is 5.00 m.

11.2 Recommendations

For this model, a constant value for ϕ_0 is chosen. This results in less geometric freedom. Besides, different values for ϕ_0, r, and ϕ can result in a similar geometric shape, because the arch shape remains still within the cross-section of the dome. For a wider range of values of these variables, design rules must be found, so there is more design freedom. Research must also be done on the building possibilities of the dome. What are the extreme limits of this? Also, the material properties of HCEB must be further investigated, so that it is clear how much horizontal load a wall can bear, and at which span extra support is required. Research should also be conducted into the inclusion of horizontal loads on walls.

The maximum span when using a corbelled dome is 5.00 m. If larger spans are wanted, ordinary domes must be considered, and which span one can achieve with them. With equal spans, a corbelled dome and an ordinary dome can be compared in terms of performance. One can look at the crack formation.

Furthermore, it can be investigated whether a round wall performs better compared to a four straight walls. In Mali, no earthquakes occur, but in other countries this can be the case. For this situation design rules can be found.

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Appendix

Appendix A

In this Appendix, a general expression for the meridian and hoop stresses is derived.

In Formula A.1, A.2, and A.3 the relationship of the radii is shown.

$$r = r_2 * \sin\phi \quad \text{[A.1]}$$

$$l_{ac} = rd\phi \quad \text{[A.2]}$$

$$l_{cd} = r_1 d\theta \quad \text{[A.3]}$$

When an axial symmetric load is applied to a shell of a revolution there are only two membrane forces, Nϕ and Nθ. There is no shear force. To determine these forces, one must set up two equilibrium equations in the direction ϕ and the normal direction r. The external forces are indicated with p, the weight per unit of area. Formula A.4 and A.5 give equations for the components of the loads in meridian and normal direction. Because of axial symmetry, there is no variation in the forces in the membrane in the direction θ. In Formula A.6 the force balance in the normal direction is shown. After dividing by $r * r_1$ one obtains Formula A.7.

$$p_\phi = p\sin\phi \quad \text{[A.4]}$$

$$p_r = -p\cos\phi \quad \text{[A.5]}$$

$$N_\phi r + N_\theta r_1 * \sin\phi - p_\phi r_1 r = 0 \quad \text{[A.6]}$$

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = p_r \quad \text{[A.7]}$$

In Formula A.8 the force balance in direction ϕ is shown.

$$\frac{d}{d\phi}\left(N_\phi r\right) - N_\theta r_1 \cos\phi + p_\phi r_1 r = 0 \quad \text{[A.8]}$$

The forces Nϕ and Nθ can be determined by solving Formula A.7 and A.8. It is also possible to solve Nϕ by solving the equilibrium equation along the vertical direction as shown in Figure A.1. The vertical force F(ϕ) is the resultant of all forces acting on the shell above angle ϕ and sustained by the resultant of the vertical components of the uniformly distributed forces Nϕ. One obtains the equation denoted in Formula A.9. A formula for Nϕ is shown in Formula A.10 and Nθ can be determined by substituting Formula A.10 in Formula A.7, resulting in Formula A.11 for Nθ. Formula A.9 shows that Nϕ is negative, which means this is a compressive force.

$$F(\phi) - N_\phi 2\pi r sin\,\phi = 0 \quad \text{[A.9]}$$

$$N_\phi = -\frac{F(\phi)}{2\pi r(sin\,\phi)^2} \quad \text{[A.10]}$$

$$N_\theta = -p\cos\phi\frac{r}{\sin\phi} + \frac{F(\phi)}{2\pi r(sin\,\phi)^2} * \frac{r}{r1} \quad \text{[A.11]}$$

The meridian and hoop stresses can be found by dividing the forces Nϕ and Nθ by thickness t. The formulas for σ_ϕ and σ_θ are shown in Formula A.12 and A.13.

$$\sigma_\phi = -\frac{F(\phi)}{2\pi r sin\,\phi\; t} \qquad [A.12]$$

$$\sigma_\theta = \left(-p\cos\phi\frac{r}{\sin\phi} + \frac{F(\phi)}{2\pi r(sin\,\phi)^2} * \frac{r}{r1}\right) * \frac{1}{t} \qquad [A.13]$$

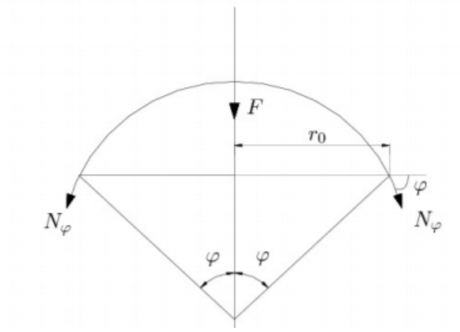

Figure A.1 (Revolution under the angle ϕ)

Appendix B

Derivation meridian and hoop stresses, using Figure 3.1. Solving Formula A.7 for Nθ and substituting this in Formula A.8, one obtains Formula B.1. Combining the two parts at the left and integrating for N ϕ an equation for Nθ is found in Formula B.2. Constant C represents the loads above $\phi=\phi_0$, which is the concentrated load F. To simplify Formula B.2, C is assumed to be zero. Later the concentrated load is added, which is equal to Formula A.10.

