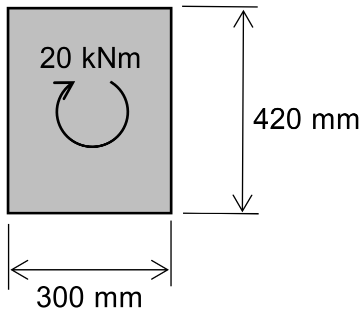
Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIEM5210-3 Advanced mechanics of structural elements
Thursday 25 June 2024, 13:30 – 16:30 hours

Choose A, B, C or D.

1 See figure. What is the largest shear stress? (0.25 point)



A 0.28 N/mm^2
B 2.00 N/mm^2
C 2.30 N/mm^2
D 2.47 N/mm^2

2 Which torsion theory includes a distributed torsion moment load [kNm/m]? (0.25 point)

A Vlasov's
B Saint Venant's
C Circulatory
D Rankine's

3 For which sections is restrained warping a potential problem? (0.25 point)

A Circular tubes
B I sections
C Multi-cell box-girders
D T sections

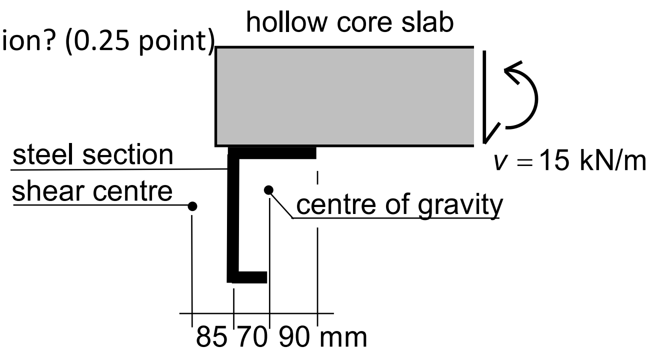
4 What is a ϕ hill? (0.25 point)

A Soap film
B Function of which the slope is the shear stress
C Membrane analogy
D Function of which the volume is the torsion moment

5 Why are offshore platforms made of tubular sections? (0.25 point)

A High out of plane bending resistance against wave impact
B No stresses due to warping restrained
C Easy to fit onto each other
D Minimal painting, easy cleaning and recyclable

6 What is the torsion moment on the steel section? (0.25 point)



A $m_x = 0.15$ kNm/m
B $m_x = 1.05$ kNm/m
C $m_x = -1.28$ kNm/m
D $m_x = -2.48$ kNm/m

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7 How is the thick plate theory different from the thin plate theory? (0.25 point)

A Strain energy is the *x*, *y* and *z* directions
B Developed by Mindlin instead of Reissner
C Plane sections do not remain plane
D Shear deformation is included

8 Why do we need such small elements, if we use the thick plate theory for a thin plate? (0.25 point)

A To compute the deflection accurately
B To compute the edge shear force and edge moments accurately
C To compute the singularities accurately
D To compute the minimally required reinforcement accurately

9 What do we do when there is a torsion moment on a plate edge? (0.25 point)

A Re-compute with more accurate elements
B Check the load. Is this edge torsion moment applied intentionally?
C Interpret as concentrated shear force
D Accept as numerical error

10 A plate corner is pulled down by a support reaction. How large is this force? (Notation as Blaauwendraad's book.) (0.25 point)

A $2m_{xy}$
B $(v_x + v_y)t$
C V
D f_x

11 Where can we expect singularities in plates? (0.25 point)

A In simply supported and fixed edges
B At the end of line supports
C Next to circular holes and openings
D In every corner

12 In a point of a plate the following membrane forces are computed.

n_{xx} = 10 kN/m, n_{yy} = -4 kN/m, n_{xy} = 3 kN/m

Which reinforcement is sufficient and smallest? (0.25 point)

A n_{sx} = 11 kN/m, n_{sy} = 5 kN/m

B n_{sx} = 12 kN/m, n_{sy} = 0 kN/m

C n_{sx} = 12 kN/m, n_{sy} = 1 kN/m

D n_{sx} = 13 kN/m, n_{sy} = 1 kN/m

A frame consists of three members (Fig. 1). The members have a strength M_p. The members are rigidly connected. The left support is fixed. The right-hand support is a hinge. The structure is loaded by line load q evenly distributed per member length (self-weight). The relation of Figure 2 exists between the plastic moment and the plastic normal force.

