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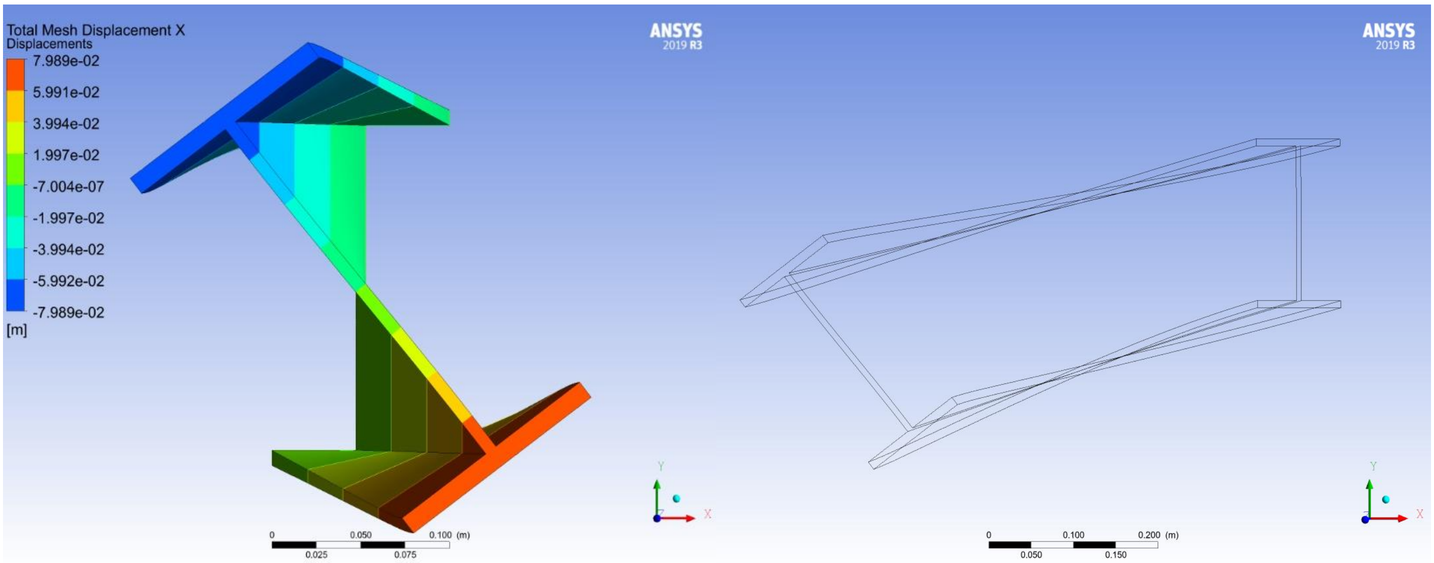
Checks of the Vlasov torsion theory



Delft University of Technology

Checks of the Vlasov torsion theory

By

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in partial fulfilment of the requirements for the degree of

Bachelor of Science
in Civil Engineering

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Preface

In front of you lies the bachelor thesis for the completion of the Bachelor in Civil Engineering from Delft University of Technology. This thesis is about checking the validity boundaries of the Vlasov torsion theory. This study is meant for students and researchers who are interested in the application of the theory of Vlasov and would like to research more about the possible limitations of the theory of Vlasov. This thesis was written during the lockdown of 2021, with much harder conditions due to academic limitations. I would like to thank my supervisors Dr. ir. P.C.J. Hoogenboom and Prof. Dr. M. Veljkovic for their valuable time and guidance during this process, despite the hardships of the pandemic and the lockdown. I would also like to thank my study advisor Anneloes Klapwijk for allowing me to work on the campus so I can focus on finishing my thesis and therefore my Bachelor.

A. Yildirim
Den Haag, April 2021

Content

Abstract

The torsion theory of Vlasov is one of the few ways to describe the torsional and warping behaviour of a beam and is mainly meant for thin-walled open cross sections. It differentiates itself from the Saint-Venant theory by taking the constraint in one end into account, whereas the Saint-Venant theory is meant for a beam which is free on both ends. According to a study performed by T. Raaphorst (2020) the theory of Vlasov is applicable for tube cross sections as well as massive rectangular cross sections for the calculations of displacement. However, it is uncertain whether the theory can be applied to every massive and tube cross sections for the calculations of the axial stresses, which indicates that there are possible limitations to the theory of Vlasov.

The goal of this study is to check the limits of the torsion theory by answering the research question ‘Are there validity boundaries to the theory of Vlasov?’. The research question is divided in sub questions, which aim to check if the theory can be applied for open cross sections and for closed cross sections. To answer this question, two different profiles are modelled in a Finite Element Analysis (FEA) program called Ansys and in the combination of Maple and ShapeDesigner, where the latter represents Vlasov’s theory.

The profiles that are modelled are Z-profiles and hollow square profiles. Each profile will have four beams that will vary in the length of the beam and in the thickness of the web and flanges, meaning that there are a total of eight beams. The length of the beams are either 0.25 m or 2 m and the thickness is either 5 mm or 20 mm. The dimension of the Z-profile is 100 mm width for the flanges and a 200 mm height for the web. The hollow square profile has a width and height of 100 mm. Beam one to four are Z-profiles with the combination the varying length and thickness. Beam one for example is a Z-profile with a length of 0.25 m and a thickness of 5mm. Beam two has a length of 2 m and a thickness of 5mm. Beam five to eight are hollow square profiles with the same varying setup as the Z-profile.

The beams are modelled in Ansys with the finest mesh possible due to licence limitations. The results are then compared with the results from the combination Maple and ShapeDesigner. Since Maple and ShapeDesigner is more accessible to the public and since the calculations from Maple and ShapeDesigner are quicker, these results are seen as the comparand for the error. Meaning; the results from Maple and ShapeDesigner are seen as the 100% value and are the denominator in the unity check. The maximum allowed margin of error is 10%, which means that the unity check should be between 0.9 and 1.1.

From this study it can be concluded that there are indeed limits to the theory of Vlasov. The results from the Z-profile were unexpected. The summary of the results can be found on page 27. The initial expectations was that the displacements would be accurate for each case and that the difference in normal stress would be higher than 10%. Against all expectations, the normal stress was accurate for all the Z-profile beams whereas the displacement for the short beams were inaccurate. The normal stresses at the fixed end was larger than at any cross sections of the beam as expected.

The shear stresses were not accurate for any Z-profile beam. The shear stresses at any cross section in the beam were also larger than at the free end, which goes against the expectation that the largest shear stresses should be at the free end. This may be the reason why the shear stresses are inaccurate. It is not clear however, whether the fault lies with Ansys or ShapeDesigner, since the shear stresses for beam 1 are the same as beam 2 in ShapeDesigner as well as for beam 3 being the same as beam 4. It may be better to judge the theory of Vlasov by judging the errors in displacements and normal stresses since the normal stresses directly come from the theory (Maple) and the normal stresses follow from the input of bi moments in ShapeDesigner, where the bi moments also follow from the theory (Maple).

Which answers the sub question ‘can the theory of Vlasov be applied to open cross sections’ as follows; yes, it can be applied to calculate the normal stresses for any length. However, to calculate the displacements, the theory can only be applied to longer beams. Where the limits for the length of the beam lie is not yet known, but they should at least be larger than 0.25 m.

The results from the hollow square profile was also unexpected and the opposite of the Z-profile. The summary of the results can be found on page 39. The displacements were accurate for all except beam 5, which is a beam that is 0.25 m long and has a thickness of 5 mm. Beam 7, which is also a short beam but has a thickness of 20 mm, was unexpectedly within allowed margin. The normal stresses were significantly different for all beams, all almost by a factor 10. It is remarkable that the normal stresses at the fixed end of the beams were larger than any cross section in the beam but still far too small compared to the results of ShapeDesigner. The shear stresses, however, were all accurate for every hollow square profile.

Which answers the sub question ‘can the theory of Vlasov be applied to closed cross sections’ as follows; no, it cannot be applied to calculate the normal stresses for any length since there is always a factor 10 difference which is too large to be neglected. However, to calculate the displacements, the theory can only be applied to most beams. ShapeDesigner can be used to calculate the shear stresses for every beam.

1 Introduction

1.1. Background information

There are a few ways to describe the torsional behaviour of a beam. One of these theories is the theory of Saint-Venant which is also known as the uniform torsion theory. However, this theory does not take the constraint of warping due to supports into account. The torsion theory of Vlasov does take the constraint of warping into account and describes the non-uniform torsional behaviour of a clamped beam. According to this theory the warping along the longitudinal axis is not constant (Hoogenboom, 2019). This theory applies to thin-walled open cross sections. According to the studies of T. Raaphorst (2020), Vlasov's theory can be applied for tube cross sections as well as for massive rectangular cross sections for the calculation of displacements. However, T. Raaphorst concludes that it is uncertain whether the theory of Vlasov can be applied to every massive and tube cross sections for the axial stresses, which indicates that there are probably limitations to the theory of Vlasov.

The differential equation of Vlasov is:

$$EC_w \frac{d^4\varphi}{dx^4} - GI_w \frac{d^2\varphi}{dx^2} = m_x \qquad \text{[Equation 1]}$$

With φ the deformation of the initial cross section in radians, EC_w the warping stiffness in Nmm^4, GI_w the torsional stiffness in Nmm'^ 2 and m_x the distributed torsional moment along the beam in Nmm/mm.

1.2. Research question

Since there is an uncertainty about the applicability of the theory of Vlasov, this study will focus on the validity of the theory. Therefore, this study aims to answer the question 'Are there validity boundaries to the theory of Vlasov?'

To answer this question, the following sub-questions are posed:

- Can the theory of Vlasov be applied to open cross sections?
- Can the theory of Vlasov be applied to closed cross sections?

1.3. Approach and layout of the study

The validity of the theory will be approached by modelling different beams with different properties. Each beam will first be modelled in a Finite Element Analysis (FEA) program. In this study the preferred FEA program is Ansys. Afterwards, the beams are calculated with the differential equation of Vlasov, with the use of Maple and a cross section designer and calculator called ShapeDesigner. Since this study is about checking the validity of the theory of Vlasov, the results from Ansys will be compared with the results from Maple. This way, the reader can replicate the calculations and later on judge whether their problems can be solved with Maple or with Ansys. If the reader does use Maple, the reader can estimate how much the error could be. The error in this study is in the form of a unity check (u.c.). An example calculation is given in equation 2. The allowed margin of error is assumed to be 10%, which means a unity check between 0.9 and 1.1.

$$Unity\ Check = \frac{Ansys}{ShapeDesigner} \qquad \text{[Equation 2]}$$

To introduce the theory of Vlasov as well as the FEA program Ansys, a test beam is modelled. This test beam is an IPE200 beam, which is also calculated in the reader "aantekeningen over wringing" by Dr. Hoogenboom in Appendix 8 on page 58. Dr. Hoogenboom compares the results from Ansys with formulas based on the theory of Vlasov. The formulas are also an approach and not a direct result of the differential equations from the Vlasov's theory. In chapter two of this study the following results will be compared: Ansys results from Dr. Hoogenboom, hand calculations (based on formulas) from Dr. Hoogenboom, Ansys results from this study and the combination of Maple and ShapeDesigner results.

Afterwards two different beam profiles will be modelled to test the theory of Vlasov. A z-profile will be modelled in chapter three. Chapter four will be about a hollow square profile. Each profile will have a length of 0.25 m and 2 m and each of the either lengths will have a thickness of 5 mm and 20 mm, which means there are in total 8 models.
A summary of the beams can be found in Table 1. The dimensions of the cross sections can be found in Figure 1. The thickness t Figure 1 are the same as in Table 1.

The conclusion of the study can be found in chapter five and chapter six is the recommendation on potential studies to follow up on this topic. A linear buckling analysis is performed on the IPE200 test beam. But since no other buckling analyses have been performed on the other beams and because it had no further relevancy to the topic of this study, the buckling analysis is moved to Appendix C. It was only performed upon request.

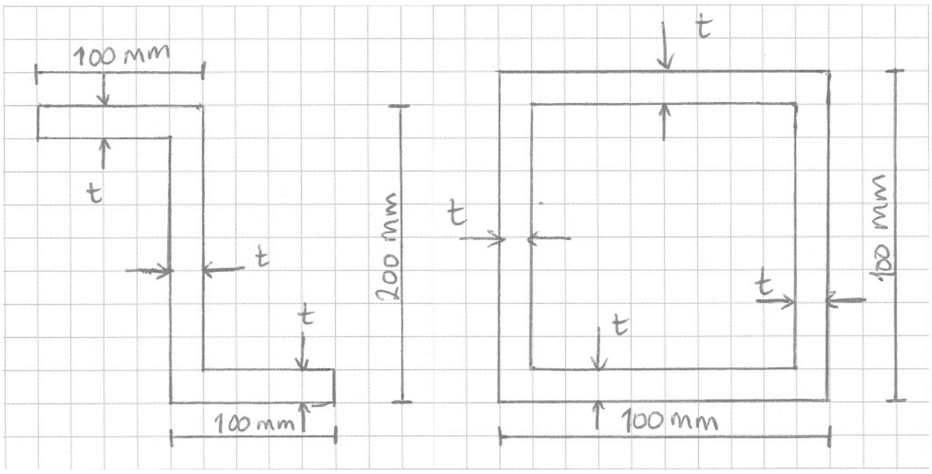

Figure 1 The dimensions of the Z profile (left) and the hollow square profile (right)

Table 1 The summary of the beams that will be modelled in chapter three and four.

Beam	Profile	Length	Thickness t
1	Z	0.25 m	5 mm
2	Z	2 m	5 mm
3	Z	0.25 m	20 mm
4	Z	2 m	20 mm
5	Hollow square	0.25 m	5 mm
6	Hollow square	2 m	5 mm
7	Hollow square	0.25 m	20 mm
8	Hollow square	2 m	20 mm

2 Test beam IPE200

In this chapter an IPE200 beam will be modelled and calculated in Ansys and compared to the Maple results from the theory of Vlasov as well as the results from Dr. Hoogenboom from Appendix 8 from the reader. The Maple scripts can be found in Appendix A.

2.1 Finite Element Analysis (FEA) in Ansys

A Finite Element Analysis is a method where a model, in this case an I-beam, is divided in smaller pieces, almost always small cubes. These pieces are called elements and the total array of the elements is called a mesh. If the elements are smaller, the mesh will be finer, which usually means more accurate results. The disadvantage of a too fine mesh is the time it takes the computer to calculate the model. Another disadvantage with too fine meshes is the occurrence of singularities. Singularities are points in model which show an unrealistically large force. These points are usually corners in crossing elements, for instance the inner corner of an I-beam in the transition point from flange to web. Since singularities are unrealistic values, these values should be ignored. When analysing a contour plot, it should be taken into account that the maximum (or minimum) shown value in the legend of the contour plot may not always be the correct maximum value since it could be a singularity. Usually, these singularities will be in the cross section at the support. Therefore, it could be a solution to look at the cross section *near* the support and not *at* the support, with a local legend.

The preferred FEA tool in this study is Ansys. Due to licence limitations, it is not possible to calculate the desired fineness for the meshes. The meshes can be created, but Ansys does not calculate the stresses, deformations or any other desired value.

2.1.1 IPE200 properties and Ansys model

The dimensions of the test beam can be found in Figure 2 down below. The test beam is a clamped IPE200 beam with a torsional load of 1.2 kNm.

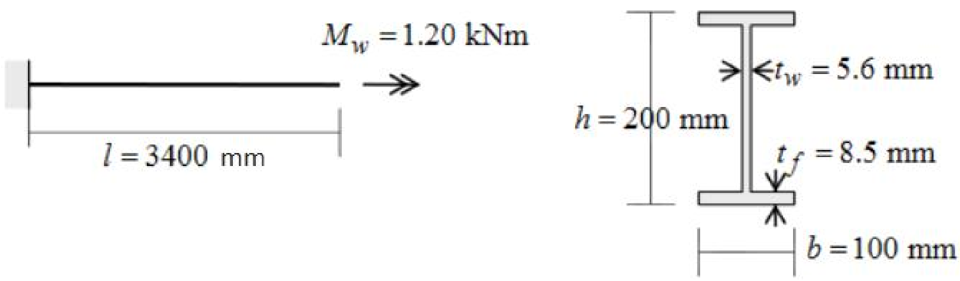

Figure 2 The clamped test beam with a length of 3.4 m undergoing a torsional load of 1.2 kNm, together with the dimensions of the test beam (Hoogenboom, 2019)

The properties below are copied from the reader of Dr. Hoogenboom which can be found in Appendix 8 on page 58.

Young's modulus	$E = 2.1\ 10^5$ N/mm²
Poisson's ratio	$\nu = 0.35$
Shear modulus	$G = \dfrac{E}{2(1+\nu)} = 77777$ N/mm²
Torsion constant	$I_w = \frac{1}{3}(h-t_f)t_w^3 + \frac{2}{3}bt_f^3 = 52152$ mm^4
Warping constant	$C_w = \frac{1}{24}t_f(h-t_f)^2 b^3 = 1299\ 10^7$ mm^6
Characteristic length	$l_c = \sqrt{\dfrac{EC_w}{GI_w}} = 820.0$ mm

Figure 3 displays the test beam modelled in Ansys. The origin of the model is at the base of the beam on the clamped side (shown with a shaded square). This is the fixed end. The torsional load of 1.2 kNm is applied on the face of the free end and is shown in red.

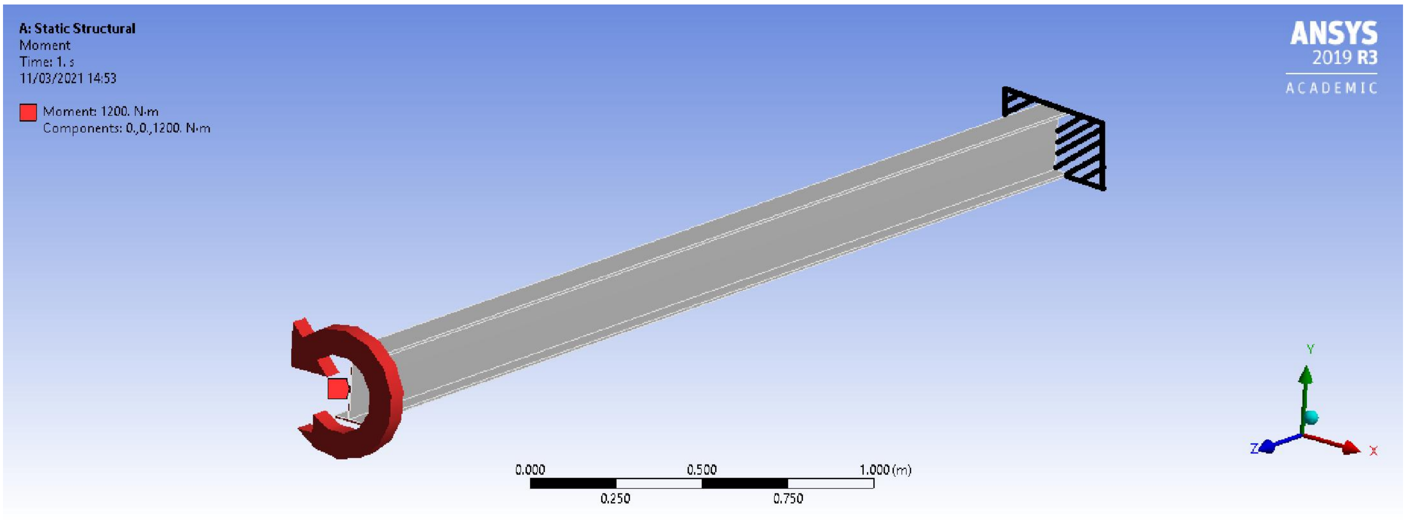

Figure 3 The test beam modelled in Ansys

The mesh size of the model is 5 mm element size with a mesh refinement on the free end, see Figure 4. If the reader wishes to recreate the model with the same setup, it is likely that the reader will not get a result. Probably due to a bug in the licensing servers, Ansys was able to calculate the model at first, but later denied the calculations since this mesh size is too fine for the student licence to be calculated.

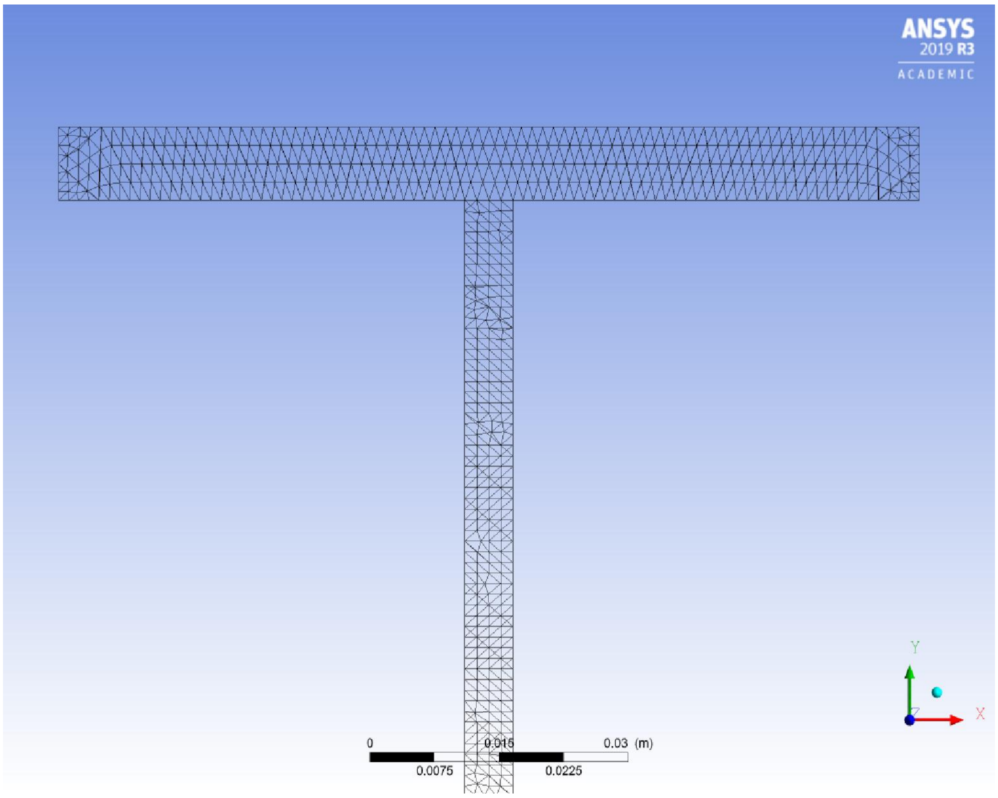

Figure 4 The mesh over the thickness of the flanges and the web on the free end

2.1.2 Displacement and rotation

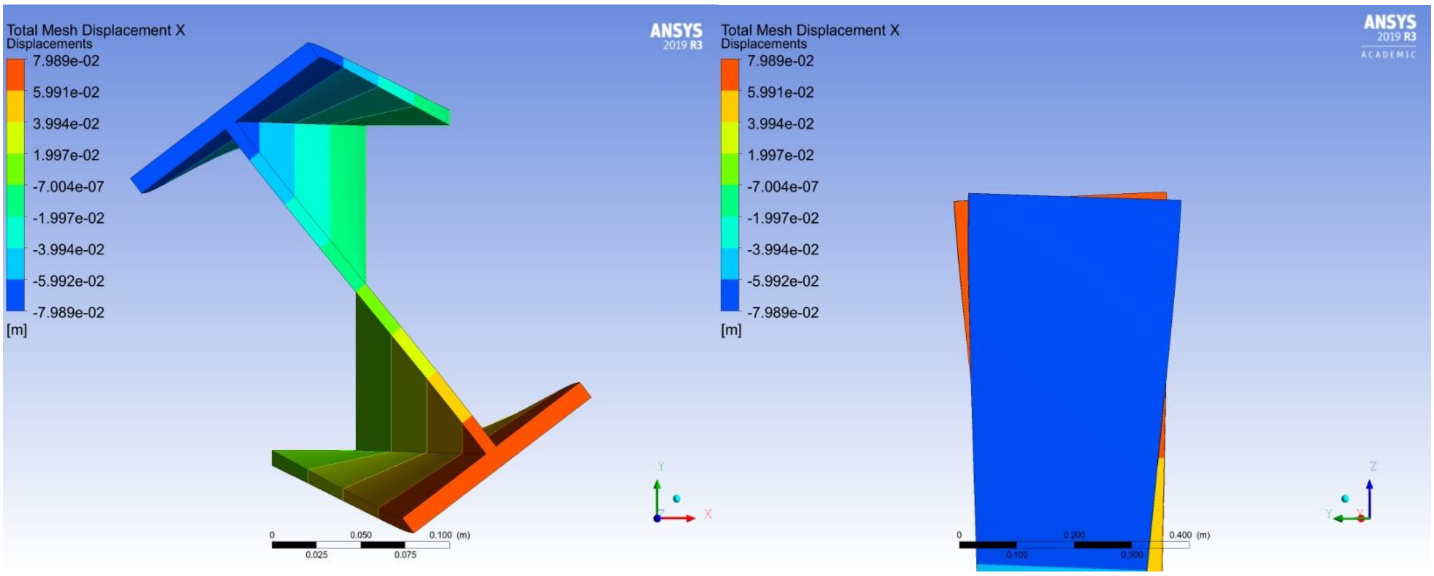

Figure 5 The contour plot of the horizontal displacement view with true scale. Left; view on z-axis. Right; The warping of the flanges on the free end

The horizontal displacement of the beam can be read in the legend in Figure 5. The maximum horizontal displacement is 79.89 mm and therefore the rotation is 0.799 rad. Dr. Hoogenboom found a displacement of 87.3 mm in Ansys, which is a rotation of 0.873 rad. The reason why the horizontal displacement is considered is because the theory of Vlasov is dependent on the X axis and calculates the rotation in the XY plane. When multiplying the height of the beam, the result would be the horizontal displacement.

