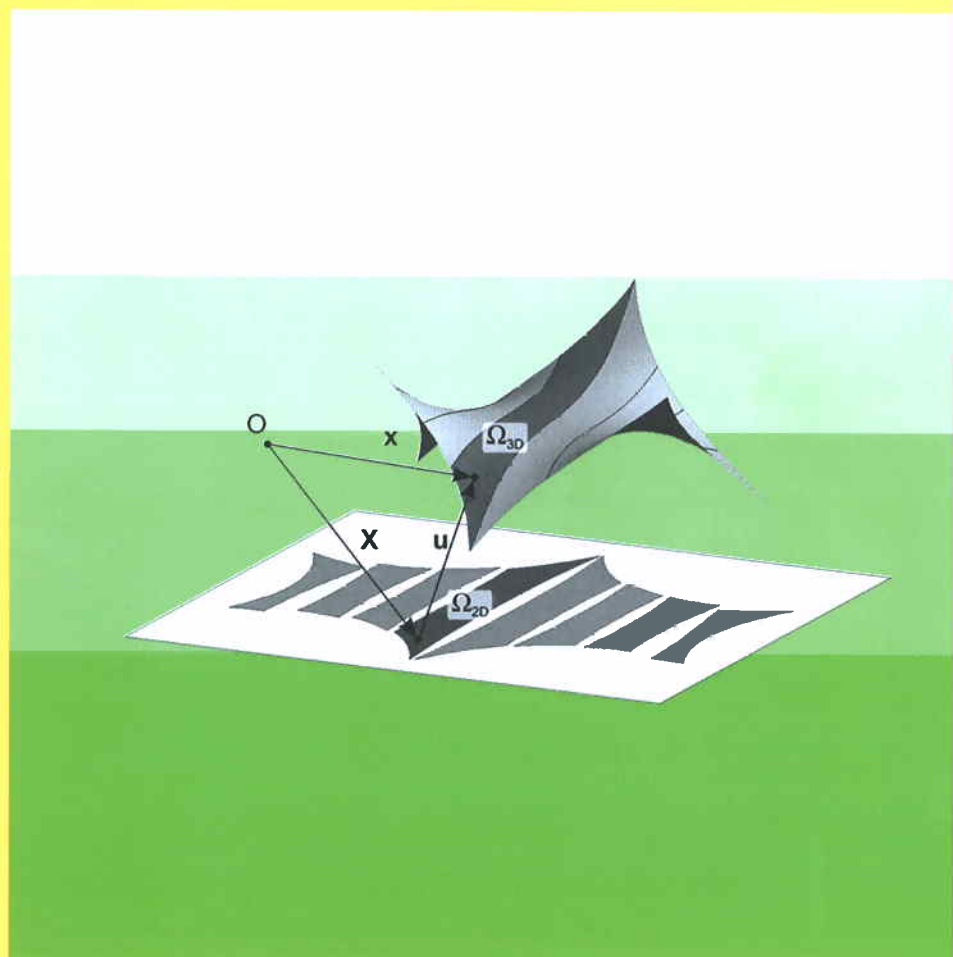




**JOURNAL OF
THE INTERNATIONAL ASSOCIATION
FOR SHELL AND SPATIAL
STRUCTURES**

FORMERLY BULLETIN OF THE INTERNATIONAL ASSOCIATION FOR SHELL AND SPATIAL STRUCTURES

Prof. D. h-C Eng .E. TORROJA, founder



Vol. 50 (2009) No. 1

April n. 160

ISSN: 1028-365X



Journal

VOL. 50 (2009) No. 1
n. 160 April

contents

Technical Papers

- | | |
|--|-----------|
| A Design Tool for Spatial Tree Structures
<i>J. Hunt, W. Haase and W. Sobek</i> | 3 |
| Iterative Geometric Design for Architecture
<i>I. Stotz, G. Gouaty and Y. Weinand</i> | 11 |
| Requirements for Cutting Patterns of Smooth Membrane Structures
<i>P.C.J. Hoogenboom, P. de Vries and R. Houtman</i> | 23 |
| Extended and Integrated Numerical Form Finding and Patterning of Membrane Structures
<i>K-U. Bletzinger, J. Linhard and R. Wüchner</i> | 35 |
| Three Pending Questions in Structural Stability
<i>A. Steinboeck, G. Hoefinger, X. Jia and H. A. Mang</i> | 51 |

COVER: Figure from paper by Bletzinger, Linhard and Wüchner

IASS Secretariat: CEDEX-Laboratorio Central de Estructuras y Materiales
Alfonso XII, 3; 28014 Madrid, Spain
Tel: 34 91 3357409; Fax: 34 91 3357422; iass@cedex.es; <http://www.iass-structures.org>

Printed by SODEGRAF

ISSN:1028-365X

Depósito legal: M. 1444-1960

REQUIREMENTS FOR CUTTING PATTERNS OF SMOOTH MEMBRANE STRUCTURES

P.C.J. HOOGENBOOM¹, P.A. DE VRIES^{1,2}, R. HOUTMAN²

¹ Delft University of Technology, Faculty of Civil Engineering and Geosciences, Delft, the Netherlands

² Tentech B.V. Utrecht, the Netherlands

Editor's Note: Manuscript submitted 22 January 2009; revision received 11 April 2009; accepted 23 April. This paper is open for written discussion, which should be submitted to the IASS Secretariat no later than December 2009.

ABSTRACT

In this paper requirements are derived for the width of fabric parts in tent cutting patterns in case the tent surface needs to be particularly smooth. The result follows from nonlinear elastic analyses of membranes with imposed Gaussian curvatures. It is shown that the maximum width depends on the prestress, the fabric stiffness, the seam stiffness and the curvature of the design surface.