The meridian and hoop stresses can be found by dividing the forces Nϕ and Nθ by thickness t.

$$\frac{d(rN_\phi)}{d\phi} sin\phi - rN_\phi cos\phi = r_1 r_2 p_r cos\phi sin\phi - r_1 r_2 p_\phi sin^2\phi \qquad [B.1]$$

$$N_\phi = \frac{1}{r_1 sin^2\phi}\left[\int r_1 r_2 (p_r cos\phi - p_\phi sin\phi) sin\phi d\phi + C\right] \qquad [B.2]$$

The unit weight p is assumed to be equal to the unit weight of the dome and the water-resisting cover, indicated by variable w. The concentrated load is indicated by variable W. The relation between r2 and constant radius a.

$$r_2 = a * (1 - \frac{sin\phi 0}{sin\phi}) \qquad [B.3]$$

The meridian forces are found by integrating Formula B.2 and the hoop forces are found by substitution of the solution in Formula A.7. The formulas for the meridian and hoop forces are shown respectively in Formula B.4 and B.5. The concentrated loads are added by assuming that W is applied at ϕ0.

$$N_\phi = \left(-wa\left(\frac{(\cos\phi - cos\phi 0) - (\phi - \phi 0)\sin\ \phi)}{(\sin\phi - \sin\phi 0)\sin\phi}\right) - \frac{W}{2\pi\, \mathrm{a}(\sin\phi - \sin\phi 0)}\right) \qquad [B.4]$$

$$N_\theta = \frac{-wa\big((\phi-\phi 0)s\,\mathrm{in}\,\phi - (\cos\varphi - \cos\phi 0)\sin\phi + (\sin\phi - \sin\phi 0)\cos\phi\sin\phi\big)}{(\sin\phi)^2} + \frac{W}{2\pi\, \mathrm{a}(\sin\phi - \sin\phi 0)} \qquad [B.5]$$

The meridian and hoop stresses can be found by dividing the forces Nϕ and Nθ by thickness t. Respectively shown in Formula B.6 and B.7.

$$\sigma_\varphi = \left(-wa\left(\frac{(\cos\phi - cos\phi 0) - (\phi - \phi 0)\sin\ \phi)}{(\sin\phi - \sin\phi 0)\sin\phi}\right) - \frac{W}{2\pi\, \mathrm{a}(\sin\phi - \sin\phi 0)}\right) * \frac{1}{t} \qquad [B.6]$$

$$\sigma_\theta = (\frac{-wa\big((\phi-\phi 0)s\,\mathrm{in}\,\phi - (\cos\varphi - \cos\phi 0)\sin\phi + (\sin\phi - \sin\phi 0)\cos\phi\sin\phi\big)}{(\sin\phi)^2} + \frac{W}{2\pi\, \mathrm{a}(\sin\phi - \sin\phi 0)}) * \frac{1}{t} \qquad [B.7]$$

