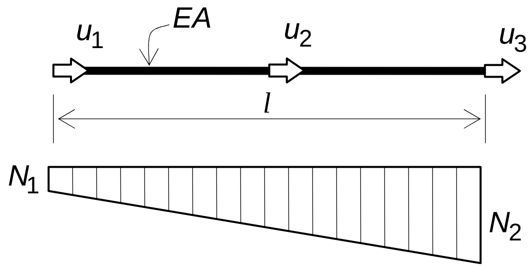
Element and Verification Manual

This report gives descriptions of the elements implemented in the procedure kernel. It also contains 25 tests and verified results that were performed to validate this software.

P.C.J. Hoogenboom, Delft, 26 February 2007

Stringer



The stringer element has three displacement degrees of freedom (dofs). l is the stringer length. EA is the cross-section extensional stiffness, which is constant over the length. The constitutive equation is $N = EA\,\varepsilon$. The normal force diagram is linear. An evenly distributed shear force n acts along the stringer length.

$$n = \frac{N_2 - N_1}{l}$$

The kinematic equations are

$$\varepsilon_1 = \frac{-4u_1 + 6u_2 - 2u_3}{l}$$
$$\varepsilon_2 = \frac{2u_1 - 6u_2 + 4u_3}{l}.$$

The stiffness matrix is

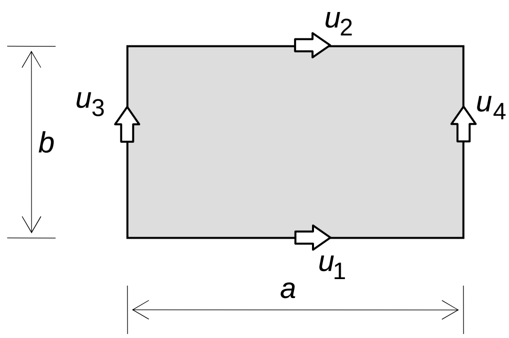
$$K = \frac{EA}{l}\begin{bmatrix} 4 & -6 & 2 \\ -6 & 12 & -6 \\ 2 & -6 & 4 \end{bmatrix}.$$

Rectangular shear panel

The rectangular panel element has four dofs. The panel has dimensions *a* x *b*. The edges are loaded by uniformly distributed shear forces causing a homogeneous shear stress in the panel. The constitutive equation is

$$n = G\,t\,\gamma\,,$$

where G is the material shear modulus, t is the panel thickness and γ is the shear strain.



The kinematic equation is

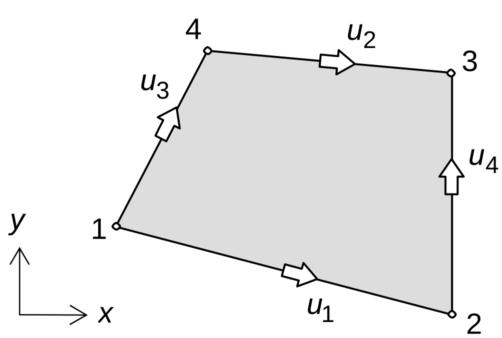
$$\gamma = \frac{u_2 - u_1}{b} + \frac{u_4 - u_3}{a}.$$

The stiffness matrix K is

$$K = Gt\begin{bmatrix} \frac{a}{b} & -\frac{a}{b} & 1 & -1 \\ -\frac{a}{b} & \frac{a}{b} & -1 & 1 \\ 1 & -1 & \frac{b}{a} & -\frac{b}{a} \\ -1 & 1 & -\frac{b}{a} & \frac{b}{a} \end{bmatrix}.$$

A deformed panel does not perfectly connect to a deformed stringer, however, the stresses between the elements are in perfect equilibrium.