Keywords: Membrane structures, cutting pattern, Gaussian curvature, elasticity theory, large deformations

1. INTRODUCTION

Some architects require very smoothly curved surfaces for their tent structures. To this end, the tent fabric needs to be cut in the right pattern, sewn together and stretched to the designed curvatures. Clearly, in normal loading conditions the fabric should not be floppy or wrinkle. Consequently, there are two requirements, 1) small deviations from the design surface and 2) only tension in the fabric. One of the ways to fulfil these requirements is a cutting pattern with small fabric widths. However, small fabric widths result in many seams and high production costs. Therefore, it is important to know which fabric width is necessary to just fulfil the design requirements.

In this paper the theory of elasticity is applied to structural membranes. It is shown that the membrane stresses depend on Young's modulus of the fabric, the curvatures of the design surface, the shape of the cutting pattern parts, the stiffening effect of the seams and the prestressing of the tent structure. The stresses in the fabric are solved analytically for a circular fabric part and a long rectangular fabric part. These solutions are interpreted to obtain remarkably simple formulas for the required width of the fabric parts of tent cutting patterns.

2. CURVATURES

Consider a particular point on a tent structure (Fig. 1). In this point a local coordinate system is applied with the z axis perpendicular to the surface. The surface curvatures k_x , k_y , k_{xy} are defined as [1]

$$k_x = \frac{\partial^2 z}{\partial x^2} \quad k_y = \frac{\partial^2 z}{\partial y^2} \quad k_{xy} = \frac{\partial^2 z}{\partial x \partial y}. \quad (1)$$

The principal curvatures are calculated with

$$k_1 = \frac{1}{2}(k_x + k_y) + \sqrt{\frac{1}{4}(k_x - k_y)^2 + k_{xy}^2}, \quad (2)$$

$$k_2 = \frac{1}{2}(k_x + k_y) - \sqrt{\frac{1}{4}(k_x - k_y)^2 + k_{xy}^2}.$$

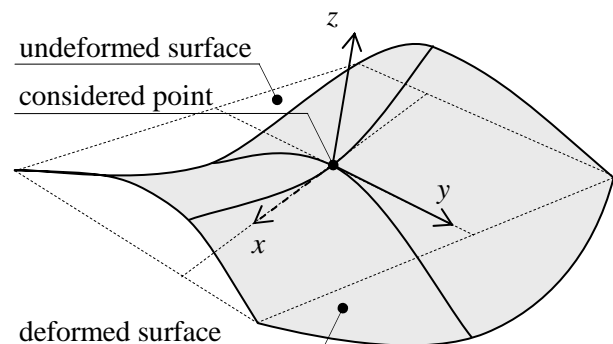


Figure 1. Local coordinate system on a curved surface

The Gaussian curvature is defined as $k_G = k_1 k_2$ which can be evaluated to

$$k_G = k_x k_y - k_{xy}^2. \quad (3)$$

For tent structures the Gaussian curvature has a value smaller than zero. For inflatable structures the Gaussian curvature can be also a value larger than zero.

3. COMPATIBILITY

The strains in an initially flat membrane are [2]

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial z}{\partial x} \right)^2, \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial z}{\partial y} \right)^2, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial z}{\partial x} \frac{\partial z}{\partial y}. \end{aligned} \quad (4)$$

Where u is the displacement of a material point in the x direction, v is the displacement of a material point in the y direction and z is the displacement of a material point in the z direction. Therefore, z is a function of x and y . This formulation includes large deformations. The displacements u , v can be eliminated from these equations which gives the compatibility equation.

$$-\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2 \quad (5)$$

From equations (1) and (3) it follows that the right-hand side of Eq. (5) is equal to the Gaussian curvature, therefore,

$$-\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = k_G. \quad (6)$$

This equation is the bases of this paper. In a flat membrane $k_G = 0$. Equation (6) shows that curvature can accommodate deformations that would be incompatible in a flat membrane.

4. STRESS FUNCTION

The constitutive equations for linear elastic behaviour are [2]

$$\begin{aligned} \varepsilon_{xx} &= \frac{1}{Et} (n_{xx} - \nu n_{yy}), \\ \varepsilon_{yy} &= \frac{1}{Et} (n_{yy} - \nu n_{xx}), \\ \gamma_{xy} &= \frac{2(1+\nu)}{Et} n_{xy}. \end{aligned} \quad (7)$$

Where E is Young's modulus, ν is Poisson's ratio, t is the fabric thickness, n_{xx} , n_{yy} , n_{xy} are cross-section forces per unit length (For example N/m). In this paper we will refer to them as the membrane stresses. Substitution of (7) in (6) gives

$$\begin{aligned} &-\frac{\partial^2 n_{xx}}{\partial y^2} + 2 \frac{\partial^2 n_{xy}}{\partial x \partial y} - \frac{\partial^2 n_{yy}}{\partial x^2} + \\ &+ \nu \left(\frac{\partial^2 n_{xx}}{\partial x^2} + 2 \frac{\partial^2 n_{xy}}{\partial x \partial y} + \frac{\partial^2 n_{yy}}{\partial y^2} \right) = Et k_G. \end{aligned} \quad (8)$$

The membrane equilibrium equations are [2]

$$k_x n_{xx} + 2k_{xy} n_{xy} + k_y n_{yy} + p_z = 0 \quad (9)$$

$$\frac{\partial n_{xx}}{\partial x} + \frac{\partial n_{xy}}{\partial y} = 0 \quad (10)$$

$$\frac{\partial n_{yy}}{\partial y} + \frac{\partial n_{xy}}{\partial x} = 0 \quad (11)$$

where it is assumed that there is only loading p_z perpendicular to the surface. Eq. (10) and (11) are fulfilled by Airy's stress function ϕ [2]. This function is defined such that

$$n_{xx} = t \frac{\partial^2 \phi}{\partial y^2}, \quad n_{yy} = t \frac{\partial^2 \phi}{\partial x^2}, \quad n_{xy} = -t \frac{\partial^2 \phi}{\partial x \partial y}. \quad (12)$$

Substitution of Eqs (12) in (8) gives the differential equation

$$\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = -E k_G. \quad (13)$$

5. CIRCULAR SHEET

Consider a thin flat sheet that is cut in a circular pattern (Fig. 2). The radius is a . The thickness is t . A Gaussian curvature k_G is imposed to the sheet.