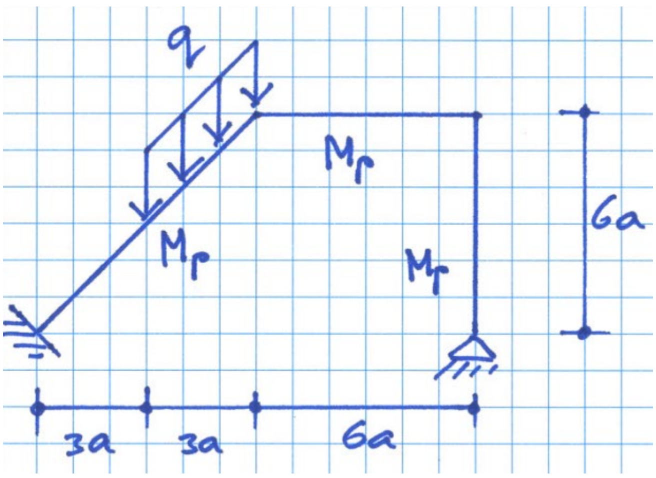


Figure 1. Frame structure

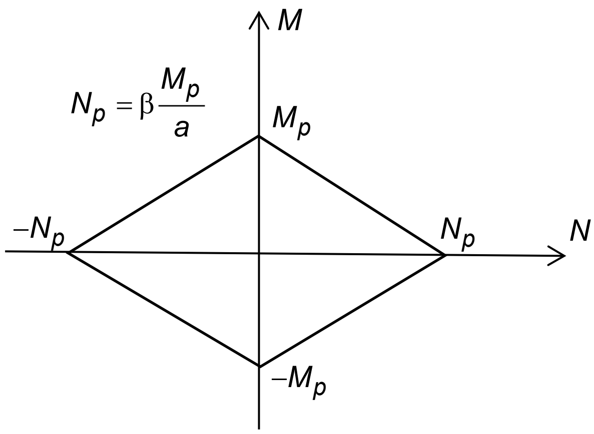


Figure 2. Yield contour

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

13 Assume $\beta \rightarrow \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

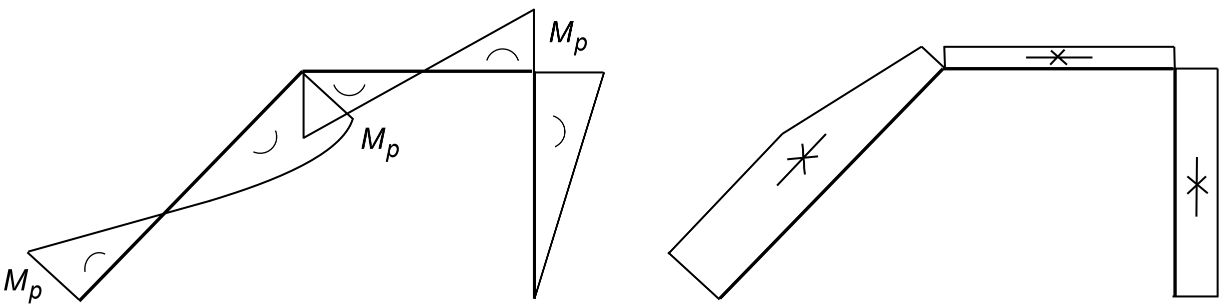


Figure 3. Moments and normal forces of the decisive mechanism

14 Assume β = 6. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

A reinforced concrete plate has simply supported edges and free edges (Fig. 4). It carries an evenly distributed load p [kN/m^2] on part of it surface. There is no other load on the plate. The plate is homogeneous and orthotropic.

