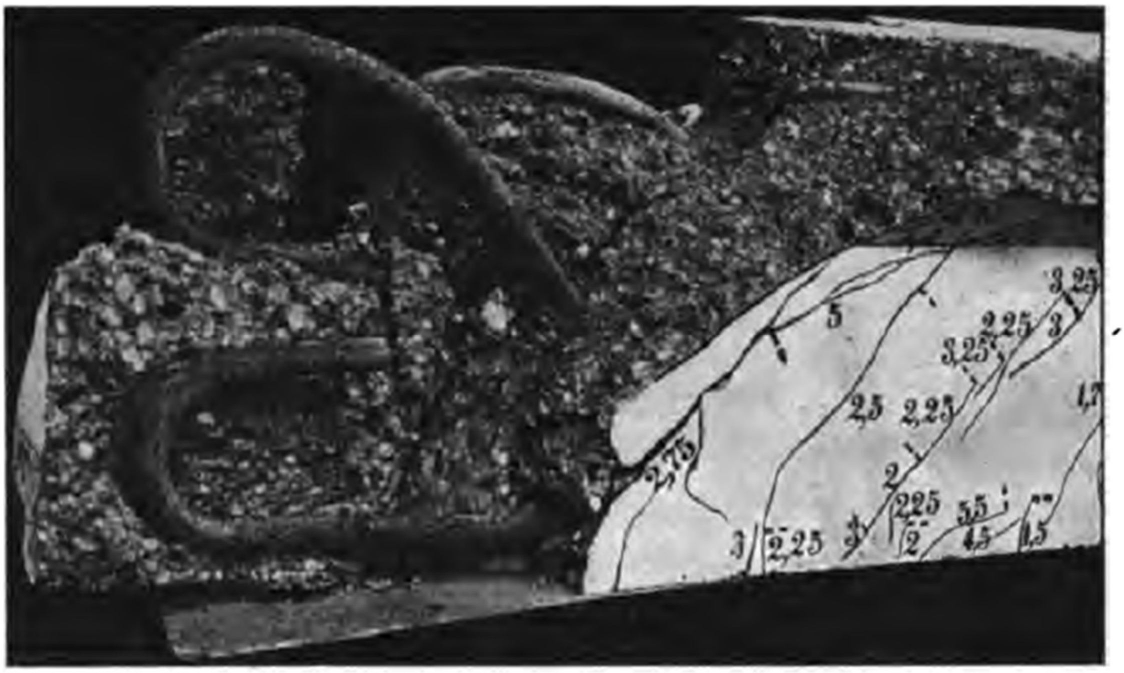
Bent up bars

Assessment and implementation of outdated
reinforcement configurations in existing concrete structures



Timo Douma
August 2023

TU Delft

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$$|\tau_{Ed}| \leq 1/3 * \tau_{Rdc,max}$$

Cover photo from:

E. Probst, “Vorlesungen über Eisenbeton”, Erster band, 1917 (p.353)

Bent up bars

Assessment and implementation of outdated reinforcement configurations in concrete structures

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PREFACE

This thesis is written as part of the Master of Science in Structural Engineering with a specialisation in Structural Mechanics at Delft University of Technology. The research project is conducted at the Dutch contractor Mobilis TBI.

Interests in the subject of outdated reinforcement layouts is raised after the viewing different drawings of concrete structures designed the past. Some layouts of reinforcement bars were not taught during the followed courses and various interesting details were discovered which are rarely applied nowadays. Primarily the detail of bent up bars, which are commonly found in older structures but can be applied in new designs again, stimulates for fascination for the topic of research: the assessment and implementation of outdated reinforcement configurations in concrete structures

Before I invite the reader to explore this report in detail, I would like to thank some special people for their support during the research process.

First of all, I would like to thank Jeroen Meijdam for his endless support and patience during the entire research project. The meetings in office Rotterdam were valuable and highly appreciated. Further, I would like to thank my colleagues at Mobilis for their support during the process. Especially the enthusiasm, feedback and discussions with Thomas van Ingen, Jonathan Siccema and Richard Westerbeek helped me a lot during the research and writing process. For his practical support during the graduation internship, I would like to thank Nick Nass.

I appreciate the efforts of the committee members and for their spend time for meetings and reading of the report.

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Timo Douma
May 2023

ABSTRACT

Introduction
Bent up bars were prescribed as shear reinforcement in the first half of the twentieth century, stirrups after the 1960's. In current Eurocode, bent up bars can be applied again but the restriction $\tau_{Ed} \leq 1/3 * \tau_{Rdc,max}$ should be followed. Goal of this report is to provide background for this shear strength restriction which should lead toward two products, namely (i) a shear strength limitation, and (ii) an assessment method for bent up bars in concrete structure. The corresponding research question is formulated as:

What is the structural limitation of bent up bars as shear reinforcement?

By formulation an answer on this question, the following sub questions are considered:
1. What prefers the application of stirrups or bent up bars as shear reinforcement?
2. What is the capacity of members reinforced with bent up bars to resist shear stresses?
3. How should structures reinforced with bent up bars be assessed?

Method
First part of the report analyses with help of truss models critical design aspects of bent up bars in the transfer of shear stresses. Secondly, shear strengths of reinforcement sections and concrete struts are examined with help of experimental specimen. In last part, the obtained insights are collected and captured into a conceptual model which describes the expected failure mechanism of bent up bars. As result, it is possible to reflect on shear strength limitations and possible assessment methods for the application bent up bars in concrete specimen. A comparison with the shear strength restriction found in the Eurocode closes the report.

Results
Part I reveals that curved sections of bent up bars influences the capacity to transmit shear stresses. While stirrups can embrace longitudinal reinforcement bars, anchoring of inclined sections of bent up bars behind parallel reinforcement bars is impossible. Equilibrium conditions in nodal regions of bent up bars are violated as the strength of struts, ties or nodal region is exceeded.

Part II shows that inclined sections can be excluded as decisive member for the shear strength of bent up bars; largest tensile stresses are found in horizontal reinforcement sections. Comparison of stresses in horizontal sections with observed failure mechanisms shows consequences of premature flexural failure or bond and anchorage failure mechanisms with bent up bars as shear reinforcement. Finally, consequences of the concrete strut supporting in curved sections of bent up bars are considered. Main finding of this examination is that curved sections of bent up bars promotes the formation of cracks in the concrete strut.

Part III focusses on consequences of crack formation on the shear strength of the concrete strut. With help of a conceptual and finite element model it is shown that the concrete strut loses its vertical support with exceeding the strength of an evolving shear friction plane around curved sections of bent up bars. The remaining capacity to transmit shear stresses via the strut depends on the combined resistance of aggregate interlocking and dowel action. By reviewing shear friction tests two design methodologies are discovered. A conservative shear strength limit of $\tau_{Ed} \leq 0.10 f_{ck}$ and a more elaborated approach with a maximum of $0.30 f_{ck}$. Validation of both methods shows that latter method predicts shear strengths of specimen with bent up bars accurately. The conservative lower bound approach is identical to the restrictions found in the Eurocode.

Discussion

The findings implies that any model based on the tensile strength of inclined members is applicable for the analysis of bent up bars as long as it is applied in combination with the restriction $\tau_{Ed} \leq 0.10 f_{cd}$. Also, identical shear stress restriction found in previous building codes shows that the rules prescribed in the current Eurocode makes sense. Finally, assessment of bent up bars requires special attention to: shear *and* flexural reinforcement inclusive designs, cover spalling mechanisms, and detailing of anchorage flexural reinforcement bars.

Conclusion

With the application of bent up bars in concrete structures, cracks are initiated in the supporting concrete strut by curved sections of bent up bars. The remaining shear strength of cracked concrete structures depends on the shear resistance of cracked concrete struts.

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GLOSSARY

Abbreviations:

BC	Bottom chord
BU	Bent up (bar)
STR	Stirrups
STM	Strut-and-tie model
TC	Top chord

Latin symbols:

A_{sl}	Area of longitudinal reinforcement bars	[m2]
A_{sw}, A_{BU}	Area of shear reinforcement/bent up bars in web of specimen	[m2]
a	Shear span	[m1]
B	Bearing load applied to concrete via reinforcement	[N]
b_w	Web width specimen	[m1]
C, C_i	(i) Compressive load (ii) A constant	[N] [-]
d	Effective depth	[m1]
E, E_{cm}	Modulus of elasticity	[Mpa]
$F_n^{(\dots)}$	Axial load in truss members of field n	[N]
f_{ck}	Characteristic compressive strength of concrete	[Mpa]
f_{cm}	Mean characteristic compressive strength of concrete	[Mpa]
f_{ywd}	Yielding limit of reinforcement	[Mpa]
i	Maximum number of fields in the shear span	[-]
k	Number of nodes in a truss model (*Dutch: "knopen"*).	[-]
M_{Ed}	Applied bending moment	[kNm]
M_{Rd}	resisting moment capacity	[kNm]
m	Ration between area of applied bent up bars and flexural reinforcement	[-]
n	Region between two truss nodes in the shear span from support to point of loading	[-]
P, P_{max}	(i) (Maximum) applied point load (ii) Total bearing load	[N] [N]
p	Bearing stress	[Mpa]
r	Number of reaction forces in a truss model (*Dutch: "reactiekrachten"*).	[-]
s, s_n	(i) Spacing between two shear reinforcing systems, (ii) Number of struts in a truss model (*Dutch: "staven"*)	[m1] [-]
T	Tensile load	[N]
V_{Ed}	Shear force	[N]
$V_{Rdc,max}$	Maximum shear strength of concrete only	[N]
z, z_n	Height of truss	[m1]

Greek symbols:

α	Angle of inclined tensile tie, to horizontal	[deg]
θ, θ_n	Angle of compressive strut diagonal, to horizontal	[deg]
μ	Friction factor for roughness (shear) plane	[-]
ρ_{sw}	Shear reinforcement ratio	[-]
ρ_v	Shear reinforcement ratio; crossing a shear plane perpendicular	[-]
σ_c	Stress in concrete	[Mpa]
σ_s	Stress in reinforcement	[Mpa]
σ_{sl}	Stress in flexural reinforcement bar	[Mpa]
τ_{Ed}	Applied shear stresses	[Mpa]
τ_{max}	Maximum applied shear stresses	[Mpa]
$\tau_{Rdc,max}$	Maximum shear strength concrete	[Mpa]
τ_{Rds}	Maximum shear strength reinforcement	[Mpa]
υ'	Strength reduction factor, material effects	[-]
ψ	Angle of inclined top chord truss, to horizontal	[deg]
ω	Shear strength factor	[-]

1 INTRODUCTION

1.1 BACKGROUND

Which layout of shear reinforcement is most suitable to resist shear stresses applied to concrete structures? This question have been subject of debate for a long period. A reflection of this discussion can be found in the changing preferences for bent up bars or stirrups in Dutch building codes.