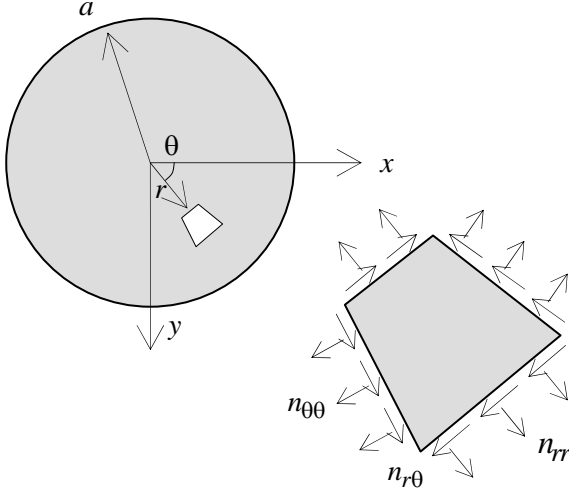


Figure 2. Circular sheet and membrane stresses on an elementary part

The boundary conditions after deformation are zero load imposed onto the edge and equilibrium in any section of the sheet. The solution to differential equation (13) and the boundary conditions is

$$\phi = -\frac{1}{64} k_G E (a^2 - x^2 - y^2)^2. \quad (14)$$

This can be written as

$$\phi = -\frac{1}{64} k_G E (a^2 - r^2)^2 \quad (15)$$

where r is the radial coordinate of the sheet. The membrane stresses are

$$\begin{aligned} n_{rr} &= \frac{t}{r} \frac{\partial \phi}{\partial r} + \frac{t}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = \frac{1}{16} E t k_G (a^2 - r^2), \\ n_{\theta\theta} &= t \frac{\partial^2 \phi}{\partial r^2} = \frac{1}{16} E t k_G (a^2 - 3r^2), \\ n_{r\theta} &= -t \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi}{\partial \theta} \right) = 0. \end{aligned} \quad (16)$$

These stresses are shown in Figure 3. Since $n_{r\theta} = 0$, the stresses n_{rr} and $n_{\theta\theta}$ are also the principal stresses.

The stresses in the middle of the sheet are

$$n_{rr} = n_{\theta\theta} = \frac{1}{16} a^2 E t k_G. \quad (17)$$

The stresses in the edge of the sheet are

$$n_{rr} = 0, \quad n_{\theta\theta} = -\frac{1}{8} a^2 E t k_G. \quad (18)$$

To compensate for compressive stresses in case of a negative Gaussian curvature, the sheet needs to be tensioned to the value $-\frac{1}{16} a^2 E t k_G$ perpendicular to the edge. To compensate for compressive stresses in case of a positive Gaussian curvature, the sheet it needs to be tensioned to the value $\frac{1}{8} a^2 E t k_G$ perpendicular to the edge.

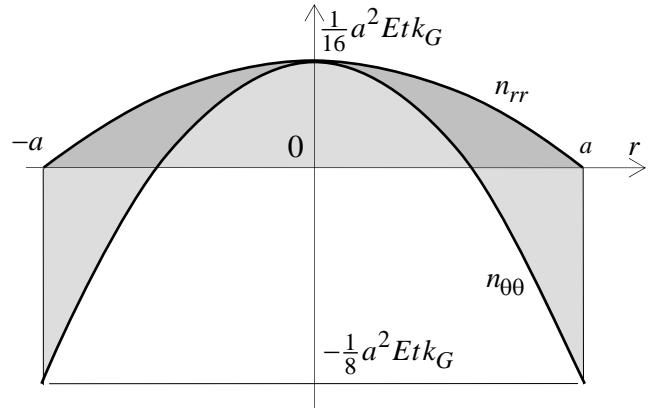


Figure 3. Membrane stresses in an initially flat circular sheet due to an imposed Gaussian curvature

The loading p_z perpendicular to the surface can be found by substituting Eqs (14) in Eqs (12). The result is substituted in Eq. (9). This gives

$$p_z = \frac{1}{8} E t k_G [k_x y^2 - 2k_{xy} xy + k_y x^2 - k_m (a^2 - r^2)],$$

where k_m is the mean curvature $k_m = \frac{1}{2}(k_1 + k_2)$. This loading is necessary to force the sheet into the curved shape. The resultant of the loading is zero.

$$\int_{x=-a}^a \int_{y=-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} p_z dy dx = 0$$

When p_z is removed from the sheet its shape will change again. To prevent a large change the sheet needs to be tensioned perpendicular to the edge.

6. CIRCULAR SHEET WITH AN EDGE RING

Consider a thin flat sheet that is cut in a circular pattern (Fig. 4). The sheet thickness is t . The sheet radius is a . The sheet has an edge ring. The ring cross-section area is A . The ring is made of the same material as the sheet. A Gaussian curvature k_G is imposed to the sheet and the edge ring. The sheet and ring are not loaded externally.