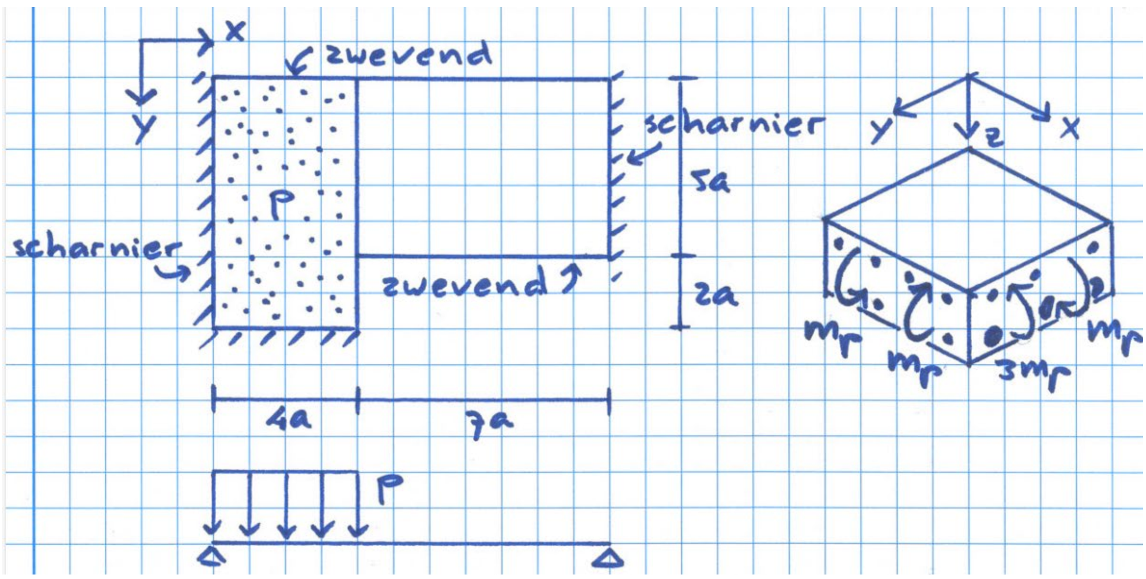


Figure 4. Plate dimensions and reinforcement

15 Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

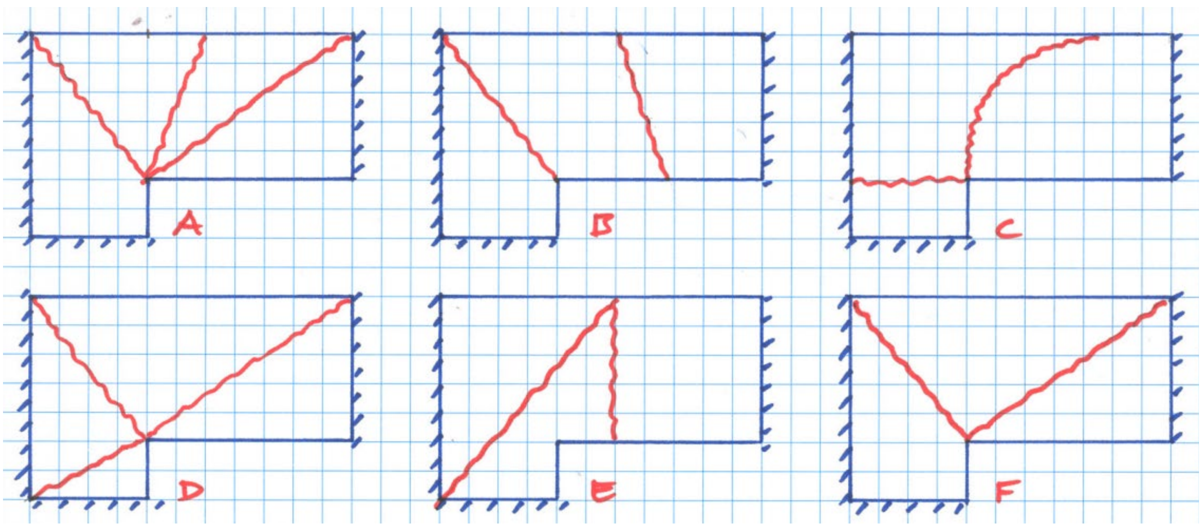


Figure 5. Yield line patterns

16 Consider the mechanism of Figure 6. Determine an upper-bound for p expressed in m_p and a (1.5 point).

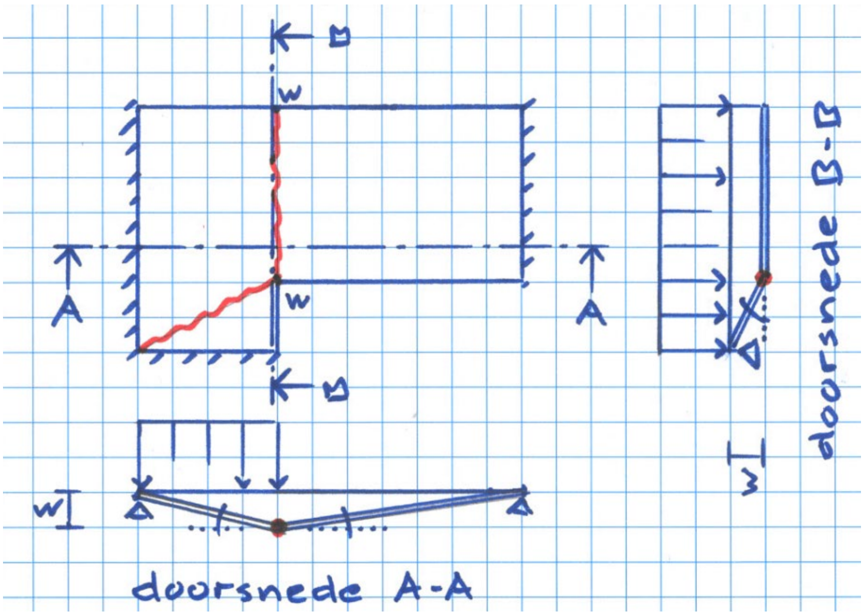


Figure 6. Plastic failure mechanism

17 Determine the largest lower-bound for p using torsion free beams (m_{xy} = 0). You need only to write down the equations and not solve the equations. (1.5 point)

Answers

1	C	71 % of the answers were correct
2	A	86 %
3	B	71 %
4	B	86 %
5	B	100 %
