Accuracy of shell finite elements in SCIA Engineer

Assignment for course CIE4143 Shell Analysis

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1. Introduction

The software SCIA Engineer computes the behaviour of structures, for example deformation, stresses, buckling load and natural frequencies. The software is used in the course Shell Analysis at Delft University of Technology [1]. The software has two types of finite element: beam elements and shell elements, with which all structures are modelled. At the start of this project, the accuracy of the shell elements was not known.

Research objective

Determine the order of the error of the shell elements in SCIA Engineer.

Approach

In this project, a shell canopy (figure 1) has been modelled in SCIA Engineer. Computed were membrane forces, moments and shear forces at four edge locations for several element sizes. From this the orders of the errors were derived.

The computation results are presented in a table so that they can be checked easily. The table will be added to the course reader for future reference.

2. Canopy dimensions

A linear elastic analysis of a canopy has been performed in this project. The canopy was assumed as reinforcement concrete (elastic material behaviour), with the material properties as:
$E = 10^7 \text{kN/m}^2, \nu = 0.15, \text{unit mass } m = 2500 \text{ kg/m}^3$.

The dimensions of the canopy are:
$\text{Radius } a = 2 \text{ m, shell, thickness } t = 0.2 \text{ m, slenderness } a/t = 2/0.2 = 10,$
$\text{length } = 6 \text{ m, width} = 2a = 4 \text{ m.}$

This is a thick shell according to reference [1], page 3. Consequently, membrane force, out of plane bending moments and out of plane shear forces occur. All associated deformations need to be included in modelling its structural behaviour including in-extensional deformation.

The load condition is:
A point load $F = 100$ kN in one corner as shown in figure 1. Self-weight is not applied. Boundary conditions on fixed curved edge have been posed as shown in figure 1. The results on the free curved edge are computed by the finite element method. The latter results need to be

checked for modelling mistakes by comparing with the theoretical derived shell boundary conditions shown in reference [1], page 82, based on Sanders-Koiter equations 6.

The model was meshed with standard shell elements (shell 98), isotropic materials and member coordinate system plane at center. Shell in this case is thick one based on geometrical analysis above shown, which means shear flexibility is quite important to be studied. However, to simplify this project as thin shell theory, bending theory performed on Kirchhoff shell analysis. Meanwhile, Shear deformation should be switched off in this case, with Kirchhoff shell theory consider vertical deformation only from bending moment. If the shear deformation is switched on, shear locking will influence on displacements of nodes as finite element error by using Kirchhoff shell theory analyze thick shell with 8-node shell elements, leading to wrong numerical results.

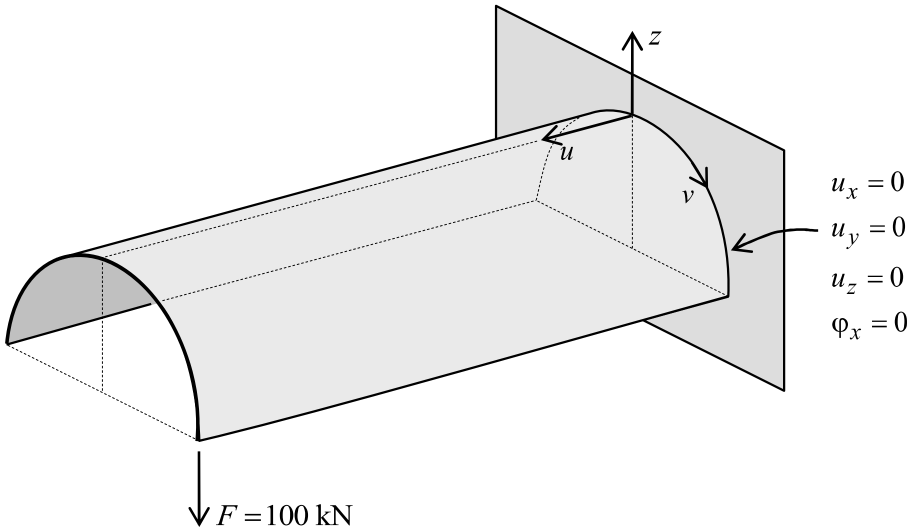

Figure 1. Canopy, point load and boundary conditions

3. Modelling checks

Element size check

According to reference [1], page 69 the influence length can be used to choose a finite element mesh. If we use elements that approximate a solution linearly, we need at least 6 elements in a length l_i in order to obtain solutions with some accuracy (see [1], figure 91). Clearly, more elements will improve the accuracy.