Shear reinforcement in Dutch building regulations

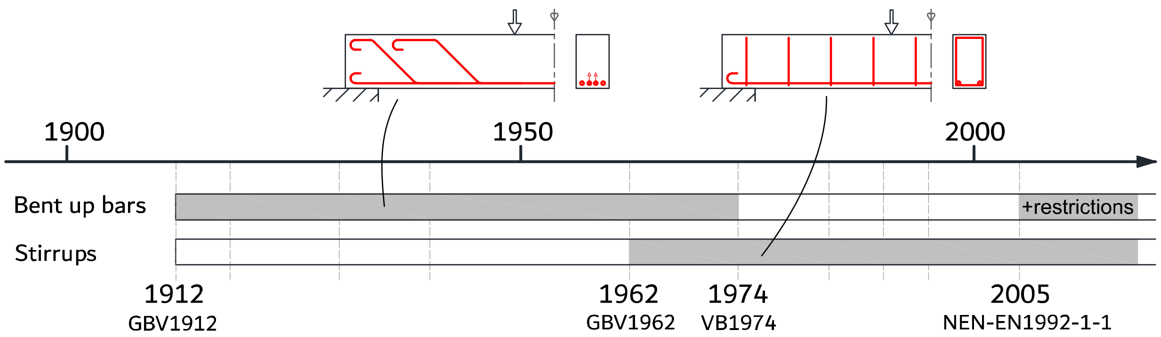
The preference for bent up bars as shear reinforcement is found explicitly in the GBV1918 [1]. In this building code, 50% of the flexural bars required at midspan should bent upwards near supports to serve as shear reinforcement. A less strict value of one-third is mentioned in the GBV1930[2] to the GBV1950[3]. In the GBV1962[4], no specific rules for bent up bars or stirrups as shear reinforcement are given.

Stirrups are prescribed as principle shear reinforcement from the VB1974[5] to the VBC1995[6]. Similar preferences for stirrups can be found in the current Eurocode [7]. Despite, shear reinforcement in slabs may consist of bent up bars only provided that the shear strength is restricted. Latter can be found in Clause 9.3.2 of the Eurocode:

In slabs, if $|V_{Ed}| \leq 1/3 * V_{Rdc,max}$, *the shear reinforcement may consist entirely of bent-up bars (…)* (cl. 9.3.2(3))[7].

Figure 1-1 shows the prescribed application of bent up bars or stirrups as shear reinforcement in Dutch building codes in graphically. Most interesting aspect of this timeline is the reintroduction of bent up bars in the current Eurocode.

Figure 1-1
Timeline with preferences for bent up bars or stirrups as shear reinforcement in Dutch building codes.



Then: rejection of bent up bars

Bent up bars can be found in building codes from the first half of the twentieth century. Arguments for the rejection of bent up bars from building codes after the sixties can be found in literature. For example, Leonhardt [8] allocated the limited shear strength of bent up bars to the development of transverse spalling stresses in curved sections. Park [9] assigned further the inferior performance of bent up bars to the premature reduction of flexural reinforcement bars and lack of cross sectional confinement. Finally, the design procedure of bent up bars is criticized by Sorenson [10] and Ertl [11]. They report that a design procedure with truss models is too conservatism and gives inaccurate results.

Currently: application of bent up bars

Despite these criticisms, bent up bars are in current Eurocode applicable as shear reinforcement again. Apparently, the defects of bent up bars as shear reinforcement are not as critical as supporters of stirrups suggest. It raises the questions: How should bent up bars being applied concrete structures? Which critical details which should be considered during the assessment of existing concrete structures with bent up bars? And, more specific, where does the shear strength restriction for bent up bars in current Eurocode come from?

1.2 Research goal

First goal of this report is to provide background for the shear strength restrictions given to bent up bars in the current Eurocode. This goal is achieved with the presentation of two products, namely (i) a shear strength limitation, and (ii) an assessment method for bent up bars in concrete structure. As second goal, the report searches for an answer on the central research question:

What is the structural limitation of bent up bars as shear reinforcement?

To answer the research question, three sub questions will be followed in the report:

1. What prefers the application of stirrups or bent up bars as shear reinforcement?
2. What is the capacity of members reinforced with bent up bars to resist shear stresses?
3. How should structures reinforced with bent up bars be assessed?

1.3 Research aims

Three aims can be distinguished in the research project:

1. First, the report searches for critical design aspects of bent up bars. A search for regions of bent up bars influencing the capacity to transmit shear stresses sharpens the scope of research.
2. Secondly, the report aims at finding decisive sections for the limited shear strength of bent up bars. By utilization existing experimental result, insights are obtained about effects of bent up bars on the distribution of exerted shear stresses over reinforcement and concrete.
3. Finally, the report is intended to develop a mechanical model for conceptualization of the failure mechanism of bent up bars. The introduced concept can be employed to elaborate on the limited shear strength of bent up bars and the required assessment procedure.

1.4 METHODOLOGY

The report is subdivided into three parts.

1.4.1 Part I: Critical design aspects of bent up bars

Part I of the report searches for critical design aspects of bent up bars which limits the effectiveness as shear reinforcement. Critical sections of bent up bars are analysed qualitatively by examination of the load path in concrete structures. These analyses will reveal defects in the transfer of shear forces applied to concrete structures. Comparison with the load paths of models with vertical stirrups gives insights in effectiveness of both types of shear reinforcement to transfer shear stresses.

The load transfer in concrete structures is evaluated with truss models primarily. Resulting loads in tensile ties and compressive members provides a global overview of the load distribution. For more detailed insights in nodal region of truss models, the strut-and-tie method is employed. With the concept of anchoring tensile members “from behind”, equilibrium conditions in nodal areas can be checked.

First section of Part I reviews the classical truss models proposed by Ritter, Mörsch and Leonhard. A review of principles behind these trusses provides insights in motivations to prefer stirrups or bent up bars as shear reinforcement. Also, two truss models representing stirrups or bent up bars can be selected. Next, the selected truss models are judged for their effectiveness in structural analysis. Outcomes of these analyses are employed in a following section with the comparison of resulting loads in truss members. Finally, the interaction of tensile and compressive loads in nodal regions is examined.

1.4.2 Part II: Shear strength of concrete elements reinforced with bent up bars

Part II assesses the shear strength of specimen reinforced with bent up bars. The capacity to resist shear stresses is examined with help of experimental results. Experimental results of beams reinforced with bent up bars are collected from experiments performed in the past. Outcomes of the analysis are utilized to, for example, (i) examine stress distribution over reinforcement and concrete, and (ii) validate design procures for inclined sections of bent up bars.

The restrictions in the Eurocode are specifically prescribed for the application of bent up bars in slabs. Despite, the dataset in this report encloses simply supported beams reinforced with bent up bars only. Main reason is that almost all shear experiments in the past were performed with simply supported beams. Also, the utilization of experiments with beams instead of plates could reveal disadvantage effects of the application of bent up bars in concrete beams. A reflection on the application of bent up bars in beams or slabs follows in the Discussion section of this report.

Part II starts with an evaluation of the relation between applied shear stresses and developing tensile stresses in inclined sections of bent up bars. Theoretical relations between applied shear stresses and tensile stresses in inclined sections are obtained from three existing design procedures. Validation of these design procedures gives insights in their accuracy to predict the shear strength of specimen reinforced with bent up bars. Secondly, effects of horizontal sections of bent up bars on the shear strength of concrete members are evaluated. After a reflection on specimen enclosed in the dataset, the report treats whether tensile stresses in reinforcement members limits the shear strength of bent up bars. Finally, the shear strength of a concrete strut and the interaction with curved sections of bent up bars is explored with help of stress measurements and experimental photographs.

1.4.3 Part III: Assessment of bent up bars

In Part III the observations of parts I and II are combined to elaborate on (i) assessment procedures for bent up bars, and (ii) corresponding shear strength limitations.

To reach this goal, a conceptual model is introduced first. This models captures the observations from parts I and II of the report for the transfer of shear stresses with bent up bars. As result, the failure mechanism of this model provides background for the maximum capacity of bent up bars to transmit shear stresses. After validation of this failure mechanisms with a finite element model, the report reflects on its consequences for the assessment and maximum shear strength of specimen reinforced with bent up bars.

The effects on the maximum shear strength and assessment method of bent up bars are worked out in more detail in the closure of this part. After a review of existing shear strength experiments, different design methods for the concrete strut are considered. Then, together with the earlier design expressions for inclined reinforcement members, the report validates the proposed methods with concrete specimen in the dataset. Finally, the obtained insights are compared to the shear strength restrictions for bent up bars found in the Eurocode.

2 PART I: CRITICAL DESIGN ASPECTS OF BENT UP BARS

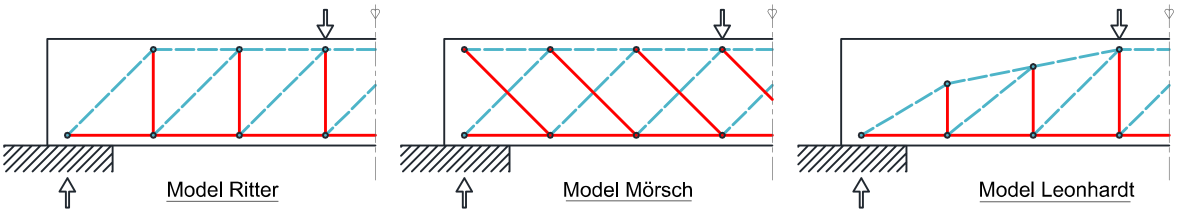
Part I examines critical design aspects of bent up bars which affects the transfer of shear stresses. First, a literature study reviews truss models proposed in the past (2.1). Then, two truss models with vertical stirrups or bent up bars are judged (2.2) for application in structural analysis. Outcomes of this evaluation are employed in member analysis (2.3) and, finally, nodal analysis (2.4) with strut-and-tie models.