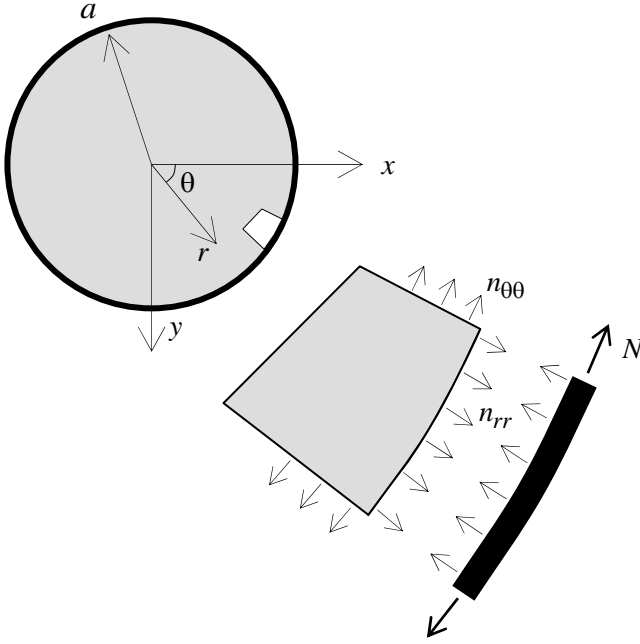


Figure 4. Circular sheet with an edge ring and the interaction between the sheet and the ring

Equilibrium of the deformed ring gives

$$N = -a n_{rr}|_{r=a}. \quad (19)$$

The deformed sheet is attached to the deformed ring. Therefore, at the edge, they have the same strain in the circumferential direction.

$$\varepsilon_{\theta\theta}|_{r=a} = \varepsilon_{ring} \quad (20)$$

The constitutive equation of the ring is $N = EA\varepsilon_{ring}$. The constitutive equation of the sheet

is $\varepsilon_{\theta\theta} = \frac{1}{Et}(n_{\theta\theta} - \nu n_{rr})$. Substitution in Eq. (20) gives

$$n_{\theta\theta}|_{r=a} - \nu n_{rr}|_{r=a} = \frac{Nt}{A}. \quad (21)$$

Substitution of (19) in (21) gives the boundary condition

$$n_{\theta\theta}|_{r=a} = \left(\nu - \frac{at}{A}\right) n_{rr}|_{r=a}. \quad (22)$$

The solution to differential equation (13) and boundary condition (22) is

$$\phi = -\frac{1}{64} k_G E \left[(a^2 - r^2)^2 - \frac{4Aa^2 r^2}{at + A(1-\nu)} \right]. \quad (23)$$

The membrane stresses are

$$\begin{aligned} n_{rr} &= \frac{1}{16} E t k_G \left(a^2 - r^2 + \frac{2a^2 A}{at + A(1-\nu)} \right), \\ n_{\theta\theta} &= \frac{1}{16} E t k_G \left(a^2 - 3r^2 + \frac{2a^2 A}{at + A(1-\nu)} \right), \\ n_{r\theta} &= 0. \end{aligned} \quad (24)$$

These stresses are shown in Figure 5. The stresses in the middle of the sheet ($r = 0$) are

$$\begin{aligned} n_{rr} = n_{\theta\theta} &= \frac{1}{16} a^2 E t k_G \frac{at + A(3-\nu)}{at + A(1-\nu)}, \\ n_{r\theta} &= 0. \end{aligned} \quad (25)$$

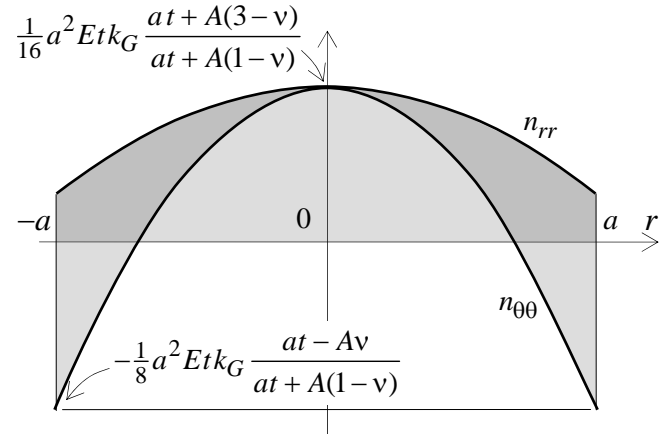


Figure 5. Membrane stresses in an initially flat circular sheet with an edge ring due to an imposed Gaussian curvature

The stresses in the edge of the sheet ($r = a$) are

$$\begin{aligned}
 n_{rr} &= \frac{1}{8}a^2 E t k_G \frac{A}{at + A(1-\nu)}, \\
 n_{\theta\theta} &= -\frac{1}{8}a^2 E t k_G \frac{at - A\nu}{at + A(1-\nu)}, \\
 n_{r\theta} &= 0.
 \end{aligned} \tag{26}$$

7. RECTANGULAR SHEET

Consider a thin flat sheet that is cut in a rectangular pattern (Fig. 6). The sheet width is $2a$. Its length is infinite. The sheet thickness is t . A Gaussian curvature k_G is imposed to the sheet. The sheet edges are not loaded.