The influence length of this canopy is:
$l_i < 3.4\sqrt[4]{atl^3} = 3.4 \times \sqrt[4]{2000 \times 200 \times 6000^2} = 6623.23$ mm, which means the maximum element size is: $h = {}^{6623.23}/_{6} = 1103.87$ mm. (see [1], page 69) Therefore, the element sizes of table 1 are suitable for linear analysis of this canopy.

Boundary condition check

The finite element results of this canopy can be seen in the following figures (with element size of 50 mm). It can be observed that, on the curved fixed edge the deformation is zero, which agrees with the specified boundary conditions. On the straight edges n_{yy}, $(n_{xy} + n_{yx})/2$ and m_{yy} are approximately zero. This is correct for these free edges. On the curved free edge n_{xx}, $(n_{xy} + n_{yx})/2$ and m_{xx} are approximately zero, which is also correct. The shear forces v_y and v_x are not zero, which can be explained as that they are equal to the gradient of m_{xy}.

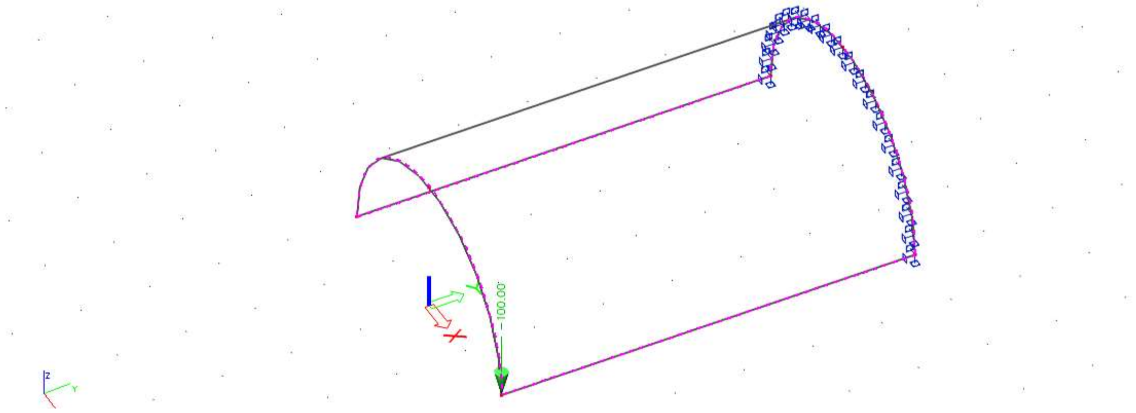

Figure 2. Definitions of model in SCIA (set local system rotated 90 degrees for set u-v system)