Quadrilateral shear panel



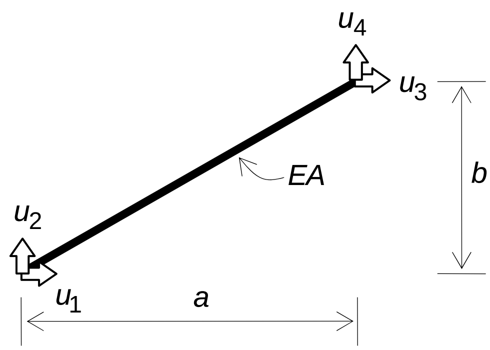
The quadrilateral shear panel element has also four dofs. The dimensions are determined by the coordinates of the vertices. The element behaviour does not depend on the position or direction of the coordinate system. The constitutive equations are

$$n_{xx} = \frac{E\,t}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy})$$
$$n_{yy} = \frac{E\,t}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx})$$
$$n_{xy} = G\,t\,\gamma_{xy}$$

where $\nu = \frac{E}{2G} - 1$. The edges are loaded by uniformly distributed shear forces. In general, the stress field is not homogeneous. The shear stress in the middle of the panel is computed. The element derivation is described in

Hoogenboom PCJ, Blaauwendraad J, "Quadrilateral Shear Panel", Engineering Structures, Elsevier Science Ltd., ISSN 0141-0296, Vol. 22 (2000), No. 12, pp. 1690-1698.
Online: http://www.mechanics.citg.tudelft.nl/~pierre/panelpaper.pdf

Two dimensional bar



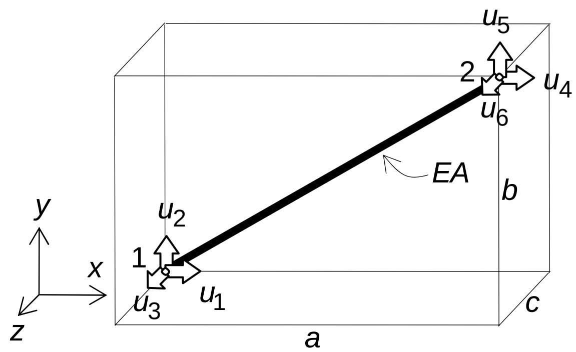
The two dimensional bar element has four dofs. The bar dimensions are *a* and *b*. *EA* is the extensional stiffness of the cross-section, which is constant over the length. The normal force *N* is constant over the bar length. The constitutive equation is $N = EA\,\varepsilon$. The kinematic equation is

$$\varepsilon = \frac{a(u_3 - u_1) + b(u_4 - u_2)}{l^2}.$$

Where *l* is the bar length. The stiffness matrix is

$$K = \frac{EA}{l^3}\begin{bmatrix} a^2 & ab & -a^2 & -ab \\ ab & b^2 & -ab & -b^2 \\ -a^2 & -ab & a^2 & ab \\ -ab & -b^2 & ab & b^2 \end{bmatrix}.$$

Three dimensional bar



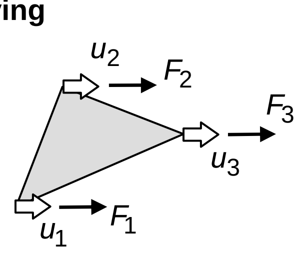
The three dimensional bar element has six dofs. The bar geometry is determined by a, b and c. EA is the extensional stiffness of the cross-section, which is constant over the length. The normal force N is constant over the bar length. The constitutive equation is $N = EA\,\varepsilon$. The kinematic equation is

$$\varepsilon = \frac{a(u_4 - u_1) + b(u_5 - u_2) + c(u_6 - u_3)}{l^2}.$$

Where l is the bar length. The stiffness matrix is

$$K = \frac{EA}{l^3}\begin{bmatrix} a^2 & ab & ac & -a^2 & -ab & -ac \\ ab & b^2 & bc & -ab & -b^2 & -bc \\ ac & bc & c^2 & -ac & -bc & -c^2 \\ -a^2 & -ab & -ac & a^2 & ab & ac \\ -ab & -b^2 & -bc & ab & b^2 & bc \\ -ac & -bc & -c^2 & ac & bc & c^2 \end{bmatrix}.$$

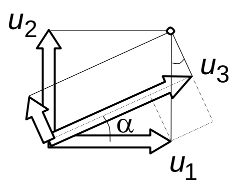
Tying

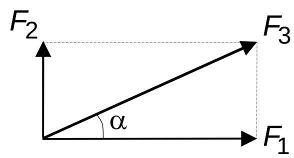


A tying is not an element. A tying couples the displacement of a dof – the slave – to two other dofs – the masters. Also the force at the slave is distributed to the masters.

$$u_3 = \alpha_1 u_1 + \alpha_2 u_2$$
$$F_1 = \alpha_1 F_3$$
$$F_2 = \alpha_2 F_3$$