The boundary condition on the edges ($y = a, y = -a$) are

$$n_{yy} = 0, \quad n_{xy} = 0. \tag{27}$$

The solution to differential equation (13) and boundary conditions (27) is

$$\phi = -\frac{1}{24}k_G E (a^2 - y^2)^2. \tag{28}$$

The membrane stresses are

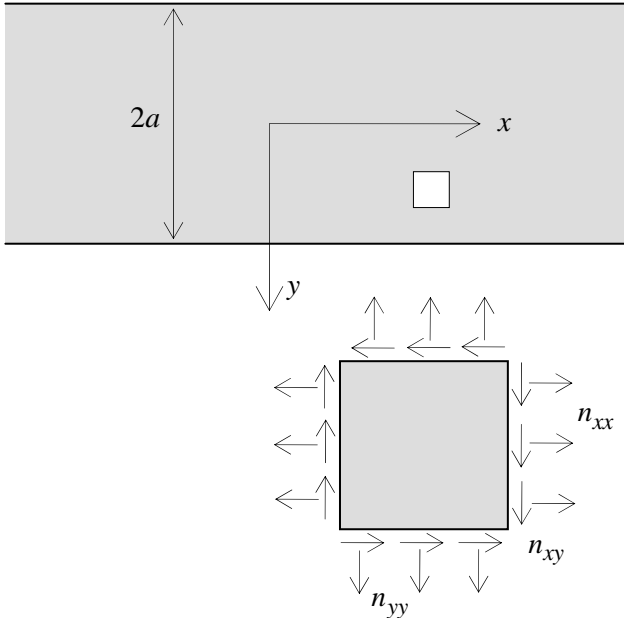


Figure 6. Rectangular sheet and membrane stresses

$$\begin{aligned}
 n_{xx} &= t \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{6} E t k_G (a^2 - 3y^2), \\
 n_{yy} &= t \frac{\partial^2 \phi}{\partial x^2} = 0, \\
 n_{xy} &= -t \frac{\partial^2 \phi}{\partial x \partial y} = 0.
 \end{aligned} \tag{29}$$

These stresses are shown in Figure 7. The stresses in the middle of the sheet ($y = 0$) are

$$n_{xx} = \frac{1}{6}a^2 E t k_G, \quad n_{yy} = 0, \quad n_{xy} = 0. \tag{30}$$

The stresses in the edges of the sheet ($y = a, y = -a$) are

$$n_{xx} = -\frac{1}{3}a^2 E t k_G, \quad n_{yy} = 0, \quad n_{xy} = 0. \tag{31}$$

Surprisingly, n_{yy} is zero everywhere in the sheet.

This result can be tested by pressing a 40 mm wide strip of aluminium foil onto a football. Doing so, wrinkles occur in the y direction along the long edges of the strip. Apparently, stresses would occur in the x direction; compression along the edges and tension in the middle. The pressed strip shows no wrinkle in the x direction. Therefore, there would be no stresses in the y direction. This simple test confirms the analytical result.

The loading p_z perpendicular to the sheet is found by substituting Eqs (29) into Eq. (9).

$$p_z = -\frac{1}{6} E t k_G k_x (a^2 - 3y^2). \tag{32}$$

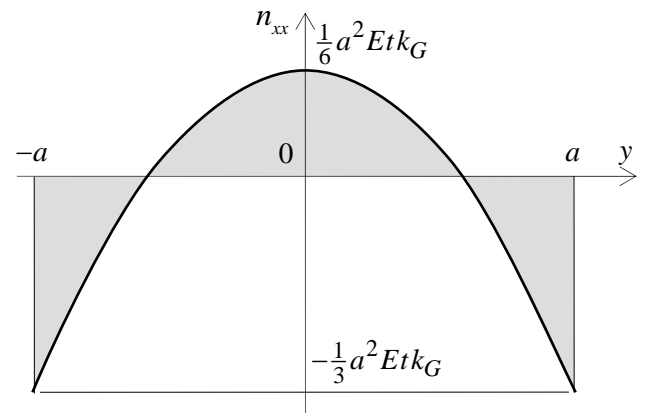


Figure 7. Membrane stresses in an initially flat rectangular sheet due to an imposed Gaussian curvature

Consider again the thin flat sheet of Figure 6. This time a Gaussian curvature is not imposed. Instead, the sheet is tensioned in the y direction and the loaded by $-p_z$. The differential equation of this situation is

$$n_{yy} \frac{d^2 z}{dy^2} = p_z. \quad (33)$$

The boundary conditions at $y = 0$ are

$$z = 0, \quad \frac{dz}{dy} = 0. \quad (34)$$

The solution to this differential equation is

$$z = -\frac{Et k_G k_x}{24 n_{yy}} y^2 (2a^2 - y^2). \quad (35)$$

The largest displacement \hat{z} (in absolute value) occurs at $y = a$ and $y = -a$,

$$\hat{z} = -\frac{Et k_G k_x a^4}{24 n_{yy}}. \quad (36)$$

8. RECTANGULAR SHEET WITH CABLE EDGES

Consider a thin flat sheet that is cut in a rectangular pattern (Fig. 8). The sheet width is $2a$. Its length is infinite. The sheet thickness is t . The sheet has cable edges. The cable cross-section area is A . The cable is made of the same material as the sheet. A Gaussian curvature k_G is imposed to the sheet and the cable. The sheet and cable are not loaded externally.

