TRANSVERSE STRESSES IN SHEAR LAG OF BOX-GIRDER BRIDGES

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1 INTRODUCTION

Box and I-beam girders are two types of bridge structure. Compared to the I-beam type, the box girder bridge is lightweight and has better torsion rigidity. Besides this, larger girders can be built with box type because of the two webs. However, high maintenance and fabrication costs make it not feasible to fully replace I-beam girders.

Although the shear lag effect was first studied by T. von Karman [1], it did not draw bridge designer's attention until a series of disasters happened from 1969 to 1971. Several steel box girder bridges collapsed or lost stability in Europe and Australia. The shear lag effect on flange plates is believed as one reason for these disasters [2].

Nowadays, when designing a bridge, no matter whether it is a box or I-beam girder, the shear lag effect is a must factor which cannot be omitted. By using the effective width of flange plate, the shear lag effect can be taken into account. This can be obtained from national design codes, for example Euro-code 3. There are different methods for solving the shear lag problem to get an approximate answer of the stresses, but almost all of them only take into account normal stress (σ_{xx}) in longitudinal direction. The transverse normal stress (σ_{yy}) is not as important as longitudinal one mainly because, this stress is generally far lower than σ_{xx} . However, recently, shear lag effect draws more attentions, as people found cracks in the middle of some concrete girder bridges. Transverse tensile stress (σ_{yy}) due to shear lag is found as the cause of the cracks.¹

The shear lag effect is not only relevant for bridges. It is relevant for any slender box element or plate structure such as airplane wings and core walls of high-rise buildings.

1.1 PROBLEM STATEMENT

In shear lag situations a transverse tensile stress or compressive stress occurs in the unloaded edge. This can lead to unexpected fatigue in steel box-girder bridges and unexpected cracks in concrete boxgirder bridges. There exists no formula to calculate this stress.

A mathematically exact solution to the shear lag problem has not been found as yet. It is not clear whether a mathematically exact solution exists.

1.2 SOLUTION METHOD

To solve the above mentioned problems the influence of shear lag has been studied on the transverse tensile stress (σ_{yy}) of the bottom flange plate of a closed box-girder (see Figure 1-1). The box-girder is fixed on one end, free on the other and loaded in pure bending. As shown in Figure 1-1, uniform shear forces from webs exert on both sides of the bottom plate. According to traditional simple beam theory, the tension stress σ_{yy} is null over the whole plate. This is not the case in reality because of shear lag effect. In this thesis, σ_{yy} of this plate has been calculated and compared with the edge load to find the relationship

¹ Personal communication with Dr. P.C.J. Hoogenboom in Sept. 2011

between them. Several factors which the author thinks could affect the value of σ_{yy} (with same external load from webs) are defined in section 3. The influence of them on σ_{yy} are determined and listed in section 3.

The finite element method has been used to calculate the stresses. The static structure package Ansys 11.0 (student edition) has been used for building the model and calculating the stresses. A formula has been determined for determining the transverse stresses. The finite elements have been made as small as possible to determine the transverse stress very accurately. The nature of the transverse stress formula (integer, rational, algebraic) gives a clue as to that whether an exact mathematical solution exists.

1.3 REPORT STRUCTURE

This report contains the following parts. Section 1 describes the problem of this thesis. In Section 2 theory background of the shear lag effect in both transverse and longitudinal direction is given and the approximate formula of solving the shear lag is introduced. Subsequently, the finite element modeling and analysis of the problem is given in Section 3. The conclusions are presented in Section 4.



Figure 1-1 Applied shear force and occurring tensile stress in a slender plate (the top flange of a box-girder)

2 SHEAR LAG EFFECT

2.1 SHEAR LAG EFFECT IN X DIRECTION

The traditional beam theory assumes that beam cross sections remain plane after bending, which results a uniform normal stress distribution in flange plates and a uniform shear flow in the cross-section. This is not the case in reality. Shear stress does not flow uniformly in the cross section at both ends of a box-girder. At the connection between web and flange, shear stress is higher than in other parts. The nonlinear shear flow in the cross-section influences the normal stress in the flange plates. At the connection of the flange and web, the normal stress is higher and it decreases away from the connection. Due to the nonlinear shear distribution over cross-section, the normal stress presents a "lag" in its distribution. This phenomenon is called shear lag.