6	C	43 % … The section will deform and the cross-section will rotate around the SC. The concrete plate will be supported by the web. Alternative answer: A conservative choice is that the concrete plate is evenly supported by the top flange. This produces answer D. This answer was a accepted too, provided that the correct calculation was included (29 %).
7	D	100 %
8	B	100 %
9	C	100 %
10	A	100 %
11	B	86 %
12	C	28 % … (12-10)(1+4)>3^2 … A (43 %) is wrong because it requires more steel than C.

13

$E := Mp \cdot t + Mp \cdot (t+t) + Mp \cdot t;$

$$E := 4\,Mp\,t$$

$A := q \cdot 3 \cdot a \cdot \mathrm{sqrt}(2) \cdot \frac{3}{2} \cdot a \cdot t;$

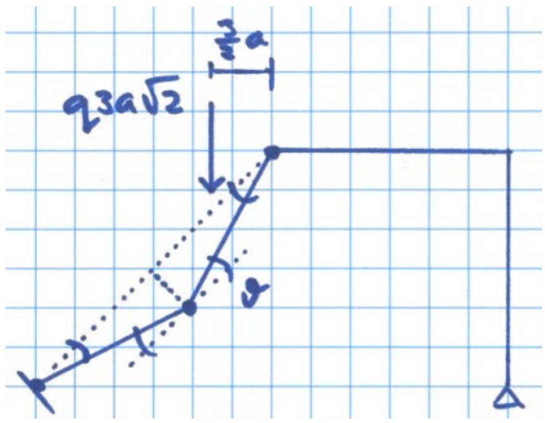
$$A := \frac{9\,q\,a^2\sqrt{2}\,t}{2}$$

$q := solve(E = A, q);$

$$q := \frac{4\,Mp\sqrt{2}}{9\,a^2}$$

$evalf(q);$

$$\frac{0.6285393608\,Mp}{a^2}$$



restart :