2.1.3 Shear stresses

Dr. Hoogenboom analyzed the shear stresses in the XZ-plane and the YZ-plane at 300 mm from the free end. In Figure 6 the results from Ansys from this study are compared with the results of Dr. Hoogenboom for the XZ-plane. Figure 7 shows the comparison for the YZ-plane.

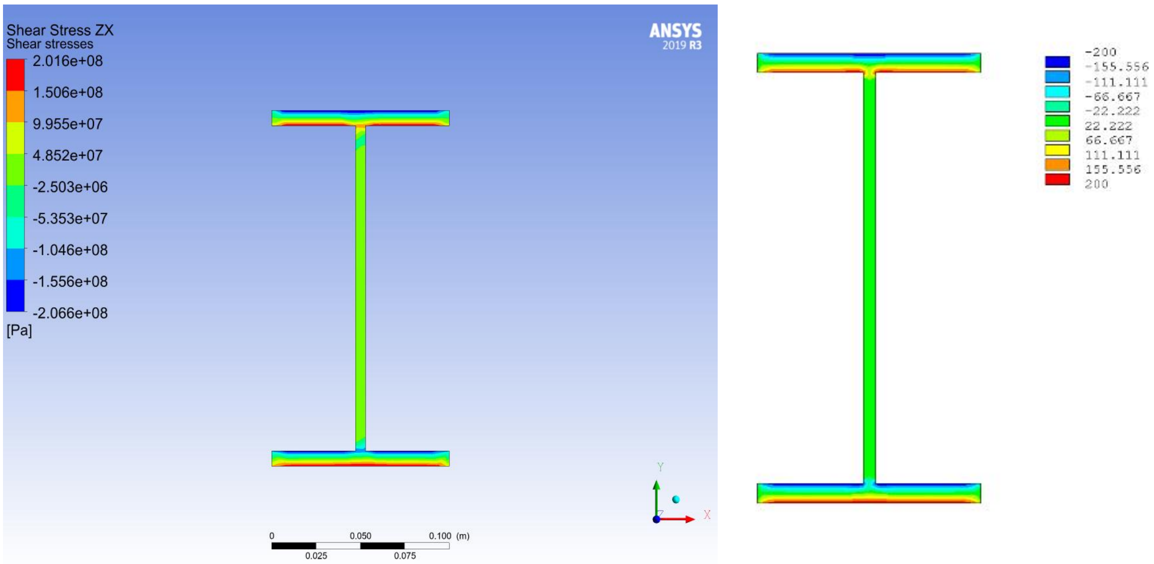

Figure 6 Contour plots for the shear stress in XZ-plane. Left; Contour plot from this study. Right; Contour plot from Dr. Hoogenboom's reader

Both contour plots in Figure 6 show similar contour plots with similar values. Dr. Hoogenboom found an absolute maximum shear value of 200 N/mm^2. The result from this study shows an absolute maximum result of 206.6 N/mm^2. This difference can be due to the difference in mesh size, since Dr. Hoogenboom has a finer mesh size and/or due to the difference in the way the torsional load is applied. Dr. Hoogenboom applied the torsion load by 6 forces on the edges of the flange and web in a counter clock-wise rotation to create the torsion.

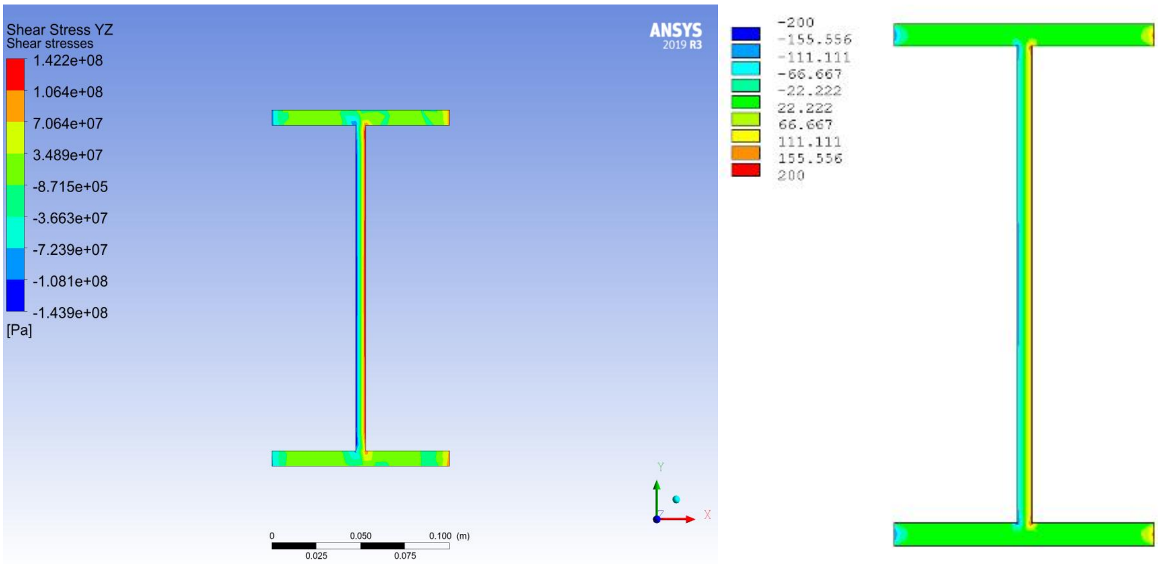

Figure 7 Contour plots for the shear stress in YZ-plane. Left; Contour plot from this study. Right; Contour plot from Dr. Hoogenboom's reader

The contour plots for the YZ-plane again show similar results, although the values in the legends do not coincide. The first difference is that the legend of the right plot is the same legend for the right plot in Figure 6. In other words, it is not a legend for this specific shear distribution, which *is* the case for the left plot in Figure 7. It can be noted that the colors of the maximum values do not show in the right plot, except for the transition corners from flange to web. However, as mentioned earlier, these points could be singularities. And therefore are not representative for the absolute maximum value for the shear stress in this cross-section.

2.1.4 Normal stresses

The normal stresses will be higher near the fixed end of the beam rather than the free end, since there is a constraint and the torsional load, which also causes normal stresses, needs to be transferred to the clamped part. The cross section is analyzed at 50 mm from the fixed end. Figure 8 shows the contour plots from this study as well as the reader.

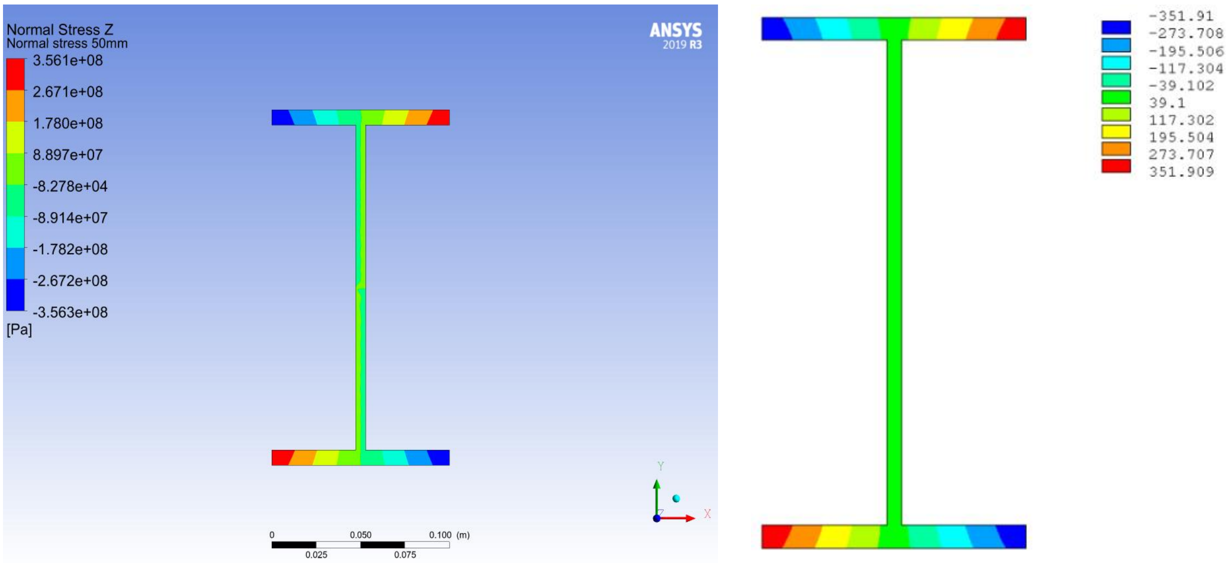

Figure 8 The contour plots of the normal stresses in Z-direction. Left; Contour plots from this study. Right; Contour plots from Dr. Hoogenboom's reader

Both contour plots show similar results with close absolute maximum values. The absolute maximum normal stress in z direction from Ansys is 356 N/mm^2. Dr. Hoogenboom got an absolute maximum normal stress of 352 N/mm^2.

2.2 Theory of Vlasov in Maple and ShapeDesigner

In this subsection the results from the FEA will be compared to the theory of Vlasov with the help of Maple and ShapeDesigner. ShapeDesigner is a program where any cross section can be modelled and together with material properties, it will calculate and return the cross section properties. It can also show contour plots for the normal stresses and deformations. Maple calculates the necessary input properties and values for ShapeDesigner.

2.2.1 Input

Firstly, the equation of Vlasov, equation 1, is put in Maple together with the properties from page 9. For the differential equations it is important to give boundary conditions, so that the equation can be calculated. Before defining the boundary conditions, other important equations will be discussed. The first important equation is the equation for the bimoments. According to Dr. Hoogenboom, the bimoments can be interpreted as the moments in the flanges times the distances between the two (bi) moments. In the case of the test beam, that would be 200 mm. The second important equation is equation for the torsional load Mw.

In the reader of Dr. Hoogenboom, the following equations have been given for the bimoments and torsional load:

$$B = -EC_w \frac{d^2\varphi}{dx^2} \qquad \text{[Equation 3]}$$

$$M_w = GI_w \frac{d\varphi}{dx} + \frac{dB}{dx} = GI_w \frac{d\varphi}{dx} - EC_w \frac{d^3\varphi}{dx^3} \qquad \text{[Equation 4]}$$

The boundary conditions for a cantilevering clamped beam can also be found in the reader and are as following:

- At the fixe end (x=0) the rotation φ = 0 and warping $\frac{d\varphi}{dx}$ = 0

- At the free end (x=l) the torsional load is equal to the applied torsional load $GI_w \frac{d\varphi}{dx} - EC_w \frac{d^3\varphi}{dx^3} = M_w$ and the bimoments $B = -EC_w \frac{d^2\varphi}{dx^2} = 0$

2.2.2 Displacement and rotation

With these boundary conditions the differential equation can be solved. The theory estimates a displacement of 77.19 mm and therefore a rotation of 0.772 rad. The hand calculations Dr. Hoogenboom estimate a displacement of 76.3 mm and a rotation of 0.763 rad. Which means a unity check of 0.988. The unity check for the displacement from Ansys from this study is 1.035. The unity check for the results between maple and Ansys from Dr. Hoogenboom is larger, namely 1.131, which is the same error as he had found between his Ansys results and hand calculations. The results from Ansys from this study and the hand calculations from Dr. Hoogenboom are within the allowed error margin, but the results from Ansys from Dr. Hoogenboom are larger than the error margin.

2.2.3 Shear stress

With the use of various formula, derived from the theory of Vlasov, the shear stresses and normal stresses can be calculated. Calculating the shear stress can be tricky and to calculate the normal stresses a parameter called the warping function ψ is needed. The warping function can be calculated with the help of ShapeBuilder or ShapeDesigner. The preferred program for this study is ShapeDesigner, which, instead of calculating the value of ψ, immediately plots the normal stress in the cross section due to the bimoments, created by the torsional load.

For this to work, the cross section is hand drawn in ShapeDesigner according to the dimensions (as given in Figure 2) together with the loads, including the maximum bimoment, which is obtained from the results of Maple.

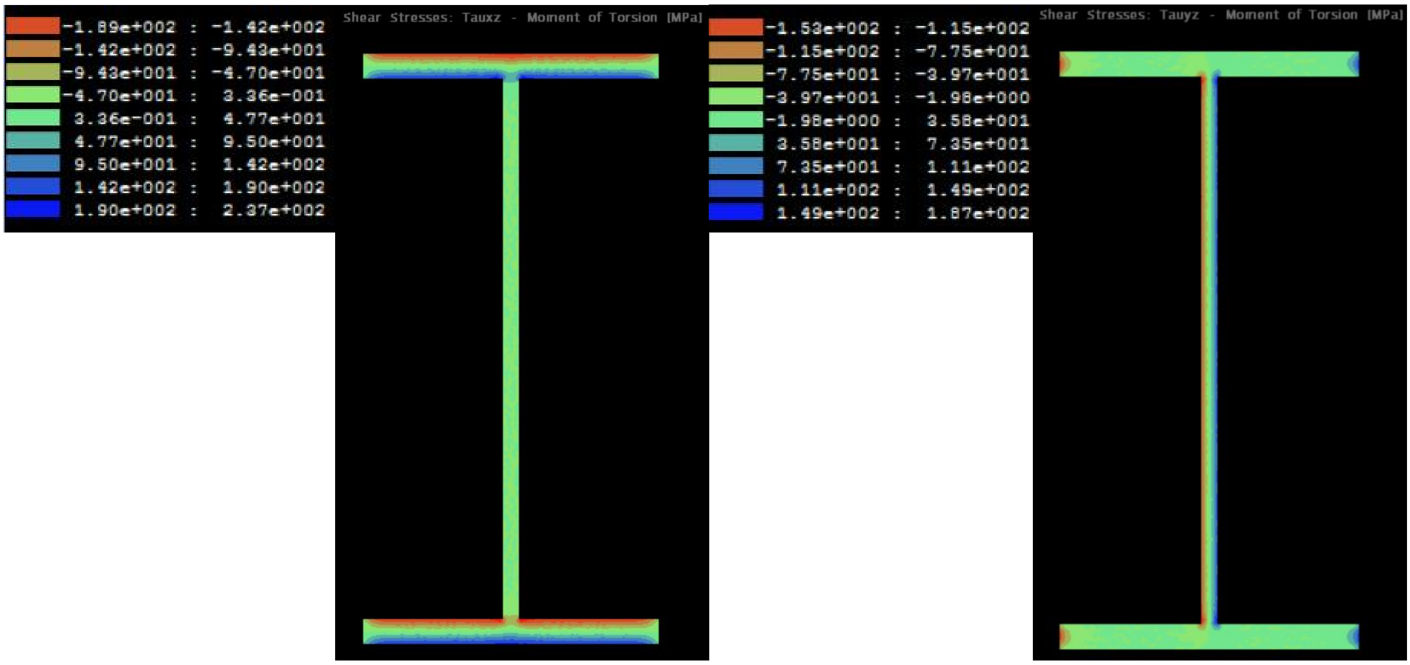

Figure 9 Left: The contour plot of the shear stress in XZ-plane. Right: The contour plot of the shear stress in YZ-plane

The contour plots in Figure 9 are similar to the contour plots in Figure 6 and 7 respectively. Since only the XZ-plane has been discussed, here too, only the XZ-plane will be compared. The absolute maximum value according to ShapeDesigner is 237 N/mm^2. The results from Ansys from this study is 206.6 N/mm^2, which means that the unity check is 0.872. But when the results of ShapeDesigner are compared with the results of Dr. Hoogenboom, who had a finer mesh, the unity check becomes 0.844. The unity checks are higher than the allowed error margin. The difference could be because ShapeDesigner does not have a length for the beam and most likely calculates the maximum shear stress possible in the beam, which is the same approach as the result of this study, where the legend (and therefore the maximum value) is given for the entire beam, whereas the result of Dr. Hoogenboom is purely for the cross section at 300 mm.

2.2.4 Normal stresses

Figure 10 show the contour plot from ShapeDesigner. The contour plot is similar to the plots in Figure 8. The absolute maximum value for the normal stress is 380 N/mm^2. The maximum results from Ansys from this study was 356 N/mm^2 which would mean a unity check of 0.937. When compared to the result from Dr. Hoogenboom, 352 N/mm^2, the unity check becomes 0.926 which is still within the margin of error (between 0.9 and 1.1).

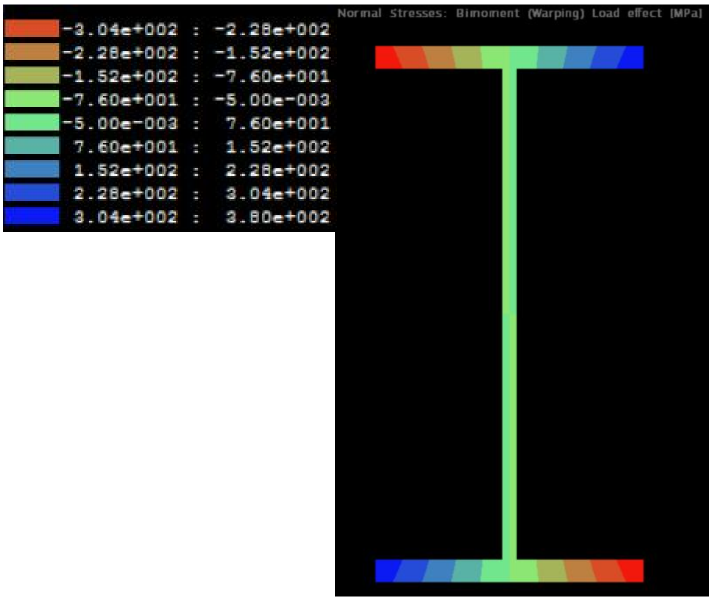

Figure 10 The contour plot for the normal stress from ShapeDesigner

2.3 Summary

In this subsection, the results are summarized in table 2 with the unity check multiplied by 100% to display the error in percentages.

Table 2 Summary of the results from Ansys, Maple and ShapeDesigner

	Maple and ShapeDesigner		Ansys This study		Ansys Dr. Hoogenboom		Hand calculations	
Displacement [mm]	77.19	100%	79.89	103.5%	87.3	113.1%	76.3	98.8%
Shear stress xz [N/mm^2]	237	100%	206.6	87.2%	200	84.4%	196	82.7%
Normal stress zz [N/mm^2]	380	100%	356	93.7%	352	92.6%	363	95.5%

2.4 Conclusion

The first observation that can be made is that the normal stresses are within the margin of error of 10%. The displacement seems to be accurate for the Ansys results from this study and the hand calculations of Dr. Hoogenboom. The Ansys results from Dr. Hoogenboom are, however, significantly larger with an error of 13.1%. He had the same difference between his hand calculations and computer calculations. Dr. Hoogenboom has explained that the large error for deformation could be because of the deformation of the flanges due to shear. The shear stresses on the other hand, seem to be inaccurate, when comparing the combination Maple and ShapeDesigner with the other tools. The possible explanation for the large error of the shear stress has already been explained in section 2.2.3, but in short: ShapeDesigner gives the maximum shear stress at any point and not specifically at 300 mm from the free end.

3 Z-profile

In this chapter, four different z-profiles will be modelled to test the theory of Vlasov. The differences between beam one to four have already been mentioned in table 1. The meshes for the beams are unfortunately not the same since there is a licence. To get the most accurate result, the number of allowed elements has been pushed for each beam, resulting in different mesh sizes per length of beam. As can be seen in table 3, the beams with a length of 0.25 m have a mesh size of 2 mm and the beams with a length of 2 m have a mesh size of 5 mm. All profiles have a mesh refinement on the fixed end and the free end of the beam. The applied force is 1.2 kNm, the same as the test beam in chapter 3.

Since there are no precalculated models, the comparison will directly be between the results from Ansys and ShapeDesigner and Maple. The Maple scripts can be found in Appendix A. The meshing of each beam can be found in Appendix B.

Table 3 An overview of the Z-profile beams that will be modelled in this chapter.

Beam	Mesh size	Refinement	Profile	Length	Thickness t
1	2 mm	Free and fixed end	Z	0.25 m	5 mm
2	5 mm	Free and fixed end	Z	2 m	5 mm
3	2 mm	Free and fixed end	Z	0.25 m	20 mm
4	5 mm	Free and fixed end	Z	2 m	20 mm

3.1 Beam 1: 0.25 m and 5 mm thickness

3.1.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 0.161 mm and therefore the rotation is 0.161/100 = 1.61 * 10^-3 rad. According to Ansys, as can be seen in Figure 11, the maximum horizontal displacement is 0.275 mm and therefore the rotation is 2.75 * 10^-3 rad. With the results from theory being the comparand, unity check is 1.708, which is unexpectedly large, especially since the theory should be accurate for rotation (and displacement) calculations.

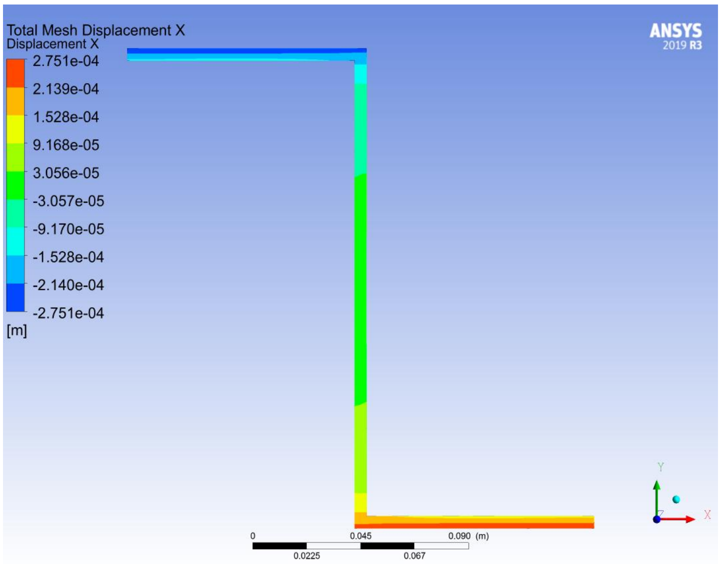

Figure 11 The horizontal displacement of beam 1 from Ansys

3.1.2 Normal stress

The cross section from Ansys is at 50 mm from the fixed end. As can be seen in Figure 12, the stresses are fairly similar. The difference is the sign, but the amount of stress does happen on the same part. The reason for this is that the bimoment needs to be put in by hand in ShapeDesigner. The bimoment according to Maple is -2.98 * 10^8 Nmm^2. If the bimoment was put in as a positive value, that the distribution would be the same except for a flipped colour scale on ShapeDesigner.

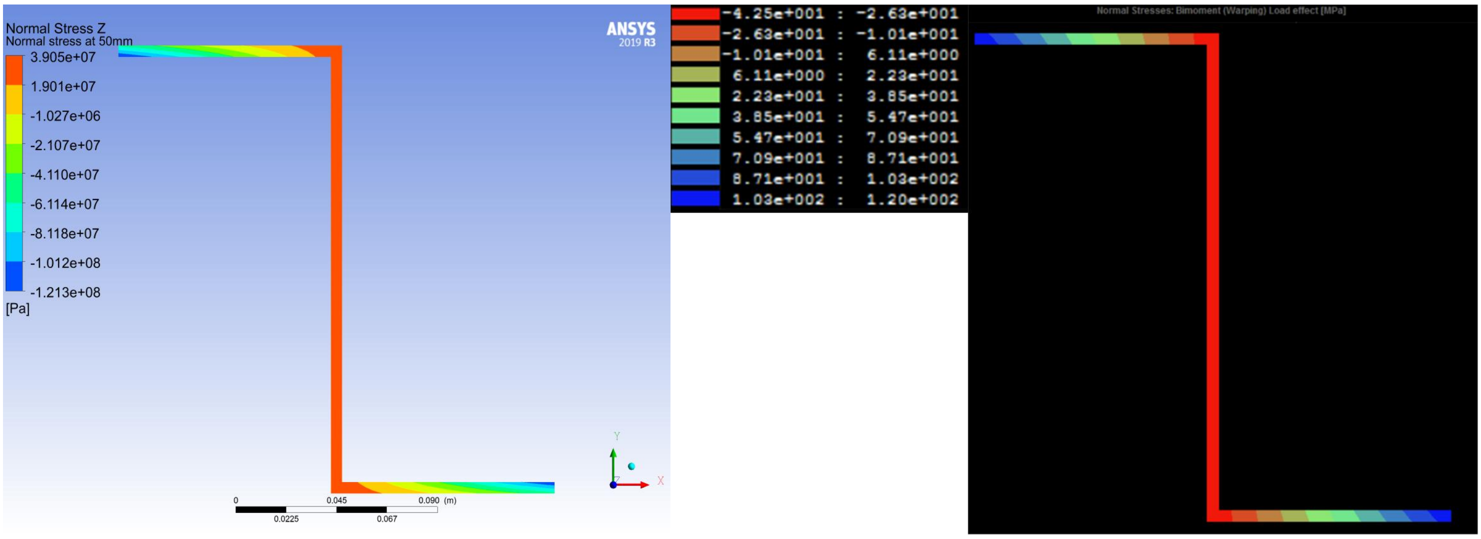

Figure 12 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress, which is more important that the minimum normal stress, in Ansys is 121 N/mm^2 and in ShapeDesigner 120N/mm^2, which means a unity check of 1.008.