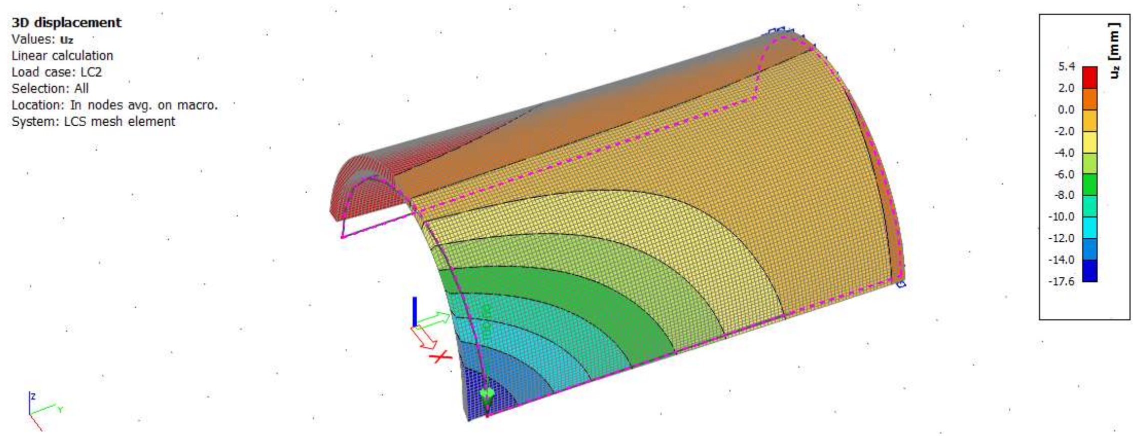

Figure 3. Vertical displacement in global Z direction [mm]
(negative is down positive is up)

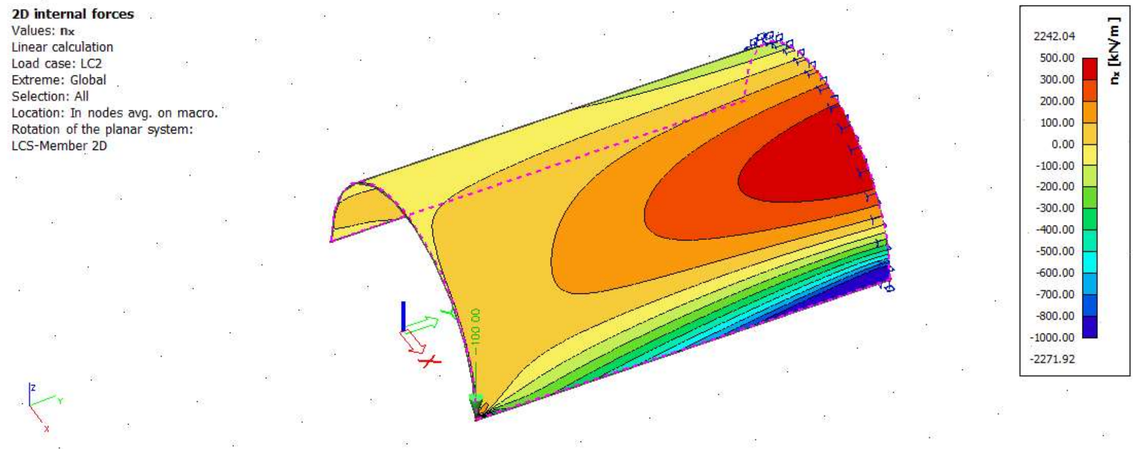

Figure 4. Normal force n_{xx} [kN/m]