The sheet and edge cable are connected. Therefore, the strain of the cable is equal to the strain of the sheet. The cable normal force is.

$$N = EA \varepsilon_{xx} \big|_{y=a} \quad (37)$$

Substitution of Eq. (7) into Eq. (37) and using $n_{yy} \big|_{y=a} = 0$ gives

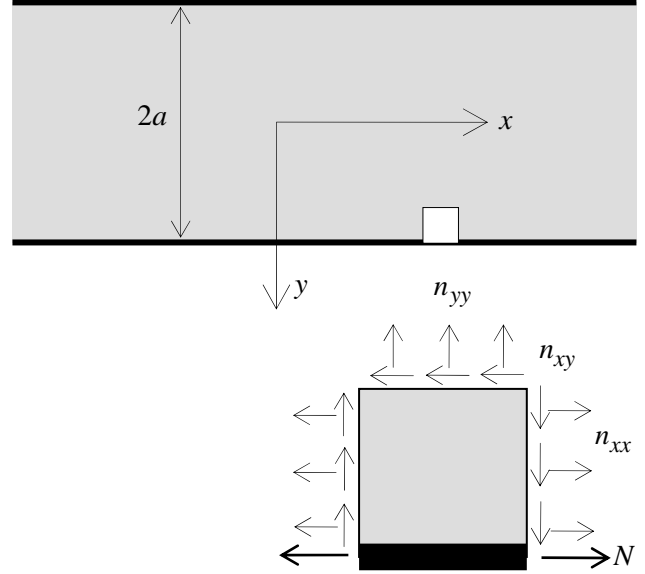


Figure 8. Rectangular sheet with edge cables and membrane stresses on an elementary part

$$\frac{N}{A} = \frac{n_{xx} \big|_{y=a}}{t}. \quad (38)$$

Since there is no edge loading, in a vertical section (x is constant) the section resultant needs to be zero.

$$2N + \int_{-a}^a n_{xx} dy = 0 \quad (39)$$

Substitution of Eq. (38) in Eq. (39) gives

$$2 \frac{A}{t} n_{xx} \big|_{y=a} + \int_{-a}^a n_{xx} dy = 0. \quad (40)$$

The solution to differential equation (13) and boundary condition (40) is

$$\phi = -\frac{1}{24} k_G E [(a^2 - y^2)^2 - \frac{4a^2 y^2 A}{at + A}]. \quad (41)$$

The membrane stresses are

$$\begin{aligned} n_{xx} &= t \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{6} Et k_G (a^2 - 3y^2 + \frac{2a^2 A}{at + A}), \\ n_{yy} &= t \frac{\partial^2 \phi}{\partial x^2} = 0, \\ n_{xy} &= -t \frac{\partial^2 \phi}{\partial x \partial y} = 0. \end{aligned} \quad (42)$$

These stresses are shown in Figure 9. The stresses in the middle of the sheet ($y = 0$) are

$$n_{xx} = \frac{1}{6}a^2 Etk_G \frac{at+3A}{at+A}, \quad (43)$$

$$n_{yy} = n_{xy} = 0.$$

The stresses in the edges of the sheet ($y = a, y = -a$) are

$$n_{xx} = -\frac{1}{3}a^2 Etk_G \frac{at}{at+A}, \quad (44)$$

$$n_{yy} = n_{xy} = 0.$$

The normal force in the edge cables is

$$N = -\frac{1}{3}a^2 Etk_G \frac{aA}{at+A}. \quad (45)$$

The largest displacement due to the loading $-p_z$ on a flat sheet is

$$\hat{z} = -\frac{Etk_G k_x a^4}{24n_{yy}} \frac{at+5A}{at+A} \quad (46)$$

9. DESIGN FORMULAS

The prestressing in a tent structure is computed in a form finding analysis or a finite element analysis. When the form is obtained also the Gaussian curvature can be computed.

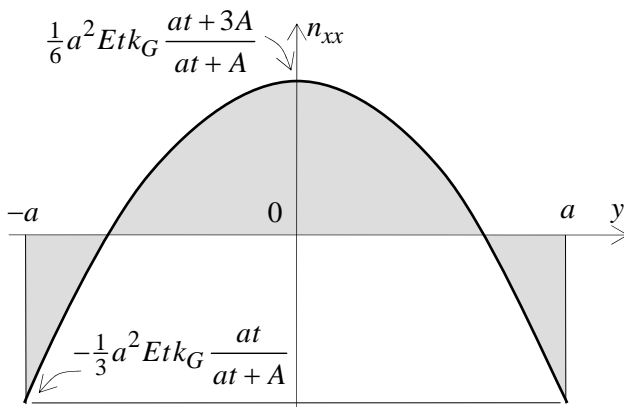


Figure 9. Membrane stresses in an initially flat rectangular sheet with edge cables due to an imposed Gaussian curvature

Therefore, in every point of the design the Gaussian curvature k_G , the principal directions and the principal membrane stresses n_1 and n_2 are known. For designing the cutting pattern the rectangular sheet is considered that is analysed in Section 7. The sheet stresses – Eqs (29) – occur due to the curvature in which it is forced. Onto these stresses the prestressing stresses are superimposed. The resulting stress needs to be larger than zero in order to prevent wrinkles and a floppy fabric.

Suppose that the Gaussian curvature is positive, for example an inflatable fabric structure. Then the largest compressive stress due to pushing the fabric into shape is $n_{xx} = -\frac{1}{3}a^2 Etk_G$ Eq. (31). Wrinkles would occur next to the seams. The prestress n_{px} in the seam direction needs to be at least $n_{px} \geq \frac{1}{3}a^2 Etk_G$ because this way, when we add n_{xx} and n_{px} there are no compressive stresses. The latter equation can be rewritten as

$$2a \leq \sqrt{\frac{12n_{px}}{Etk_G}}.$$

$2a$ is the width of the rectangular sheet. Therefore, this provides the maximum width w_m of the fabric cutting pattern.

$$w_m = \sqrt{\frac{12n_{px}}{Etk_G}} \quad (47)$$

This width is a local value since the curvature k_G and the prestress n_{px} are local values (Fig 10).