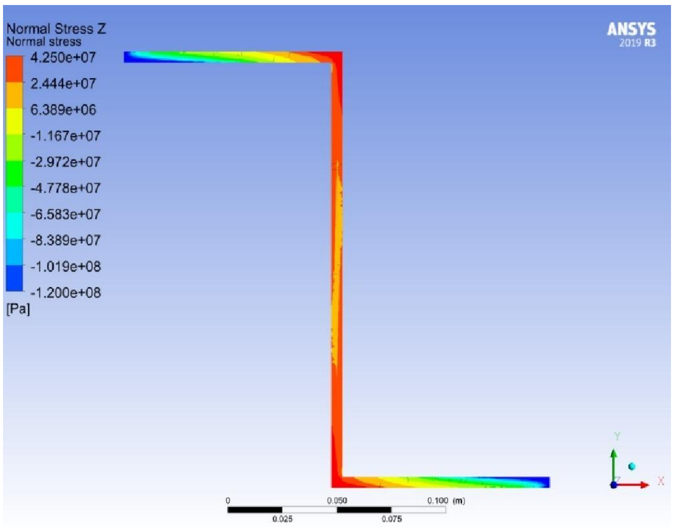

Figure 13 The normal stress in Z-direction at the fixed end with a rescaled legend

The normal stress is also analysed at the fixed end, with a legend rescaled to the legend of ShapeDesigner. In Figure 13, it can be seen that the contour plot is fairly similar to the right plot in Figure 12 (ShapeDesigner). However, there are colour shades darker than the shown legend scale (in Figure 13, they are at the corners of the web and flanges), indicating that these points have stresses outside the specified range, but these points could be singularities.

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3.1.3 Shear stress

The cross section for both shear stresses from Ansys is at 200 mm from the free end. The results from the shear stress analysis are concerningly different. Figure 14 and 15 show a drastic difference between the absolute maximum shear stresses in the cross section.

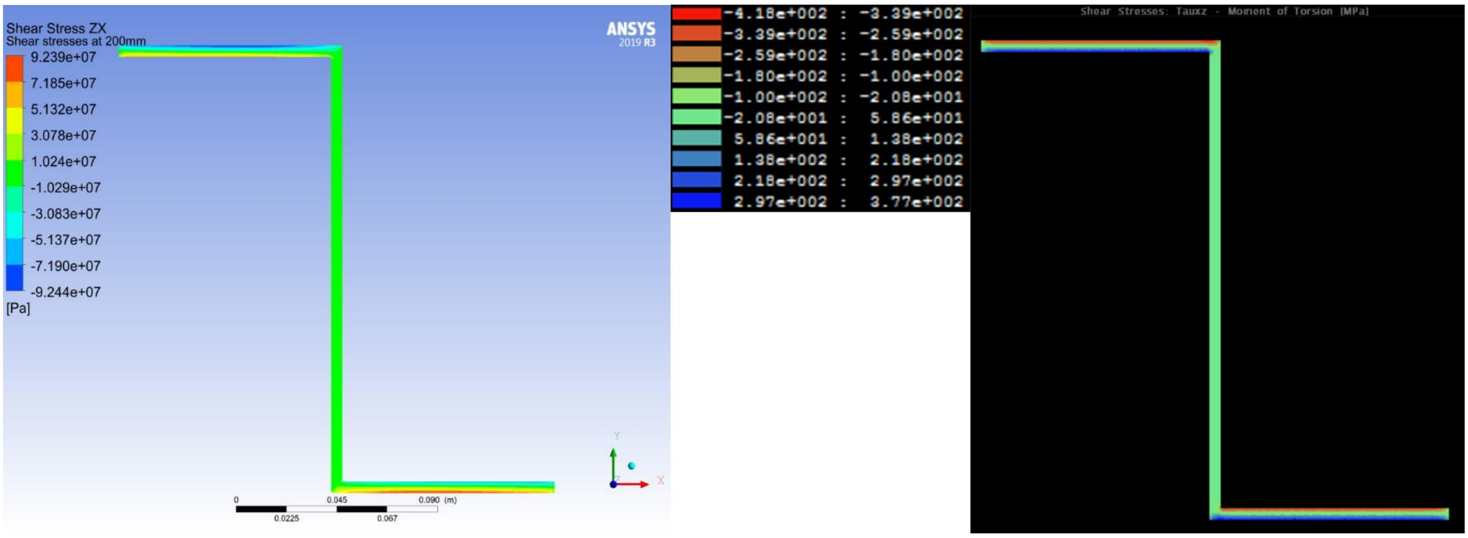

Figure 14 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 92.4 N/mm^2 and in ShapeDesigner 418 N/mm^2. The results from ShapeDesigner are a factor 4.5 larger than the results from Ansys. The unity check is 0.221. The reason is for now unclear. It could be because the beam is too short for the theory, but the theory does not specify a limit to the length of the beam for when it is applicable. The only conditions for the theory are that it should be applicable for thin-walled open cross sections, which is the case for this beam.

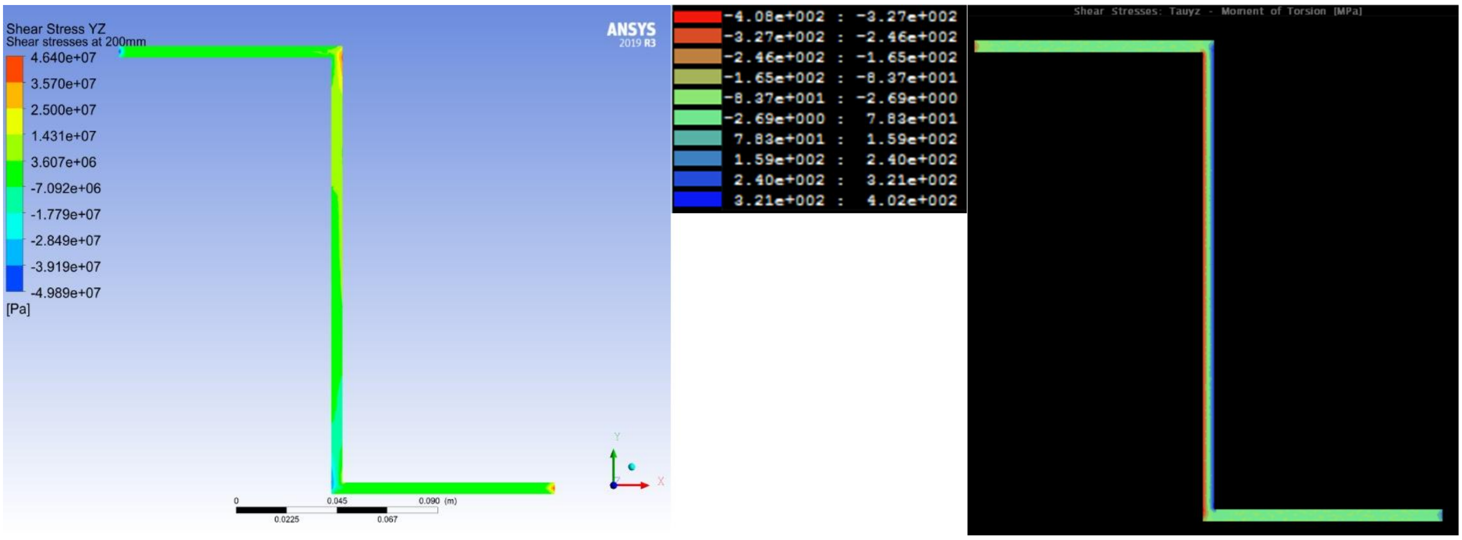

Figure 15 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The same large difference can be seen for the shear stresses in the YZ plane. The absolute maximum shear stress in Ansys is 49.9 N/mm^2 and in ShapeDesigner 408 N/mm^2. The shear stress in ShapeDesigner is a factor 8 larger than the results from Ansys. The unity check is 0.122.

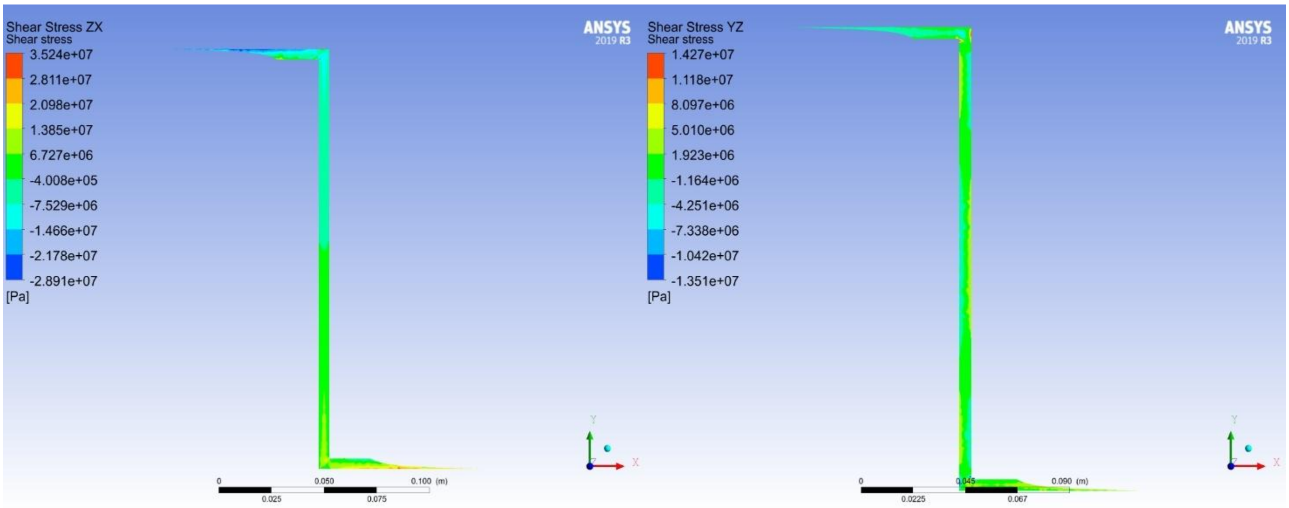

Figure 16 The shear stresses at the free end. Left; XZ plane. Right; YZ plane.

Because the beam is short, it is interesting to look at how Ansys implements the shear stresses at the free end. Figure 16 shows the contour plots for the shear stresses at the free end. The applied torsion moment of 1.2 kNm is changed into shear stresses by Ansys.

The reason why the entire beam is not visible is because of the shape of the elements. The elements are not cubes but rather tetrahedrons. Therefore, some elements are cut off creating a different cross section. The same reason applies to the other short beams.

It is strange that the shear stresses at the free end are smaller than the stresses from Ansys at 50 mm from the free end. As stated on page 58 from the reader of Pierre Hoogenboom, the shear stresses should be larger at the free end, since there are not constraints to warping. The normal stresses are higher at the fixed end, since there is a constraint against warping. It could mean that the conversion from torsion to shear stresses are incorrect in Ansys, since there is a large difference in results for the shear stresses in XZ and YZ plane (Figure 14 and 15). The only results that resemble each other was the normal stress.

3.1.4 Remodel

Since beam 1 showed a lot of differences, this beam was remodelled to make a finer mesh. The mesh size could not be chosen smaller and therefore remained 2 mm. The refinement on the free end was removed and the refinement on the fixed end was increased from level 1 to level 2. The reason why the mesh size was not smaller and why the refinement was not on the highest level, level 3, is because of the limits of the licence. The result, and therefore the conclusion, did not vary much however. The differences between the first model and the remodel were negligible, therefore the contour plots were not added to the study.

3.2 Beam 2: 2 m and 5 mm thickness

3.2.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 54.07 mm and therefore the rotation is 54.07/100 = 0.541 rad. According to Ansys, as shown in Figure 17, the maximum horizontal displacement is 54.76 mm and therefore the rotation is 0.548 rad. The unity check is 1.013, which is well within the allowed error margin.

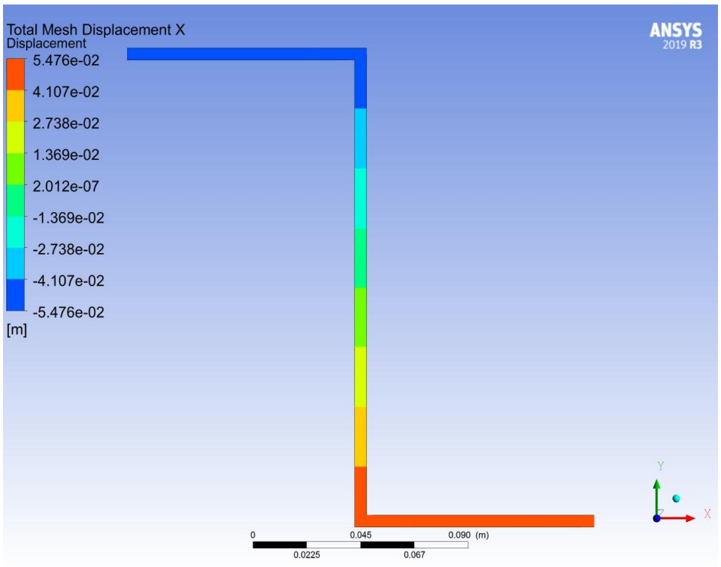

Figure 17 The horizontal displacement of beam 2 from Ansys

3.2.2 Normal stress

The cross section from Ansys is at 50 mm from the fixed end. As can be seen in Figure 18, the stresses are fairly similar. For this beam, there is also a sign difference just like beam 1. Because the explanation for the reason of the sign change is done in section 3.1.2 it will not be explained here nor in the rest of the study, unless there is a different reason.

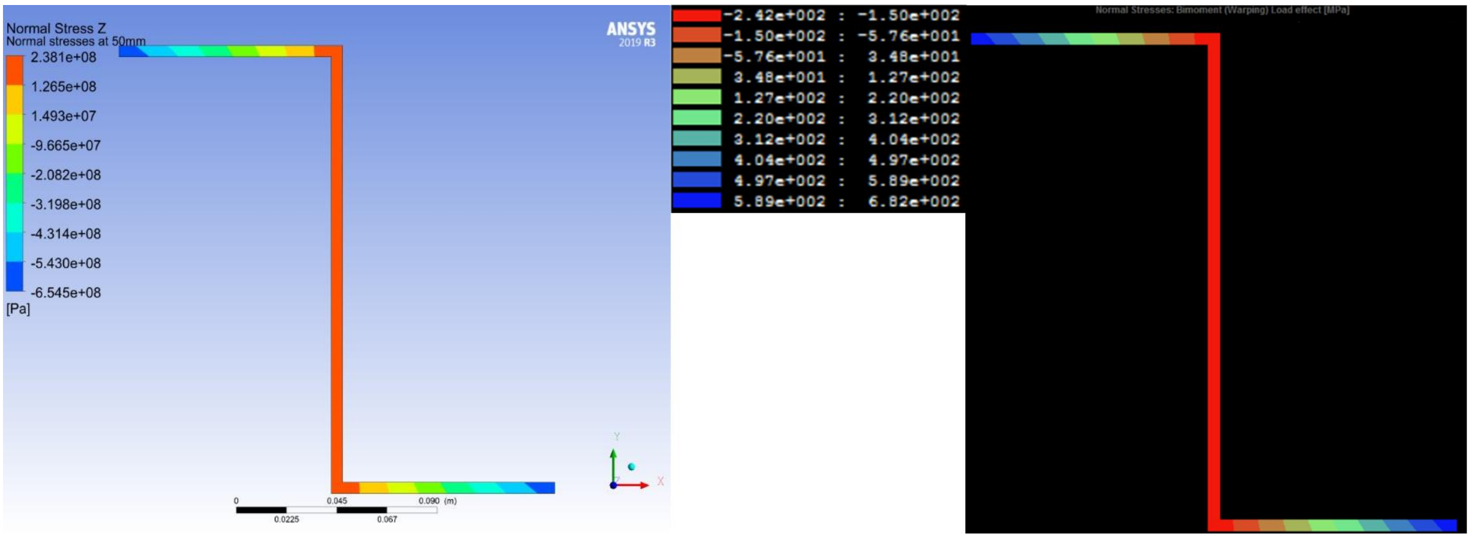

Figure 18 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 655 N/mm^2 and in ShapeDesigner 682 N/mm^2, which means a unity check of 0.96.

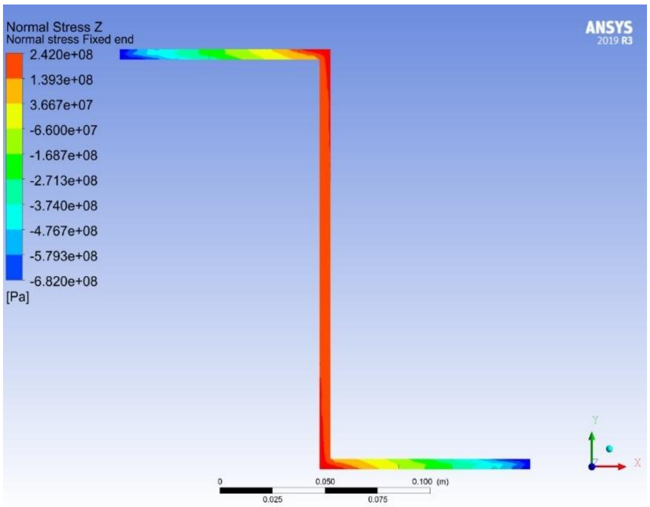

Figure 19 The normal stress in Z-direction at the fixed end with a rescaled legend

The normal stress is also analysed at the fixed end, with a legend rescaled to the legend of ShapeDesigner. In Figure19, it can be seen that the contour plot is fairly similar to the right plot in Figure 20 (ShapeDesigner). However, the same overly shaded parts are visible here as well. But as indicated before, these points could be singularities.

3.2.3 Shear stress

The cross section from Ansys is at 170 mm from the free end. The results from the shear stress analysis are concerningly different. Figure 20 and 21 also show an unreasonable difference between the absolute maximum shear stresses in the cross section.

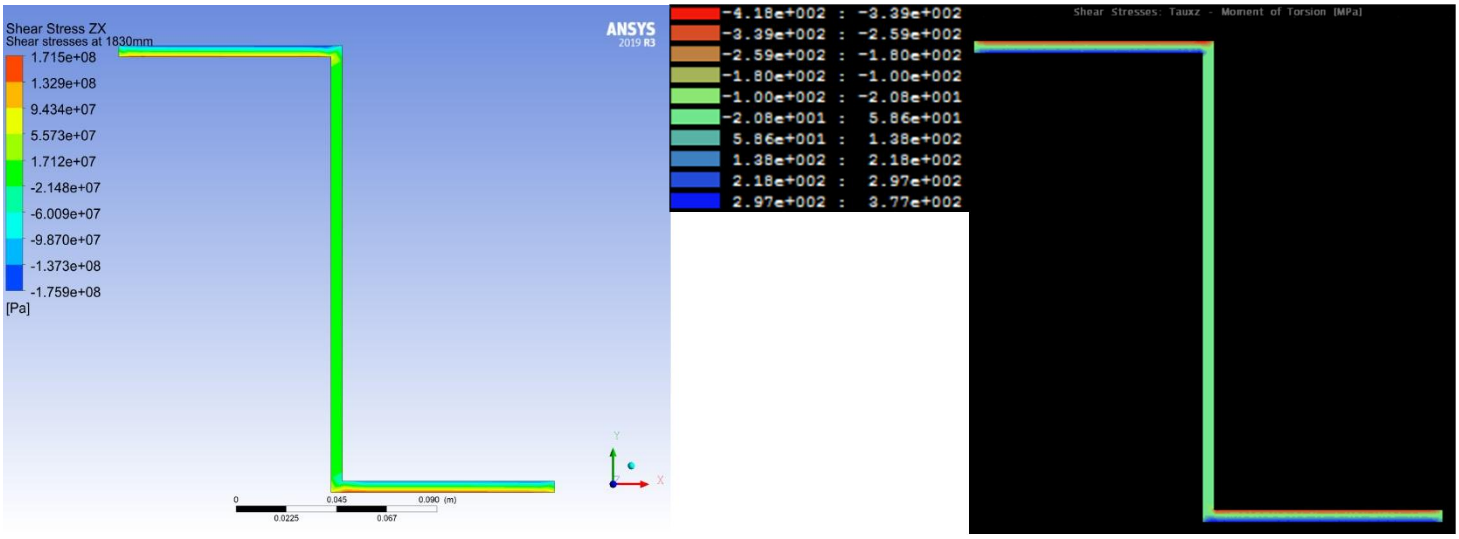

Figure 20 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 176 N/mm^2 and in ShapeDesigner 418 N/mm^2. The results from ShapeDesigner are a factor 2.4 larger than the results from Ansys or expressed in the unity check as 0.421.

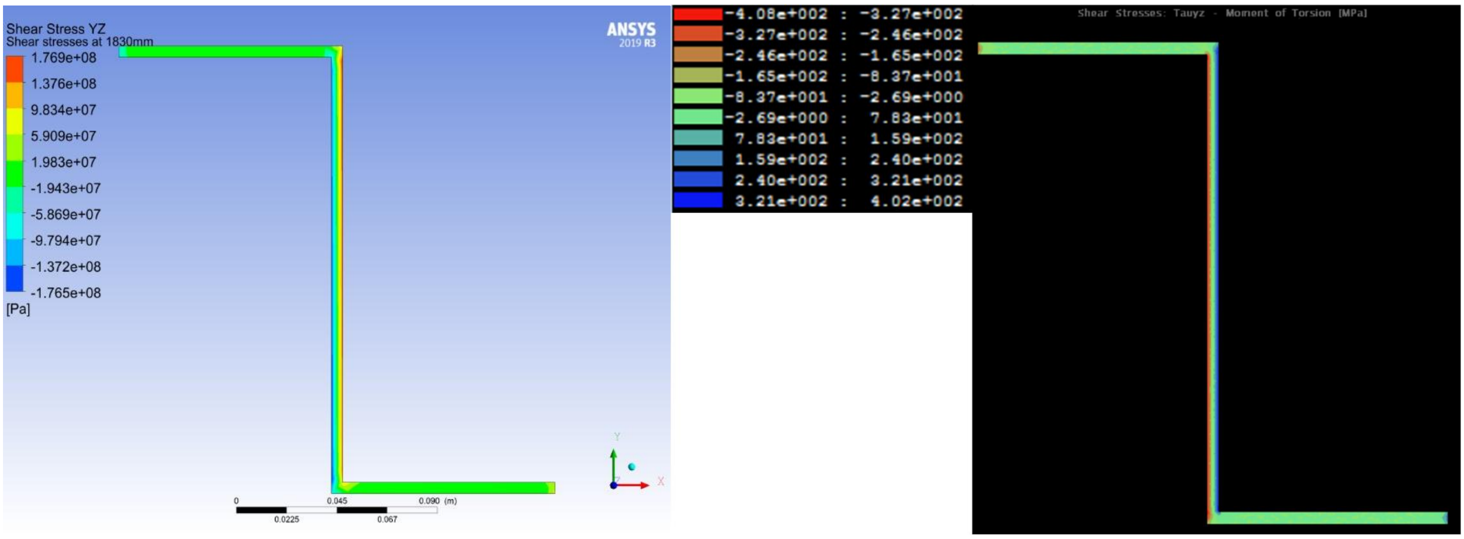

Figure 21 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The same large difference can be seen for the shear stresses in the YZ plane. The absolute maximum shear stress in Ansys is 177 N/mm^2 and in ShapeDesigner 408 N/mm^2. The shear stress in ShapeDesigner is a factor 2.31 larger than the results from Ansys. The unity check, as consistently is expressed for the error, is 0.434.

3.3 Beam 3: 0.25 m and 20 mm thickness

3.3.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 0.0512 mm and therefore the rotation is 0.0512/100 = 5.12 * 10^-4 rad. According to Ansys, as can be seen in Figure 22, the maximum horizontal displacement is 0.06393 mm and therefore the rotation is 6.39 *10 ^-4 rad. With the results from theory being the comparand, the unity check is 1.248 which is, similarly to beam 1, above the allowed error margin. For now, it may be the case that it could be because the length of the beam is short.