2.1 TRUSS MODELS DEVELOPED IN THE PAST

Truss models are traditionally used as representation of the complex stress distribution in concrete structures. Ritter [12] is recognized to be the first who uses a truss to conceptualize the load transfer in concrete structures. With this concept, the resulting compressive stresses in reinforced concrete are compared to the compressive struts of a truss. The function and application of shear reinforcement in concrete structures are considered equivalents to tensile ties truss models [13].

Ritter, Mörsch and Leonhardt have proposed different types of truss model layouts in the past. While the models of Ritter and Leonhardt encloses vertical stirrups, Mörsch' truss model includes bent up bars. A graphical representation of the layouts of tensile ties and compressive truss members can be found in Figure 2-1[1]. Main principles behind these classical truss models are discussed in a separate section.

Figure 2-1
Visualization of the truss models proposed by Ritter, Mörsch and Leonhardt.



2.1.1 Principles behind classical truss models

Ritter (1899), stirrups

The truss model proposed by Ritter has vertical stirrups and inclined compressive struts with an angle of 45 degrees with respect to horizontal. His idea about the function of stirrups is (i) resistance to tensile stresses and (ii) postponing cracking behaviour. Further, Ritter mentioned that struts and ties are most effective in the directions of principle compressive and tensile stress trajectories [12].

Mörsch (1906), bent up bars

A few years later, Mörsch introduces a truss model with diagonal tensile members and diagonal compressive struts. Both truss members are inclined over an angle of 45 degrees to horizontal axis. From theoretical and experimental perspective, Mörsch argued that bent up bars are most efficient as shear reinforcement; the inclined shear cracks at neutral axis level at 45 degrees found in experiments confirms the principle directions according to the theory of elasticity [14].

[1] In this report, tensile truss members are presented by continuous red lines. Compressive truss members are visualized with blue fragmented lines.

Leonhardt (1962), stirrups
In the sixties a new truss model for the transfer of internal stresses was introduced by Leonhardt. The model is similar to Ritter's but includes an inclined top chord. The inclined top chord in Leonhardt' truss model is added due to renewed experimental and scientific insights. Leonhardt reports the beneficial effects of (i) direct load transfer via arching action, and (ii) the contribution of uncracked concrete above the neutral axis to the shear strength of concrete structures. Both effects are included with the inclined top chord [15].

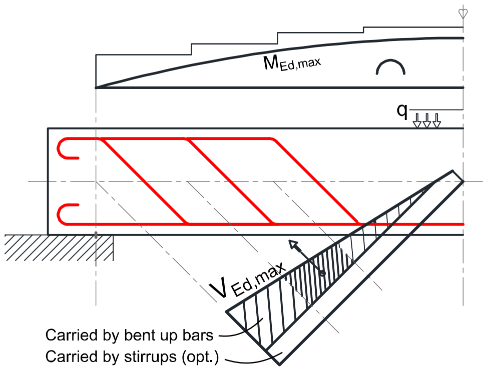
2.1.2 Trusses in shear reinforcement design

Besides conceptualization of the stresses distribution, truss models can also being employed for the design of stirrups.

The VB1974 [5] to VBC1995 [6] bases their shear reinforcement design on truss models with stirrups and horizontal top chord. The shear strength of uncracked concrete is been taken into account via an additional concrete term. Alternatively, the contribution of concrete to the shear strength can be introduced in a truss model with parallel chords and variable concrete strut angles [16][17]. Latter model is found in current Eurocode (Clause 6.2.3(1), fig 6.5)[7] and is shown to be accurate for the application of stirrups (Figure B1 of Annex B).

Design of bent up bars
Bent up bars were designed with help of a graphical method in the past. Principle of this methodology is based on an equal distribution of applied shear forces over a number of bent up bars. An overview of the design procedure is presented in Figure 2-2 based on design recommendations by Mörsch [14]. Interestingly, this design methodology is later criticized by Leonhardt [15] and others [18][10] for its conservatism.

Figure 2-2
Overview of the graphical design methodology for bent up bars.



Note: Adapted from *Concrete-Steel Construction* (figure 172,p.188) by Mörsch, 1909 [14].

2.2 CRITICAL REVIEW OF CLASSICAL TRUSS MODELS

In this section the truss model proposed by Mörsch (bent up bars) and Leonhardt (vertical stirrups) are assessed for application in structural analysis. Leonhardt' truss is selected specifically to asses the effects of the inclined top chord on the calculation procedure. Moreover, the assessment criteria for both truss selected models are primary based on simplification aspects of the calculation process, like required efforts to obtain member forces.

2.2.1 Mörsch’ truss model

The requirements with respect to static equilibrium are considered first. This prerequisite can be validated for truss models from the number of knowns and unknowns with expression Eq. 2-1 [19]. For application to Mörsch’ truss model, the number of nodes k, reaction forces r and truss members s are expressed in terms of a field number n. While the followed process is exemplified in Figure 2-3, the resulting expressions can be found in equation Eq. 2-2.

$$s = 2 * k - \mathrm{r} \qquad \textbf{Eq. 2-1}$$

$$s = 4n$$
$$k = 2n + 2$$
$$r = 3$$

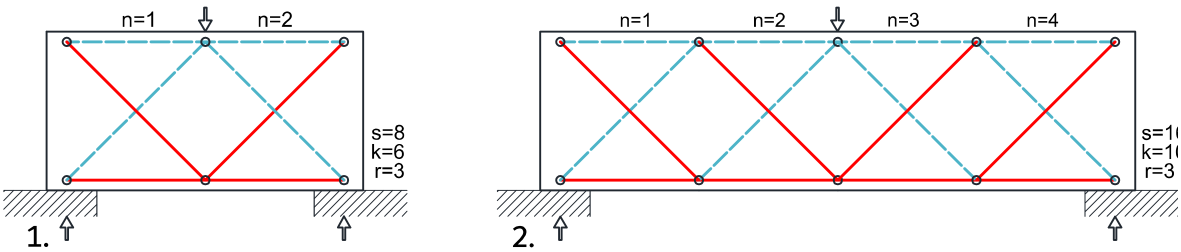
$$(4n) < 2(2n + 2) - 3 = 4n + 1 \qquad \textbf{Eq. 2-2}$$

As result, for any number of fields n, the number of reaction forces r or truss members s is to small for the number of nodes k. Consequently, a layout of truss members according to Mörsch’ model results in the development of a mechanism, see Figure 2-4(1). In order to obtain a static system, two solutions are explored in more detail:

1. An increase of the number of reaction forces r;
2. An increase of the number of truss members s.

Figure 2-3

Two trusses with members arranged according to Mörsch' truss model with different number of fields.



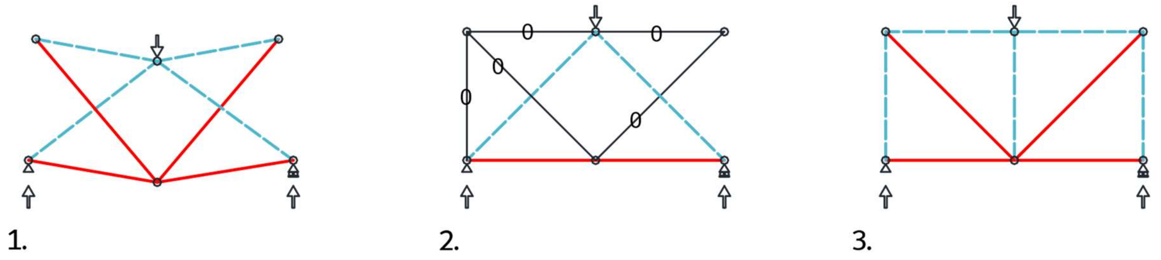
<u>Solution 1 - increase of reaction forces</u>

In expression Eq. 2-2 internally statically determinant trusses are considered. An increase of the number of reaction forces will result in a statically indeterminate model. Consequence of this system is that axial loads in the truss members must be found with additional constitutive and kinematic relations. For relations between loads and displacements in each truss member, geometrical dimensions and material properties are needed.

Cross sectional dimensions and material properties of reinforcing ties can be implemented in the model relative easily. Definition of these parameters for concrete struts is more challenging. First, the dimensions of the struts are unknown and depends on the state of cracking. Secondly, the moduli of elasticity E of concrete is non-linear and has time-dependent characteristics [20]. Moreover, stiffness differences between members of statically indeterminant structures affecting the distribution of loads in truss members directly. The selection of representative values for E-modulus of concrete is therefore critical for the outcomes of the structural analysis.

Unknown cross-sectional areas and the variations in moduli of elasticity of concrete reduces the beneficial effect of model simplification. Further, they restrict the practical application of statically indeterminant trusses in structural analysis. A static determinant truss model is therefore preferred, thus ($r = 3$).

Figure 2-4
Solutions considered for Mörsch' truss model in order to obtain a static system. 1) Mörsch' truss as mechanism, 2) a model with the addition of one vertical member, or 3) a model with inclined and vertical members.



Solution 2 - Additional truss members
A second solution for a static system is the addition of one truss member s. With the addition of one truss member, two variants can be proposed. First model is composed of crossing struts and addition of one vertical truss member, Figure 2-4(2). Another model is formed by reducing the number of inclined truss members and application of vertical members over the entire span, see Figure 2-4(3).

Effects of both variants on the distribution of loads in truss members can be with the help of structural analysis. In model (2), no axial loads can be found in inclined tensile members. Since inclined members in this model represents inclined section of bent up bars, structural comparison of unloaded bent up bars with stirrups is undesirable. In model (3), tensile loads can be found in all inclined tensile members. An arrangement of truss members according to model (3) is therefore preferred above Mörsch' truss model for analysis of the load transfer with bent up bars.