A master can be used as a slave in another tying provided that this tying is processed after the tying of the just mentioned master. A master can have an imposed displacement. A slave cannot have an imposed displacement.
Consider also the following example.





$$u_3 = \cos\alpha\, u_1 + \sin\alpha\, u_2$$
$$F_1 = \cos\alpha\, F_3$$
$$F_2 = \sin\alpha\, F_3$$

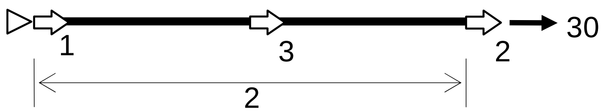
Imposed displacement (Support)

An imposed displacement is a fixed value of a dof. Often the imposed displacement will be zero. In this way the structural model is supported. Slaves cannot have an imposed displacement.

Force (Loading)

At any degree of freedom (dof) a force can be imposed. For every dof at which no force is specified the program assumes a force of zero value. If a force is imposed at a fixed dof, this force is simply added to the reaction force.

Test 1 Stringer

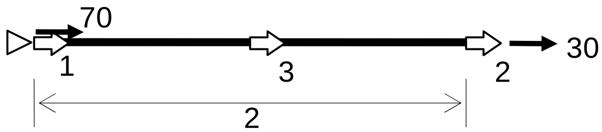


input
$u1$ = 0.050 m
l = 2 m
$F2$ = 30 kN
EA = 2.1e3 kN

output
$N1 = N2$ = 30 kN
$R1$ = -30 kN
$u2$ = 0.06429 m
$u3$ = 0.07857 m

$$u = u_1 + \frac{Fl}{EA} = 0.05 + \frac{30 \times 2}{2.1 \times 10^3} = 0.07857$$

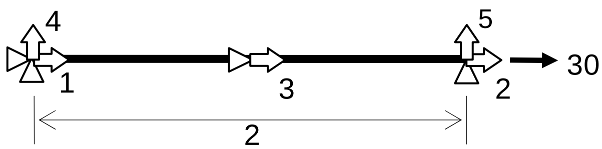
Test 2 Stringer



input
$u1$ = 0.050 m
l = 2 m
F1 = 70 kN
$F2$ = 30 kN
EA = 2.1e3 kN

output
$N1 = N2$ = 30 kN
$R1$ = -100 kN
$u3$ = 0.06429 m
$u2$ = 0.07857 m

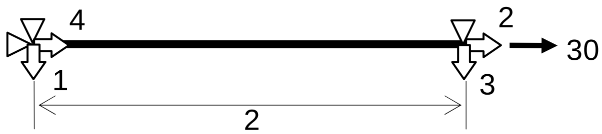
Test 3 2D bar



input
$u1$ = 0.050 m
$u3 = u4 = u5$ = 0 m
a = 2 m
b = 0 m
$F2$ = 30 kN
EA = 2.1e3 kN

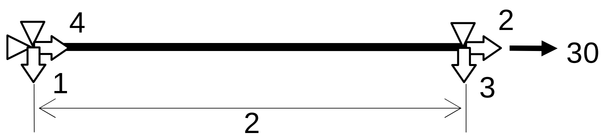
output
$N1 = N2$ = 30 kN
$R1$ = -30 kN
$R3 = R4 = R5$ = 0
$u2$ = 0.07857 m

Test 4 2D bar



input	output
*u*4 = 0.050 m	*N*1 = *N*2 = 30 kN
*u*1 = *u*3 = 0 m	*R*4 = -30 kN
a = 0 m	*R*1 = *R*3 = 0
b = 2 m	*u*2 = 0.07857 m
*F*2 = 30 kN	
EA = 2.1e3 kN	

Test 5 3D bar



input	output
*u*4 = 0.050 m	*N*1 = *N*2 = 30 kN
*u*1 = *u*3 = 0 m	*R*4 = -30 kN
a = 0 m	*R*1 = *R*3 = 0
b = 2 m	*u*2 = 0.07857 m
*F*2 = 30 kN	
EA = 2.1e3 kN	

Test 6 3D bar

Test 7 3D bar

Test 8 3D bar

Test 9

Test 10

Test 11

Test 12

Test 13

Test 14

Test 15

Test 16

Test 17

Test 18

Test 19

Test 20

Test 21

Test 22

Test 23

Test 24

Test 25