Appendix C

Python script for meridian, hoop, and shear stresses.

```
# # conoidal shape
# FC1/2 mer and hoop stresses
hoek0 = 15
r = 1
b = 60

e = 0
alpha = np.arange(e, b)
alpha1 = np.arange(hoek0, b)

w = 10.5
W = 0

sigma_mer = np.zeros([b-hoek0,b-e])
sigma_hoop = np.zeros([b-hoek0,b-e])
sigma_cr = np.zeros([b-hoek0,b-e])
c = np.ones([b-e])*r/(np.sin(b*np.pi/180)-np.sin(hoek0*np.pi/180))
a = np.zeros(b)

for i in range(len(alpha1)):
    a[i] = r/(np.sin(alpha[i]*np.pi/180)-np.sin(hoek0*np.pi/180))
    for j in range(len(alpha)):
        sigma_mer[i,j] = (-w*a[i]*((np.cos(hoek0*np.pi/180)-np.cos(alpha[j]*np.pi/180))-(alpha[j]*np.pi/180-hoek0*np.pi/180)*np.sin(hoek0*np.pi/180))/((np.sin(alpha[j]*np.pi/180)-np.sin(hoek0*np.pi/180))*np.sin(alpha[j]*np.pi/180))-(W/(2*np.pi*a[i]*(np.sin((alpha[j]-hoek0)*np.pi/180))**1)))/(0.295*1000)
        sigma_hoop[i,j] = ((-w*a[i]*((alpha[j]*np.pi/180-hoek0*np.pi/180)*np.sin(hoek0*np.pi/180)-(np.cos(hoek0*np.pi/180)-np.cos(alpha[j]*np.pi/180))+(np.sin(alpha[j]*np.pi/180)-np.sin(hoek0*np.pi/180))*np.cos(alpha[j]*np.pi/180)*np.sin(alpha[j]*np.pi/180))/(np.sin(alpha[j]*np.pi/180))**2)+(W/(2*np.pi*a[i])*(np.sin((alpha[j]-hoek0)*np.pi/180))**1))/(0.295*1000)
        sigma_cr[i] = -440/(c[i]*1000)

sigma_merb = sigma_mer[b-hoek0-1]
sigma_hoopb = sigma_hoop[b-hoek0-1]

plt.plot(alpha[hoek0:], sigma_merb[hoek0:])

plt.plot(alpha[hoek0:], sigma_hoopb[hoek0:])
plt.plot(alpha[hoek0:], sigma_cr, 'r')
plt.xlabel('angle \u03C6 [\N{DEGREE SIGN}]')
plt.ylabel('stresses [N/mm2]')
plt.title('FC1: Meridian and hoop stresses \u03C6=60\N{DEGREE SIGN} for \u03C60=15\N{DEGREE SIGN}')
plt.legend(['\u03C3_\u03C6: meridian stresses', '\u03C3_\u03B8: hoop stresses', 'Critical buckling strength'])
```

```python
# FC1/2 shear strength
r = 1

hoek0 = 15

w = 9.4
W = 2.3

c = 0
d = 60
alpha = np.arange(c, d)
alpha1 = np.arange(hoek0, d)

sigma_mer = np.zeros([d-hoek0, d-c])
tau_hor = np.zeros([d-hoek0, d-c])
tau_str = np.zeros([d-hoek0, d-c])

a = np.zeros(d-hoek0)

for i in range(len(alpha1)):
    a[i] = r/np.sin(alpha1[i]*np.pi/180)
    for j in range(len(alpha)):
        sigma_mer[i,j] = (-w*a[i]*((np.cos(hoek0*np.pi/180)-np.cos(alpha[j]*np.pi/180))-(alpha[j]*np.pi/180-hoek0*np.pi/180)*np.sin(hoek0*np.pi/180))/((np.sin(alpha[j]*np.pi/180)-np.sin(hoek0*np.pi/180))*np.sin(alpha[j]*np.pi/180))-(W/(2*np.pi*a[i]*(np.sin(alpha[j]*np.pi/180)-np.sin(hoek0*np.pi/180)))))/(0.295*1000)
        tau_str[i,j] = (-sigma_mer[i,j]*np.sin(alpha[j]*np.pi/180)*0.4 + 0.2)/1.7
        tau_hor[i,j] = -sigma_mer[i,j] * np.cos(alpha[j]*np.pi/180)

tau_hor_b = tau_hor[d-hoek0-1]
tau_str_b = tau_str[d-hoek0-1]

plt.plot(alpha[hoek0:], tau_hor_b[hoek0:]);
plt.plot(alpha[hoek0:], tau_str_b[hoek0:], 'r')
plt.xlabel('angle \u03C6 [\N{DEGREE SIGN}]')
plt.ylabel('stresses [N/mm2]')
plt.title('FC2: Shear stress and shear strength for \u03C6=30\N{DEGREE SIGN} and \u03C60=0\N{DEGREE SIGN}')
plt.legend(['\u03C4:shear stress', 'fvd:design shear strength'])
```

Appendix D

In this appendix for both load combination the meridian stresses, hoop stresses, and shear stresses in the dome are calculated. The strength parameters result in boundaries for angle ϕ for different spans. This results in a relation between angle ϕ and the base radius r. For various angles ϕ_0 a relation can be found. The chosen angles for ϕ_0 are 0° and 5°.

D.1 Segmental dome

A segmental dome is a dome as introduced in Section 7.1 with a ϕ_0 = 0°. This dome is horizontal at the crown and therefore not ideal for a corbelled dome as the horizontal stacking of the bricks results in a pointed top. The dome becomes more horizontal in the middle, therefor the adjacent bricks must overlap as little as possible. This can be difficult as the bricks can turn over by their weight. For this shape, an ordinary dome is more ideal, because this building method is more efficient in following the shape. With a corbelled dome, it is possible the dome cannot follow the shape exactly, possibly resulting in a pressure line outside the cross-section.

First, the meridian and hoop stresses are determined for a base radius of r=1 m. The relation between the magnitude of these stresses and the and the angle ϕ is determined for the two load combinations. Next, the relation between angle ϕ and radius r is determined by setting the shear stress equal to the shear strength. These two steps result in an upper and lower boundary for the relation between angle ϕ and radius r.

Meridian and hoop stresses

The meridian and hoop stresses are determined for a base radius of r = 1 m. The relation between the magnitude of these stresses and the and the angle ϕ is determined for the two load combinations. Load combination 1 resulted in a uniform load of 10.5 kN/m^2. For ϕ is 30° and 60° the meridian and hoop stresses. These are shown in Figures D.1 and D.2. In both figures can be seen that the hoop stresses become tensile stresses at ϕ = 51.8°. This angle can be found by setting the Formula 7.2 equal to zero. For a segmental dome, the angle at which tensile stresses occur is independent of the loads and base radius. This is the first boundary for the dome design. The base angle cannot be smaller than 51.8°. A difference between the results is that with a higher base angle the stresses at the top of the dome are lower. In Figure D.3 the stresses are shown for a base radius of r=4 m and ϕ is 30. Comparing Figure D.1 and D.3, a bigger base radius results in bigger stresses all over the dome.