2.2.2 Leonhardt's truss model
For the analysis of Leonhardt's truss model, a truss with vertical tensile members and diagonal compressive struts is defined, see Figure 2-5. In contrast with Mörsch' truss, this model is a static and statically determinant structure ($s = 4n + 1, k = 2n + 2$). Therefore, it is possible to focus on the consequences of the inclined top chord on the structural analysis.

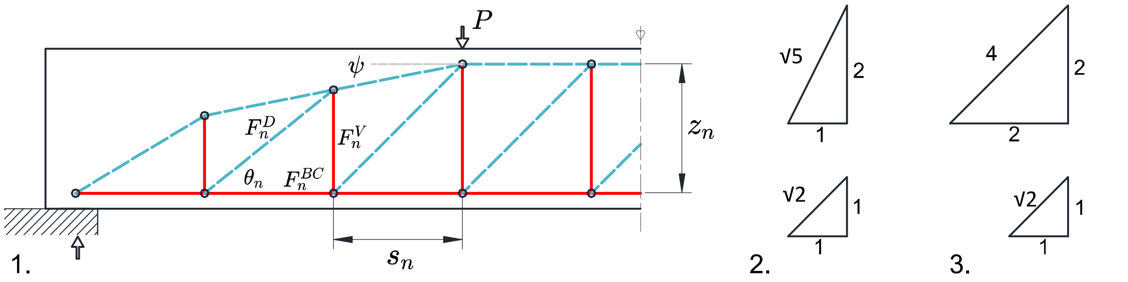
As result of the addition of an inclined top chord, the height of the vertical members vary over the shear span. Given that member heights $z_n(x)$ are known, two different layouts can be employed for analysis (equation Eq. 2-3):

1. An arrangement with constant member spacing;
2. An arrangement with constant compressive strut angle.

$\tan(\theta_n) = \frac{z_n(x)}{s_n} = \frac{F_n{}^V}{F_n{}^{BC}}$	**Eq. 2-3**

Figure 2-5

Truss model employed for the review of Leonhardt' truss layout. 1) Global overview of the considered model, (2) effects of constant member spacing, and (3) effects of constant compressive strut angles on axial loads in truss members.



<u>Arrangement 1 – constant member spacing s_n</u>

In first layout a constant member spacing is considered. Combined with reducing member heights, the inclined top chord will lead to alternating compressive strut angles over the shear span. Structurally, this influences ratios between loads in horizontal, vertical and inclined member over the shear span, see Figure 2-5(2). As result, different member forces can be found throughout the shear span. The application of Arrangement 1 therefore increases calculation efforts to obtain member forces.

<u>Arrangement 2 – constant strut angle θ_n</u>

Second arrangement consists of a constant compressive strut angle. Benefit of this arrangement is a constant ratios between loads in vertical and horizontal members over the shear span. Disadvantage is a deviating load in diagonal member over the shear span, see Figure 2-5(3). Moreover, this arrangement requires a discontinue member spacing s_n which increases calculation efforts to obtain loads in horizontal, and consequently, vertical members. Thus, also Arrangement 2 requires more efforts to find loads in horizontal, vertical and diagonal members.

In conclusion, the inclined top chord of Leonhardt' truss increase calculation efforts and diminishes the benefits of truss models for analyses of internal stresses. On top of that, the shape and position of the top chord, represented by $z_n(x)$, can be point of discussion. The calculation procedure can be simplified by considering a combination of (i) constant compressive strut angle, and (ii) a constant vertical member spacing. Both aspects can be reached simultaneously by application of a horizontal top chord. For structural analysis of vertical stirrups, a truss model with horizontal top chord is preferred, thus $(\psi = 0)$.

2.3 Loads in truss members

Based on observations from previous sections two truss models are derived. The trusses represents beams reinforced with vertical stirrups or bent up bars. Comparison of resulting loads in both trusses reveals benefits or consequences of both types of shear reinforcement on the transfer of applied shear forces.

2.3.1 Member analysis

Model A (stirrups) is a truss model with diagonal compressive struts and vertical tensile members, Model B (bent up bars) a truss with inclined tensile ties and vertical compressive struts, see Figure 2-6. For clearness of the figure, a total number of three fields $(i = 3)$ is drawn.
Resulting loads in truss members located in the shear span are determined first. Generalized expressions for both models can be found in Equations Eq. 2-4 till Eq. 2-11.

Model A

$$F_n{}^{V} = +P \quad \text{Eq. 2-4}$$

$$F_n{}^{D} = -\frac{P}{\sin(\theta)} \quad \text{Eq. 2-5}$$

$$F_n{}^{TC} = -\frac{P*a}{z}*\left(\frac{n-1}{i}\right) \quad \text{Eq. 2-6}$$

$$F_n{}^{BC} = +\frac{P*a}{z}*\left(\frac{n}{i}\right) \quad \text{Eq. 2-7}$$

Model B

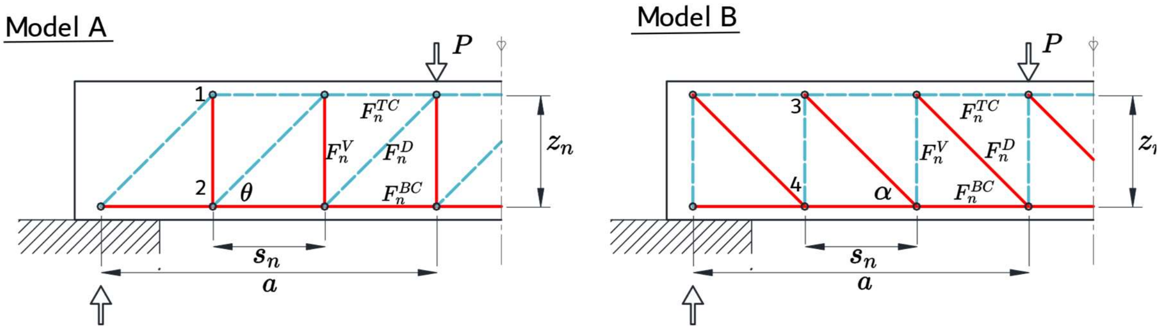
$$F_n{}^{V} = -P \quad \text{Eq. 2-8}$$

$$F_n{}^{D} = +\frac{P}{\sin(\alpha)} \quad \text{Eq. 2-9}$$

$$F_n{}^{TC} = -\frac{P*a}{z}*\left(\frac{n}{i}\right) \quad \text{Eq. 2-10}$$

$$F_n{}^{BC} = +\frac{P*a}{z}*\left(\frac{n-1}{i}\right) \quad \text{Eq. 2-11}$$

Figure 2-6
Truss models with vertical stirrups (model A) or bent up bars (model B) applied in structural analysis.



Comparison loads in vertical and inclined members

Identical loads can be found in diagonal members of trusses A and B ($\alpha = \theta$) except from differences in compression or tension. Similarly, vertical members are, in absolute sense, equally loaded.

In both models largest loads can be observed in the inclined members. In model A, the compressive strut is subjected to largest loads. In model B, largest loads develop in the inclined tensile members (bent up bars). As result, the utilization of the concrete strut in model B (stirrups) is larger than model A for identical shear force P. Similarly, the utilization factor of bent up bars is larger than with the application of vertical tensile members.

Latter suggests that the strength of inclined members of model B (bent up bars) is decisive for the transfer of applied shear force P. The capacities of tensile ties and compressive struts to transmit loads, however, depends largely on the cross sectional areas and strengths of steel and concrete respectively. Although the area of reinforcement bars is relative small compared to the area of the struts, the tensile strength of reinforcement steel is ~15 times larger than the compressive strength of concrete (FE500B,C30). Whether tensile ties or compressive struts are governing for the transfer of shear force P is therefore difficult to determine.

Thus, while differences between resulting loads in vertical stirrups and bent up bars can be observed with these trusses, finding a critical section of bent up bars in the transfer of applied shear force P has been unsuccessful.

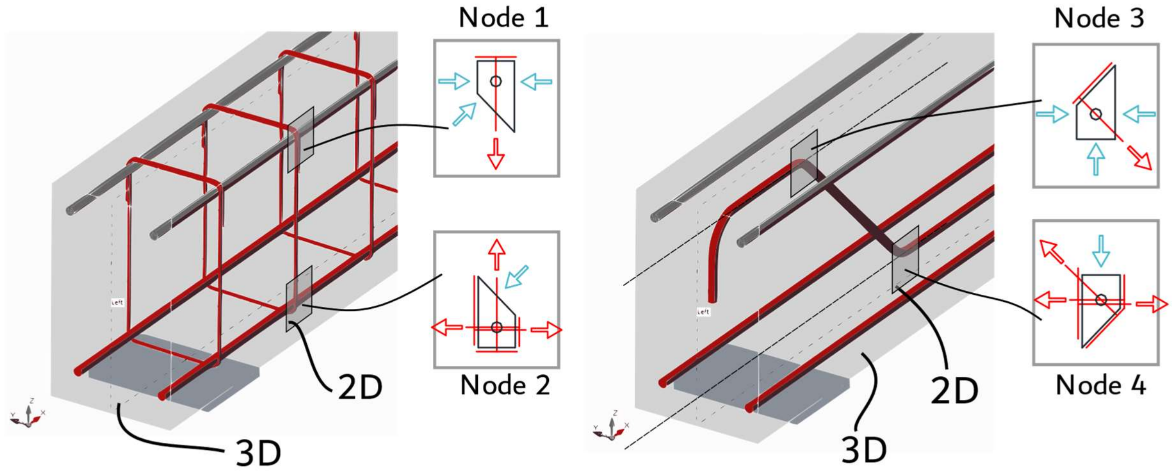
2.4 LOADS ACTING AT NODAL REGIONS

In more detail, the interaction of loads in nodal regions is explored. The equilibrium conditions in nodal regions are examined with help of the strut-and-tie methodology. Special attention is given to the anchorage of tensile members[2] in these regions.