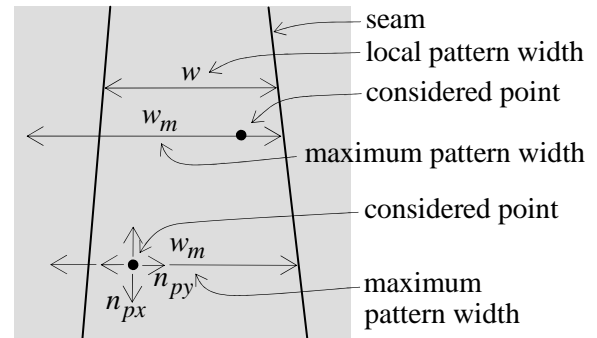


Figure 10. Interpretation of the derived design formula

The seams are stiffer than the fabric and do not deform into the desired shape. Consequently, for positive Gaussian curvatures the seams need to be as small as possible. If possible they should be on the inside of the structure because there they deform less than the fabric.

The sheet solutions are also valid for negative Gaussian curvatures. Then the fabric tends to be floppy in the midst between the seams. For this case Eq. 30 is rewritten to obtain the maximum fabric width.

$$w_m = \sqrt{\frac{-24n_{px}}{Et k_G}} \quad (48)$$

The seams that join the fabric parts have an influence. The membrane stress due to forcing into the Gaussian curvature is $n_{xx} = \frac{1}{6}a^2 E t k_G \frac{at+3A}{at+A}$ (Eq. 42). The average prestress in the fabric direction is n_{px} . Part of this is carried by the seams. The real prestress in the fabric is $n_{px} \frac{at}{at+A}$. To prevent a floppy fabric it needs to be tensioned. The requirement to fulfil is

$$n_{px} \frac{at}{at+A} \geq -\frac{1}{6}a^2 E t k_G \frac{at+3A}{at+A}.$$

This can be rewritten as

$$w_m = \sqrt{\frac{-24n_{px}}{Et k_G} \frac{w_m}{w_m + 6\frac{A}{t}}}. \quad (49)$$

Note that w_m occurs both on the left-hand side and on the right-hand side of the equation. It can be solved iteratively, starting with an estimate of w_m – Eq. (48) – and applying Eq. (49) as many times as needed.

A loading p_z is needed to impose a Gaussian curvature onto the flat sheet. In a tent structure this loading does not occur. When the load is removed the sheet will deform again. This deformation can be calculated with Eq. (36). Suppose that the architect accepts a deviation \hat{z} from the design surface. Eq. (36) can be rewritten to obtain the

maximum fabric width that fulfils this requirement.

$$w_m = \left(\frac{384n_{py} \hat{z}}{Et |k_G k_x|} \right)^{\frac{1}{4}}. \quad (50)$$

This formula is valid for both positive and negative Gaussian curvatures. The influence of the seams can be included by rewriting Eq. (46)

$$w_m = \left(\frac{384n_{py} \hat{z}}{Et |k_G k_x|} \frac{w_m + 2\frac{A}{t}}{w_m + 10\frac{A}{t}} \right)^{\frac{1}{4}} \quad (51)$$

The formulas show that the maximum width w of a fabric part does not depend on Poisson's ratio. Poisson's ratio of fabrics can be very high and depends strongly on the loading state [3]. In fact, many fabrics cannot be modelled as a linear elastic material. Instead an advanced model needs to be used that includes crimp interchange. Fortunately, in the previous analysis Poisson's ratio drops out of the equations and has no influence on the determined maximum fabric width. Therefore, there is no compelling need for using an advanced model. Moreover, the advantage of a higher accuracy would be lost in far more complicated formulas.

10. EXAMPLE

The cutting pattern of a tent needs to be determined such that the fabric will accurately follow the smooth design surface (Fig. 11, 12). A deviation of just 3 mm is acceptable. In a particular point the Gaussian curvature is $k_G = -0.111 \text{ m}^{-2}$. The curvature in the direction of the seams is $k_x = -0.205 \text{ m}^{-1}$. The principal membrane stress in this point is $n_{px} = 1.07 \text{ kN/m}$ in the direction of the seams and $n_{py} = 1.37 \text{ kN/m}$ perpendicular to the seams. The stiffness of the fabric is $Et = 1000 \text{ kN/m}$.

The tension requirement Eq. (48) gives

$$w_m = \sqrt{\frac{-24n_{px}}{Et k_G}} = \sqrt{\frac{-24 \times 1.07}{1000 \times -0.111}} = 0.481 \text{ m}.$$

The seams consist of 4 layers of fabric of 10 mm width (Fig. 13). The area that stiffens the fabric is $A = 3 \times 0.005 \text{ m} \times t = 0.015 t \text{ m}$. Applying Eq. (49)



Figure 11. Canopy at the European patent office in Rijswijk (Tentech)



Figure 12. Detail of the canopy of Figure 11 (Tentech)

we obtain

$$w_m = \sqrt{\frac{-24n_{px}}{Et k_G} \frac{w_m}{w_m + 6 \frac{A}{t}}} = \sqrt{\frac{-24 \times 1.07}{1000 \times -0.111} \frac{0.481}{0.481 + 6 \times 0.015}} = 0.441 \text{ m.}$$

An extra iteration of the latter equation gives $w_m = 0.438 \text{ m}$. The influence of the seams is 9% in this example.

The shape requirement Eq. (50) gives

$$w_m = \left(\frac{384 n_{py} \hat{z}}{Et |k_G k_x|} \right)^{\frac{1}{4}} = \left(\frac{384 \times 1.37 \times 0.003}{1000 \times 0.111 \times 0.205} \right)^{\frac{1}{4}} = 0.513 \text{ m.}$$

Applying Eq. (51) we obtain

$$w_m = \left(\frac{384 n_{py} \hat{z}}{Et |k_G k_x|} \frac{w_m + 2 \frac{A}{t}}{w_m + 10 \frac{A}{t}} \right)^{\frac{1}{4}} = \left(\frac{384 \times 1.37 \times 0.003}{1000 \times 0.111 \times 0.205} \frac{0.513 + 2 \times 0.015}{0.513 + 10 \times 0.015} \right)^{\frac{1}{4}} = 0.488 \text{ m.}$$

An extra iteration of the latter equation gives $w_m = 0.487 \text{ m}$. Consequently, the tension requirement is decisive in this situation.