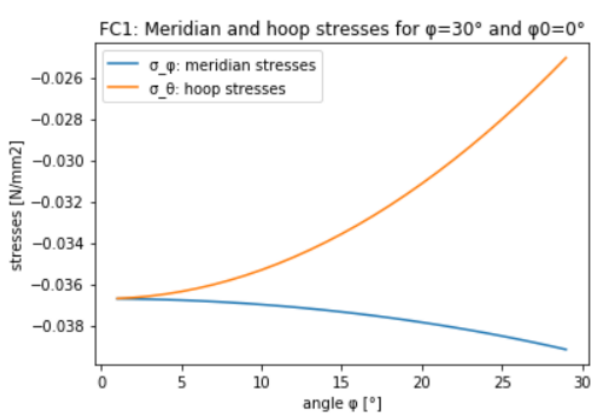

Figure D.1

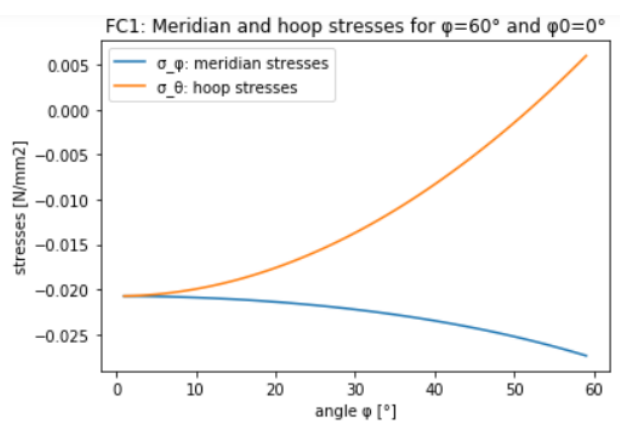

Figure D.2

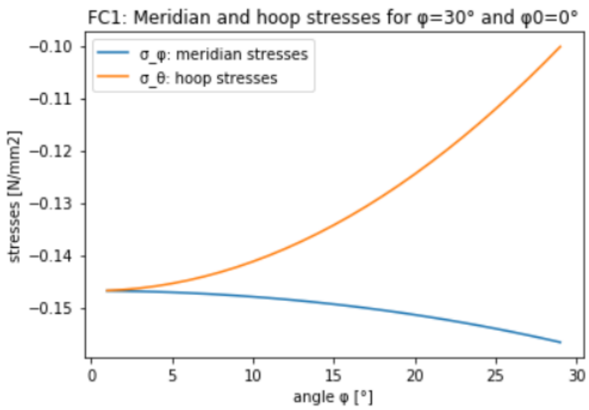

Figure D.3

Load combination 2 results in a uniform load of 9.4 kN/m^2 and a concentrated load at the crown of 2.3 kN. The meridian and hoop stresses for r=1 m and ϕ is 30° and 60° are shown in Figure D.4 and D.5. Because of the concentrated load, the meridian stresses approach infinite at the crown of the dome. Besides this phenomenon, no major difference is visible between the stresses due to load combination 1 and 2. The angle at which hoop stresses become tensile stresses is not 51.8° exact but varies around this value for different base angles. These are minor variations.

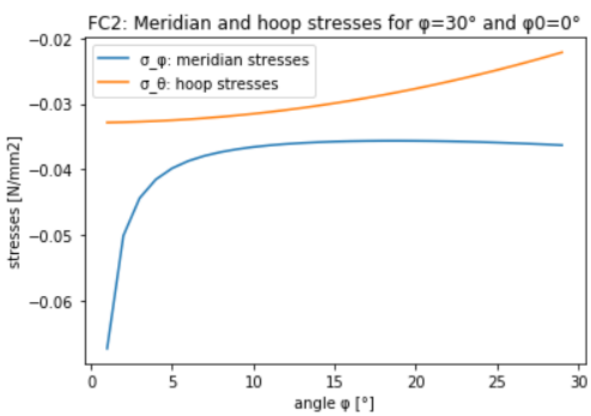

Figure D.4

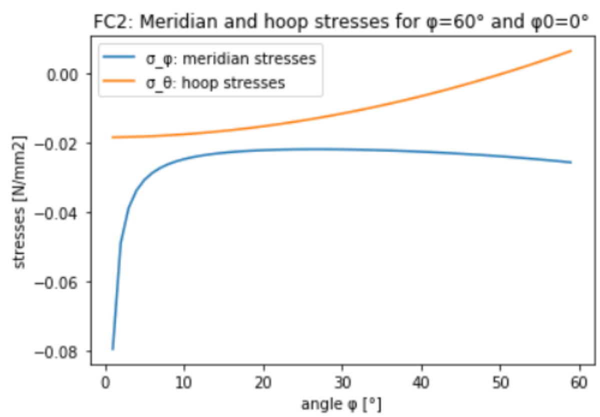

Figure D.5

Shear stresses

For load combination 1, the shear stresses and shear strength for a base radius r=1m and ϕ is 30° and 60° are shown in Figure D.6 and D.7. In these figures is visible that the shear stresses do not exceed the shear strength for these combinations of ϕ and radius r. In Figure D.7 the stresses are shown for r = 4 m and with these values for variables r and ϕ the shear stress is higher than the strength over the entire dome.