Nodes 1 and 2 are extracted from truss model A (stirrups), nodes 3 and 4 from model B (bent up bars). The positions of these nodes in top and bottom chord are projected on a three-dimensional model in Figure 2-7.

Figure 2-7

Three-dimensional projections of analysed nodal regions on concrete beams reinforced with stirrups or bent up bars.



2.4.1 Nodal analysis – nodes 1 and 2

Nodes 1 and 2 represents nodal regions of truss model A in top and bottom chord respectively. For simplification of analysis, the trapezoidal areas are subdivided into hydrostatic triangular nodes with three loads and prismatic nodes loaded by two horizontal loads as shown in Figure 2-8.

<u>Equilibrium conditions in nodes 1 and 2</u>

The vertical tensile member in the top chord node (node 1) is equilibrated by the vertical component of the diagonal strut. With this compressive counterpart, anchorage of the tensile member “from behind” is established and prevents ripping apart of the nodal region.

In practice, top chords of reinforced concrete beams contains longitudinal reinforcement. These reinforcement bars must be applied to compensate for, for example, moments due to partial fixity, torsional effects and improvement of cross-sectional rigidity [7][3]. Additional benefit of these longitudinal reinforcement bars is the possibility of physical anchorage of vertical tensile members. With application of hooks to the vertical tensile members and embracing the top reinforcement, loads can be transferred in this nodal region even after failure of the compressive strut.

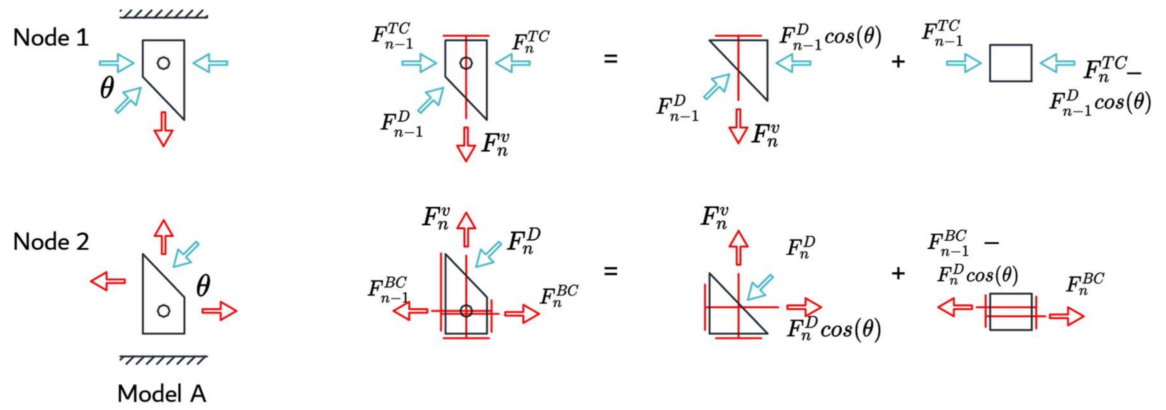
Vertical equilibrium in node 2 is established similarly by the vertical tensile members and vertical components of the compressive strut. Also here, hooks can be added to these vertical ties and positioned around flexural reinforcement bars in the bottom chord to reduce premature loss of equilibrium with failure of the diagonal strut.

[2] Tensile loads acting on nodal regions should exert a compressive force to that region (p.105)[54]. The concept is in this report visualized with fictious anchorage plates.

Thus, with the application of hooks around top and bottom chord reinforcement a secondary load path is available in nodes 1 and 2 after exceeding strength of the concrete strut. As result, the development of more ductile failure mechanisms is promoted.

The two horizontal tensile loads in node 2 are introduced via flexural reinforcement bars and balances each other.

Figure 2-8
Nodal analysis of Nodes 1 and 2 (truss model A).



2.4.2 Nodal analysis – nodes 3 and 4

Nodal regions of model B (bent up bars) are represented by nodes 3 and 4 respectively, see Figure 2-9.

<u>Equilibrium in nodes 3 and 4</u>

Bent up bars originates from flexural reinforcement bars. While part of the bars remain straight up to the supports (ongoing reinforcement bars), part of the flexural bars is bend upwards to serve as shear reinforcement (inclined reinforcement bar).

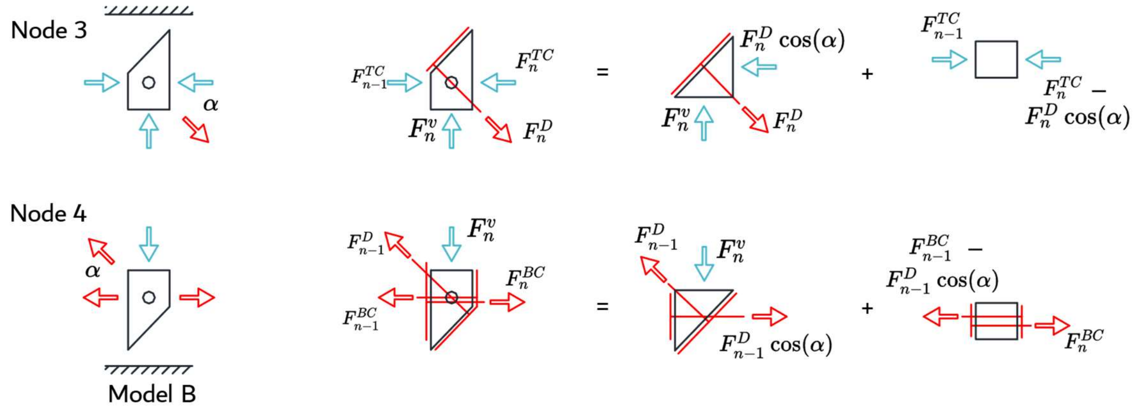
This characteristic of bent up bars results in a significant difference in the load transfer compared to stirrups. While hooks on stirrups can embrace top and bottom chord reinforcement bars, with bent up bars the physically anchorage of diagonal tensile members behind parallel reinforcement bars is impossible. As result, the transfer of compressive and tensile loads in nodal regions of bent up bars is based on equilibrium only.

Thus, equilibrium conditions should be respected to avoid failure of the nodal regions. In nodes 3 and 4, horizontal equilibrium is established by balancing tensile loads in inclined and horizontal sections of bent up bars and. Vertical equilibrium results from loads in the compressive strut and the vertical component of the inclined member.

Both equilibrium conditions are respected as long as the compressive strength of concrete strut, the yielding strength of tensile ties, or the effective strength of the nodal region are not exceeded. Thus, the maximum load transmitted via inclined members of bent up bars is limited by (i) the strength of compressive strut, (ii) the tensile strength of adjacent reinforcement sections, or (iii) the strength of the nodal regions.

Figure 2-9

Nodal analysis of Nodes 3 and 4 (truss model B).



2.5 FINDINGS

After a selection and judgement of suitable truss models, member analysis and nodal analysis, the following insights related to critical design aspects of bent up bars are obtained:

- The preferences for bent up bars in early reinforced concrete design were primarily based on visual crack observations and agreements with trajectories of principle tensile stresses according to linear elasticity;

- While physically anchorage of the tensile members around reinforcement bars in fictious top and bottom chord is possible for stirrups with hooks, this detail is not applicable for bent up bars.

- The load transfer via inclined members of bent up bars is limited by (i) the strength of compressive strut, or (ii) the tensile strength of adjacent reinforcement bars, or (iii) the strength of the nodal region;

- Critical regions of bent up bars for the transfer of shear forces are situated the curved sections of bent up bars.

3 PART II: SHEAR STRENGTH OF BENT UP BARS

This part treats the shear strengths of reinforcing sections of bent up bars and concrete struts. The capacity of the individual members to transmit shear stresses is evaluated with help of experimental results. An overview of the collected experiments is included in Annex A. By analysing the distribution of stresses over the bent up bar and concrete strut insights are obtained about decisive sections for design.

The shear strengths of inclined (3.1) and horizontal (3.2) reinforcement sections of bent up bars are treated first. Then, the strength of concrete struts and the interaction with curved sections of bent up bars (3.3) is considered.

3.1 SHEAR STRENGTH OF INCLINED SECTIONS BENT UP BARS

In Part I relations between applied shear forces and tensile loads in inclined sections have been derived with truss models. Here, the relation between exerted shear stresses and tensile stresses in inclined section of bent up bars is examined in with help of existing design procedures. After reviewing and validation of three design procedures, the developed tensile stresses in inclined sections are inspected with stress measurements.

3.1.1 Three design procedures

Three methods for the design of inclined members of bent up bars are selected.

First method selected is the graphical method. This method is used for the design of bent up bars in the early twentieth century and was introduced in Figure 2-2. Corresponding relation between applied shear stresses τ_{Ed} and shear strength $\tau_{Rd,s}$ in inclined sections of bent up bars is shown in Equation Eq. 3-1. A second approach is found in a report published by Sorenson (eq.8)[10] and presented in this report by Equation Eq. 3-2. Finally, the third method can be found in current Eurocode (eq.(6.13), eq.(9.4))[7]. Equilibrium of a truss node with variable angles α and θ result in Equations Eq. 3-3. In Annex B a more detailed derivation of latter expression can be found.

Equation Eq. 3-4 represents the shear reinforcement ratio ρ_{sw} independent of shear reinforcement angle α and is identical to (eq.9.14) found in the Eurocode [7]. The independency with angle α results to an additional sine-term expression in Eq. 3-3.

$$\tau_{Rd,s} \geq \tau_{Ed}$$

$$\tau_{Rd,s} = \frac{f_{ywd} * A_{sw}}{b_w * z} * \frac{1}{\sqrt{2}} \quad \textbf{Eq. 3-1}$$

$$\tau_{Rd,s} = \frac{f_{ywd} * A_{sw}}{b_w * s} = f_{ywd} * \rho_{sw} * \sin(\alpha) \quad \textbf{Eq. 3-2}$$

$$\tau_{Rd,s} = f_{ywd} * \rho_{sw} * \sin^2(\alpha)\,[\cot(\alpha) + \cot(\theta)] \quad \textbf{Eq. 3-3}$$

$Where$:

$$\rho_{sw} = \frac{A_{sw}}{b_w * s * \sin(\alpha)} \quad \textbf{Eq. 3-4}$$

Comparison of design procedures

The graphical method gives most conservative designs compared to Sorenson' method. First method requires 41% more shear reinforcement area A_{sw} than application of Sorenson' expression for equal shear stresses τ_{Ed} $(\alpha = 45°, z/s = 1.00)$.