$E := Mp \cdot t + Mp \cdot (t+t) + Mp \cdot (t+t);$

$$E := 5\,Mp\,t$$

$A := q \cdot 3 \cdot a \cdot \mathrm{sqrt}(2) \cdot \frac{9}{2} \cdot a \cdot t;$

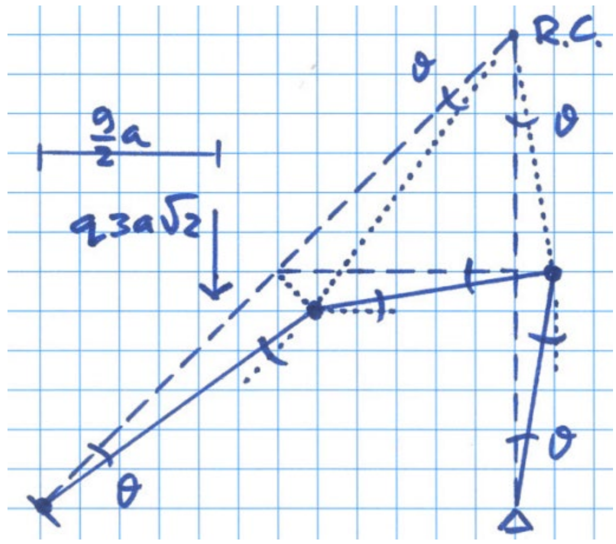
$$A := \frac{27\,q\,a^2\sqrt{2}\,t}{2}$$

$q := solve(E = A, q);$

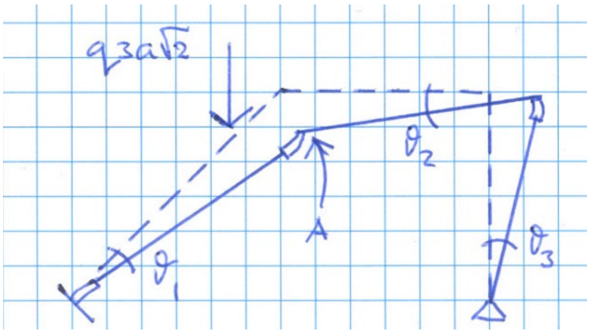
$$q := \frac{5\,Mp\sqrt{2}}{27\,a^2}$$

$evalf(q);$

$$\frac{0.2618914004\,Mp}{a^2}$$



14 Upperbound



$b := 6.:$

$eq1 := t1 \cdot 6 \cdot a - t1 \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} - (t1 + t2) \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} = t3 \cdot 6 \cdot a:$

$eq2 := t1 \cdot 6 \cdot a + t1 \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} + (t1 + t2) \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} = (t2 + t3) \cdot \frac{a}{b} + t2 \cdot 6 \cdot a:$

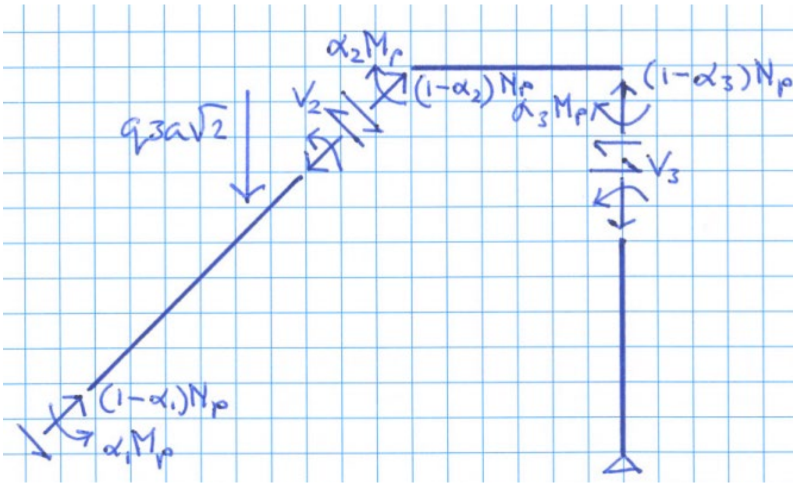
$E := Mp \cdot t1 + Mp \cdot (t1 + t2) + Mp \cdot (t2 + t3):$