With a constant radius, the angle in the dome at which the shear stress is equal to the shear strength increases with an increasing base angle. the shear stress must never exceed the shear strength in the dome. This first occurs at the top of the dome, at ϕ = 0°. For this angle, the shear stresses must be equal to the shear strength. The angle and base radius for which this applies can be found by setting Formula 7.4 equal to Formula 7.5 and for σ_ϕ substituting Formula 7.2. One now found a formula for variable a, which gives a relation between base angle ϕ and base radius r. Variable a can be found by setting ϕ approaching zero. For load combination 1 this results in a value for a is 6.61. This means that shear stress does not exceed the shear strength if r/sin(ϕ) < 6.61m. This is the lower boundary for the dome design.

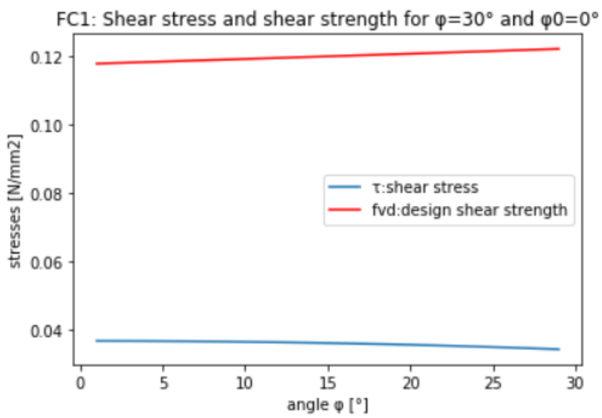

Figure D.6

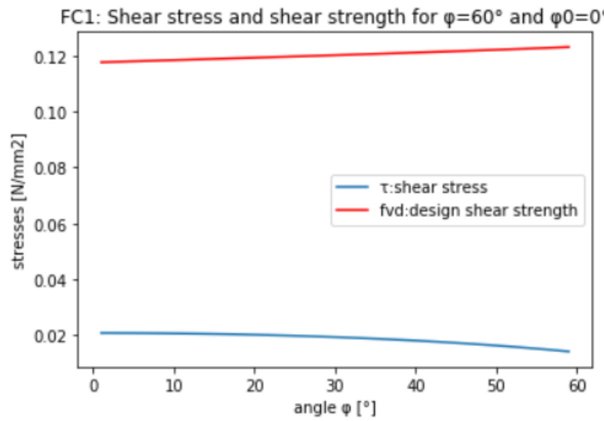

Figure D.7

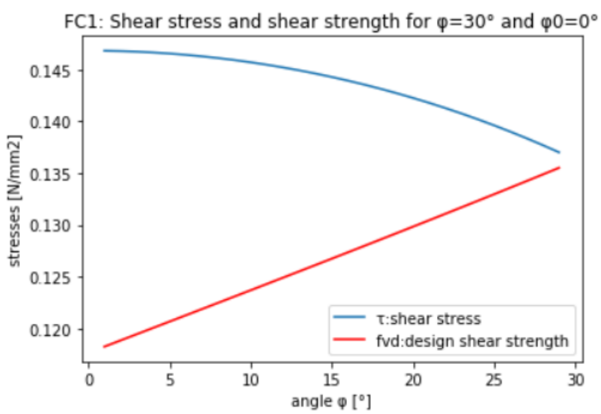

Figure D.8 Stresses at radius r = 4m

For load combination 2, the shear stresses and shear strength for a base radius r = 1 m and ϕ is 30 and 60 are shown in Figure D.9 and D.10. In this case, the value found for a is infinite, because of the concentrated load. For a small value of ϕ = 0.4 a lower boundary is r/sin(ϕ) < 5.36 m. Comparing the two load combinations, a point load results in a higher dome, which is a more conservative design. In Figure D.11 the shear stress and shear strength are shown for r = 4 m for ϕ = 30. With a bigger base radius and a constant base angle, the shear stresses are much higher. This is because the dome with the bigger radius is shallower, compared to the smaller radius, resulting in more shear stresses, especially at the top of the dome.