Difference between the expressions proposed by Sorensen and the truss model approach is the dependency of compressive strut angle θ and shear inclination angle α. Sorenson mentioned in his report that the shear strength is independent of compressive strut angle θ and shear inclination angle α. Main reason for this statement is the notion that loads in horizontal sections of bent up bars are greater than loads in inclined members [10].

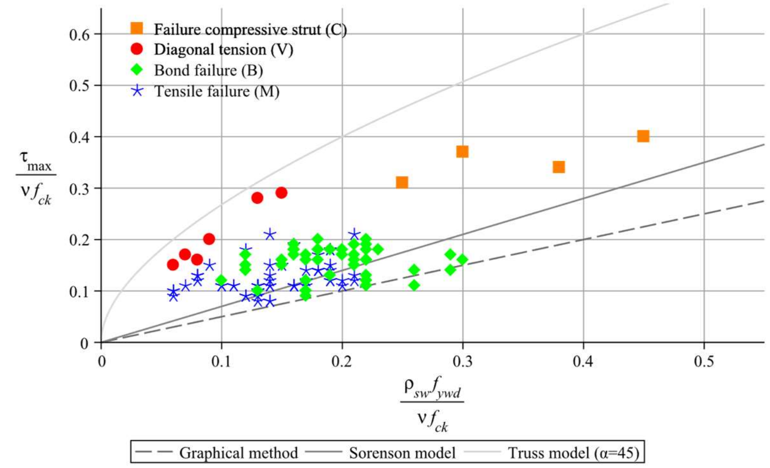
Sorenson also reports that the design of bent up bars with traditional truss models *"results in an … overestimation of the contribution of the bent up bars to the shear strength"* (p. 120)[10]. Similar conclusions are drawn by others [15][18][21][22]. The inaccuracy of the truss model design procedure for inclined sections $(\alpha = 45°)$ is confirmed with the collection of experiments in Figure 3-1.

In latter figure, the functions are presented by expressions Eq. 3-1 to Eq. 3-3. The data points in the figure are calculated from experimental outcomes and layouts taken from a set of experiments (see Annex A). Factor ν' is determined with the expression (eq.(6.6N)) in the Eurocode [7], introduced in this report as equation Eq. 3-5.

$$\nu' = 1 - \frac{f_{ck}}{250} \qquad \textbf{Eq. 3-5}$$

Figure 3-1

Experimental validation of three design expressions for inclined sections of bent up bars.



Note: Data points in the graph are calculated from experimental results and specimen layout. The data is taken from Bach [23](outcomes from [24]), Saliger[25][26], Richart[21] and Shoukry et al. [27].

3.1.2 Stresses around curved section of bent up bars

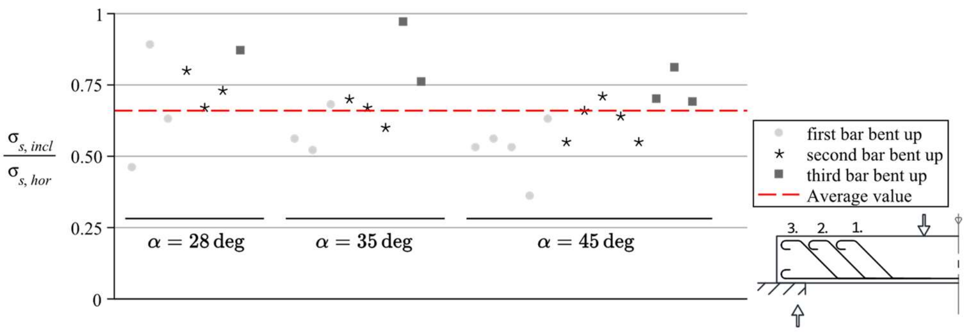
The notion of larger stresses in horizontal sections is an interesting and importing finding in the research process. The development of larger stresses in horizontal sections of bent up bars will namely exclude inclined sections as decisive members for design.

As additional check, stress measurement around curved sections of bent up bars are searched for. In Richart [21] tensile stresses in horizontal and inclined section around the curved sections are found for specimen with different numbers of bent up bars and shear inclination angles α. From these data, ratios between stresses are calculated and plotted in Figure 3-2.

Despite variations in ratios, tensile stresses in inclined sections of bent up bars are on average 66% of the stress values found in horizontal sections of these specimen, see Figure 3-2. This value is consistent with the value of 0.7 $(\approx \sqrt{2})$ mentioned by Sorenson [10] and others [22][28][18]

Figure 3-2

Ratios between stress measurements in inclined and horizontal sections of bent up bars around curved sections of bent up bars.



Note: Data points are calculated from measurements found in Richart (table 8, p.66)[21]. Measurements are taken at 0.8Pmax

3.2 Horizontal sections of bent up bars

Effects of larger stresses in horizontal sections of bent up bars on the strength of concrete specimen are studied first. Then, the section discusses consequences of horizonal sections of bent up bars on the observed failure mechanisms.

3.2.1 Stresses in horizontal reinforcement bars

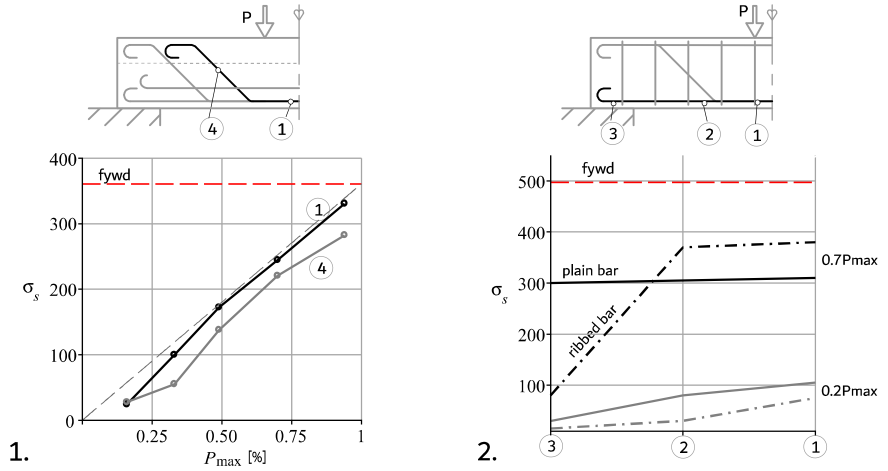
The development of tensile stresses in horizontal sections of bent up bars with increasing shear force was examined by Uchimura [29] and Walther [30].

The stress distribution presented in Uchimura [29] is shown in adapted form in Figure 3-3a for two distinct points. As expected, largest tensile stresses are found at midspan while the development of smaller stresses inclined sections is in line with previous discussion. As result of the larger stresses found at midspan, the yielding limit is reached here first. A corresponding tensile failure mechanism due to yielding of the reinforcement bars at midspan is confirmed for this specimen.

Walther [30] measured different stress levels near end anchorages of ongoing reinforcement bars. For plain bars, identical stresses are observed near the supports or at midspan. A significantly reduction of stresses near supports is found with the application of ribbed bars. Figure 3-3b therefore indicates that largest stresses near supports are introduced to concrete by plain bars. It can be expected that detailing of end anchorages with plain bars is more critical than for ribbed bars. For the two specimen in Figure 3-3b, bond failure is observed with application of plain bars, tensile failure with ribbed bars [30].

Figure 3-3

Relations between applied load and measured stresses in horizontal sections of bent up bars. a) Stress measurement at two distinct points along bent up bars, and b) stress measurement in ongoing reinforcement bars.



Note: Figure 1: Data points in the graph are taken from Uchimura (diagram, p.224)[29]. Figure 2: Adapted from Walther (fig.11, p.13)[30].

3.2.2 Reflection on validity experiments

Stress measurements examined in previous section clarifies why (i) tensile failure, or (ii) bond failure mechanisms near end anchorages can be observed for specimen reinforced with bent up bars. Both mechanisms, however, are primarily related to design and detailing of horizontal flexural reinforcement bars.

<u>Criticisms about experiments performed in the past</u>

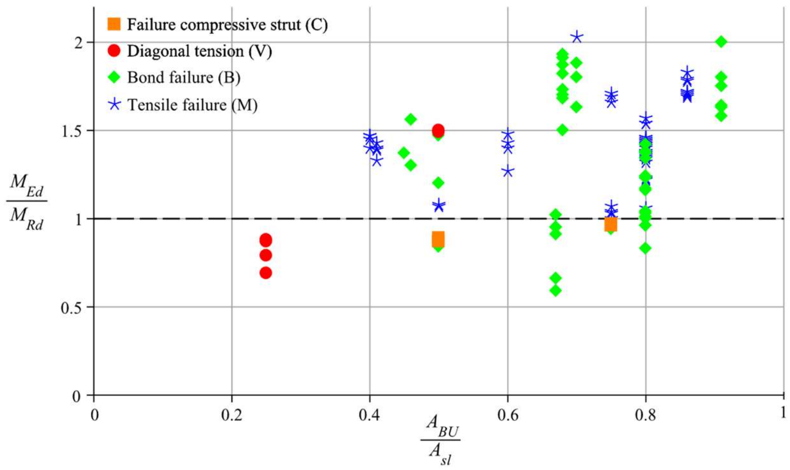
Similar to the specimen of Figure 3-3, tension or bond failure mechanisms are observed for most specimen in Figure 3-1. An explanation for this notion is related to the philosophy generally adopted in the first decades of the twentieth century. In these years, experiments were designed according to the philosophy that shear failure mechanisms "…*should never occur in reinforced concrete structures*" (p.21)[13][31].