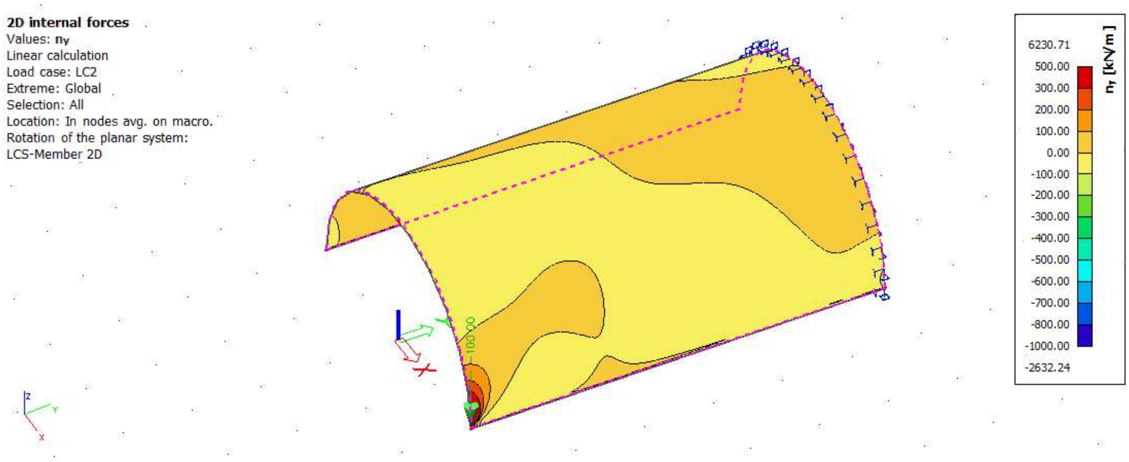

Figure 5. Normal force n_{yy} [kN/m]

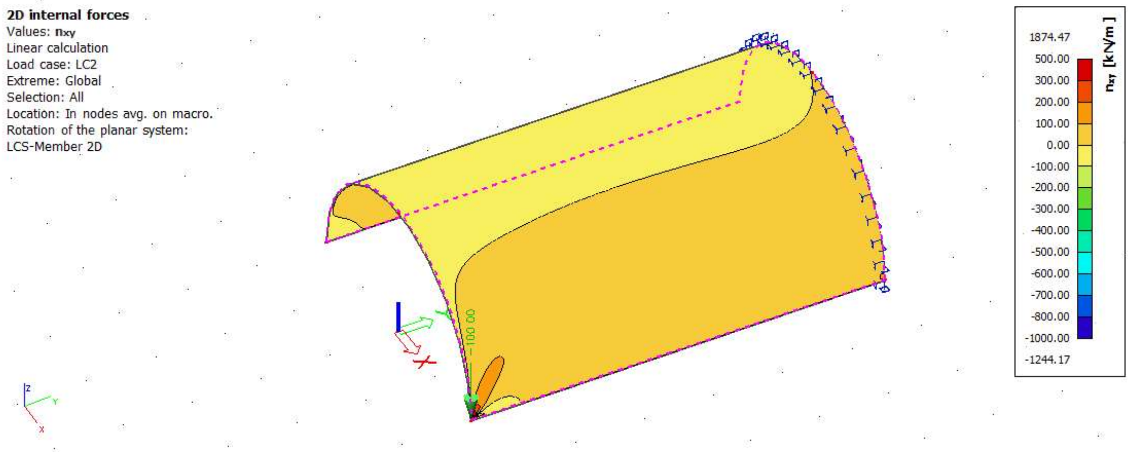

Figure 6. In plane shear force $\frac{n_{xy}+n_{yx}}{2}$ [kN/m]

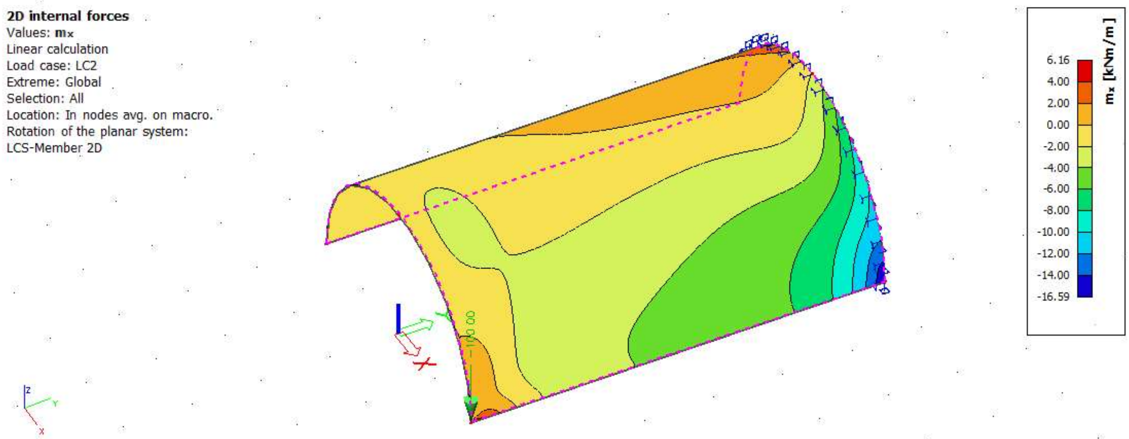

Figure 7. Bending moment m_{xx} [kNm/m]