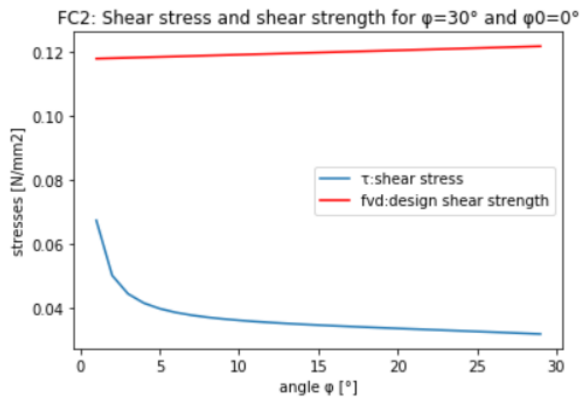

Figure D.9

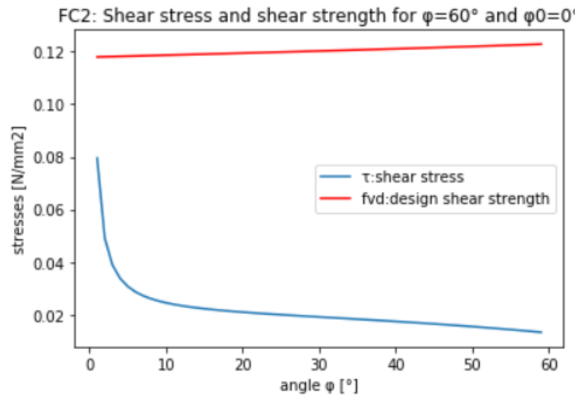

Figure D.10

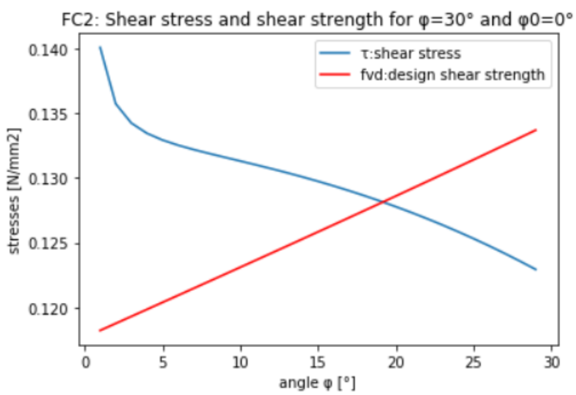

Figure D.11 stresses at radius r = 4m

D.2 Pointed dome

A pointed dome is a dome with a ϕ_0 higher than zero. This dome is pointed at the crown and therefore can be easily be built with a corbelling technique because the shape of a corbelling dome is always pointed at the crown and therefore the middle section always lays within the thickness of the dome.

Meridian and hoop stresses

Like with the segmental dome the meridian and hoop stresses are determined, but now for a $\phi_0 = 5$, a base radius of r = 1 m and the two load combinations. Again, for ϕ = 30° and 60°, the stresses are calculated and shown below in figures D.12 till D.15. Figures D.12 and D.13 show load combination 1 and figure D.14 and D.15 show load combination 2.