The calculation was performed for several critical points in the design. The cutting pattern width is smaller or just smaller than required by Eq. 48 and Eq. 50 everywhere. No wrinkles were observed during serviceability loading. No flapping was observed in moderate to strong winds.

11. IMPLICATION

In most tent structures the warp and weft directions of the fabric will be the directions of the principal curvatures. Consider the situation of Figure 14 in which the weft direction has a radius of curvature b and the warp direction has a radius of curvature c . The angle α is the angle of the tangents at the seams.

In this situation $w = \alpha \pi / 180^\circ c$ and $k_G = -1/(bc)$. When these are substituted in Eq. 47 the maximum

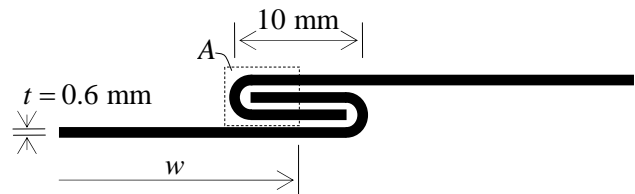


Figure 13. Seam cross-section

angle α_m can be solved

$$\alpha_m = \frac{180^\circ}{\pi} \sqrt{\frac{24n_{px}}{Et} \frac{b}{c}}.$$

For most tent structures the prestress n_{px} varies between 1.0 and 2.0 kN/m. The membrane stiffness Et varies between 500 and 1500 kN/m. The ratio b/c varies between 1/5 and 5. Substitution in the latter equation gives $3^\circ < \alpha_m < 40^\circ$. Consequently, if the angle α is smaller than 3° the cutting pattern fulfils the tension requirement. This provides a quick geometrical check for tent cutting patterns that can be carried out without calculations. However, any angle up to 40° can provide a satisfactory design. Therefore, a 3° limit would be very conservative in general.

12. CONCLUSIONS

An initially flat fabric will follow a Gaussian curvature with a deviation less than \hat{z} if the width of the cutting pattern in a particular point is less than

$$w_m = \left(\frac{384 n_{py} \hat{z}}{Et |k_G k_x|} \right)^{\frac{1}{4}},$$

where n_{py} is the principal membrane prestress perpendicular to the seams (N/m). Et is the fabric stiffness (N/m) k_G is the Gaussian curvature (m^{-2}) and k_x is the curvature in the direction of the seams (m^{-1}).

An initially flat fabric deformed in a positive Gaussian curvature will be tensioned everywhere when the width of the cutting pattern in a particular point is less than

$$w_m = \sqrt{\frac{12 n_{px}}{Et k_G}},$$

where n_{px} is the principal membrane prestress in the direction of the seams. A wider cutting pattern will give wrinkles in the fabric next to the seams.

An initially flat fabric deformed in a negative Gaussian curvature will be tensioned everywhere if the width of the cutting pattern in a particular point is less than

$$w_m = \sqrt{\frac{-24 n_{px}}{Et k_G}}.$$

A wider cutting pattern will result in floppy fabric in the middle between the seams. The influence of the seams can be included in the formulas.

Experiments are planned to further validate the derived design rules.

REFERENCES

- [1] do Carmo M.P., "Differential Geometry of Curves and Surfaces", 1976.
- [2] Timoshenko S.P., Woinowsky-Krieger S., "Theory of Plates and Shells", second edition, 1976.
- [3] Van Asselt P.H., Coenders J.L., Hoogenboom P.C.J., De Vries P.A., Houtman R., "Analysis of stressed membrane structures, Implementation of non-linear material behaviour in structural analysis", *Symposium IASS-SLTE 2008*, Oct. 27-31, Acapulco, Mexico.

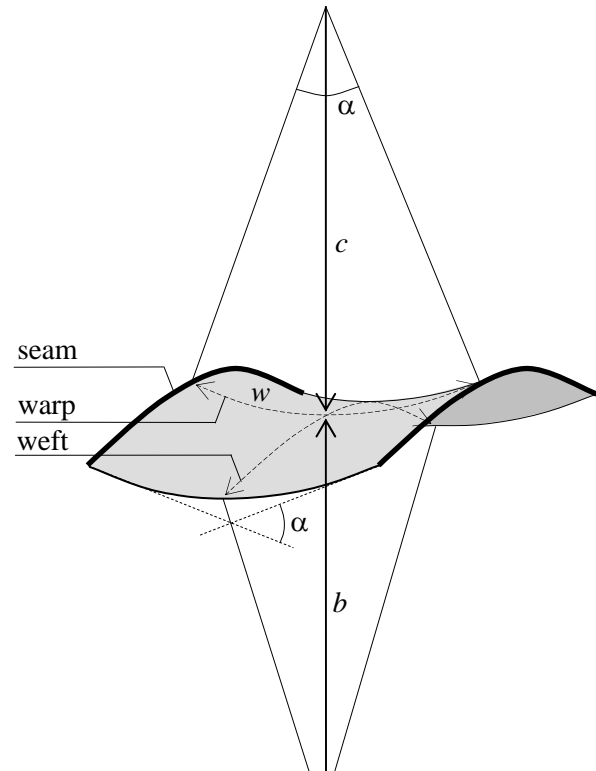


Figure 14. Typical fabric part of tent a tent structure