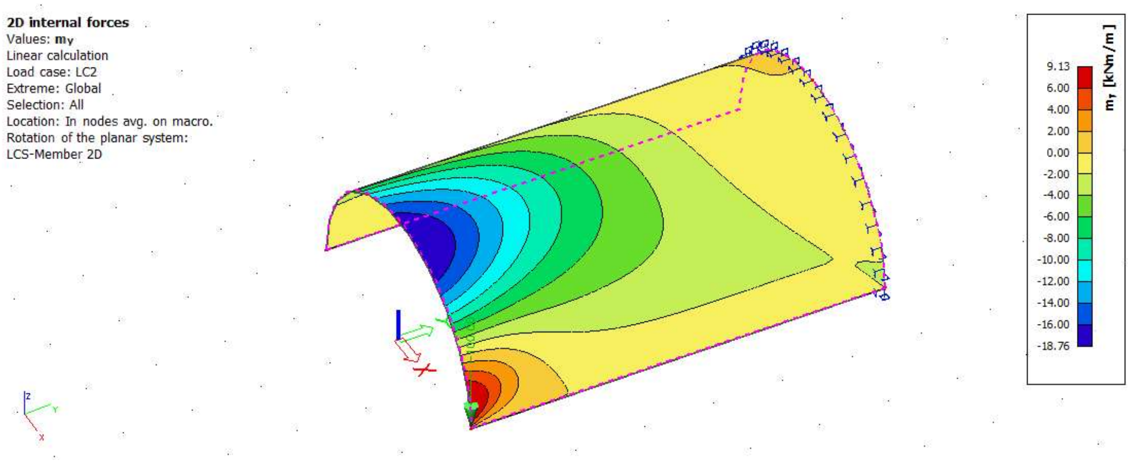

Figure 8. Bending moment m_{yy} [kNm/m]

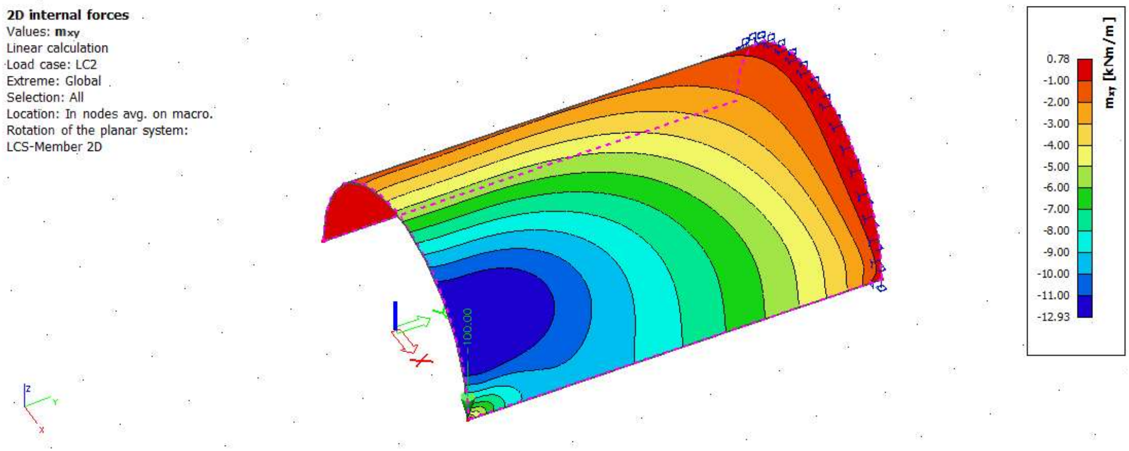

Figure 9. Torsion moment m_{xy} [kNm/m]