$A := q \cdot 3 \cdot a \cdot \mathrm{sqrt}(2) \cdot \left(t1 \cdot \frac{9}{2} \cdot a + t1 \cdot \frac{a}{b} \cdot \frac{1}{\mathrm{sqrt}(2)} \right):$

$solve(\{eq1, eq2, E = A\}, \{t1, t3, q\});$

$$\left\{ q = \frac{0.2527023582\ Mp}{a^2},\ t1 = 0.9950554563\ t2,\ t3 = 0.9363241313\ t2 \right\}$$

14 Lowerbound



$Np := 6. \cdot \frac{Mp}{a}:$

$eq1 := (1 - a1) \cdot Np = q \cdot 3 \cdot a + (1 - a2) \cdot Np:$

$eq2 := a1 \cdot Mp = q \cdot 3 \cdot a \cdot \mathrm{sqrt}(2) \cdot \frac{9}{2} \cdot a - a2 \cdot Mp - V2 \cdot 6 \cdot a \cdot \mathrm{sqrt}(2):$

$eq3 := (1 - a2) \cdot Np \cdot \frac{1}{\mathrm{sqrt}(2)} - V2 \cdot \frac{1}{\mathrm{sqrt}(2)} + (1 - a3) \cdot Np = 0:$

$eq4 := (1 - a2) \cdot Np \cdot \frac{1}{\mathrm{sqrt}(2)} + V2 \cdot \frac{1}{\mathrm{sqrt}(2)} = V3:$

$eq5 := a2 \cdot Mp = (1 - a3) \cdot Np \cdot 6 \cdot a - a3 \cdot Mp:$

$eq6 := a3 \cdot Mp = V3 \cdot 6 \cdot a:$

$solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{V2, V3, a1, a2, a3, q\});$

$$V2 = \frac{0.3430462888\ Mp}{a},\ V3 = \frac{0.1575674082\ Mp}{a},\ a1 = 0.8936842081,\ a2 = 1.020035387,\ a3 = 0.9454044490,\ q = \frac{0.2527023582\ Mp}{a^2} \}$$

There is something wrong in this solution: $a2$ is larger than 1. We have moved out of the yield contour. Apparently, the normal force $(1 - a2)Np$ has become tension instead of compression. Now the solution is

$$Np := 6.\cdot\frac{Mp}{a}:$$

$$eq1 := (1 - a1)\cdot Np = q\cdot 3\cdot \mathrm{a} - (1 - a2)\cdot Np:$$

$$eq2 := a1\cdot Mp = q\cdot 3\cdot a\cdot \mathrm{sqrt}(2)\cdot\frac{9}{2}\cdot a - a2\cdot Mp - V2\cdot 6\cdot a\cdot \mathrm{sqrt}(2):$$

$$eq3 := -(1 - a2)\cdot Np\cdot\frac{1}{\mathrm{sqrt}(2)} - V2\cdot\frac{1}{\mathrm{sqrt}(2)} + (1 - a3)\cdot Np = 0:$$

$$eq4 := -(1 - a2)\cdot Np\cdot\frac{1}{\mathrm{sqrt}(2)} + V2\cdot\frac{1}{\mathrm{sqrt}(2)} = V3:$$

$$eq5 := a2\cdot Mp = (1 - a3)\cdot Np\cdot 6\cdot a - a3\cdot Mp:$$

$$eq6 := a3\cdot Mp = V3\cdot 6\cdot a:$$

$$solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{V2, V3, a1, a2, a3, q\});$$

$$V2 = \frac{0.3386652192\ Mp}{a},\ V3 = \frac{0.1577444301\ Mp}{a},\ a1 = 0.8948836672,\ a2 = 0.9807365155,\ a3 = 0.9464665807,\ q = \frac{0.2487596345\ Mp}{a^2}\}$$