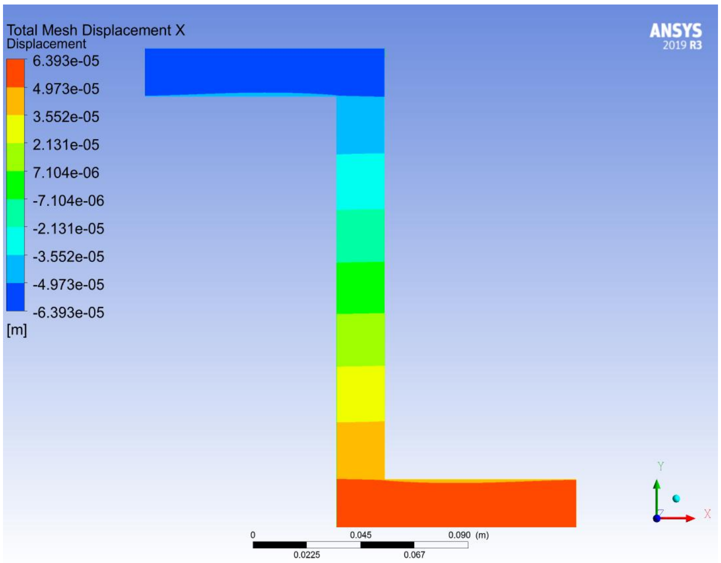

Figure 22 The horizontal displacement of beam 3 from Ansys

3.3.2 Normal stress

The cross section from Ansys is at 25 mm from the fixed end. As can be seen in Figure 23, the stresses are fairly similar. The contour plots show a similar distribution, the only difference is at the web of the cross section.

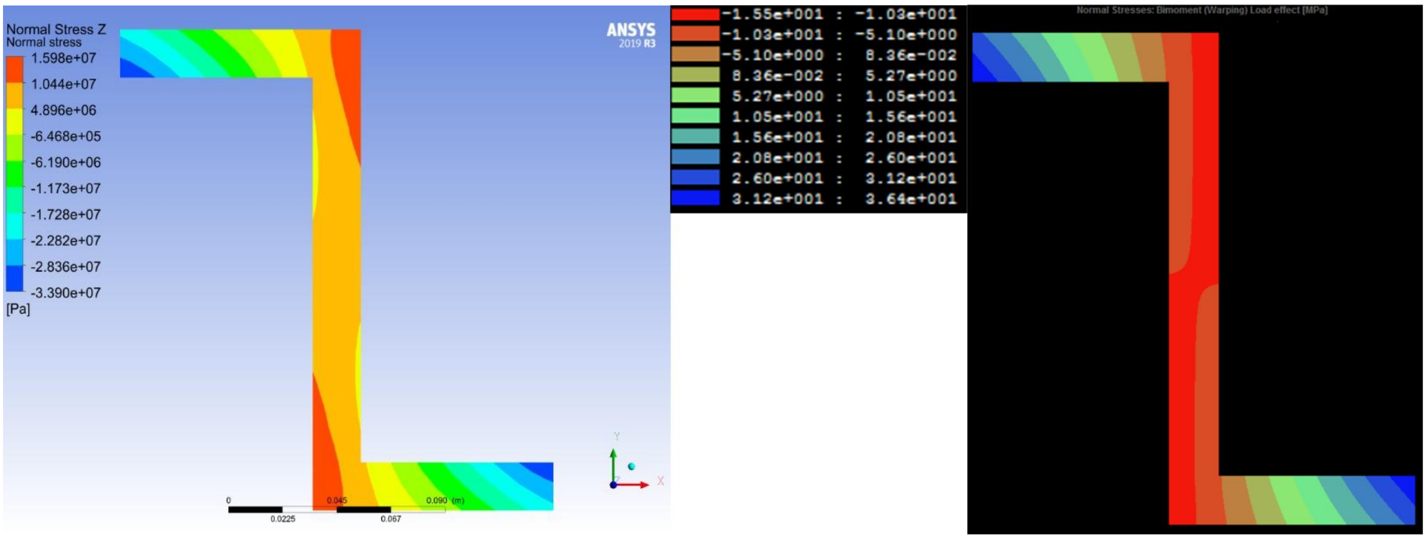

Figure 23 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 33.9 N/mm^2 and in ShapeDesigner 36.4 N/mm^2, which means a unity check of 0.931.

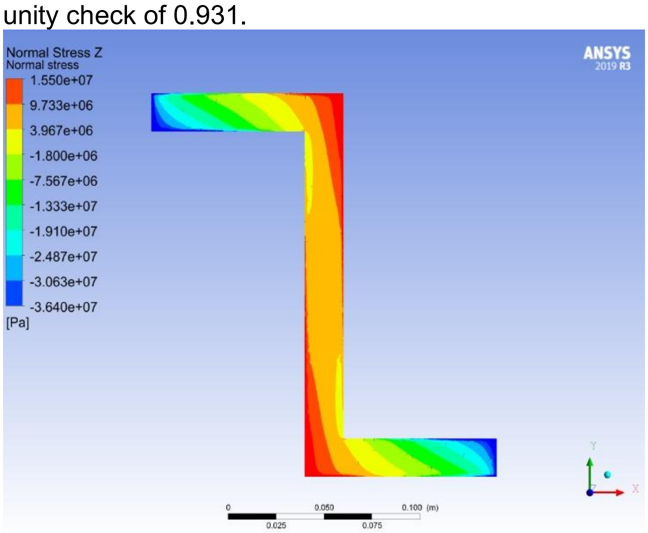

Figure 24 The normal stress in Z-direction at the fixed end with a rescaled legend

The normal stress is also analysed at the fixed end, with a legend rescaled to the legend of ShapeDesigner. In Figure 24, it can be seen that the contour plot is fairly similar to the right plot in Figure 23 (ShapeDesigner). However, just like beam 1 and beam 2, there are overly shaded parts, although it is less severe unlike beam 1 and 2.

3.3.3 Shear stress

The cross section from Ansys is at 50 mm from the free end. The same pattern of large differences between results also continue for this beam for both shear stresses as can be seen in the difference between the contour plots and the maximum values in Figure 25 and 26.

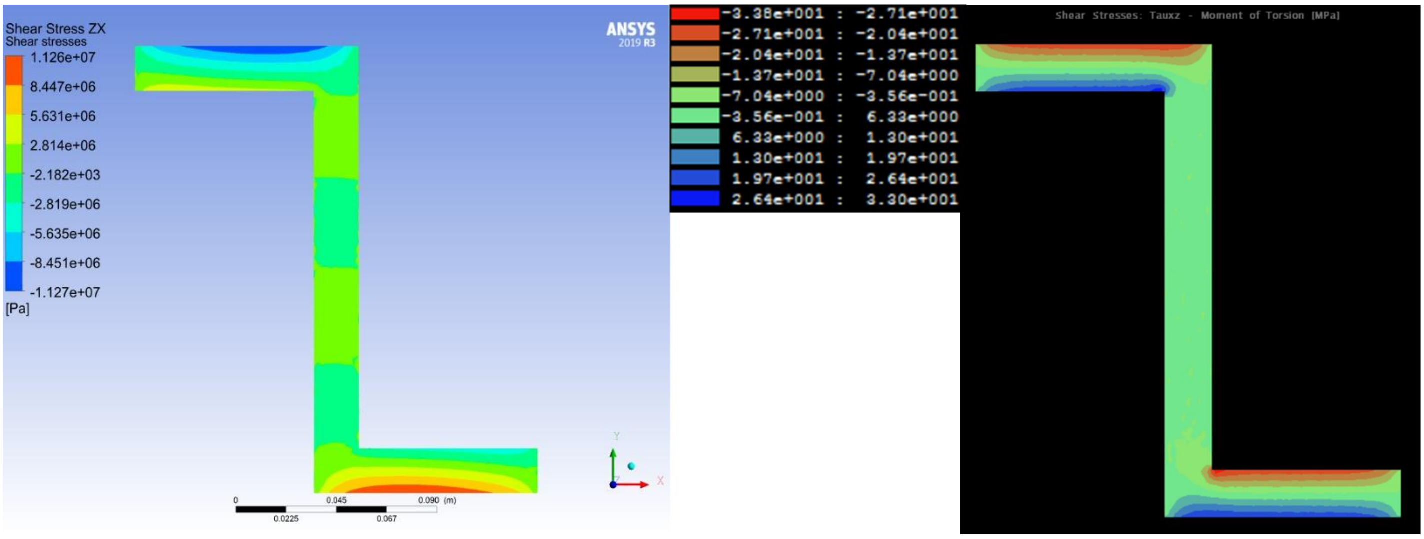

Figure 25 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 11.3 N/mm^2 and in ShapeDesigner 33.8 N/mm^2. The results from ShapeDesigner are a factor 2.99 larger than the results from Ansys. The unity check is 0.334.

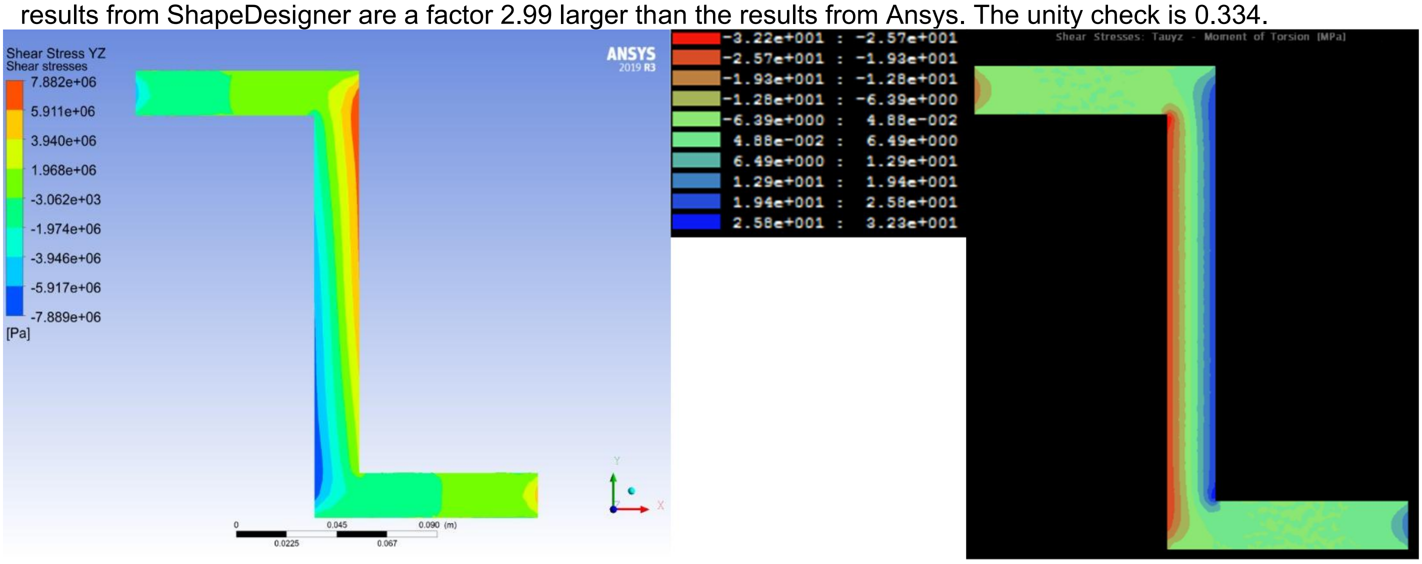

Figure 26 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 7.9 N/mm^2 and in ShapeDesigner 32.3 N/mm^2. The shear stress in ShapeDesigner is a factor 4.09 larger than the results from Ansys. The unity check is 0.241.

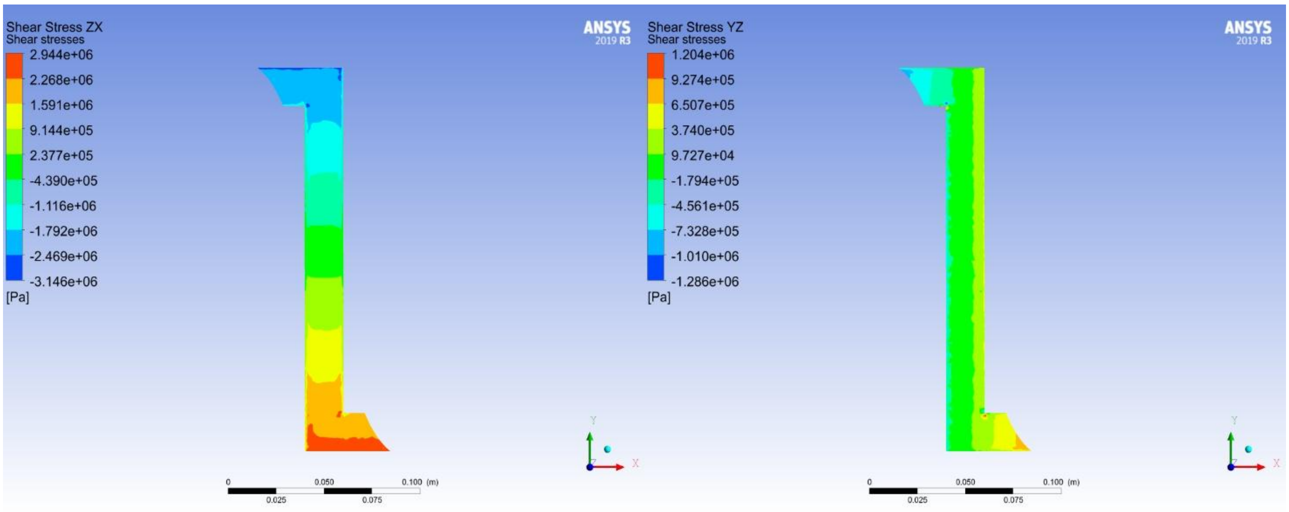

Figure 27 The shear stresses at the free end. Left; XZ plane. Right; YZ plane.

For the same reason as beam 1, here too, the shear stresses at the free end are analysed. Just like in beam 1, the shear stresses at the free end (Figure 27) are smaller than the stresses from Ansys at 50 mm from the free end. Similarly, for beam 3, the shear stresses do not resemble and the reason for this, a problem with the conversion from torsion to shear stresses, could be the same as explained in section 3.1.3.

3.4 Beam 4: 2 m and 20 mm thickness

3.4.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 2.62 mm and therefore the rotation is 2.62/100 = 0.0262. According to Ansys, as can be seen in Figure 28, the maximum horizontal displacement is 2.68 mm and therefore the rotation is 0.0268 rad. With the results from theory being the comparand, the unity check is 1.023, which is well within the error margin.

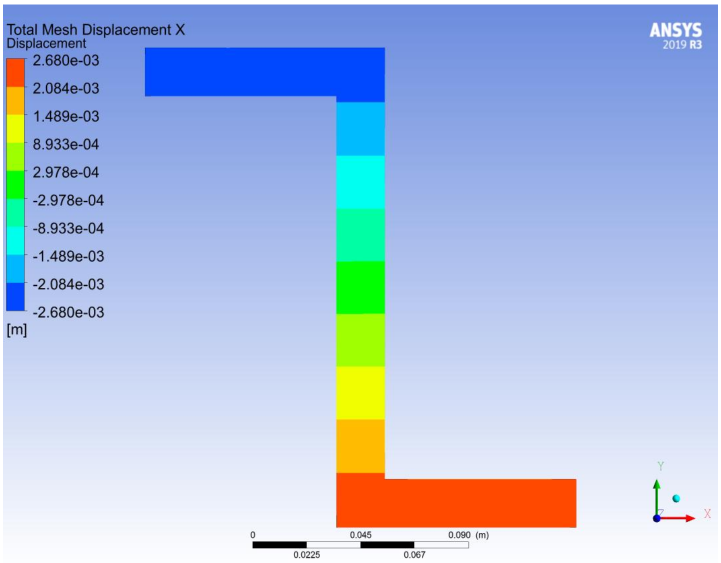

Figure 28 The horizontal displacement of beam 4 from Ansys

3.4.2 Normal stress

The cross section from Ansys is at 25 mm from the fixed end. As can be seen in Figure 29, the stresses are fairly similar. The contour plots show a similar distribution, just like beam 3, there is a difference at the web of the cross section.

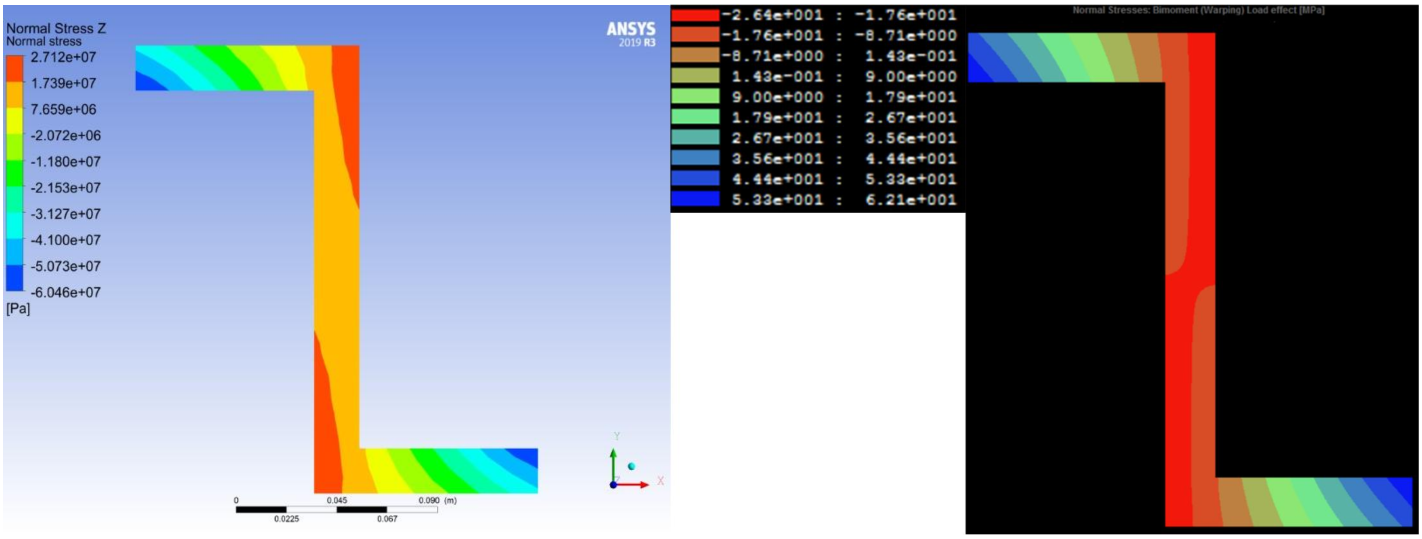

Figure 29 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 60.5 N/mm^2 and in ShapeDesigner 62.1 N/mm^2, which means a unity check of 0.974.

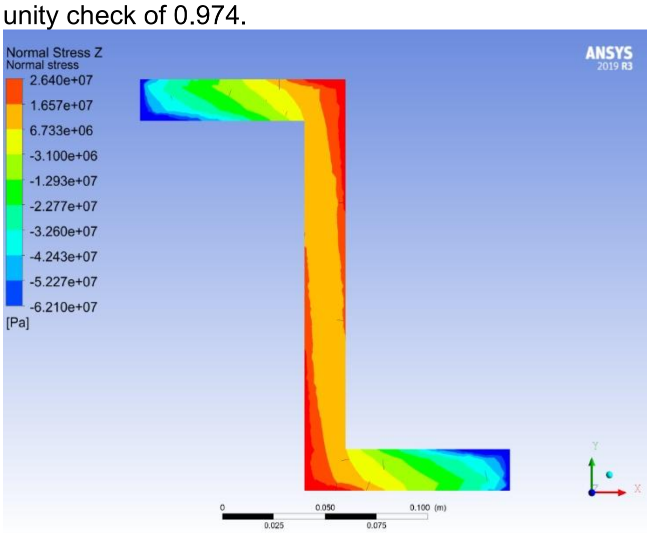

Figure 30 The normal stress in Z-direction at the fixed end with a rescaled legend

The normal stress is also analysed at the fixed end, with a legend rescaled to the legend of ShapeDesigner. In Figure 30, it can be seen that the contour plot is again similar to the right plot in Figure 29 (ShapeDesigner). And just like the previous beams, there are some overshaded parts and just like beam 3, the difference is less severe than beam 1 and 2.

3.4.3 Shear stress

The cross section from Ansys is at 200 mm from the free end. Figure 31 and 32 show that the difference between the contour plots and the results for the shear stresses for beam 4 are closer to each other.

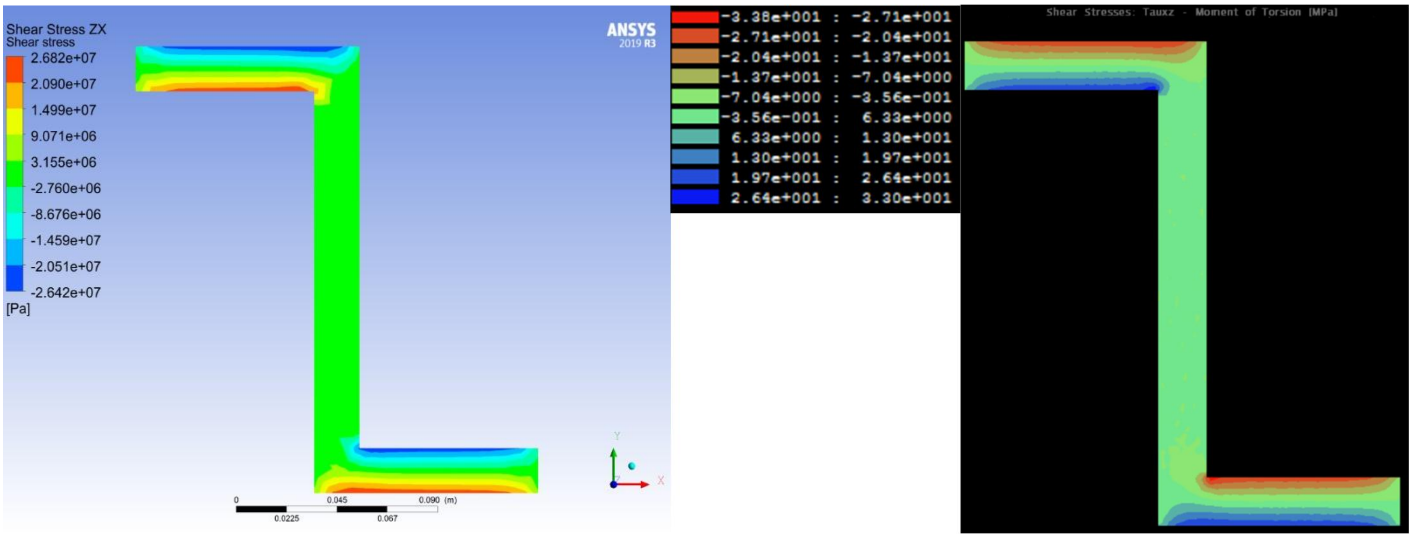

Figure 31 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 26.4 N/mm^2 and in ShapeDesigner 33.8 N/mm^2 occurring at the top part of the flanges. The unity check is 0.781, which is less than the errors for shear stresses in the previous beams but still above the allowed error margin of 10%.

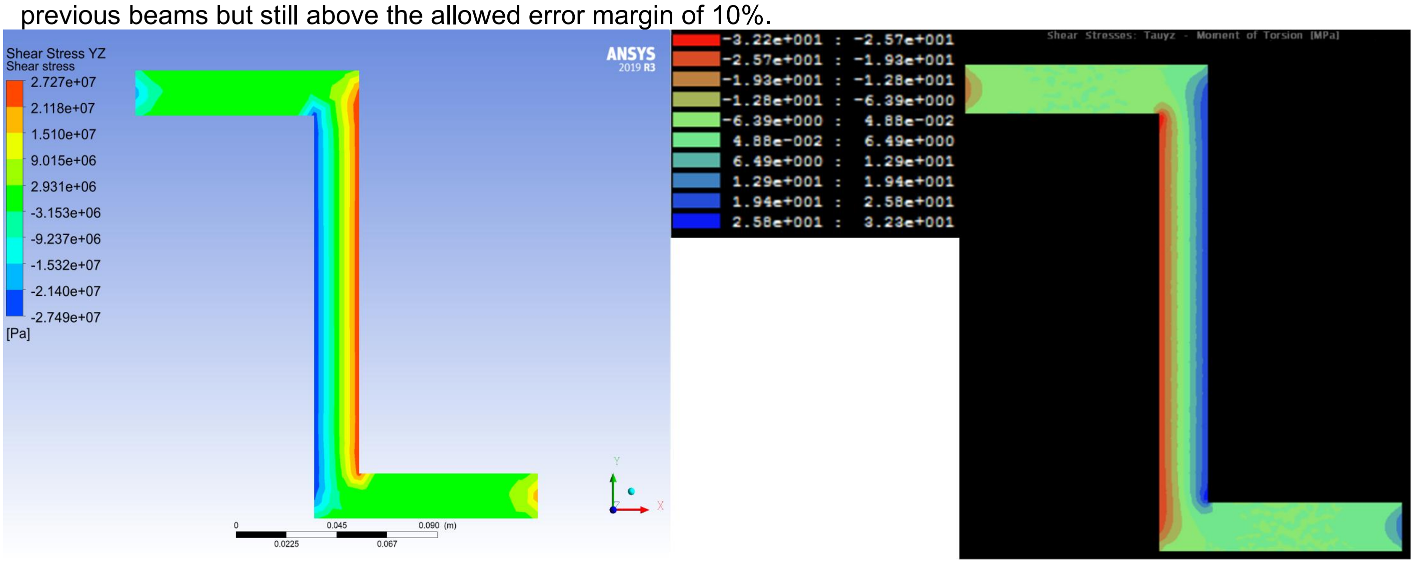

Figure 32 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 27.3 N/mm^2 and in ShapeDesigner 32.3 N/mm^2 occurring at the right part of the web. The unity check is 0.845, which again is less of an error than the previous beams but still above the allowed error margin.