Consequence of this design approach is that experimental specimen where heavily reinforced in shear in the past. The dataset confirms this finding by the notion that more than 80% of the specimen in the dataset is designed with ≥50% of the flexural reinforcement bars bent upwards. Figure 3-5 indicates that for these large percentages of bent up bars, specimen will fail in prematurely by tension or bond before the tensile strength of bent up bars is utilized fully. Additionally, recalculation of unity check values for the specimen in the dataset confirms the development of tensile failure mechanisms, see Figure 3-4.

Another consequence of this point of view is the application of beams with large top flanges. T-shaped specimen with large top flanges and web widths were initially applied to avoid brittle failure mechanisms of the compressive zone height [13]. Later experiments [15], however, shows the significant beneficial effects of these flanges on the contribution of concrete to the shear strength.

Thus, most examined specimen with bent up bars in the past could hardly fail in shear due to excessive amounts of shear reinforcement and inefficient geometry. Searching for defects of bent up bars as shear reinforcement with these specimen is undesirable.

Figure 3-4
Relations between observed failure mechanisms, applied percentages of bent up bars and calculated unity check values flexural reinforcement bars.



Note: Data points in the graph are calculated from experimental results and specimen layout. The data is taken from Bach [23](outcomes from Probst [24]), Saliger [25][26], Richart [21] and Shoukry et al. [27].

3.2.3 Experimental series by Saliger

In an important experimental series performed by Saliger [26][25] different types of failure mechanisms develop. For larger percentages of bent up bars, the specimen fail premature via a flexural or bond failure mechanism. Small percentages of bent up bars leads to shear or compressive strut failure mechanisms. From this series, the specimen for which shear failure mechanisms are observed are of special interest (specimen 23, 24 in Figure 3-5):

Are stresses in *horizontal* sections of bent up bars decisive for the observed shear failure?

The amount of flexural reinforcement bars at midspan, and consequently, the moment resisting capacity is identical for all beams examined by Saliger. For some specimen, however, premature shear mechanisms (23,24) or compressive strut failure mechanisms (26,28) develops before the yielding strength of horizontal sections is reached. Thus, tensile stresses in horizontal sections of bent up bars cannot be decisive for the observed failure mechanisms in these specimen.

Then, are stresses in *inclined* sections of bent up bars decisive for the observed shear failure?

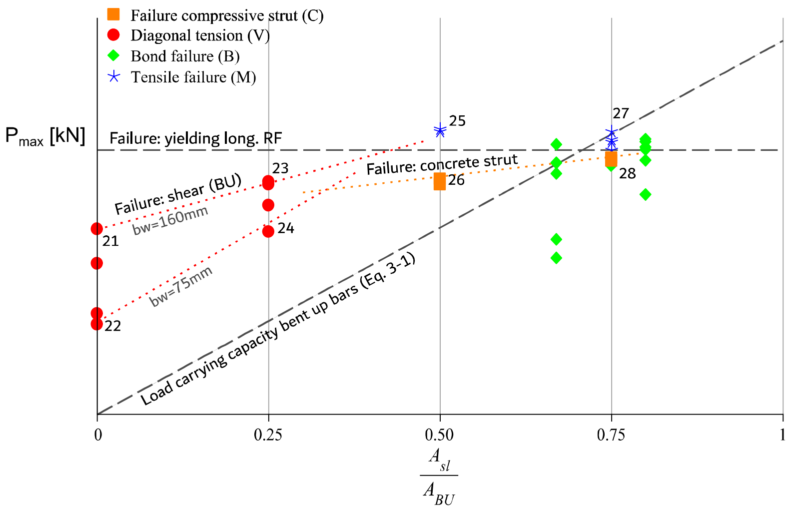
Section 0 reflects on the notion of smaller stresses in inclined sections of bent up bars. This effect can be found both in T-beams (Figure 3-3a a/d=1.7)[29] and rectangular beams (Figure 3-3b, a/d=1.6)[30] with small a/d-ratios. For large a/d-ratios, the reduced stresses in inclined sections are presented for the rectangular beams of Richart in Figure 3-2 (a/d=3.60-4.75)[21]. Thus, independent of the geometry of the cross section and the a/d-ratio, smallest stresses are measured in inclined sections of bent up bars.

By considering similar effects for Saliger' T-shaped beams (a/d=3.25), tensile stresses in inclined sections are expected to be smaller than in horizontal sections. Although yielding of inclined bent up bars can still be achieved after the formation of shear cracks, yielding of these sections do not initiate shear failure mechanisms.

In conclusion, tensile stresses in inclined and horizontal sections are therefore not decisive for the limited shear strengths of concrete members reinforced with bent up bars.

Figure 3-5

Relations observed failure mechanisms, applied percentages of bent up bars and applied shear force.



Note: Adapted from Saliger (fig.8, p.10)[26]. Figure includes additional data from earlier series by Saliger [25].

3.3 SUPPORTING CONCRETE STRUT

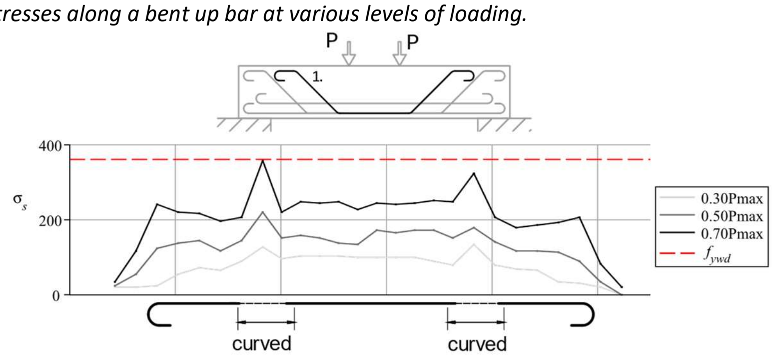
This section examines the interaction between the concrete strut and reinforcement bars in curved section of bent up bars. The distribution of tensile stresses along bent up bars is studied first. A reflection on interacting mechanisms between concrete strut and curved sections of bent up bars follows. Validation with Saliger' specimen closes this section.

3.3.1 Distribution of tensile stresses along bent up bars

The distribution of tensile stresses along bent up bars is taken from Uchimura [29]. Uchimura calculated the developed tensile stresses directly from strain measurements for different levels of loading. This methodology leads incorrectly to stress values which exceeds the yielding limit for large load levels. While this section is interested in the distribution of stresses only, three distributions with stresses below the yielding limit are selected, converted by units and presented in Figure 3-6.

Figure 3-6

Distribution of tensile stresses along a bent up bar at various levels of loading.



Note: Adapted from Uchimura (diagram, p.224)[29]. Stresses are converted to Mpa.

From the presented distribution of stresses, stress concentrations can be observed around the curved sections of bent up bars. These peaks can already been observed from small loading levels and results in large stress deviations in the reinforcement bar. From mechanic courses is known that stress (or strain) deviations in reinforcement bars indicates the development of interaction mechanism between reinforcement and concrete [20][3].

3.3.2 Interaction mechanisms in curved sections of bent up bars

Research towards the stress concentrations in curved sections of reinforcement bars delivers insight in interaction mechanisms with the surrounded concrete. Inspired by figures from Thompson (fig.2-23,p.26) [33] and Monney et al. (fig.17. p.14) [32], new figures are drawn for bent up bars specifically. The result is shown in Figure 3-7.

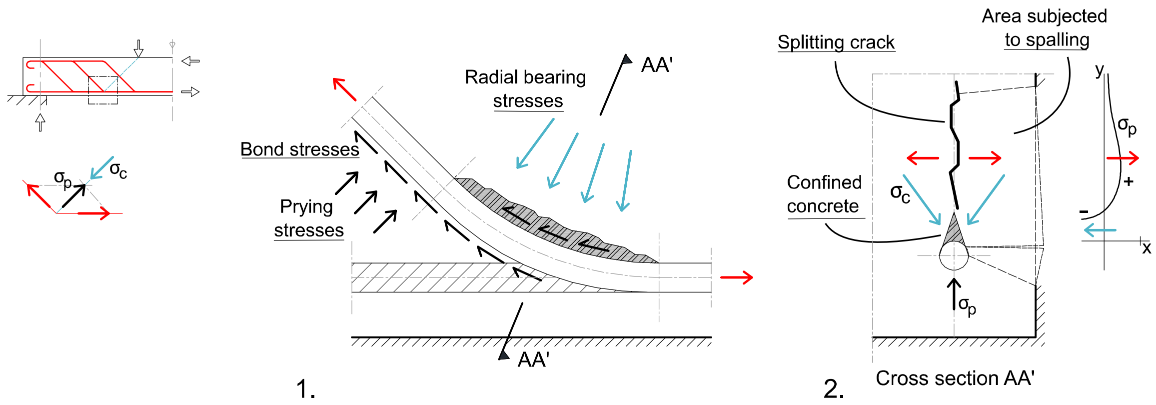
Interaction mechanisms

Three types of interaction mechanisms can be distinguished in curved sections of bent up bars. Firstly, bond stresses develops along the circumference of reinforcement bars. Secondly, prying stresses in the tail of the reinforcement bar tends to straighten the bar. Finally, radial bearing stresses develops within curved sections. The combined strengths of these stresses equilibrates the tensile load in the reinforcement bar [32][33]

Bearing stresses in curved sections of reinforcement bars are of special interest. Independent of the layout of curved reinforcement bars, tensile stresses in the reinforcement bar introduces a bearing force P to the concrete strut. As result, close to the reinforcement bar a region of confined concrete is formed. Beyond, transverse tensile stresses develops, see Figure 3-7b. These tensile stresses initiates the formation of splitting cracks in the concrete strut.

Figure 3-7

Overview of interaction mechanism between reinforcement bars and concrete strut in curved sections of bent up bars. 1) developed stresses in curved sections of bent up bars, and 2) the mechanism of spalling.



Note: Figures inspired by Thompson (fig.2-23,p.26) [33] and Monney et al.(fig.17. p.14) [32]

Transverse spalling mechanism

Splitting cracks developed in the concrete strut promotes transverse spalling mechanisms. In this mechanism, the confined wedge penetrates into the concrete strut with increasing load in reinforcement bars. Penetration of the compressive wedge enlarges splitting cracks and promotes transverse cover spalling [34]. The danger of cover spalling can be reduced with various measures, like (i) increasing concrete cover, (ii) increasing mandrel diameter, and (iii) reduction of bending angles α, see for example figure 9 of [32].