Shear lag was first analyzed by T. von Karman in 1923 [1] by using an analytic method. Since then, several scholars solved the shear lag problem in different structures by using different methods. Such as the analysis of the shear lag in box beams by the principle of minimum potential energy of E. Reissner [3]. He proposes the following approximation to the flange displacement *u* in the *x* direction.

$$u(x, y) = \pm h \left[\frac{dz}{dx} + \left(1 - \frac{y^2}{w^2} \right) U(x) \right]$$

in which,

h half of the web height,

z the beam deflection,

w half of the flange width,

U(x) a function to be determined by the principle of minimum potential energy.

The second term on the right hand side of the equation represents the correction due to shear lag. Important to this project is that Reissner's method provides an approximate solution. Apparently, the mathematically exact solution has not been found.

In practice, for closed box girders, the shear lag coefficient is used for showing degree of shear lag:

$$\lambda = \frac{\overline{\sigma}_{xx}}{\sigma_{xx}}$$

where, σ_{xx} is the stress calculated by the beam theory and $\overline{\sigma}_{xx}$ is the largest stress with shear lag effect. Obviously, a wider flange and a less deep web make the shear lag effect more prominent. It also has a relation with the box girder span if this span is small.

The shear lag effect in x direction and the definition of the effective width for I-section girders and box girders are also covered in Eurocode 3 part 1-5.

2.2 SHEAR LAG EFFECT IN Y DIRECTION

To analyze the stress in the *y* direction (transverse stress), the plate is simplified as a truss structure shown in Figure 2-2. The external shear forces from webs are substituted by two equal point loads which act on two corners of the left truss bar. The reaction force from the fixed end is replaced by two equal point loads on two corners of the right truss bar. Due to the shear lag effect, the four bars truss does not have a rectangular shape but a trapezoidal shape. Manual calculation of this truss structure with four external loads shows that there is a tension force in left truss bar which means, instead of zero transverse stress, the free end of the plate has transverse a tension stress σ_{yy} . The distribution of this tensile stress is shown in Figure 2-3.



Figure 2-2 Strut-and-tie model of the shear lag situation



Figure 2-3 Typical normal stresses in a longitudinal section

3 MODELING

The bottom flange plate of a cantilever girder is chosen for analysis. Shear forces from web to flange are applied as external uniform distributed load. Eight parameters are involved which can affect the max σ_{yy} . They are described in next sections. Four kinds of models have been built in this project:

Model 1

Model 1 has a regular mesh shown in Figure 3-1. It has been used in most of the analysis. In Section 3.2 all of the models are built as Model 1. The finite elements have been given the following widths 5, 2.5, 1.25, 0.625, 0.3125, 0.15625 m. Further smaller elements were not possible because the software (Ansys 11.0 student edition) can process at most 32000 nodes.



Figure 3-1 Model 1

Model 2

Model 2 is a symmetrical model (see Figure 3-2). Compared with Model 1, Model 2 has fewer elements. Therefore, it takes less time to run the model. In Model 2 the values of σ_{xy} and σ_{yy} are the same as those in Model 1 (See Section 3.3). The reason for Model 2 was that even smaller finite elements could be used. However, an element size of 0.078125 m resulted in still too many degrees of freedom for the software.



Figure 3-2 Model 2

Model 3

In Model 3 the element width of the free end is smaller than elsewhere. The mesh width is 0.1 m at the free end and 0.15625 m at the fixed end.