However, the normal force in the horizontal beam was calculated and it is larger than the normal force capacity of $(1\text{-}a2)Np$. Consequently, the middle plastic hinge should not be in the inclined beam but in the horizontal beam. The highest lowerbound is

$$Np := 6.\cdot\frac{Mp}{a}:$$

$$eq1 := (1 - a1)\cdot Np = q\cdot 3\cdot \mathrm{a} - (1 - a2)\cdot Np:$$

$$eq2 := a1\cdot Mp = q\cdot 3\cdot a\cdot \mathrm{sqrt}(2)\cdot\frac{9}{2}\cdot a - a2\cdot \mathrm{Mp} - \mathrm{V2}\cdot 6\cdot \mathrm{a} - (1 - \mathrm{a2})\cdot Np\cdot 6\cdot a:$$

$$eq3 := V2 = (1 - a3)\cdot Np:$$

$$eq4 := (1 - a2)\cdot Np = V3:$$

$$eq5 := a2\cdot Mp = (1 - a3)\cdot Np\cdot 6\cdot a - a3\cdot Mp:$$

$$eq6 := a3\cdot Mp = V3\cdot 6\cdot a:$$

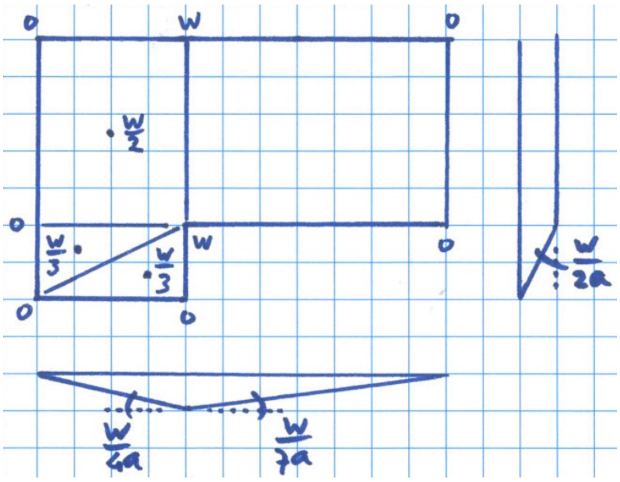
$$solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{V2, V3, a1, a2, a3, q\});$$

$$V2 = \frac{0.3200601052\ Mp}{a},\ V3 = \frac{0.1577761082\ Mp}{a},\ a1 = 0.9020859751,\ a2 = 0.9737039820,\ a3 = 0.9466566491,\ q = \frac{0.2484200859\ Mp}{a^2}\}$$

15 A, B, D

3 right = 0.0 point
4 right = 0.5 point
5 right = 0.8 point
6 right = 1.0 point

16



$$E := 3\cdot mp\cdot 5\,a\cdot\left(\frac{w}{4\cdot a}+\frac{w}{7\cdot a}\right)+3\cdot mp\cdot 2\cdot a\cdot\frac{w}{4\cdot a}+mp\cdot 4\cdot a\cdot\frac{w}{2\cdot a};$$

$$E := \frac{263\,mp\,w}{28}$$

$$A := p\cdot 5\cdot a\cdot 4\cdot a\cdot\frac{w}{2}+p\cdot\frac{4\cdot a\cdot 2\cdot a}{2}\cdot\frac{w}{3}+p\cdot\frac{4\cdot a\cdot 2\cdot a}{2}\cdot\frac{w}{3};$$