3.5 Observations and conclusion

The results for each beam were unexpected. In general, the expectations were that the results for displacement would be around the same and that the values for the normal stresses would be different. However, judging from the summary tables 4 to 7, that is not the case. It is notable that for the short beams, Table 4 and 6, the only accurate value is the Normal stress. The shear stresses and, strangely, the displacement are well above the error margin of 10%, especially the shear stresses. When the fixed end of each beam was analysed, it was noted that the normal stresses were larger than the results from Ansys. This is according to the expectation that the largest normal stresses would occur at the fixed end.

The displacement become more accurate when the beam is longer. It could be that short beams cause problems for the calculations of the displacement with the theory of Vlasov. It still remains the case, as shown in Table

5 and 7, that the shear stresses still are inaccurate. The reason could be, as analysed for the short beams, that the conversion from torsion to shear stress is inaccurate or entirely wrong. The expectation is that the largest shear stresses would occur at the free end. That is not the case. The stresses at the free end were smaller than the shear stresses at the cross section near the free end. However, the error becomes significantly less for beam 4 (table 7).

It also seems that the shear stress results from the beams with same dimensional profile but different lengths, beam 1 and beam 2 for instance, have the exact same distribution and results in ShapeDesigner. Even though it is possible to put in the length of the beam in ShapeDesigner, it might be that the program does not take it into account. Which could be an explanation for the unreasonably large differences between Ansys and ShapeDesigner, even though the differences still hold for the longer beam. So, it is uncertain for which lengths the results of Ansys and ShapeDesigner will be similar.

In short:

- The results for the normal stresses with the combination of ShapeDesigner and Maple are within the allowed error margin.
- The normal stress at the fixed end is larger than at the cross section, as expected.
- The results for displacements only are within the margin for the longer beams. It seems that short beams do cause problems and inaccuracies.
- The results for the Shear stresses never seem to be the same. The lowest errors are in beam 4 (2m with 20mm flanges), but the others beams seem to cause a significant error.
- The shear stress at the free end are less than at the cross section, which is against expectations.
- The shear stresses are the identical for same dimensional profiles. Results from ShapeDesigner for the shear stresses for beam 1 and 2 are the same and for beam 3 and 4 as well.

Table 4 Summary results beam 1

Beam 1	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.161	0.275	1.708	16
Normal stress	120	121	1.008	17
Shear stress XZ	418	92.4	0.221	18
Shear stress YZ	408	49.9	0.122	18

Table 5 Summary results beam 2

Beam 2	ShapeDesigner	Ansys	U.C.	Page
Displacement	54.07	54.76	1.013	19-20
Normal stress	682	655	0.960	20
Shear stress XZ	418	176	0.421	21-22
Shear stress YZ	408	177	0.434	21-22

Table 6 Summary results beam 3

Beam 3	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.0512	0.0639	1.248	22
Normal stress	36.4	33.9	0.931	23
Shear stress XZ	33.8	11.3	0.334	24
Shear stress YZ	32.8	7.9	0.241	24

Table 7 Summary results beam 4

Beam 4	ShapeDesigner	Ansys	U.C.	Page
Displacement	2.62	2.68	1.023	25
Normal stress	62.1	60.5	0.974	26
Shear stress XZ	33.8	26.4	0.781	27
Shear stress YZ	32.3	27.3	0.845	27

4 Hollow square profile

In this chapter, the theory of Vlasov will be tested on the hollow square profile, just like in chapter three. The mesh sizes and refinement application, together with the length and thickness can be found in Table 8. The meshing of each beam can be found in Appendix B.

Table 8 An overview of the hollow square profile beams that will be modelled in this chapter

Beam	Mesh size	Refinement	Profile	Length	Thickness t
5	2 mm	Free and fixed end	Hollow square	0.25 m	5 mm
6	5 mm	Free and fixed end	Hollow square	2 m	5 mm
7	2 mm	Free and fixed end	Hollow square	0.25 m	20 mm
8	5 mm	Free and fixed end	Hollow square	2 m	20 mm

4.1 Beam 5: 0.25m and 5 mm thickness

4.1.1 Horizontal displacement and rotation

The maximum horizontal displacement according to the theory of Vlasov should be 0.0435 mm and therefore the rotation is 0.0435/50 = 8.70 * 10^-4 rad. According to Ansys, as can be seen in Figure 33, the maximum horizontal displacement is 0.0672 mm and therefore the rotation is 1.34 * 10^-3 rad. The unity check is 1.545, which is way over the error margin.

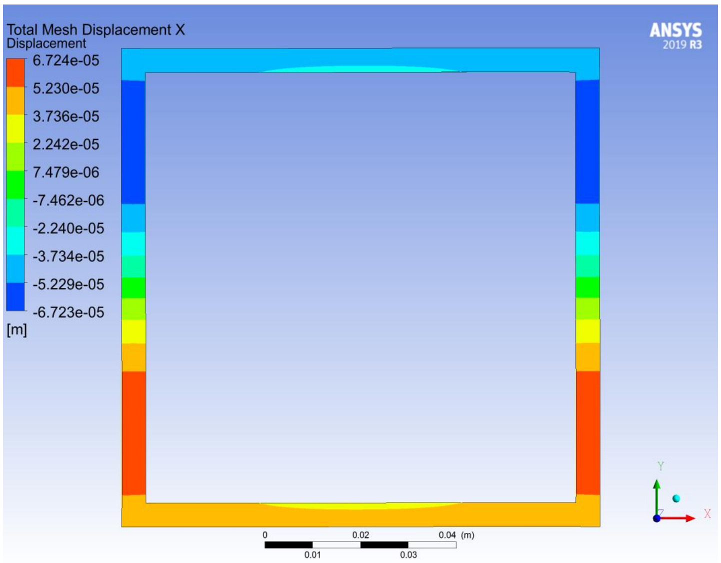

Figure 33 The horizontal displacement of beam 5 from Ansys

4.1.2 Normal stress

The cross section of the model in Ansys is at 7.3 mm from the fixed end. As can be seen in Figure 34, the contour plot show similar results, but the values are not the same. Here too, there is a sign switch, but the reason for that is explained before.

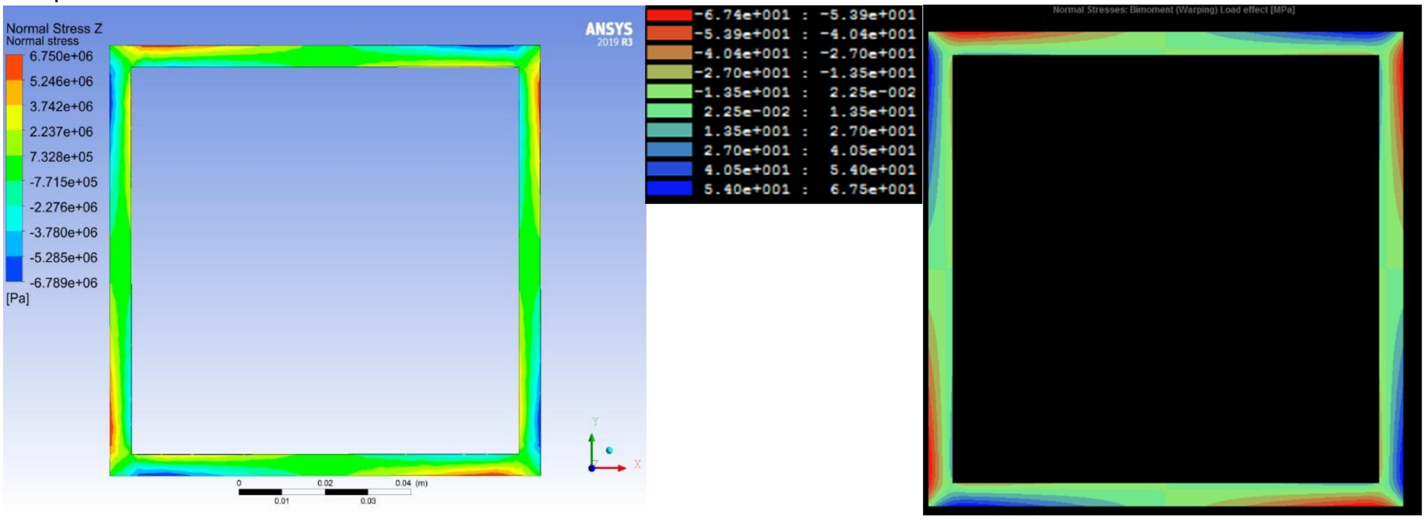

Figure 34 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 6.79 N/mm^2 and in ShapeDesigner 67.5 N/mm^2. The normal stress in ShapeDesigner is a factor 9.94 larger than the normal stress in Ansys. The unity check is 0.101. For the Z-profiles it was the case that the normal stress results from ShapeDesigner and Ansys would be similar, but that does not seem to be the case for beam 5.

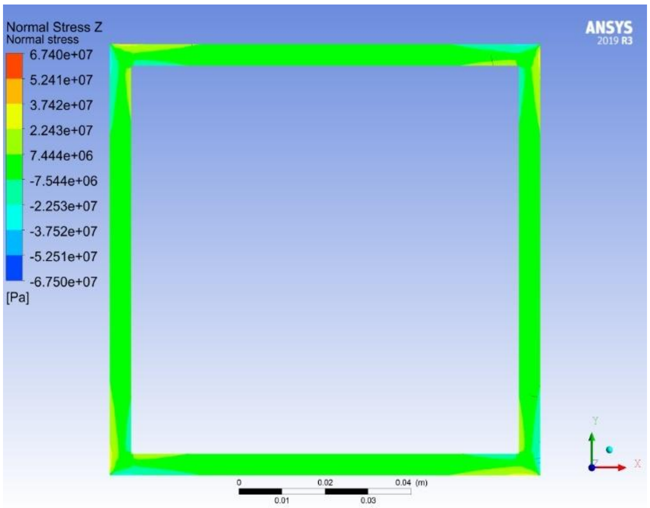

Figure 35 The normal stress in Z-direction at the fixed end with a rescaled legend

Figure 35 shows the normal stress in the fixed end of beam 5 with a legend rescaled to the legend of ShapeDesigner in Figure 34. Even at the fixed end, where the normal stress should be the highest, the blue colour or orange colour cannot be seen in the contour plot, meaning that these forces do not occur. The absolute maximum value in the plot is the yellow colour, meaning around 37.42 N/mm^2. Which even then, is significantly less than the forces in ShapeDesigner, namely a unity check of 0.554.

4.1.3 Shear stresses

The cross section from Ansys is at 25 mm from the free end. Figure 36 and 37 show the difference between the contour plots.

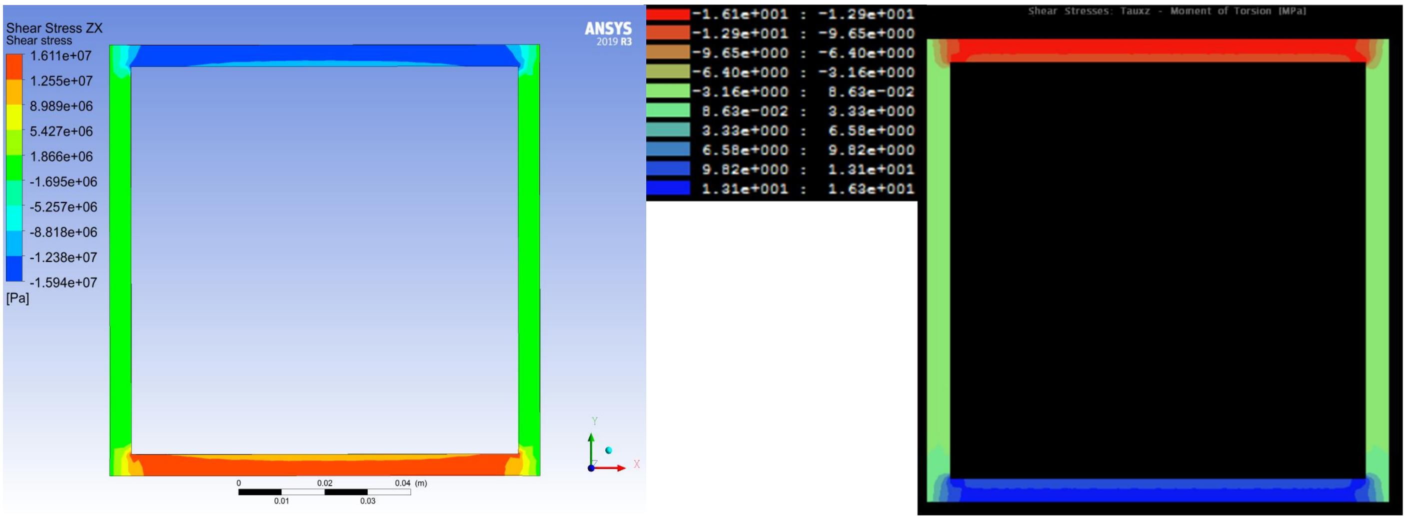

Figure 36 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 16.11 N/mm^2 and in ShapeDesigner 16.3 N/mm^2. The unity check is 0.988, which is well within the allowed error margin of 10%.

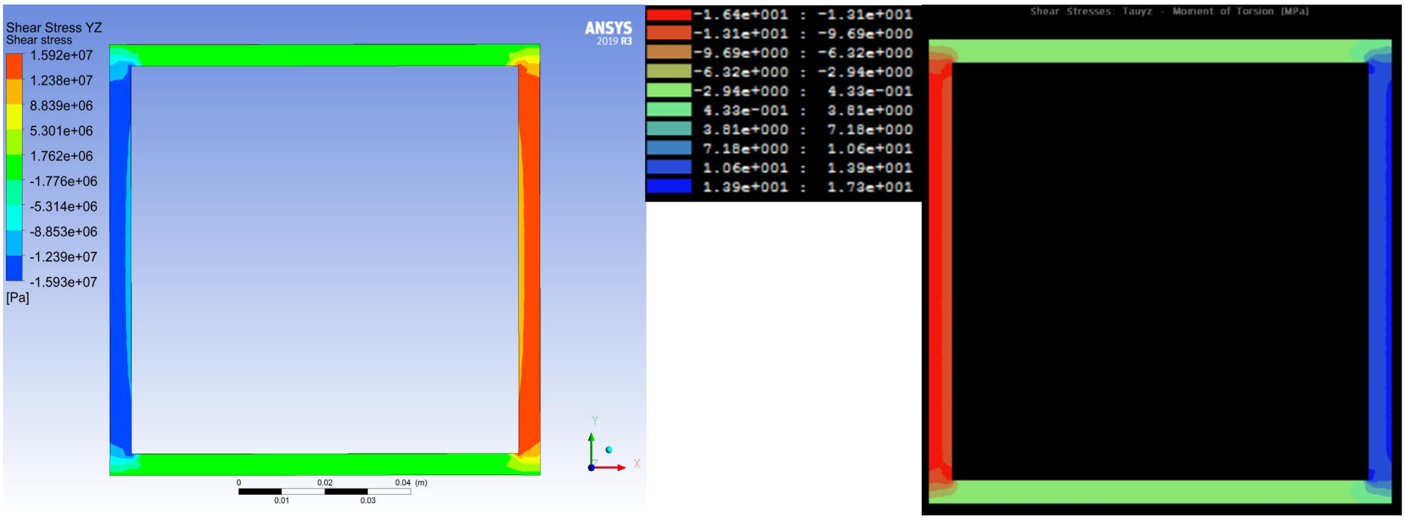

Figure 37 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 15.9 N/mm^2 and in ShapeDesigner 17.3 N/mm^2. The unity check is 0.919, which is within the allowed error margin of 10%.

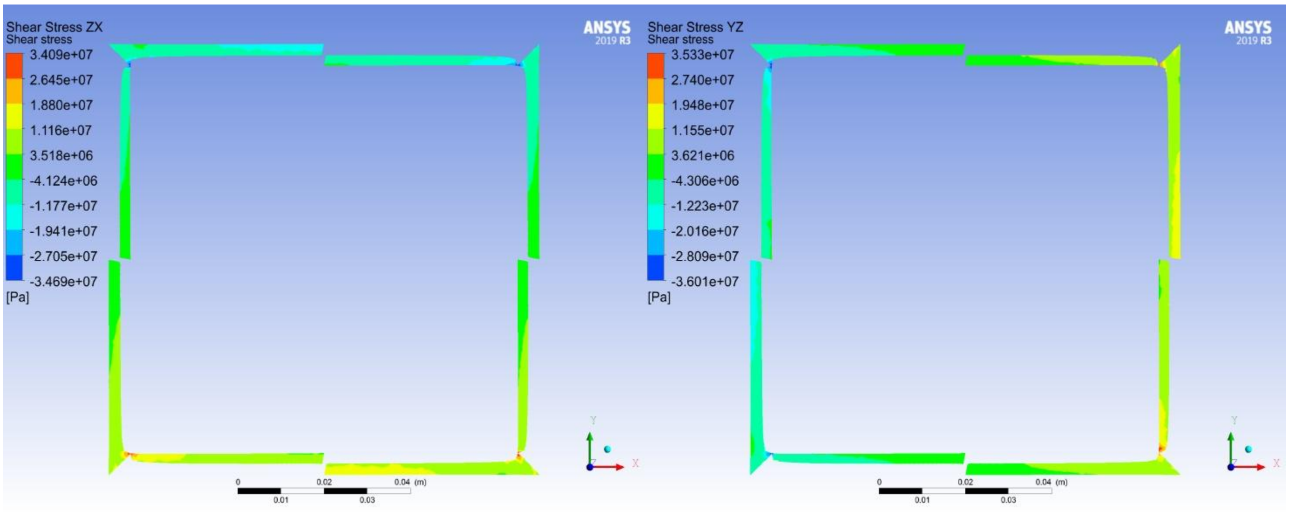

Figure 38 The shear stresses at the free end. Left; XZ plane. Right; YZ plane.

In Figure 38, the shear stress distribution at the free end is shown for both the XZ and YZ plane. It is strange to see that the stresses at the free end for beam 1 and 3 were smaller than their related cross sections in ShapeDesigner, but for beam 5, as shown in Figure 36 and 37, the shear stresses at the free end are larger than the shear stresses in the cross sections, as was expected with the theory from the reader. It is notable that when the shear stresses at the free end are larger than the stresses at the cross section, the results from ShapeDesigner and Ansys are more comparable (and therefore more accurate). With beam 1 and 3, the shear stresses at the free end are lower than the stresses at the cross section but are also less accurate.

4.1.4 Remodel

Since beam 5 showed a lot of differences, this beam was remodelled to make a finer mesh. The mesh size could not be chosen smaller and therefore still remains 2 mm. The refinement on the free end was removed and the refinement on the fixed end was increased from level 1 to level 2. The reason why the mesh size was not smaller and why the refinement was not on the highest level, level 3, is because of the limits of the licence. The result, and therefore the conclusion, did not vary much, however. The differences between the first model and the remodel were negligible, therefore the contour plots were not added to the study.

4.2 Beam 6: 2m and 5 mm thickness

4.2.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 0.350 mm and therefore the rotation is 0.350/50 = 7 * 10^-3 rad. According to Ansys, as can be seen in Figure 39, the maximum horizontal displacement is 0.355 mm and therefore the rotation is 7.1 * 10^-3 rad. With the results from theory being the comparand, the unity check is 1.014, which is well within the error margin.

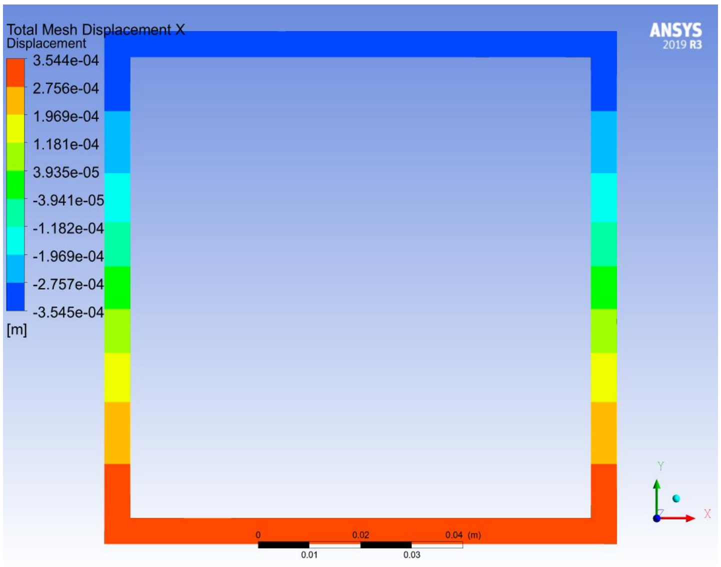

Figure 39 The horizontal displacement of beam 6 from Ansys

4.2.2 Normal stress

The cross section of the model in Ansys is at 7.8 mm from the fixed end. As can be seen in Figure 40, the contour plot show similar results, but the values are not the same, just like in Figure 34 for beam 5.

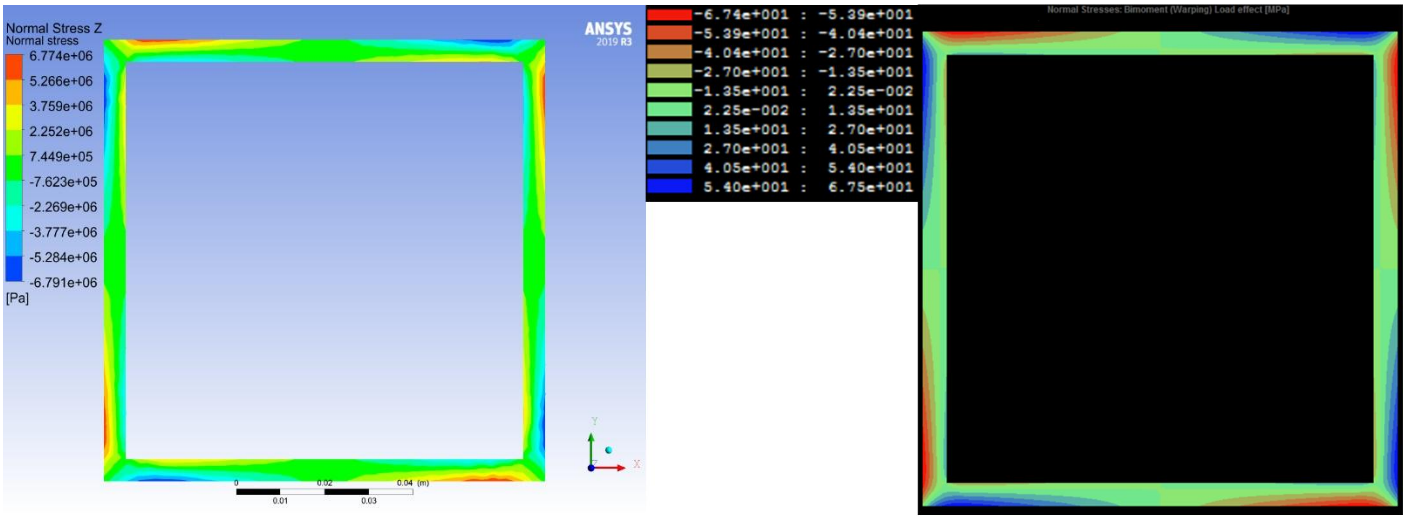

Figure 40 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 6.79 N/mm^2 and in ShapeDesigner 67.5 N/mm^2. The normal stress in ShapeDesigner is a factor 9.94 larger than the normal stress in Ansys. The unity check is 0.101. The results for beam 6 and beam 5 are identical.