Figure 3-3 Mesh of Model 3

Model 4

In Model 4 each node has two degrees of freedom (dofs) instead of six in the other models (See Section 3.3). To this end element 'Plane82' has been chosen. Figures 3.4 and 3.5 show the geometries of the element types 'Shell63', 'Shell93' and 'Plane82' [4]. The dofs of 'Shell63' and 'Shell93' are Ux,_Uy, Uz, ROTx, ROTy and ROTz. The dofs of 'Plane82' are Ux and Uy only. The reason for Model 4 was that even smaller finite elements could be used. However, an element size of 0.078125 m resulted in still too many degrees of freedom for the software.



Triangular Option



3.1 **ANALYSIS METHODOLOGY**

Before building the model, the following parameters are selected:

X (or Radial)

1

Table 3-1 Parameters of model

Parameters	Value
Width	10m
Length	100m
Thickness	0,5m
Shear force	1kN/m ²
Yong's modulus	200Gpa
Poisson ratio	0,27
Element type	variable
Mesh size	variable

A model with these parameters has been built in ANSYS in the following steps:

- 1) Model a rectangular plate with certain width, length, thickness, Young's modulus, Poisson's ratio and shear force.
- 2) Choose the element type, for example first 'Shell63' is chosen in the model.
- 3) Change one parameter, keep the others constant.
- 4) Get the results of *yy* component of stress of the free end and the *xy* shear stress in the long edge.

3.2 MODEL 1 WITH PARAMETERS

The results from different parameters are shown in the below paragraphs.

3.2.1 MODEL WITH ELEMENT TYPE 'SHELL63'

For the first modelling with Ansys, the element type 'Shell63' is chosen (see

Figure **3-6**). 'Shell63' has 4 nodes which have 6 degrees of freedom respectively. The model is built with the parameters shown in Figure 3-6. Figure 3-7, Figure 3-8 and Figure 3-9 show the stress contour plot and the displacement plot, which are obtained with 'Shell63' element and an element width of 5 meter.



Figure 3-6 Finite element model with mesh, forces and supports



Figure 3-7 yy component of the stress contour plot



Figure 3-8 xy shear stress contour plot



Figure 3-9 Displacement plot

Subsequently, all parameters are kept constant but the mesh size is changed.

Table 3-2 shows the results. Figure 3-10 shows the relation between the ratio σ_{yy} / σ_{xy} and the element width.

Table 3-2 Result for 'Shell63'

Mesh	σ_{vv}	σ _{xy}	Ratio σ_{yy}/σ_{xy}
5	0.26933	0.2	1.346665
2.5	1.26719	0.6	2.111983333
1.25	2.81677	1.4	2.011978571
0.625	5.76526	3	1.921753333
0.3125	11.595	6.2	1.87016129
0.15625	23.2219	12.6	1.843007937



Figure 3-10 Ratio σ_{yy} / σ_{xy} as a function of the element width for element type 'Shell63' (100*10)

3.2.2 MODEL WITH ELEMENT TYPE 'SHELL93'

The element type is changed to 'Shell93', which has 8 nodes. Due to the additional nodes, the result must be more accurate than 'Shell63'. For the models of 'Shell93' the dimensions are changed so that the more results are obtained. The results are shown in the following subsections.

3.2.2.1 Model with dimension 100*10m

In this subsection the dimensions are the same as the 'Shell63' model (100*10m).

Table 3-2 shows the results. Figure 3-11 shows ratio σ_{yy}/σ_{xy} as a function of the element width for element type 'Shell93'.

Mesh	σ _{yy}	σ _{xy}	Ratio σ_{yy}/σ_{xy}
5	1.497	0.799979	1.871299122
2.5	2.819	1.6	1.761875
1.25	5.76	3.2	1.8
0.625	11.603	6.4	1.81296875
0.3125	23.2047	12.8	1.812867188
0.15625	46.4628	25.6	1.814953125







3.2.2.2 Models with dimension 100*5m

In the following table and figure the results of the 'Shell93' models with dimension 100*50m are shown.