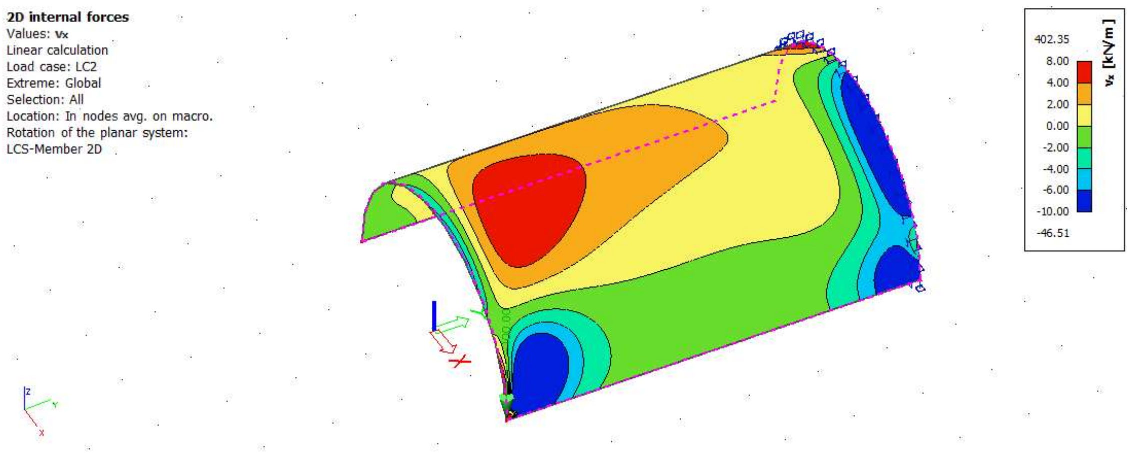

Figure 10. Out of plane shear force v_x [kN/m]

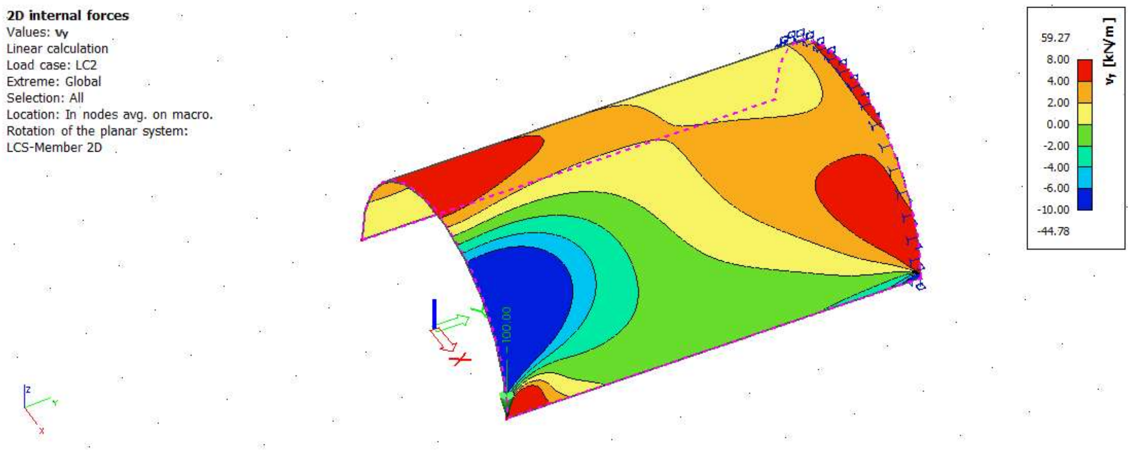

Figure 11. Out of plane shear force v_y [kN/m]

Singularity check

Figures 4, 5, 6, 10 and 11 show that singularities occur at the position of point load, which is at the location of (u, v) = (6, π) as well as the corners of connection where (u, v) = (0, ±π).