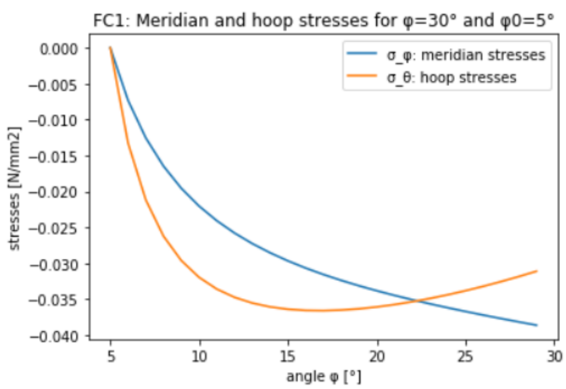

Figure D.12

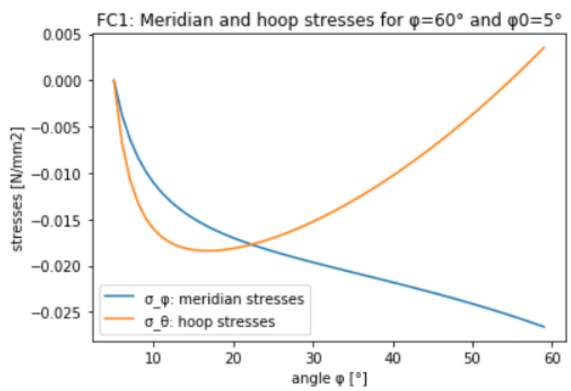

Figure D.13

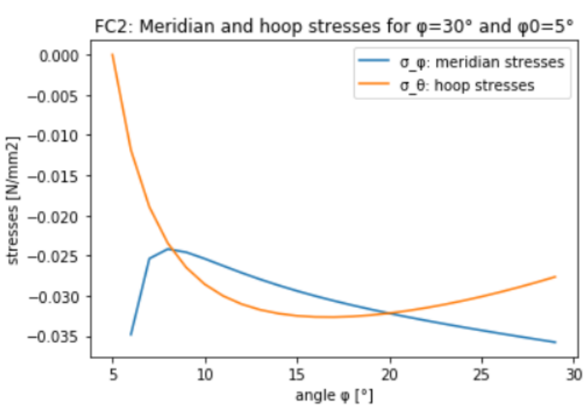

Figure D.14

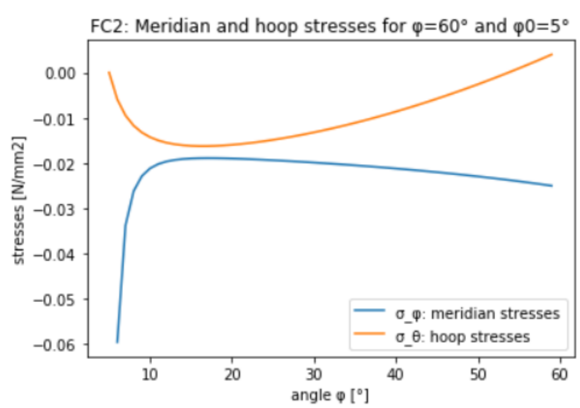

Figure D.15

Figure D.12 shows that the meridian and hoop stresses start at zero at the top of the dome. This differs from the stresses for $\phi_0 = 0°$, because these stresses can compensate at the top of the dome as the angle is zero. At the base of the dome, there are no major differences in the magnitude of the stresses because the stresses are equal at the base for Figures D.1 and D.12. The angle at which tensile hoop stresses occur is bigger with ϕ_0 = 5° than with ϕ_0 = 0°. In Figure D.13 is visible that this angle is 55°. This angle depends mostly on the angle ϕ_0 and less on the load combination and base radius.

For load combination 2, visible in Figure D.14 and D.15, the meridian stresses are big at the crown due to the concentrated load. At ϕ at the base angle, the meridian stresses are both smaller for load combination

2 than for load combination one. Load combination 1 is normative for the compressive strength and the angle at which hoop stresses become tensile stresses.

Shear stresses

The shear stresses and shear strength are calculated for radius r, ϕ_0 = 5° and ϕ is 30° and 60°. In Figure D.16 and D.17, the stresses are shown for load combination 1. The shear stresses are significantly lower than the shear strength. This also applies to load combination 2 shown in Figure D.18 and D.19. In Figure D.20 the stresses are calculated for r = 4 m. With a bigger radius and a constant base angle, this results in a higher shear strength compared to Figure D.10.