Table 3-4 Results for 'Shell93' (100*5m)

Mesh	σ_{vv}	σ _{xv}	Ratio σ_{yy}/σ_{xy}
5	0.715982	0.806577	0.88767966
2.5	2.993	1.6	1.870625
1.25	5.637	3.2	1.7615625
0.625	11.538	6.4	1.8028125
0.3125	23.115	12.8	1.80585938
0.15625	46.4546	25.6	1.81463281



Figure 3-12 Ratio σ_{yy} / σ_{xy} as a function of the element width for element type 'Shell93' (100*5m)

3.2.2.3 Model with dimension 50*10m

Now keep the width 10 meter and let the length be 50 meter. The results are shown in

Table 3-5 and in Figure 3-13.

Table 3-5 Results for 'Shell93' (50*10m)

Mesh	σ _{yy}	σ _{xy}	Ratio $\sigma_{yy} / \sigma_{xy}$
5	1.49969	0.800326	1.873848907
2.5	2.81851	1.59997	1.76160178
1.25	5.37587	3.19991	1.680006625
0.625	11.603	6.39992	1.812991412
0.3125	23.2273	12.8	1.814632813
0.15625	46.4628	25.6	1.814953125





3.2.3 COMPARISON OF THE RESULTS

In Table 3-6 the results from 'Shell63' (100*10m), 'Shell93' (100*10m), 'Shell93' (100*5m) and 'Shell93' (50*10m) are shown. It can be observed that changes in the length and width of the plate do not change the transverse tensile stress at an element size of 0.15625 m. The results show that at element width 0.15625 the ratio of σ_{yy}/σ_{xy} is 1.815. The accuracy is 4 significant digits.



Table 3-6 Comparison of the results

3.3 PARAMETERS STUDY

In this section all models have an element size of 0.15625 m. The other parameters have been changed. The results are shown in Table 3-7. In addition, Table 3-7 shows the results of Models 2, 3, and 4. It is noted that Model 3 gives less accuracy despite of the smaller elements. This can be caused by the fact that the elements are no longer square. It can also be caused by the loss of accuracy due to the large number of equations that are solved.

3.4 EXACT MATHEMATICAL SOLUTION

If an exact mathematical solution to the shear lag problem would exist then the factor 1.815 would be the evaluation of a fraction or a mathematical constant. It has been tried to find an expression which is equal

to 1.814 Many combinations of integers, , e, and $\sqrt{}$ have been tried. The best results found are $\frac{49}{27}$

=1.818 and $\frac{\pi}{\sqrt{3}}$ = 1.813... Clearly, neither of these can be the exact factor. Therefore, it is concluded

that an exact mathematical solution to the shear lag problem is not likely to exist.

Table 3-7 Parameter study results

Model	No.	Element	Mesh width	Length	Width	Thickness	Possion's	Young's	Shear	σ _{γγ}	σ _{xy}	σ _{yy} / σ _{xy}
		type					ratio	modulus	force			
Model1	1	Shell 63	0.15625	100	10	0.5	0.27	2.00E+08	1	23.2219	12.6	1.843007937
	2	Shell 93	0.15625	100	10	0.5	0.27	2.00E+08	1	46.4628	25.6	1.814953125
	3	Shell 93	0.15625	100	5	0.5	0.27	2.00E+08	1	46.4546	25.6	1.814632813
	4	Shell 93	0.15625	50	10	0.5	0.27	2.00E+08	1	46.4628	25.6	1.814953125
	5	Shell 93	0.15625	100	10	0.02	0.27	2.00E+08	1	1161.57	640	1.814953125
	6	Shell 93	0.15625	100	10	0.5	0.3	2.00E+08	1	46.4628	25.6	1.814953125
	7	Shell 93	0.15625	100	10	0.5	0.27	4.00E+07	1	46.4628	25.6	1.814953125
Model2	8	Shell 93	0.15625	100	5	0.5	0.27	2.00E+08	1	46.4628	25.6	1.814953125
Model3	9	Shell 93	0.15625(0.1	100	10	0.5	0.27	2.00E+08	1	46.4512	25.611	1.813720667
			for free end)									
Model4	10	Plane82	0.15625	100	10	0.5	0.27	2.00E+08	1	23.2314	12.8	1.814953125

4 CONCLUSIONS

Based on the results in section 3, following conclusions are obtained:

1. Finite element analyses show that the perpendicular stress due to shear lag is a factor 1.815 larger than the edge shear stress. This factor does not depend on Young's modulus, Poisson's ratio, the plate thickness, the plate width and the plate length, provided that the plate is much longer than wide and the shear stress is uniformly distributed on the long plate edges.