4. Analyses

Four finite element analyses were performed with each a different finite element size. The results in four locations were recorded (figure 12) These results are presented in in table 1.

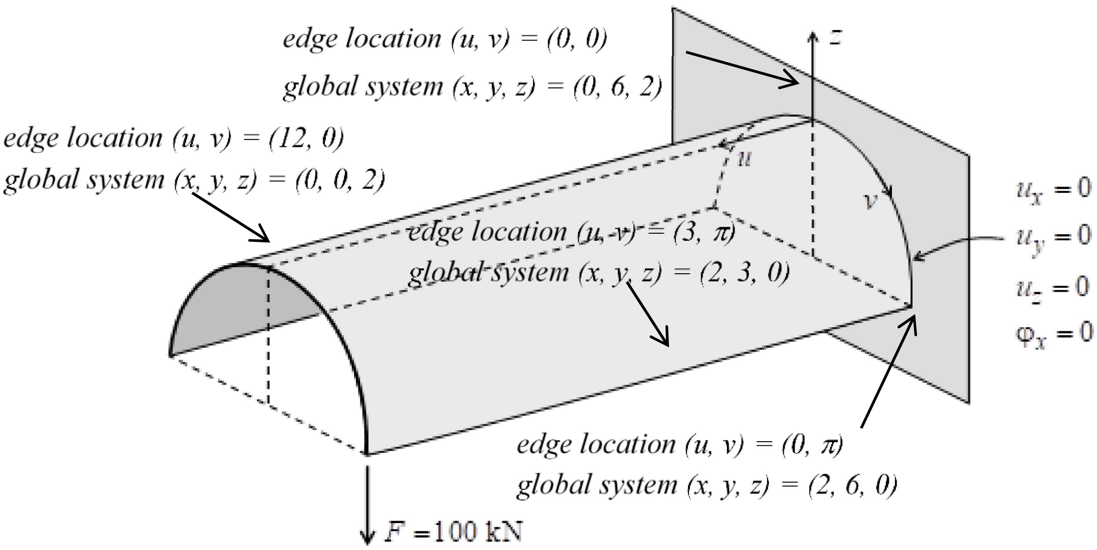

Figure 12. Check point locations in model

Table 1. Computation results at four edge locations for four element sizes

element size	n_{xx}	n_{yy}	$\frac{1}{2}(n_{xy}+n_{yx})$	m_{xx}	m_{yy}	m_{xy}	v_x	v_y
	[kN/m]	[kN/m]	[kN/m]	[kN]	[kN]	[kN]	[kN/m]	[kN/m]
edge location $(u, v) = (0, 0)$								
200 mm	124.34	19.41	1.18	-2.09	-0.34	-0.04	-6.31	5.24
100 mm	124.26	18.80	1.32	-2.11	-0.33	-0.02	-6.34	5.61
50 mm	124.27	18.66	1.34	-2.12	-0.32	-0.01	-6.33	5.76
25 mm	124.27	18.63	1.35	-2.12	-0.32	-0.01	-6.33	5.82
edge location $(u, v) = (0, \pi)$								
200 mm	-1553.23	-254.39	176.30	-16.97	-1.93	0.11	-29.49	4.71
100 mm	-1680.81	-281.00	212.40	-16.65	-1.75	0.51	-35.79	-3.93
50 mm	-1836.26	-315.82	248.41	-15.59	-1.53	0.78	-24.80	-28.81
25 mm	-2025.54	-355.75	286.32	-13.86	-1.27	0.94	39.09	-89.53
edge location $(u, v) = (3, \pi)$								
200 mm	-623.36	-0.77	0.06	-4.42	-0.07	-8.21	-0.51	-1.53
100 mm	-624.47	-0.18	-0.42	-4.37	-0.04	-8.19	-0.51	-1.51
50 mm	-624.75	-0.05	-0.35	-4.34	-0.02	-8.18	-0.51	-1.51
25 mm	-624.82	-0.01	-0.21	-4.33	-0.01	-8.18	-0.51	-1.51
edge location $(u, v) = (6, 0)$								
200 mm	0.07	-56.89	-5.24	-0.04	-17.23	-5.08	-4.11	5.53
100 mm	0.07	-55.85	-5.01	-0.01	-17.23	-5.06	-4.37	5.41
50 mm	0.03	-55.57	-4.99	0.00	-17.24	-5.05	-4.42	5.35
25 mm	0.01	-55.50	-5.01	0.00	-17.24	-5.05	-4.43	5.33