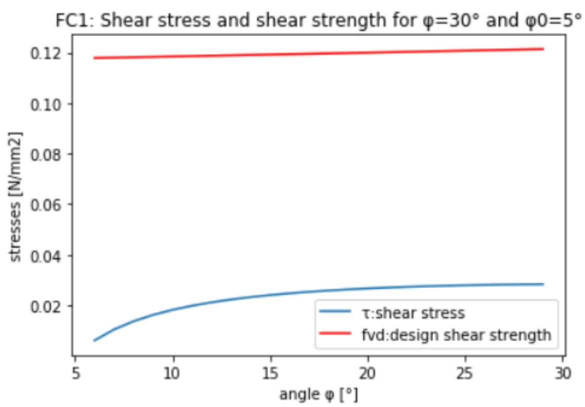

Figure D.16

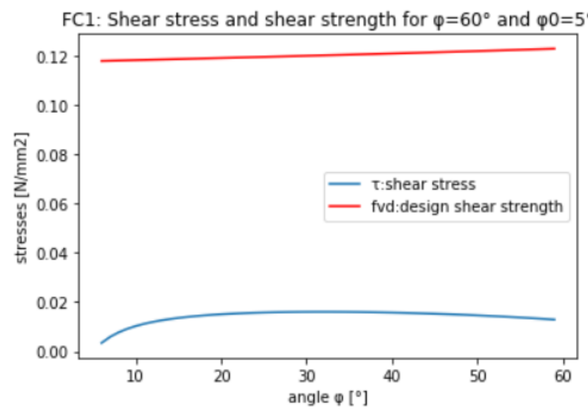

Figure D.17

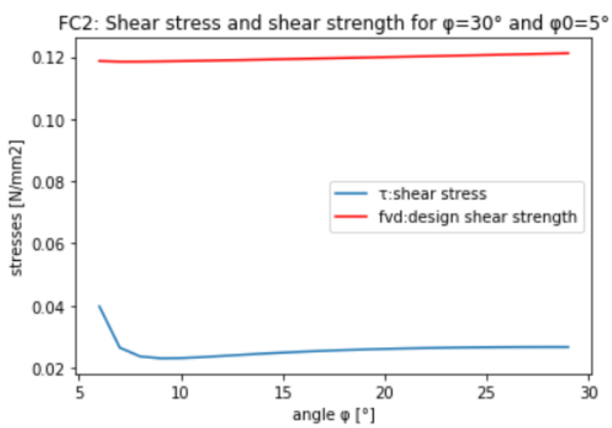

Figure D.18

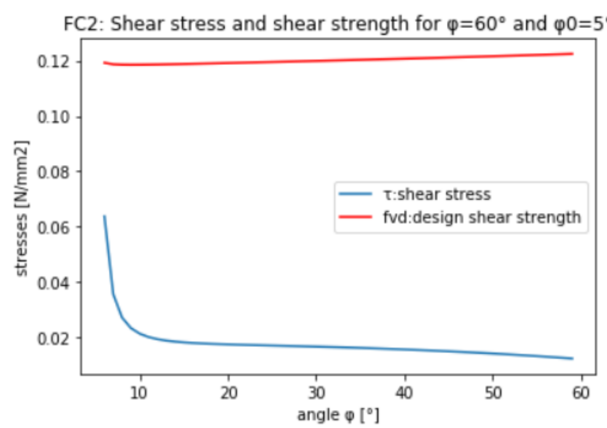

Figure D.19

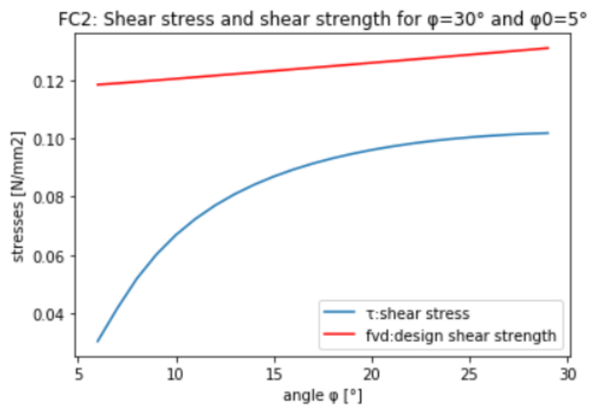

Figure D.20 Stresses at radius r = 4m

Appendix E

In this Appendix, the stresses are given for the maximal span found in Chapter 9. In Figure E.1, the decisive values for meridian and hoop stresses are shown for all angles ϕ from $\phi = \phi_0$ is 15° until ϕ = 60° for radius r is 2.5 m. In Figure E.2 the shear stress is shown for this dome design. For Figure E.1 load combination 1 results in the highest stresses and for the shear stresses in Figure E.2 this is load combination 2.

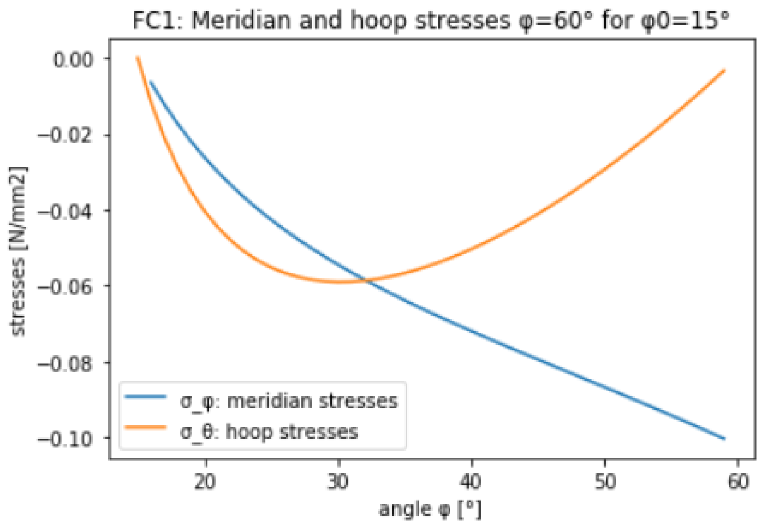

Figure E.1 Meridian and hoop stresses for r = 2.5 m

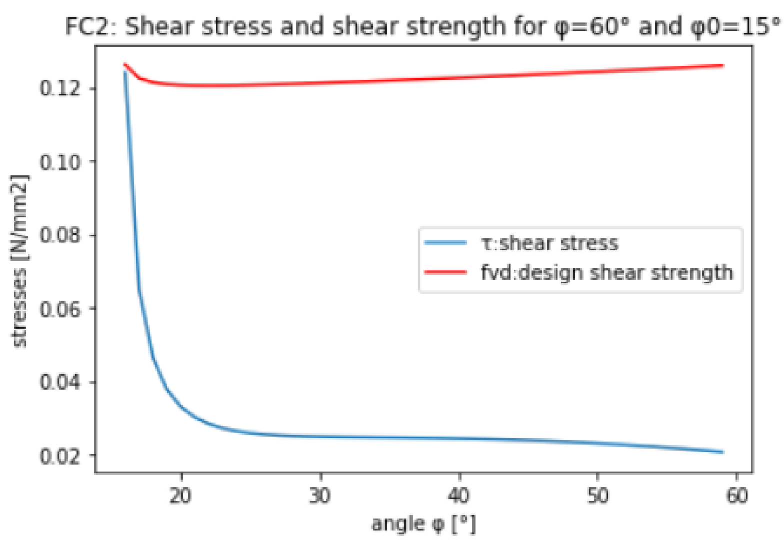

Figure E.2 Shear stresses for r = 2.5 m