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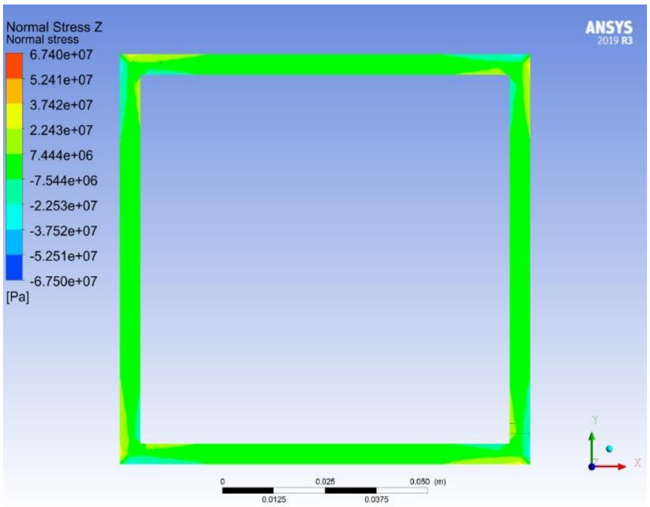

Figure 41 The normal stress in Z-direction at the fixed end with a rescaled legend

Figure 41 shows the normal stress in the fixed end of beam 6 with a legend rescaled to the legend of ShapeDesigner in Figure 40. Even at the fixed end, where the normal stress should be the highest, the blue colour or orange colour cannot be seen in the contour plot, meaning that these forces do not occur, just like beam 5. The difference between the maximum possible normal stress (at the fixed end) and the results from ShapeDesigner are still far too large.

4.2.3 Shear stresses

The cross section from Ansys is at 250 mm from the free end. Figure 42 and 43 show the difference between the contour plots.

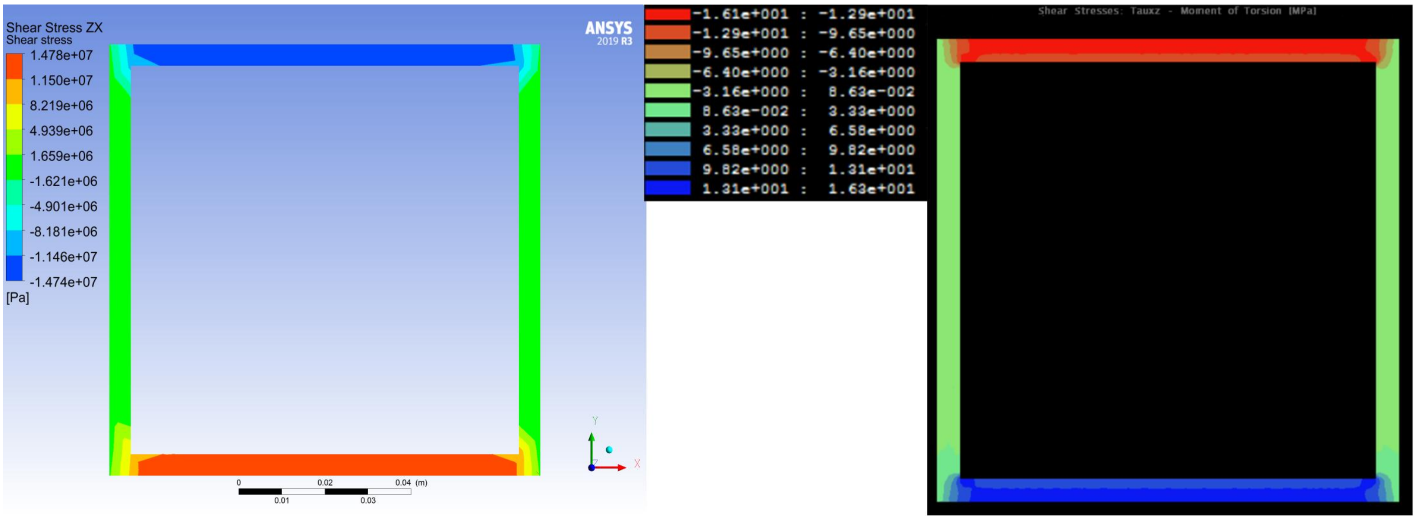

Figure 42 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 14.8 N/mm^2 and in ShapeDesigner 16.3 N/mm^2. The unity check is 0.908, which is within the allowed error margin of 10%.

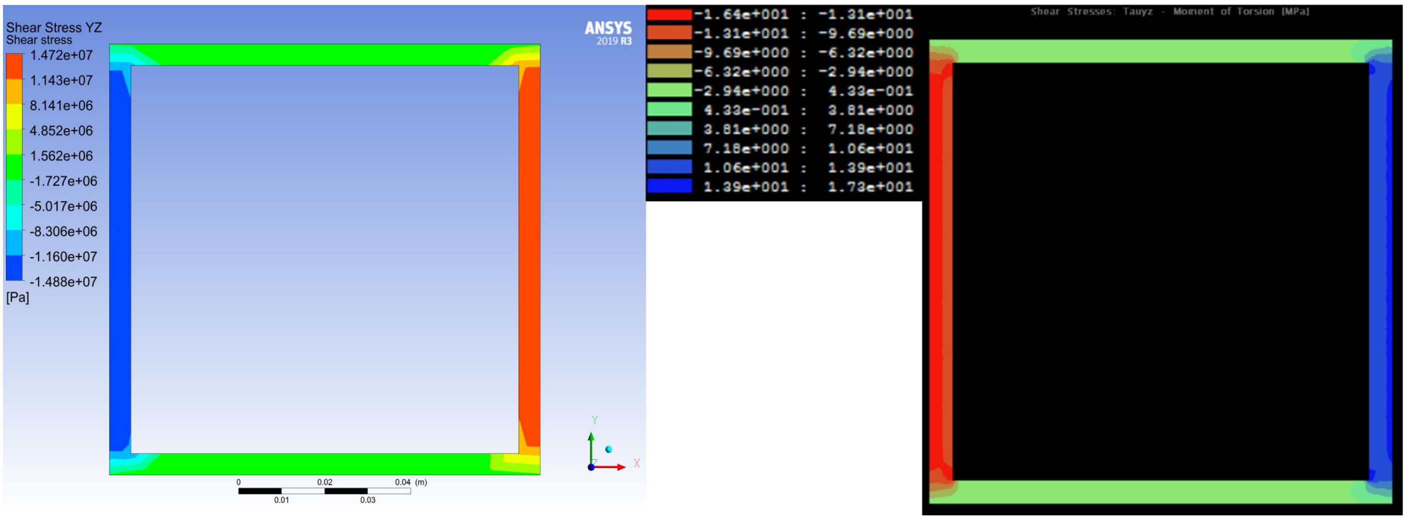

Figure 43 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 14.7 N/mm^2 and in ShapeDesigner 17.3 N/mm^2. The unity check is 0.850, which is above error margin of 10%.

4.3 Beam 7: 0.25m and 20 mm thickness

4.3.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 0.0160 mm and therefore the rotation is 0.0160/50 = 3.2 * 10^-4 rad. According to Ansys, as can be seen in Figure 44, the maximum horizontal displacement is 0.0167 mm and therefore the rotation is 3.34 * 10^-4 rad. With the results from theory being the comparand, the unity check is 1.044, which is within the error margin.

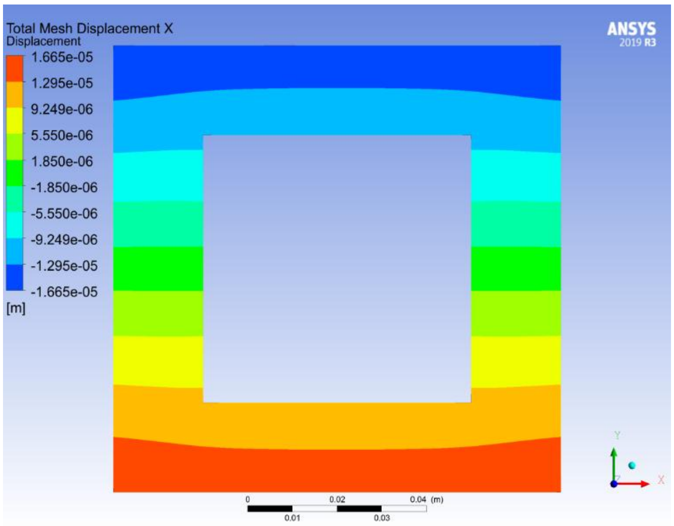

Figure 44 The horizontal displacement of beam 7 from Ansys

4.3.2 Normal stress

The cross section of the model in Ansys is at 16.8 mm from the fixed end. As can be seen in Figure 45, the contour plots show similar results, but the values are off.

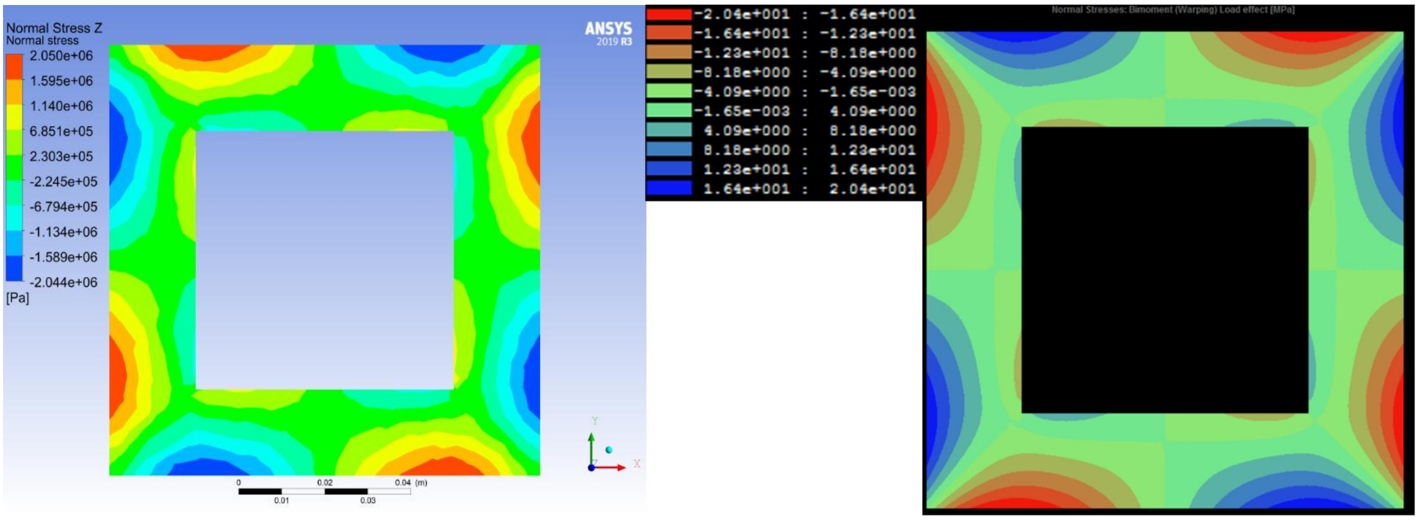

Figure 45 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 2.05 N/mm^2 and in ShapeDesigner 20.4 N/mm^2. The normal stress in ShapeDesigner is a factor 9.95 larger than the normal stress in Ansys. The unity check is 0.100, which is way over the margin. However, it is similar to the unreasonably large differences with beam 5 and 6.

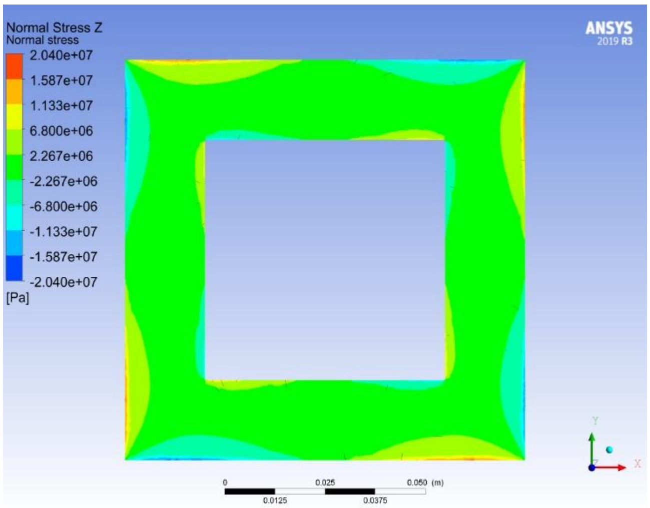

Figure 46 The normal stress in Z-direction at the fixed end with a rescaled legend

Figure 46 shows the contour plot for the normal shear stress at the fixed end for beam 7, rescaled to the legend of ShapeDesigner. Here too, the orange colour and blue colour are barely visible, meaning that the absolute maximum stresses from the legend do not occur. The maximum forces that appear are 15.87 N/mm^2, which means that the unity check then is 0.778, which is still over the allowed error margin.

4.3.3 Shear stresses

The cross section from Ansys is at 200 mm from the free end. Figure 47 and 48 show the difference between the contour plots.

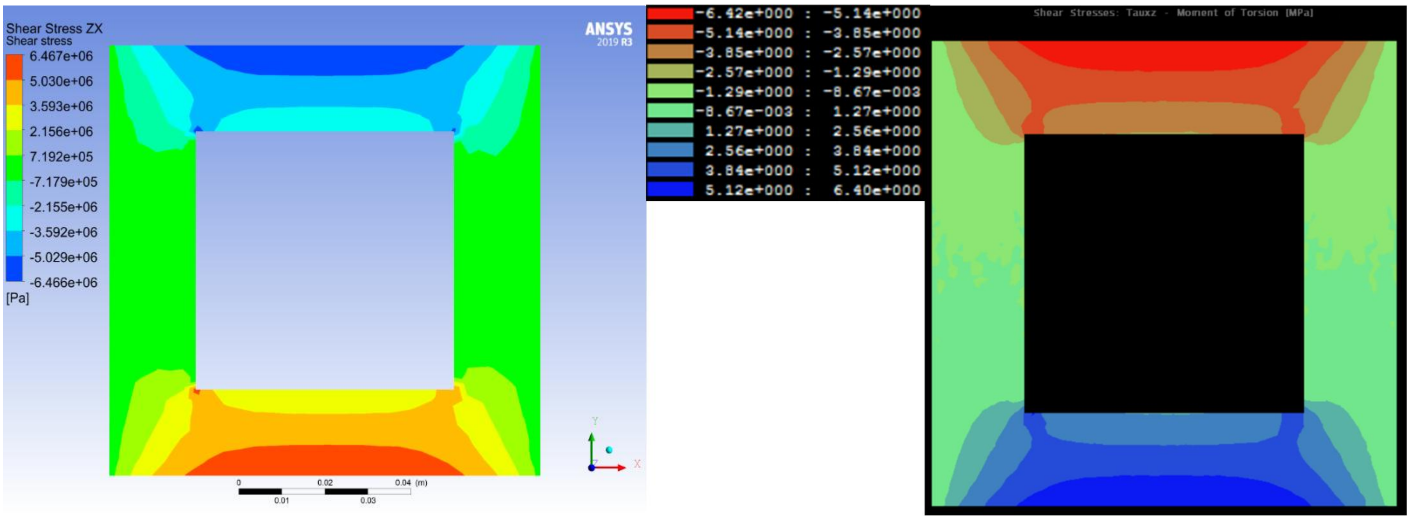

Figure 47 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 6.47 N/mm^2 and in ShapeDesigner 6.42 N/mm^2. The unity check is 1.008, which is well within the allowed error margin of 10%.

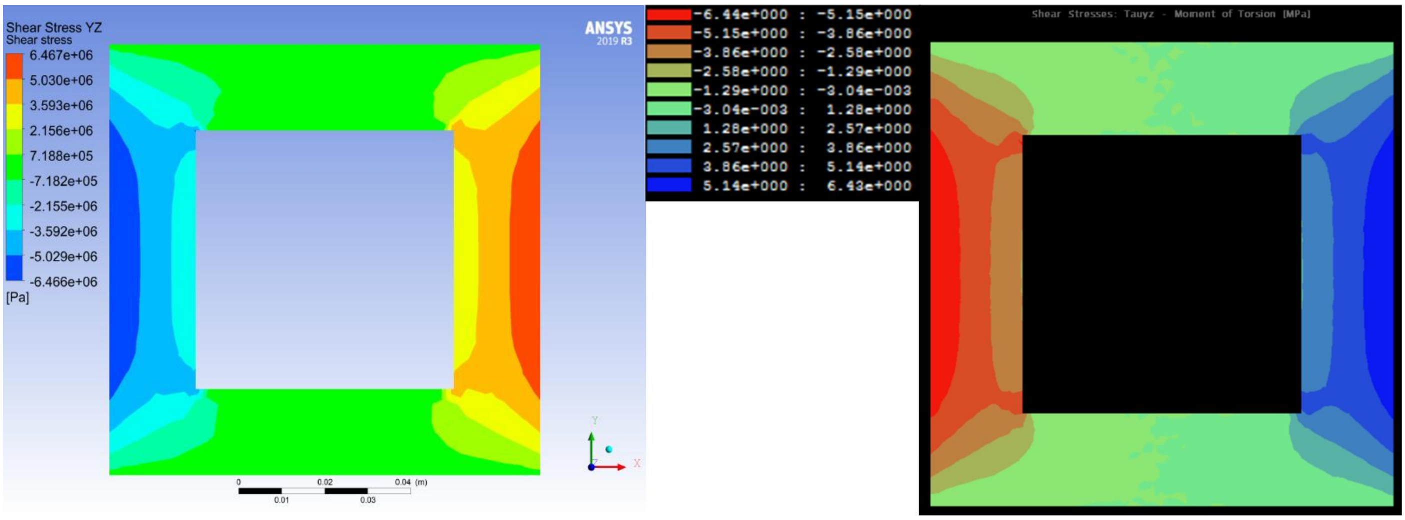

Figure 48 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 6.47 N/mm^2 and in ShapeDesigner 6.44 N/mm^2. The unity check is 1.005, which is also well within the allowed error margin of 10%.

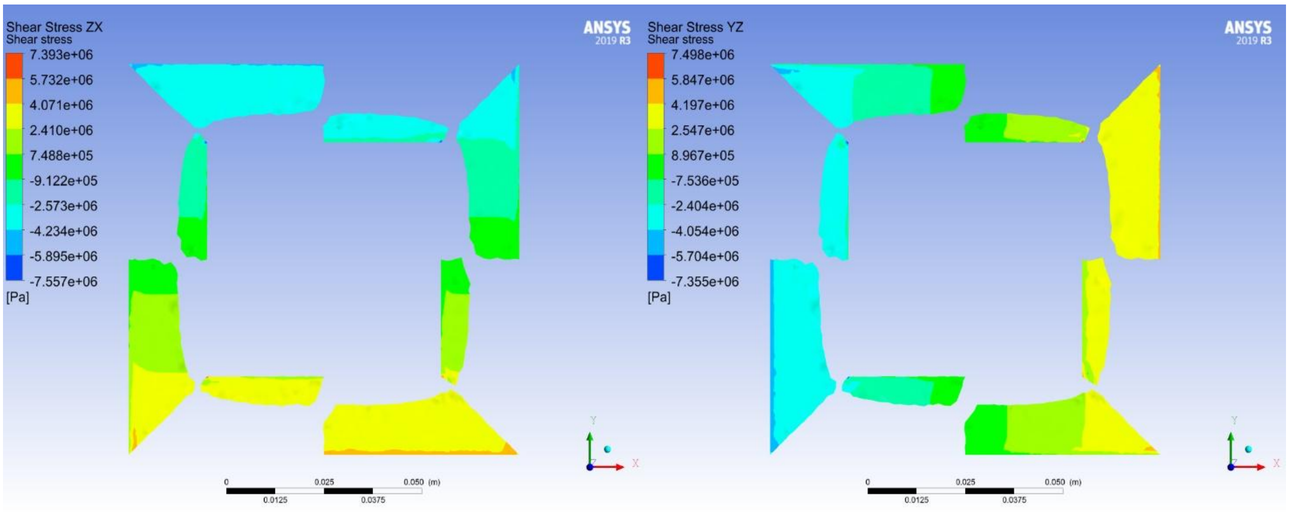

Figure 49 The shear stresses at the free end. Left; XZ plane. Right; YZ plane.

In Figure 49, the shear stress distribution at the free end is shown for both the XZ and YZ plane. The exact same observation from beam 5 can be made for beam 7; larger shear stresses at the free end (compared to the cross section from Ansys), which is expected. The results from Ansys at the cross-section c and ShapeDesigner are more comparable and therefore more accurate.

4.4 Beam 7: 2 m and 20 mm thickness

4.4.1 Horizontal displacement and rotation

The maximum horizontal displacement (in x direction) according to the theory of Vlasov should be 0.130 mm and therefore the rotation is 0.130/50 = 2.6 * 10^-3 rad. According to Ansys, as can be seen in Figure 50, the maximum horizontal displacement is 0.131 mm and therefore the rotation is 2.61 * 10^-3 rad. With the results from theory being the comparand, the unity check is 1.008, which is well within the error margin.

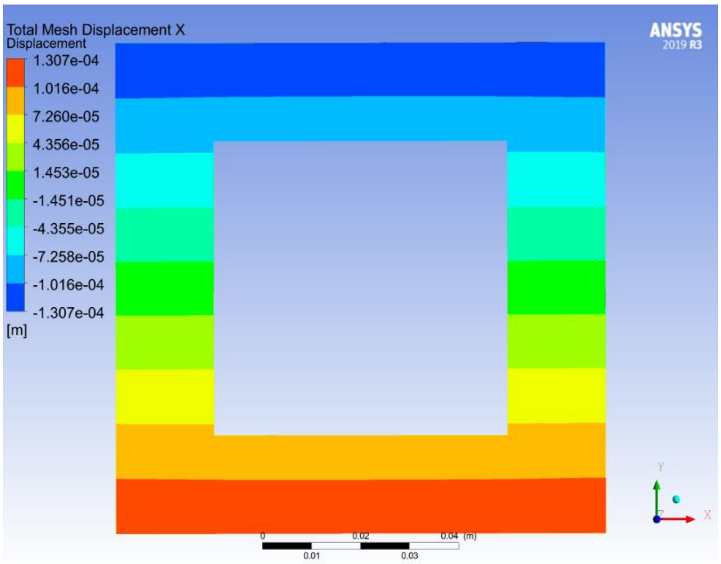

Figure 50 The horizontal displacement of beam 8 from Ansys

4.4.2 Normal stress

The cross section of the model in Ansys is at 25 mm from the fixed end. As can be seen in Figure 51, the contour plots show similar results, but the values are off.

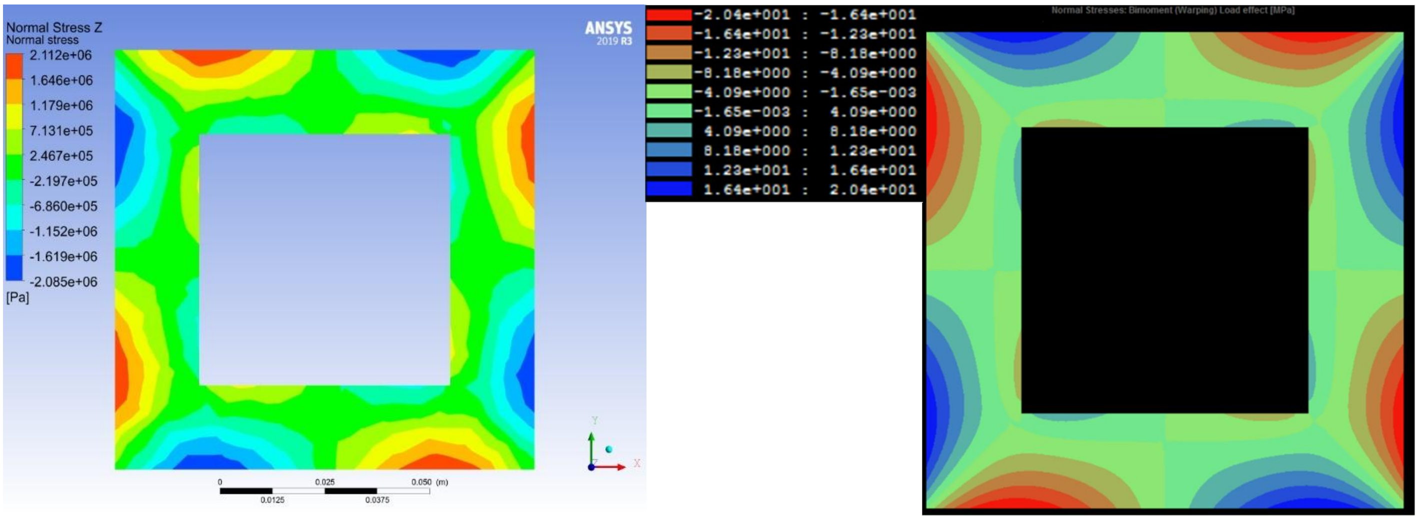

Figure 51 The contour plots of the normal stresses in Z-direction. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum normal stress in Ansys is 2.11 N/mm^2 and in ShapeDesigner 20.4 N/mm^2. The normal stress in ShapeDesigner is a factor 9.67 larger than the normal stress in Ansys. The unity check is 0.103, which is way over the margin. It seems that the normal stress does not compare at all with all the hollow square beams.