4. Calculation of the orders

According to reference [1] page 78, the order of the error in the displacement can be solved as:

$$b = \log_2\left(\frac{u_2 - u_1}{u_3 - u_2}\right),$$

where u_1, u_2 and u_3 are the displacements for different element sizes. We can use the same equation to calculate the order of errors of table 1. The result can be seen in table 2 , where $\Delta_1 = \frac{n_{100}-n_{200}}{n_{50}-n_{100}}, \Delta_2 = \frac{n_{50}-n_{100}}{n_{20}-n_{50}}, b_1 = \log_2(\Delta_1), b_2 = \log_2(\Delta_2)$. The normal forces *n* in the equations were replaced by moments *m* and by shear forces *v*.

Values of b_i, always are integer, should always be positive as well. To achieve this requirement, values of Δ_1 or/and Δ_2 considered as integer as well. However, A position without value or with zero value, means the values did not change by changing the element size, where values of b_i at those position should be ∞. This happens when the arithmetic accuracy is too small and when the number of displayed digits is too small.

Table 2. Calculation results of order of errors

Calculation	Components of internal forces							
	n_{xx}	n_{yy}	$\frac{1}{2}(n_{xy}+n_{yx})$	m_{xx}	m_{yy}	m_{xy}	v_x	v_y
edge location $(u, v) = (0, 0)$								
Δ_1	8	4	7	2	1	2	3	2
Δ_2	∞	5	2	∞	∞	∞	∞	2
b_1	3	2	3	1	0	1	2	1
b_2	∞	2	1	∞	∞	∞	∞	1
edge location $(u, v) = (0, \pi)$								
Δ_1	1	1	1	0	1	2	1	0
Δ_2	1	1	1	1	1	2	0	0
b_1	0	0	0	∞	0	1	0	∞
b_2	0	0	0	0	0	1	∞	∞
edge location $(u, v) = (3, \pi)$								
Δ_1	4	4	7	2	2	2	∞	∞
Δ_2	4	3	1	3	2	∞	∞	∞
b_1	2	2	3	1	1	1	∞	∞
b_2	2	2	0	2	1	∞	∞	∞
edge location $(u, v) = (6, 0)$								
Δ_1	0	4	12	3	0	2	5	2
Δ_2	2	4	1	∞	∞	∞	5	3
b_1	∞	2	4	2	∞	1	2	1
b_2	1	2	0	∞	∞	∞	2	2

5. Conclusions

A shell structure has been analysed with the software SCIA Engineer. The displacements, membrane forces, moments and shear forces appear to correctly fulfil the applied boundary conditions. The results in four edge points have been recorded for four element sizes.

In location (0, π) the results diverge for smaller elements due to a singularity. The order of the error could not be determined there. An exception is m_{xy} which does converge with an error $O(h)$.

In the other three locations the errors vary between $O(1)$ and $O(h^4)$. This does not agree with reference [1] page 78, where it is stated that the error is $O(h^2)$ for all results with reduced solid element applied. A possible explanation is that small computation inaccuracies have a large influence on the perceived order of an error. More research is required to explain the difference.

It is recommended that in further research larger loads or a larger number of displayed digits are used.

Literature

[1] P.C.J. Hoogenboom, *Notes on shell structures*, Reader Delft University of Technology, online (August 2020): http://homepage.tudelft.nl/p3r3s/b17_schedule.html