2. The factor 1.815 cannot be expressed as a simple fraction or a mathematical constant. Therefore, an exact mathematical solution to the shear lag problem is not likely to exist.

3. The student edition of Ansys can process models up to 32000 nodes. With this a stress accuracy of four significant digits can be obtained.

References

- [1] T. von Kármán, "Festschrift Angust Föppl", p. 114, 1923
- [2] Zhang Shiduo, Deng Xiaohua, Wang Wenzhou. Shear lag effect of thin-wall box beams [M]. China Communications Press, 1998. (In Chinese)
- [3] E. Reissner, Analysis of shear lag in box beams by the principle of minimum potential energy, The Quarterly of Applied Mathematics, Vol. 4, Oct. 1946, pp. 268-278
- [4] Ansys11.0 Tutorial, Ansys Inc. 2011

Appendix A APDL LANGUAGE CODE OF MODEL 1

FINISH \$/CLEAR /FILNAM, MODEL93 /PREP7	! SECIFY JOBNAME ! ENTER PREPROCESSOR
W=100 CHANGED H=10	! DIMENSIONS RECTANGLE, THE PARAMETERS CAN BE
D=0.5 MW=0.15625 F1=1 F2=0.5 E=200E6 PR=0.27	! MESHSIZE REDUCE FROM 5 TO 0.15625
K,1,0,0 K,2,W,0 K,3,0,H K,4,W,H	! CREATE KEYPOINTS
L,1,2 L,1,3 L,2,4 L,3,4	! CREATE LINES
AL,1,2,3,4	! CREATE AREAS
ET,1,93	! SELECT ELEMNT TYPE AS SHELL63/OR93
R,1,D	! SPECIFY REAL CONSTANT THICKNESS
MP,EX,1,E MP,NUXY,1,PR	! SPECIFY ELASTIC MODULUS ! SPECIFY POISSON'S RATIO
ESIZE,MW AMESH,ALL	! CREATE MESH WITH ELEMENT SIZE
LSEL,S,LOC,X,W DL,ALL,,ALL,0	! SELECT THE LINES
NSEL,S,LOC,Y,0 NSEL,A,LOC,Y,H NSEL,U,LOC,X,0 F,ALL,FX,F1	SELECT THE NODES
NSEL,S,LOC,X,0 NSEL,R,LOC,Y,0 F,ALL,FX,F2	
NSEL,S,LOC,X,0 NSEL,R,LOC,Y,H F,ALL,FX,F2	

ALLSEL

/SOLU SOLVE

Appendix B APDL LANGUAGE CODE OF MODEL 2

FINISH \$/CLEAR /FILNAM, MODELSHELL93 /PREP7	! SECIFY JOBNAME ! ENTER PREPROCESSOR
W=100 H=5 D=0.5	! DIMENSIONS RECTANGLE
MW=0.15625 F1=1 F2=0.5 E=200E6 PR=0.27	! MESHSIZE REDUCE FROM 5 TO 0.15625
K,1,0,0 K,2,W,0 K,3,0,H K,4,W,H	! CREATE KEYPOINTS
L,1,2 L,1,3 L,2,4	! CREATE LINES
AL,1,2,3,4	! CREATE AREAS
ET,1,93	! SELECT ELEMNT TYPE AS SHELL63/OR93
R,1,D	SPECIFY REAL CONSTANT THICKNESS
MP,EX,1,E MP,NUXY,1,PR	! SPECIFY ELASTIC MODULUS ! SPECIFY POISSON'S RATIO
ESIZE,MW AMESH,ALL	! CREATE MESH WITH ELEMENT SIZE
LSEL,S,LOC,X,W DL,ALL,,ALL,0 LSEL,S,LOC,Y,H DL,ALL,,UY,0 NSEL,S,LOC,Y,0 NSEL,U,LOC,X,0 F,ALL,FX,F1 NSEL,S,LOC,X,0 NSEL,R,LOC,Y,0 F,ALL,FX,F2 ALLSEL	! FOR SYMETERSCHE MODEL
/SOLU SOLVE	