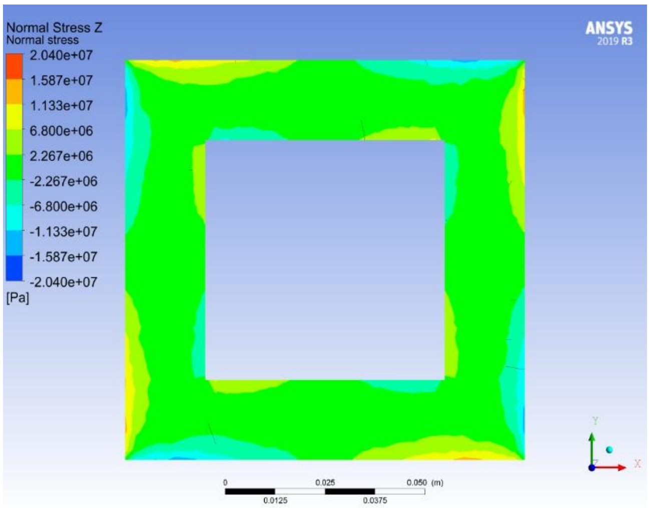

Figure 52 The normal stress in Z-direction at the fixed end with a rescaled legend

Figure 52 shows the normal stress in the fixed end of beam 8 with a legend rescaled to the legend of ShapeDesigner in Figure 51. Even at the fixed end, where the normal stress should be the highest, the blue colour or orange colour cannot be seen in the contour plot, meaning that these forces do not occur, just like beam 7. The difference between the maximum possible normal stress (at the fixed end) and the results from ShapeDesigner are still far too large. Which is exactly the same conclusion for beam 6. It seems that the normal stresses, just as proven with the comparison at the cross section, is wrong entirely. The maximum normal stress that occurs in the beam is still smaller than the results from ShapeDesigner. The reason, however, is unknown.

4.4.3 Shear stresses

The cross section from Ansys is at 90 mm from the free end. Figure 53 and 54 show the difference between the contour plots.

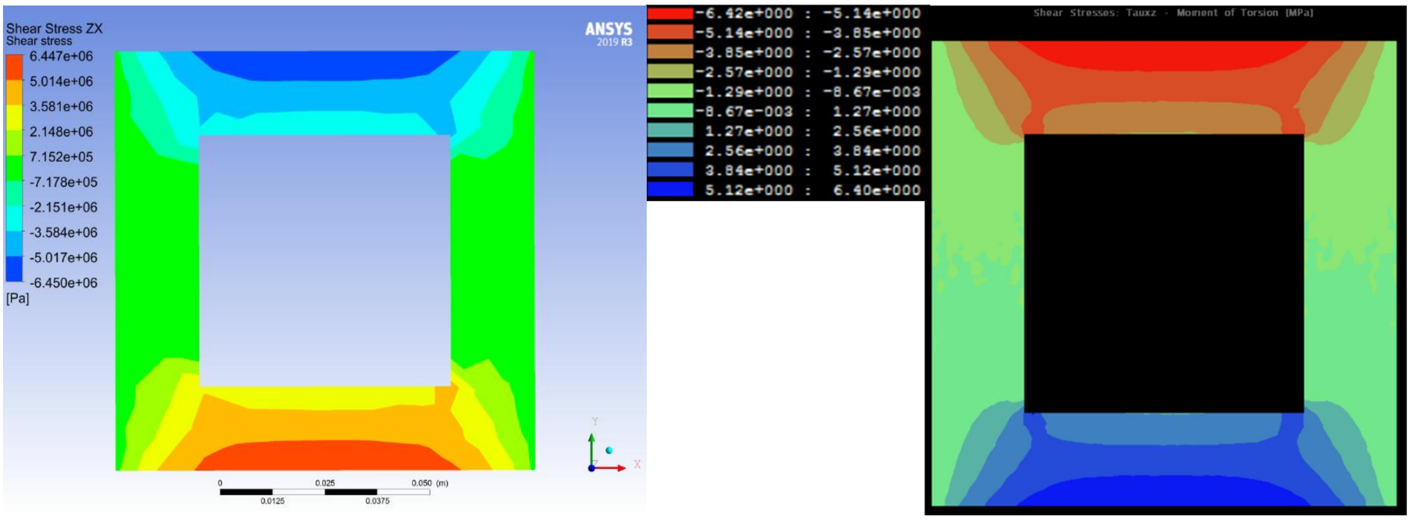

Figure 53 The contour plots of the shear stresses in the XZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in XZ plane in Ansys is 6.45 N/mm^2 and in ShapeDesigner 6.42 N/mm^2. The unity check is 1.005 which is well within the allowed error margin of 10%.

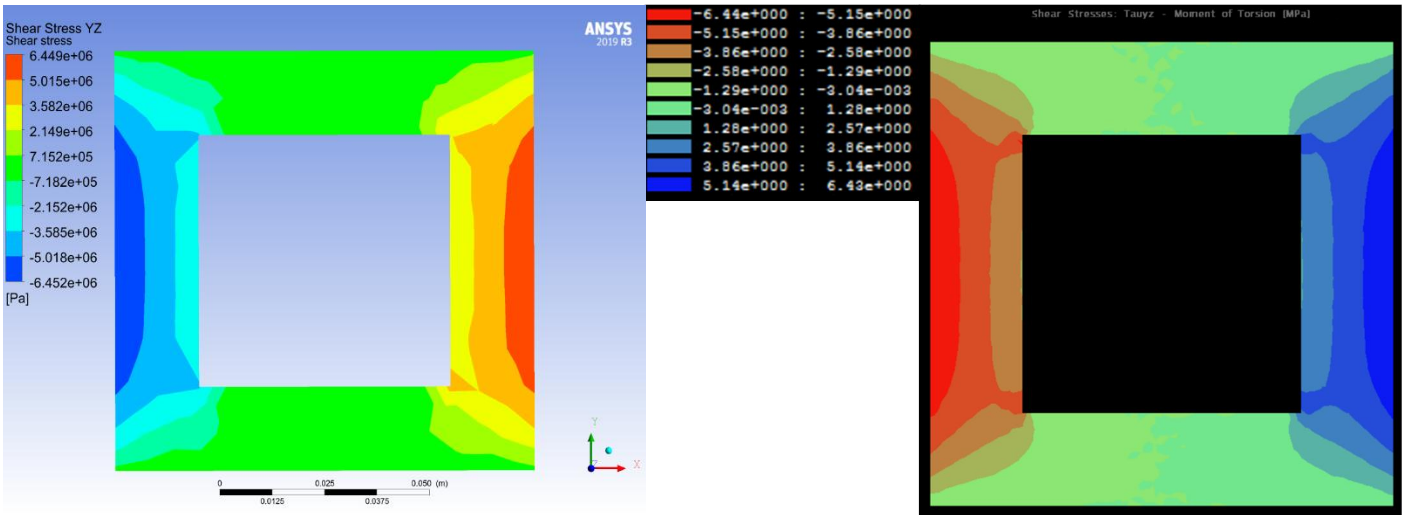

Figure 54 The contour plots of the shear stresses in the YZ plane. Left; Results from Ansys. Right; Results from ShapeDesigner

The absolute maximum shear stress in YZ plane in Ansys is 6.45 N/mm^2 and in ShapeDesigner 6.44 N/mm^2. The unity check is 1.002, which is also well within the allowed error margin of 10%.

4.5 Observations and conclusion

The results are starting to be confusing. With the results for the Z profile in mind, it was expected that at least the normal stresses would be within the error margin and that the displacements would be correct for the longer beams. However, judging from the summary tables 9 to 12, that is not the case. The first and foremost remarkable result is the normal stress. In every beam, the results are almost a factor 10 too small (an error around 90%). The applied moment in ShapeDesigner was checked, just in case it was a factor 10 smaller, but that is not the case. The normal stresses at the fixed end were larger than the cross section, but still significantly smaller than the stresses in ShapeDesigner.

The displacements, however, seem all to be accurate except for beam 5, where the error is 54.5%. But the other beams, even the short beam 7, seem to be accurate.

Also, unexpectedly, every result for the shear stresses in both XZ and YZ plane are within the error margin and therefore are all acceptable, except for the shear stress in YZ plane for beam 6. The error is 15.0% and therefore above the allowed margin of error. The shear stresses at the free end were, as expected, larger than the shear stresses at the cross section. For the Z-profile this was not the case and as the results from Ansys were not comparable with the results from ShapeDesigner.

Lastly to note; The results for the normal stress and the shear stresses across the same dimensional profile (beam 5 and 6, beam 7 and 8) all have the same result. This was also noted for the Z-profile for the shear stresses. For the hollow square profile, it also seems to be the case for the normal stress. That is because the calculated bimoment from Maple is the same for beam 5 and 6 as well as for beam 7 and 8.

In short:

- The results for the normal stresses with the combination of ShapeDesigner and Maple are all almost a factor 10 too small.
- The normal stresses at the fixed end are larger than the stresses at the cross section, but still significantly smaller than ShapeDesigner.
- The results for displacements only are within the margin for beam 6, 7 and 8. Beam 5 is significantly over the error margin.
- The results for the Shear stresses for both planes are all acceptable since they are well within the error margin. This could be because the shear stresses at the free end were expectedly larger than the cross section, which seemed to be the opposite for the Z-profile.
- The normal stress and shear stresses are identical for the same dimensional profiles. Results from ShapeDesigner for the stresses for beam 5 and 6 are the same and for beam 7 and 8 as well.

Table 9 Summary results beam 5

Beam 5	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.0435	0.0672	1.545	29
Normal stress	67.5	6.79	0.101	29-30
Shear stress XZ	16.3	16.11	0.988	30-31
Shear stress YZ	17.3	15.9	0.919	30-31

Table 10 Summary results beam 6

Beam 6	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.350	0.355	1.014	32
Normal stress	67.5	6.79	0.101	32
Shear stress XZ	16.3	14.8	0.908	33-34
Shear stress YZ	17.3	14.7	0.850	33-34

Table 11 Summary results beam 7

Beam 7	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.0160	0.0167	1.044	34
Normal stress	20.4	2.05	0.100	35
Shear stress XZ	6.42	6.47	1.008	36-37
Shear stress YZ	6.44	6.47	1.005	36-37

Table 12 Summary results beam 8

Beam 8	ShapeDesigner	Ansys	U.C.	Page
Displacement	0.130	0.131	1.008	37
Normal stress	20.4	2.11	0.103	38
Shear stress XZ	6.42	6.45	1.005	39
Shear stress YZ	6.44	6.45	1.002	39

6 Conclusion

From this study it can be concluded that there are indeed limits to the theory of Vlasov. The results from the Z-profile were unexpected. The summary of the results can be found on page 27. The initial expectations was that the displacements would be accurate for each case and that the difference in normal stress would be higher than 10%. Against all expectations, the normal stress was accurate for all the Z-profile beams whereas the displacement for the short beams were inaccurate. The normal stresses at the fixed end was larger than at any cross sections of the beam as expected.

The shear stresses were not accurate for any Z-profile beam. The shear stresses at any cross section in the beam were also larger than at the free end, which goes against the expectation that the largest shear stresses should be at the free end. This may be the reason why the shear stresses are inaccurate. It is not clear however, whether the fault lies with Ansys or ShapeDesigner, since the shear stresses for beam 1 are the same as beam 2 in ShapeDesigner as well as for beam 3 being the same as beam 4. It may be better to judge the theory of Vlasov by judging the errors in displacements and normal stresses since the normal stresses directly come from the theory (Maple) and the normal stresses follow from the input of bi moments in ShapeDesigner, where the bi moments also follow from the theory (Maple).

Which answers the sub question 'can the theory of Vlasov be applied to open cross sections' as follows; yes, it can be applied to calculate the normal stresses for any length. However, to calculate the displacements, the theory can only be applied to longer beams. Where the limits for the length of the beam lie is not yet known, but they should at least be larger than 0.25 m.

The results from the hollow square profile was also unexpected and the opposite of the Z-profile. The summary of the results can be found on page 39. The displacements were accurate for all except beam 5, which is a beam that is 0.25 m long and has a thickness of 5 mm. Beam 7, which is also a short beam but has a thickness of 20 mm, was unexpectedly within allowed margin. The normal stresses were significantly different for all beams, all almost by a factor 10. It is remarkable that the normal stresses at the fixed end of the beams were larger than any cross section in the beam but still far too small compared to the results of ShapeDesigner. The shear stresses, however, were all accurate for every hollow square profile.

Which answers the sub question 'can the theory of Vlasov be applied to closed cross sections' as follows; no, it cannot be applied to calculate the normal stresses for any length since there is always a factor 10 difference which is too large to be neglected. However, to calculate the displacements, the theory can only be applied to most beams. ShapeDesigner can be used to calculate the shear stresses for every beam.

7 Recommendations

Since there seem to be limits to the theory of Vlasov for the closed cross section, it could be interesting to look at how Ansys converts the torsional load into shear stresses and what may be the reason that there is a constant difference of a factor 10 between the results of Ansys and the combination of Maple and ShapeDesigner.

It is also remarkable that the theory of Vlasov seems to be inaccurate for the displacements for the shorter beams. The limits for the length could also be analysed. This can be done by modelling various beams with slightly increasing lengths but with the same cross sectional properties (which means also the same thickness). To test if the limits are applicable for every open cross section, the results could be applied to the same cross section but with a larger thickness or it could also be applied to different open cross sections (different profiles).

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Raaphorst, T.B. (2020, 20 June). Bsc Report. ”De wringvervormingstheorie van Vlasov; Geldigheid voor massieve en kokervormige doorsneden”. Accessed on January 7, 2021 via: http://homepage.tudelft.nl/p3r3s/BSc_projects/eindrapport_raaphorst.pdf

Appendix A: Maple Sheets

Maple Sheet Test Beam

```
> restart :
> l := 3400 :                      # [mm]
  ECw := 210000·1299·10^7 :         # [Nmm4]
  GIw := 77777·51467 :             # [mm2]
  mx := 0 :                        #`[Nmm/mm]
  T := 1.2·10^6 :       # [Nmm]
> print(T) :
```

$$1.2000000\ 10^{6}$$

```
> with(DEtools) :
> Vlasov := ECw·diff(phi(x), x, x, x, x) - GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 2727900000000000\ \frac{d^4}{dx^4}\ \phi(x) - 4002948859\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
> bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) - ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 4002948859\ D(\phi)(3400) - 2727900000000000\ D^{(3)}(\phi)(3400) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(3400) = 0$$

```
> Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -0.2473406961 + 0.0002997789985\,x - 0.00006546331515\ e^{0.001211367165\,x} + 0.2474061596\ e^{-0.001211367165\,x}$$

```
> assign(Sol); phi := phi(x)
```

$$\phi := -0.2473406961 + 0.0002997789985\,x - 0.00006546331515\ e^{0.001211367165\,x} + 0.2474061596\ e^{-0.001211367165\,x}$$

```
> x := l : phi_max := phi;
```

$$phi\_max := 0.7719078989$$

```
> u_max := phi_max·0.5·200
```

$$u\_max := 77.19078989$$

```
> x :='x';
```

$$x := x$$

```
> B := -ECw·diff(phi, x, x)
```

$$B := 262046.3023\ e^{0.001211367165\,x} - 9.903542031\ 10^{8}\ e^{-0.001211367165\,x}$$

```
> x := 0; B; x := 'x';
  plot(B, x = 0 ..l);
```

$$x := 0$$

$$-9.900921568\ 10^{8}$$

$$x := x$$

Maple Sheet Beam 1

```
restart :
l := 250 :                      # [mm]
ECw := 210000·18.356 ·10^9 :    # [Nmm4]
GIw := 77777·16.668·10^3 :      # [mm2]
mx := 0 :                       #`[Nmm/mm]
T := 1.2·10^6 :       # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 3.854760000\ 10^{15}\ \frac{d^4}{dx^4}\ \phi(x) - 1.296387036\ 10^{9}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 1.296387036\ 10^{9}\ D(\phi)(250) - 3.854760000\ 10^{15}\ D^{(3)}(\phi)(250) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(250) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -0.2298045178 + 0.0009256494910\ x - 0.6831804530\ e^{0.0005799207759\ x} + 0.9129849708\ e^{-0.0005799207759\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -0.2298045178 + 0.0009256494910\ x - 0.6831804530\ e^{0.0005799207759\ x} + 0.9129849708\ e^{-0.0005799207759\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.0016078550$$

```
u_max := phi_max·0.5·200
```

$$u\_max := 0.1607855000$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 8.856662824\ 10^{8}\ e^{0.0005799207759\ x} - 1.183581880\ 10^{9}\ e^{-0.0005799207759\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$
$$-2.979155976\ 10^{8}$$
$$x := x$$

Maple Sheet Beam 2

```
restart :
l := 2000 :                     # [mm]
ECw := 210000·18.356 ·10^9 :    # [Nmm4]
GIw := 77777·16.668·10^3 :      # [mm2]
mx := 0 :                       #`[Nmm/mm]
T := 1.2·10^6 :       # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 3.854760000\ 10^{15}\ \frac{d^4}{dx^4}\ \phi(x) - 1.296387036\ 10^{9}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 1.296387036\ 10^{9}\ D(\phi)(2000) - 3.854760000\ 10^{15}\ D^{(3)}(\phi)(2000) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(2000) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -1.310433038 + 0.0009256494910\ x - 0.1428661929\ e^{0.0005799207759\ x} + 1.453299231\ e^{-0.0005799207759\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -1.310433038 + 0.0009256494910\ x - 0.1428661929\ e^{0.0005799207759\ x} + 1.453299231\ e^{-0.0005799207759\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.5408659441$$

```
u_max := phi_max·0.5·200
```

$$u\_max := 54.08659441$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 1.852098803\ 10^{8}\ e^{0.0005799207759\ x} - 1.884038282\ 10^{9}\ e^{-0.0005799207759\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$
$$-1.698828402\ 10^{9}$$
$$x := x$$

Maple Sheet Beam 3

```
restart :
l := 250 :                          # [mm]
ECw := 210000·49.296·10^9 :   # [Nmm4]
GIw := 77777·95.793·10^4 :        # [mm2]
mx := 0 :                           #`[ Nmm/mm ]
T := 1.2·10^6 :         # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 1.035216000\ 10^{16}\ \frac{d^4}{dx^4}\ \phi(x) - 7.450492161\ 10^{10}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 7.450492161\ 10^{10}\ D(\phi)(250) - 1.035216000\ 10^{16}\ D^{(3)}(\phi)(250) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(250) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -0.003514740654 + 0.00001610631854\ x - 0.001244481803\ e^{0.002682730170\ x} + 0.004759222458\ e^{-0.002682730170\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -0.003514740654 + 0.00001610631854\ x - 0.001244481803\ e^{0.002682730170\ x} + 0.004759222458\ e^{-0.002682730170\ x}$$

```
phi_max := maximize(phi, x = 0 .. l);
```

$$phi\_max := 0.0005118389792$$

```
u_max := phi_max·0.5·200
```

$$u\_max := 0.05118389792$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 9.272001926\ 10^{7}\ e^{0.002682730170\ x} - 3.545854963\ 10^{8}\ e^{-0.002682730170\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-2.618654770\ 10^{8}$$

$$x := x$$

Maple Sheet Beam 4

```
restart :
l := 2000 :                         # [mm]
ECw := 210000·49.296·10^9 :   # [Nmm4]
GIw := 77777·95.793·10^4 :        # [mm2]
mx := 0 :                           #`[ Nmm/mm ]
T := 1.2·10^6 :         # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 1.035216000\ 10^{16}\ \frac{d^4}{dx^4}\ \phi(x) - 7.450492161\ 10^{10}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 7.450492161\ 10^{10}\ D(\phi)(2000) - 1.035216000\ 10^{16}\ D^{(3)}(\phi)(2000) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(2000) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -0.006003441801 + 0.00001610631854\ x - 1.312291035\ 10^{-7}\ e^{0.002682730170\ x} + 0.006003573032\ e^{-0.002682730170\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -0.006003441801 + 0.00001610631854\ x - 1.312291035\ 10^{-7}\ e^{0.002682730170\ x} + 0.006003573032\ e^{-0.002682730170\ x}$$

```
phi_max := maximize(phi, x = 0 .. l);
```

$$phi\_max := 0.02620919528$$

```
u_max := phi_max·0.5·200
```

$$u\_max := 2.620919528$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 9777.214075\ e^{0.002682730170\ x} - 4.472957385\ 10^{8}\ e^{-0.002682730170\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-4.472859613\ 10^{8}$$

$$x := x$$

Maple Sheet Beam 5

```
restart :
l := 250 :                        # [mm]
ECw := 210000·43.269·10^5 :   # [Nmm4]
GIw := 77777·44.024·10^5 :       # [mm2]
mx := 0 :                         # `[ Nmm/mm ]
T := 1.2·10^6 :       # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 9.086490000\ 10^{11}\ \frac{d^4}{dx^4}\ \phi(x) - 3.424054648\ 10^{11}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 3.424054648\ 10^{11}\ D(\phi)(250) - 9.086490000\ 10^{11}\ D^{(3)}(\phi)(250) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(250) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -5.709108432\ 10^{-6} + 3.504616963\ 10^{-6}\ x - 2.868552762\ 10^{-139}\ e^{0.6138641447\ x} + 5.709108432\ 10^{-6}\ e^{-0.6138641447\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -5.709108432\ 10^{-6} + 3.504616963\ 10^{-6}\ x - 2.868552762\ 10^{-139}\ e^{0.6138641447\ x} + 5.709108432\ 10^{-6}\ e^{-0.6138641447\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.0008704451324$$

```
u_max := phi_max·0.5·100
```

$$u\_max := 0.04352225662$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 9.822081428\ 10^{-128}\ e^{0.6138641447\ x} - 1.954829928\ 10^{6}\ e^{-0.6138641447\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-1.954829928\ 10^{6}$$

$$x := x$$

Maple Sheet Beam 6

```
restart :
l := 2000 :                        # [mm]
ECw := 210000·43.269·10^5 :   # [Nmm4]
GIw := 77777·44.024·10^5 :       # [mm2]
mx := 0 :                         # `[ Nmm/mm ]
T := 1.2·10^6 :       # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 9.086490000\ 10^{11}\ \frac{d^4}{dx^4}\ \phi(x) - 3.424054648\ 10^{11}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 3.424054648\ 10^{11}\ D(\phi)(2000) - 9.086490000\ 10^{11}\ D^{(3)}(\phi)(2000) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(2000) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -5.709108432\ 10^{-6} + 3.504616963\ 10^{-6}\ x - 2.319134375\ 10^{-1072}\ e^{0.6138641447\ x} + 5.709108432\ 10^{-6}\ e^{-0.6138641447\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -5.709108432\ 10^{-6} + 3.504616963\ 10^{-6}\ x - 2.319134375\ 10^{-1072}\ e^{0.6138641447\ x} + 5.709108432\ 10^{-6}\ e^{-0.6138641447\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.007003524818$$

```
u_max := phi_max·0.5·100
```

$$u\_max := 0.3501762409$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 7.940842843\ 10^{-1061}\ e^{0.6138641447\ x} - 1.954829928\ 10^{6}\ e^{-0.6138641447\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-1.954829928\ 10^{6}$$

$$x := x$$

Maple Sheet Beam 7

```
restart :
l := 250 :                    # [mm]
ECw := 210000·90.035·10^6 :   # [Nmm4]
GIw := 77777·11.836·10^6 :    # [mm2]
mx := 0 :                     # [Nmm/mm]
T := 1.2·10^6 :     # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 1.890735000\ 10^{13}\ \frac{d^4}{dx^4}\ \phi(x) - 9.205685720\ 10^{11}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 9.205685720\ 10^{11}\ D(\phi)(250) - 1.890735000\ 10^{13}\ D^{(3)}(\phi)(250) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(250) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -5.907617411\ 10^{-6} + 1.303542220\ 10^{-6}\ x - 7.192863744\ 10^{-54}\ e^{0.2206544753\ x} + 5.907617411\ 10^{-6}\ e^{-0.2206544753\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -5.907617411\ 10^{-6} + 1.303542220\ 10^{-6}\ x - 7.192863744\ 10^{-54}\ e^{0.2206544753\ x} + 5.907617411\ 10^{-6}\ e^{-0.2206544753\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.0003199779376$$

```
u_max := phi_max·0.5·100
```

$$u\_max := 0.01599889688$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 6.621524303\ 10^{-42}\ e^{0.2206544753\ x} - 5.438366923\ 10^{6}\ e^{-0.2206544753\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-5.438366923\ 10^{6}$$

$$x := x$$

Maple Sheet Beam 8

```
restart :
l := 2000 :                   # [mm]
ECw := 210000·90.035·10^6 :   # [Nmm4]
GIw := 77777·11.836·10^6 :    # [mm2]
mx := 0 :                     # [Nmm/mm]
T := 1.2·10^6 :     # [Nmm]
print(T) :
```

$$1.2000000\ 10^{6}$$

```
with(DEtools) :
Vlasov := ECw ·diff(phi(x), x, x, x, x) -GIw·diff(phi(x), x, x) = mx;
```

$$Vlasov := 1.890735000\ 10^{13}\ \frac{d^4}{dx^4}\ \phi(x) - 9.205685720\ 10^{11}\ \frac{d^2}{dx^2}\ \phi(x) = 0$$