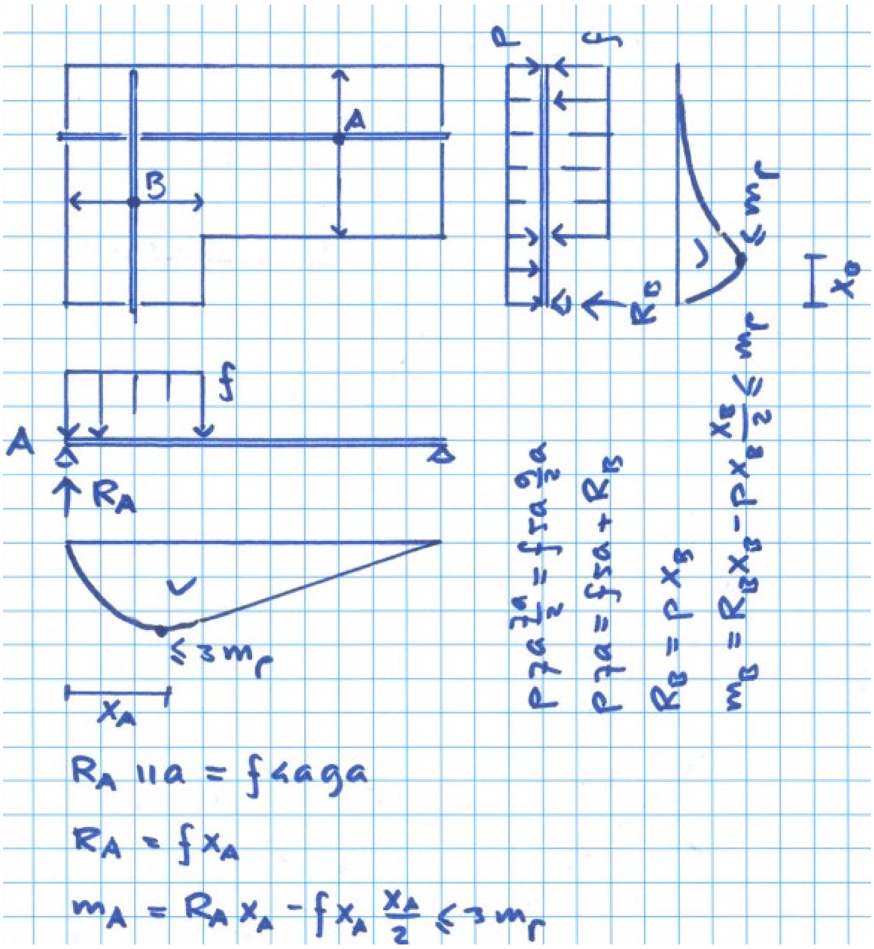
$$A := \frac{38\,p\,a^2\,w}{3}$$

$p := solve(E = A, p);\ evalf(p);$

$$p := \frac{789\,mp}{1064\,a^2}$$

$$\frac{0.7415413534\,mp}{a^2}$$

17



$eq1 := RA \cdot 11 \cdot a = f \cdot 4 \cdot a \cdot 9 \cdot a :$

$eq2 := RA = f \cdot xA :$

$eq3 := RA \cdot xA - f \cdot xA \cdot \frac{xA}{2} = 3 \cdot mp :$

$eq4 := p \cdot 7 \cdot a \cdot \frac{7 \cdot a}{2} = f \cdot 5 \cdot a \cdot \frac{9}{2} \cdot a :$

$eq5 := p \cdot 7 \cdot a = f \cdot 5 \cdot a + RB :$

$eq6 := RB = p \cdot xB :$

$eq7 := RB \cdot xB - p \cdot xB \cdot \frac{xB}{2} < mp :$

$opl := solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{RA, RB, f, p, xA, xB\}); assign(opl) :$

$$opl := \left\{ RA = \frac{11\, mp}{6\, a}, RB = \frac{605\, mp}{756\, a}, f = \frac{121\, mp}{216\, a^2}, p = \frac{605\, mp}{1176\, a^2}, xA = \frac{36\, a}{11}, xB = \frac{14\, a}{9} \right\}$$

$eq7;$

$$\frac{605\, mp}{972} < mp$$

$evalf(p);$

$$\frac{0.5144557823\, mp}{a^2}$$