Appendix C APDL LANGUAGE CODE OF MODEL 3

FINISH\$/CLEAR /FILNAM, MODELSHELL93 /PREP7	! SECIFY JOBNAME ! ENTER PREPROCESSOR
W=50 H=5 D=0 5	! DIMENSIONS RECTANGLE
MW1=0.1 MW2=0.15625 F1=1 F2=0.5 E=200E6 PR=0.27	! MESHSIZE FOR FREE END ! MESHSIZE REDUCE FROM 5 TO 0.15625
K,1,0,0 K,2,W,0 K,3,0,H K,4,W,H	! CREATE KEYPOINTS
L,1,2 L,1,3 L,2,4 L,3,4	! CREATE LINES
AL,1,2,3,4	! CREATE AREAS
ET,1,93 R,1,D MP,EX,1,E MP,NUXY,1,PR	! SELECT ELEMNT TYPE AS SHELL63/OR93 ! SPECIFY REAL CONSTANT THICKNESS ! SPECIFY ELASTIC MODULUS ! SPECIFY POISSON'S RATIO
LSEL,S,LOC,X,0 MESH SIZE	! CREATE MESH AT THE FREE END WITH SMALLER
LESIZE, ALL, MVV1,, AESIZE, 1, MW2 MESH SIZE MSHAPE,0 AMESH, ALL LSEL, S, LOC, X, W DL, ALL,, ALL,0 NSEL, S, LOC, Y,0 NSEL, A, LOC, Y,0 F, ALL, FX, F1 NSEL, S, LOC, X,0 NSEL, R, LOC, Y,0 F, ALL, FX, F2 NSEL, S, LOC, X,0 NSEL, R, LOC, Y, H F, ALL, FX, F2 ALL, FX, F2 ALL, SEL	! CREATE MESHES ON THE AREA WITH BIGGER

/SOLU SOLVE

Appendix D APDL LANGUAGE CODE OF MODEL 4

FINISH \$/CLEAR /FILNAM, MODEL82 /PREP7 W=100 H=10 D=0.5	! SECIFY JOBNAME ! ENTER PREPROCESSOR ! DIMENSIONS RECTANGLE
MW=0.15625 F1=1 F2=0.5 E=200E6 PR=0.27	! MESHSIZE REDUCE FROM 5 TO 0.15625
K,1,0,0 K,2,W,0 K,3,0,H K,4,W,H	! CREATE KEYPOINTS
L,1,2 L,1,3 L,2,4	! CREATE LINES
AL,1,2,3,4	! CREATE AREAS
ET,1,82 R,1,D MP,EX,1,E MP,NUXY,1,PR	! SELECT ELEMNT TYPE AS PLAN 82 ! SPECIFY REAL CONSTANT THICKNESS ! SPECIFY ELASTIC MODULUS ! SPECIFY POISSON'S RATIO
ESIZE,MW AMESH,ALL LSEL,S,LOC,X,W DL,ALL,,ALL,0 NSEL,S,LOC,Y,0 NSEL,A,LOC,Y,0 F,ALL,FX,F1 NSEL,U,LOC,X,0 F,ALL,FX,F2 NSEL,R,LOC,Y,0 F,ALL,FX,F2 NSEL,S,LOC,X,0 NSEL,R,LOC,Y,H F,ALL,FX,F2 ALLSEL /SOLU SOLVE	! CREATE MESH WITH ELEMENT SIZE