```
bound_con := phi(0) = 0, D(phi)(0) = 0, GIw·D(phi)(l) − ECw·(D@@3)(phi)(l) = T, (D@@2)(phi)(l) = 0;
```

$$bound\_con := \phi(0) = 0, D(\phi)(0) = 0, 9.205685720\ 10^{11}\ D(\phi)(2000) - 1.890735000\ 10^{13}\ D^{(3)}(\phi)(2000) = 1.2000000\ 10^{6}, D^{(2)}(\phi)(2000) = 0$$

```
Sol := evalf(dsolve({Vlasov, bound_con}, {phi(x)}));
```

$$Sol := \phi(x) = -5.907617409\ 10^{-6} + 1.303542220\ 10^{-6}\ x - 2.853173893\ 10^{-389}\ e^{0.2206544753\ x} + 5.907617411\ 10^{-6}\ e^{-0.2206544753\ x}$$

```
assign(Sol); phi := phi(x)
```

$$\phi := -5.907617409\ 10^{-6} + 1.303542220\ 10^{-6}\ x - 2.853173893\ 10^{-389}\ e^{0.2206544753\ x} + 5.907617411\ 10^{-6}\ e^{-0.2206544753\ x}$$

```
x := l : phi_max := phi;
```

$$phi\_max := 0.002601176823$$

```
u_max := phi_max·0.5·100
```

$$u\_max := 0.1300588412$$

```
x :='x';
```

$$x := x$$

```
B := -ECw·diff(phi, x, x)
```

$$B := 2.626542217\ 10^{-377}\ e^{0.2206544753\ x} - 5.438366923\ 10^{6}\ e^{-0.2206544753\ x}$$

```
x := 0; B; x := 'x';
plot(B, x = 0 ..l);
```

$$x := 0$$

$$-5.438366923\ 10^{6}$$

$$x := x$$

Appendix B: Meshing of each beam

Since all beams have refinement on both ends of the beam, the mesh is the same except mirror for the other end. Therefore only one figure will be shown for each beam.

Beam 1

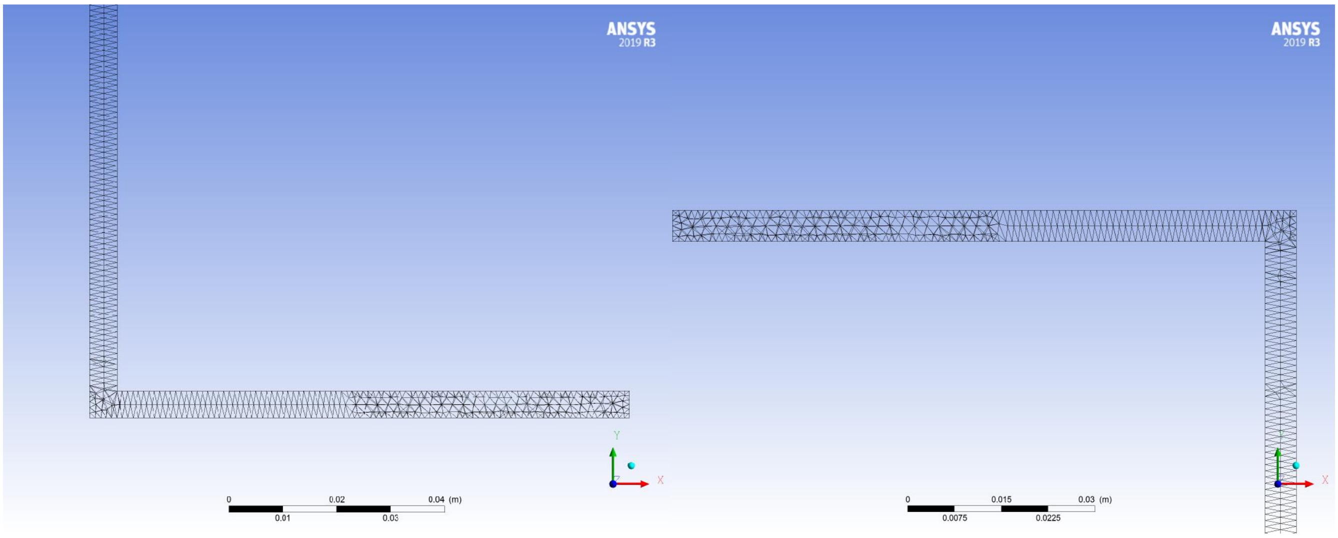

Figure B1 Meshing of beam 1. Left; Bottom flange. Right; Top flange

Beam 2

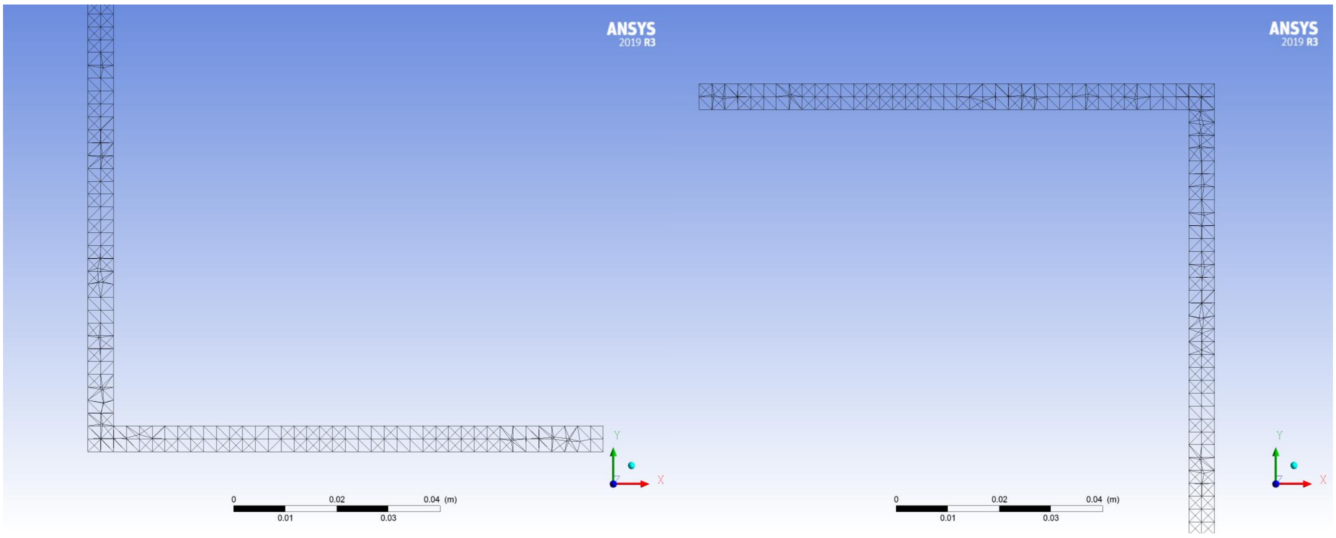

Figure B2 Meshing of beam 2. Left; Bottom flange. Right; Top flange

Beam 3

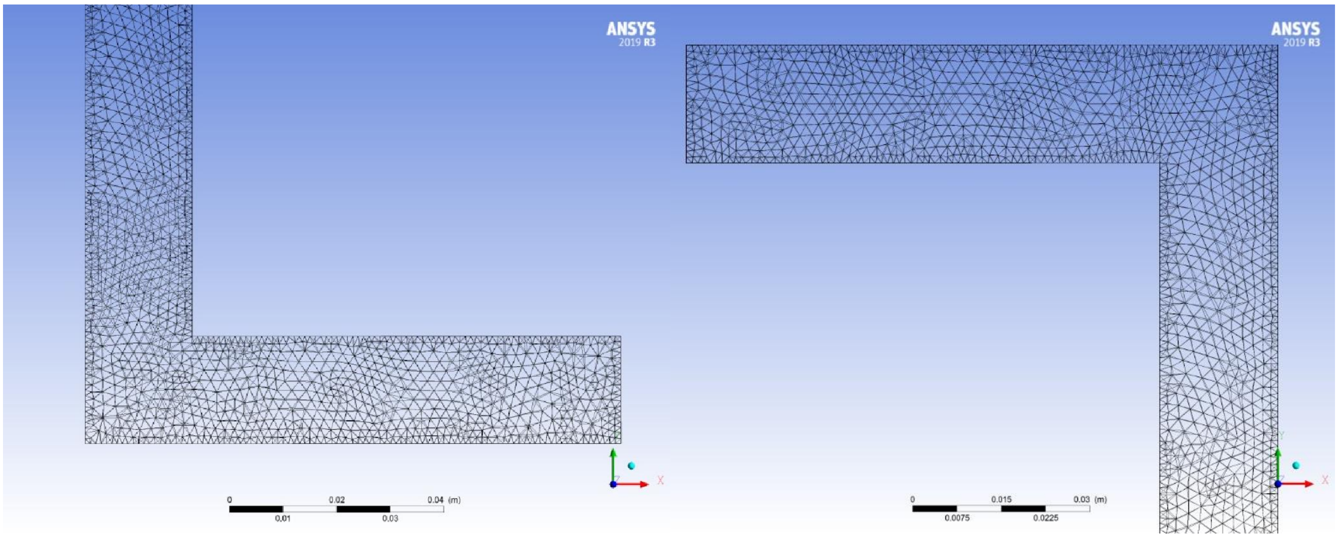

Figure B3 Meshing of beam 3. Left; Bottom flange. Right; Top flange

Beam 4

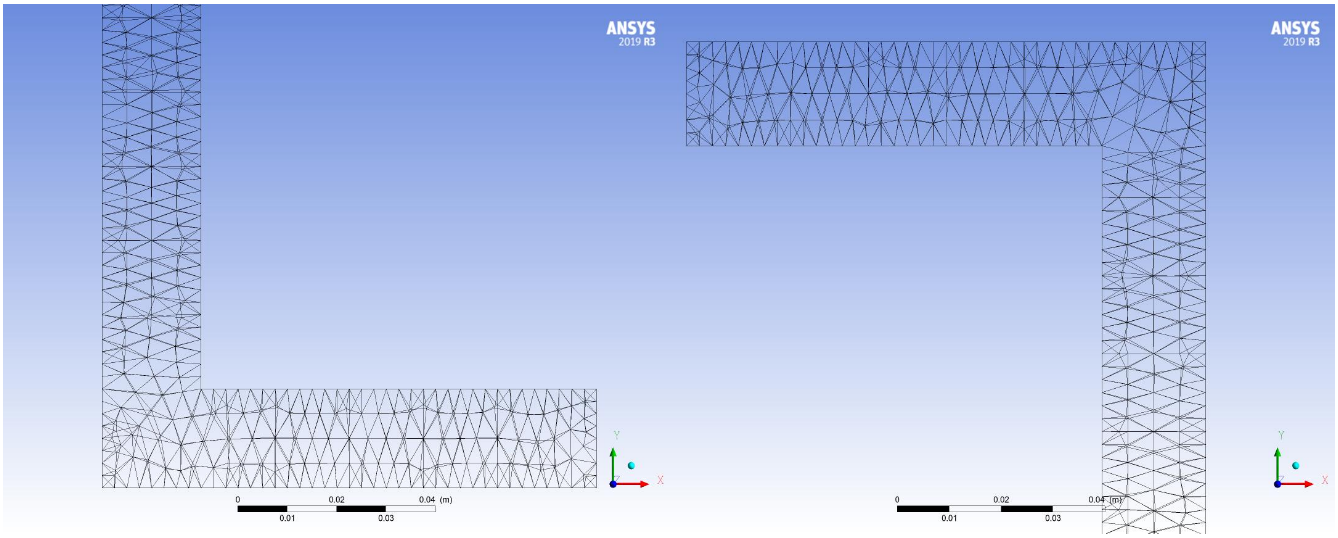

Figure B4 Meshing of beam 4. Left; Bottom flange. Right; Top flange

Beam 5

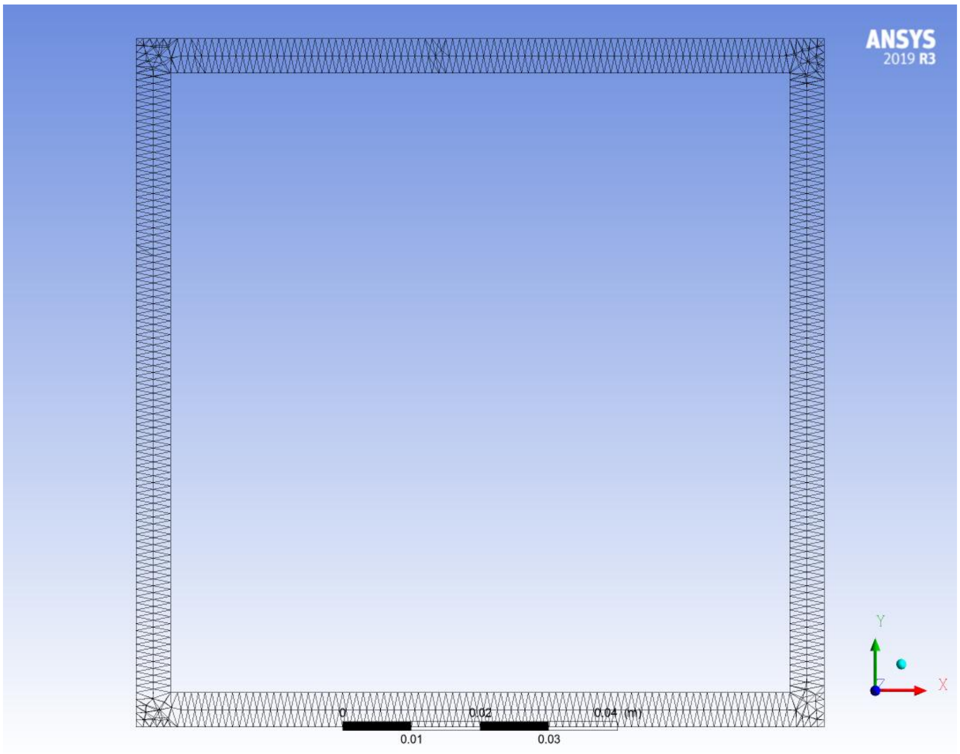

Figure B5 Meshing beam 5

Beam 6

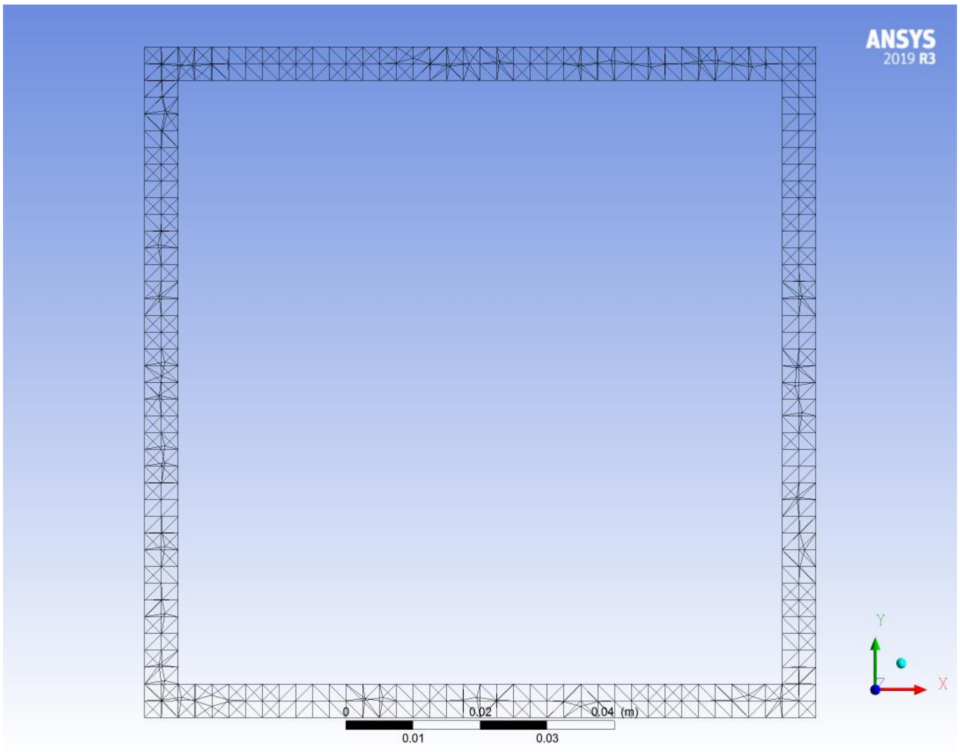

Figure B6 Meshing beam 6

Beam 7

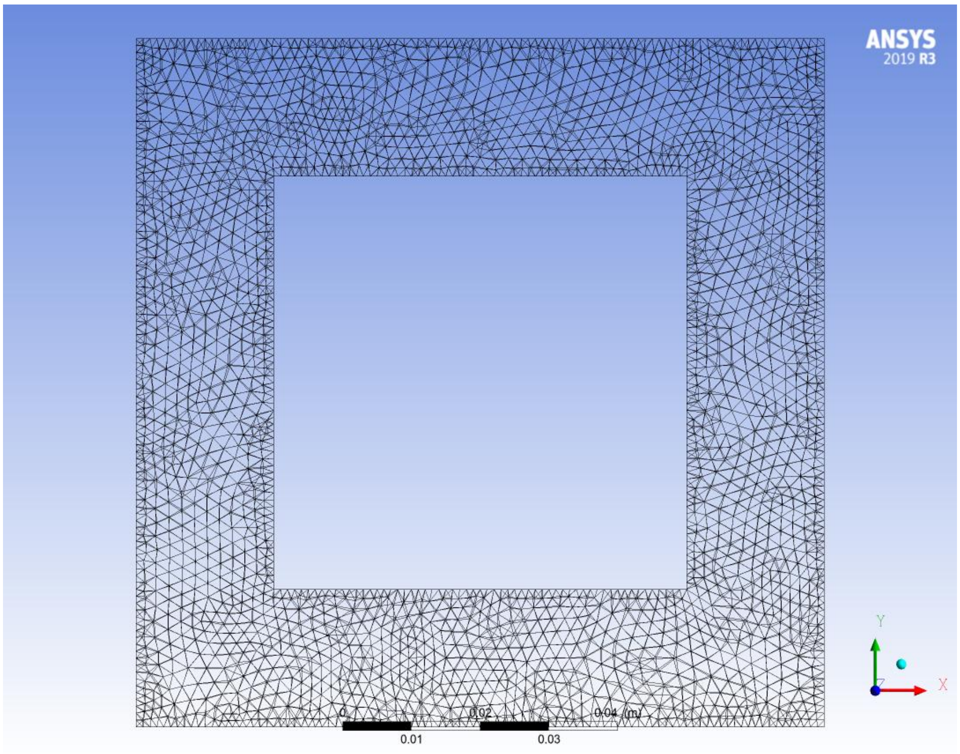

Figure B7 Meshing beam 7

Beam 8

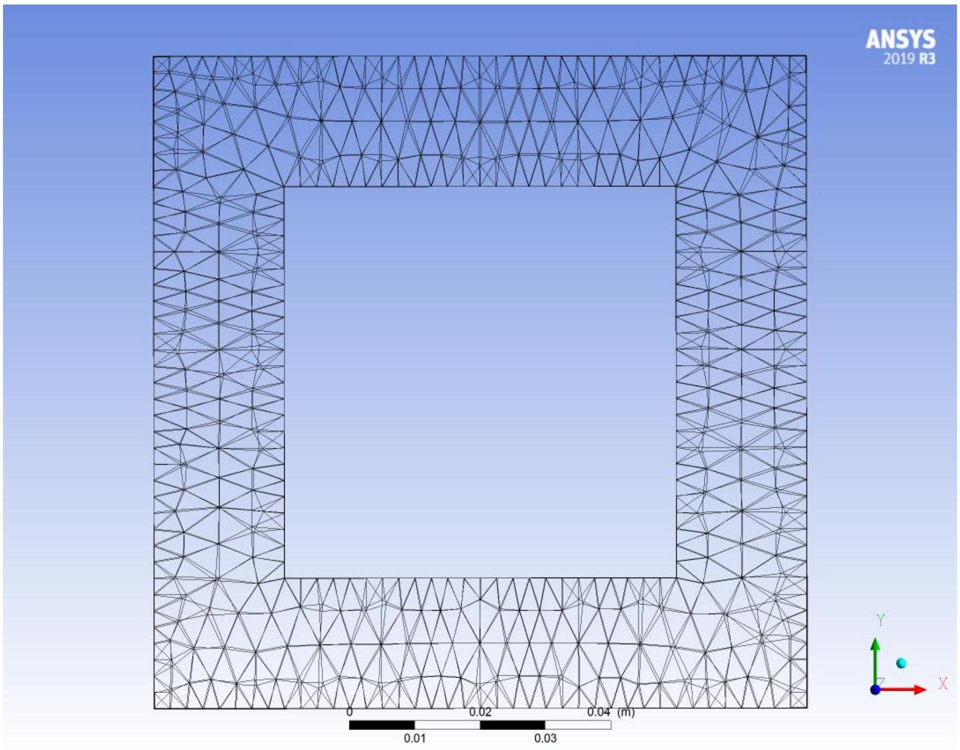

Figure B8 Meshing beam 8

Appendix C: Linear buckling analysis

The same IPE200 test beam with the same torsional load will be analysed for its linear buckling behaviour with the help of Ansys. The way the beam buckles can be seen in Figure C1.

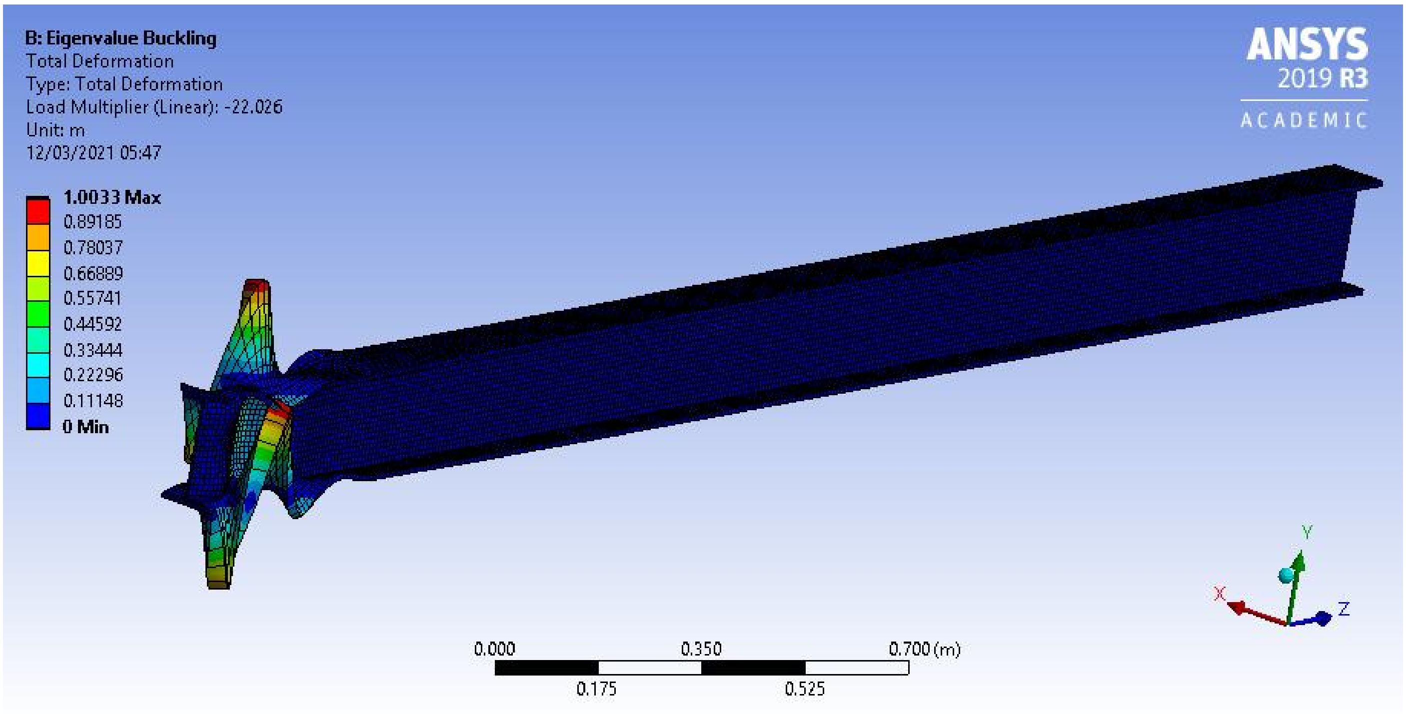

Figure C1 The linear buckling (Eigenvalue buckling) of the test beam undergoing a torsional load. The deformations are displayed with a 0.17 factor.

The beam shows a deformation at the fixed end. The load multiplier according to Ansys is -22.026. What does that mean? It means that the critical load, the load in this case being a torsional load, is equal to the load multiplier times the applied load. Therefore, the critical (torsional) load on the beam is -22.026*1.20 [kNm] = -26.4 [kNm]