However, although measures against transverse spalling mechanisms can be taken, these measures does not avoid the formation of splitting cracks.

3.3.3 Experimental validation

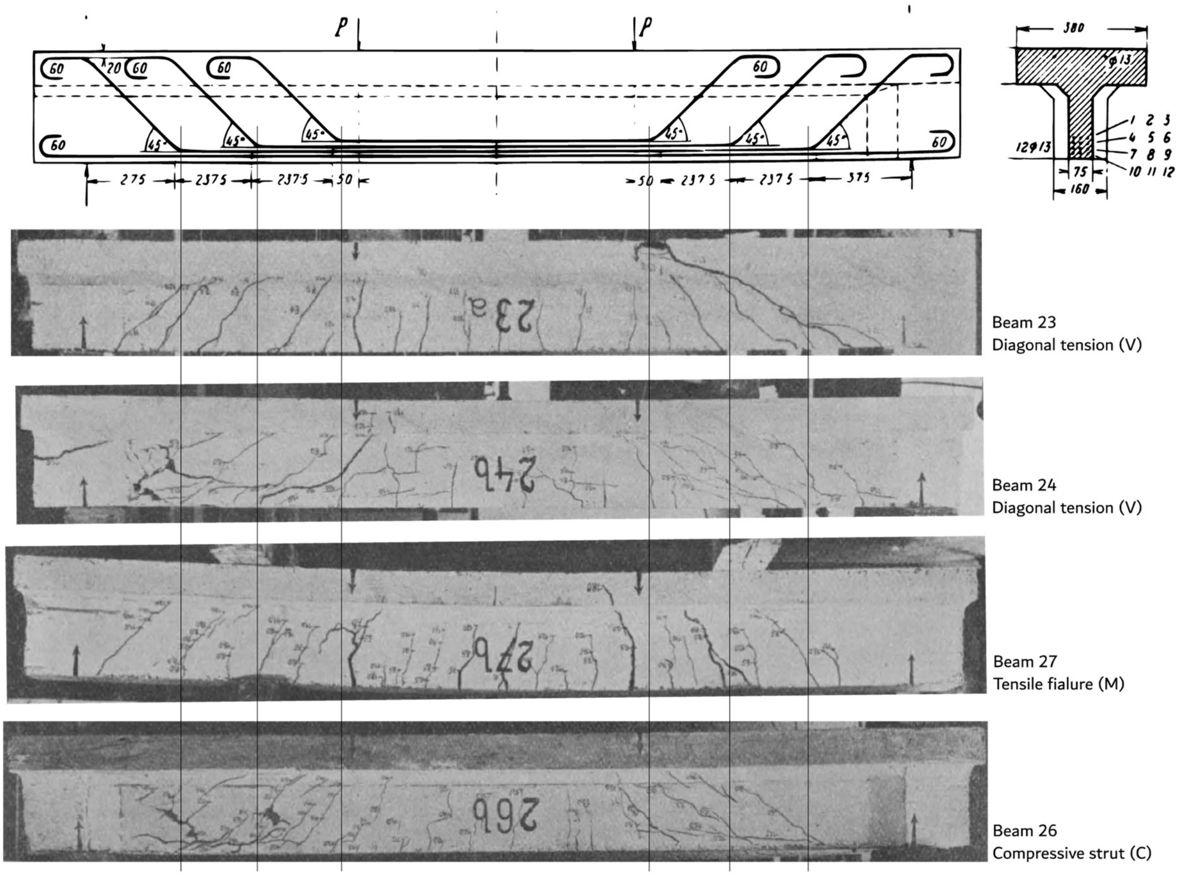
The effects of crack formation in curved sections of bent up bars can also been found from the specimen examined by Saliger. An overview of the position of bent up bars and photographs of four specimen with final crack patterns are taken from Saliger [26] and included in Figure 3-8.

Decisive shear cracks are observed at locations of curved regions of bent up bars for specimen which fail by shear failure mechanisms (23,24). Moreover, leading cracks and crack patterns of specimen which fail via flexural failure (27) or compressive strut failure (26) can be traced back to the position of curved sections of bent up bars.

Thus, curved sections of bent up bars initiates the formation of cracks in concrete. Consequences of the crack formation on the shear strength of concrete strut should be considered within the assessment procedure of bent up bars.

Figure 3-8

Final crack patterns in concrete beams reinforced with bent up bars examined by Saliger.



Note: Figures taken from Saliger (figures 4-5,p.9)[26].

3.4 FINDINGS

After evaluation of design procedures for bent up bars, a reflection on specimen in the dataset and the stresses in reinforcement and concrete, the following finding are presented:

- A design procedure of bent up bars with truss models ($\alpha = 45^0$) overestimates the capacity of bent up bars to resist shear stresses;
- While inclined sections are considered in the design of shear reinforcement, largest tensile stresses can be found in horizontal sections of bent up bars;
- Plain reinforcement bars are more vulnerable for bond and anchorage mechanisms than ribbed bars;
- Premature failure via (i) bond and anchorage, or (ii) tensile failure mechanisms limits the utilization of bent up bars and strength of concrete structures;
- Localized stress concentrations are found around curved sections of bent up bars;
- Bearing stresses applied to concrete in curved sections causes confinement of concrete near the reinforcement bar. Beyond, transverse tensile stresses are exerted to concrete which initiates the formation of splitting cracks.
- Curved sections of bent up bars initiates the formation of cracks into concrete structures, independent of the failure mechanisms.

4 PART III: ASSESSMENT OF BENT UP BARS

Aim of this section is to reflect on the reduced shear strength of bent up bars and supply recommendations for de assessment procedure of bent up bars.

To that aim, a model is introduced first (4.1) to conceptualize the developing failure mechanism of a concrete strut subjected to cracking. The obtained insights are utilized in an assessment (4.2) of the maximum shear strength of concrete struts supporting in curved sections of bent up bars. Finally, section (4.3) elaborates on the shear restriction of bent up bars found in the Eurocode, namely:

$$|\tau_{Ed}| \leq 1/3 * \tau_{Rdc,max} \qquad \textbf{Eq. 4-1}$$

4.1 CONCEPTUAL MODEL: SHEAR FAILURE MECHANISM OF THE CONCRETE STRUT

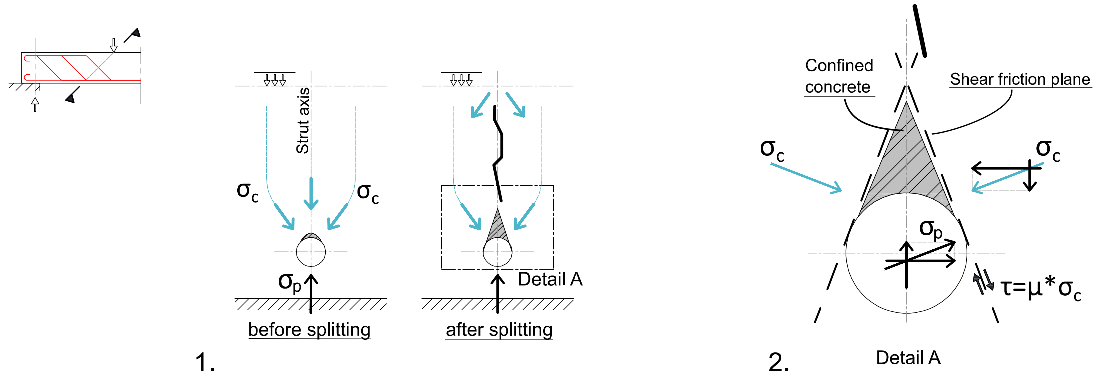
Effects of developing splitting cracks on the distribution of stresses in concrete struts are studied first with help of a conceptual model. After validation of the considered failure mechanism in a finite element analyses, consequences for the shear strength of the concrete strut are considered.

4.1.1 Interaction of stresses in curved sections of bent up bars

Based on Figure 3-7, the interaction between bearing stresses σ_p and stresses σ_c in a concrete strut is worked out in more detail. The resulting figures of Figure 4-1 are largely inspired by (fig.17b) by Monney et al.[32] who utilizes this figure to describe the mechanism of spalling failure. For the purpose of this report, however, the figure from latter report is redesigned and redrawn to visualize effects of splitting by curved sections of bent up bars specifically.

Figure 4-1

Consequences of the interaction between stresses in curved sections of bent up bars and stresses in the supporting strut. 1) Effects of the formation of a splitting crack on the stress distribution in the concrete strut, and 2) the development of an (idealized) shear friction plane.



Note: Figures inspired Monney et al.(fig.17. p.14) [32]

Radial bearing stresses σ_p in curved sections of bent up bars are introduced locally in the concrete strut. With an increase of stresses in the reinforcement bar, splitting cracks start to develop in the concrete strut. Consequently, the distribution of compressive stresses in the concrete strut is affected, as indicated in Figure 4-1a. Two changes can be observed from the distribution of stresses in the concrete stut after splitting cracks have been formed:

1. The development of a region with confined concrete;
2. The development of an (idealized) shear friction plane.

The development of a region with confined concrete

Close to the reinforcement bar, an area with confined concrete is formed in the concrete strut. From all sides this region is subjected to compression stresses. These stresses originates from bearing stresses σ_p from the reinforcement bar and stresses σ_c from the concrete strut. In out of plane direction, this region is confined by adjacent radial bearing stresses.

As results of the multiaxial confinement, the compressive strength of concrete increases. Multiaxial compression tests shows that the strength of confined concrete increases up to multiple times the uniaxial strength of concrete [9][32]. The strength outside the confined region, however, still equals compressive strength f_{ck} of concrete. Therefore, the compressive strength of this confined region is considered less critical in the shear strength of the compressive strut.

The development of a shear friction plane

Another effect of the developing splitting cracks is the changing angle at which compressive stresses are applied to the reinforcement bars. Before splitting crack develops, bearing stresses can be introduced directly to concrete in front of the reinforcement bar. After cracking, splitting cracks in front of the reinforcement bar hinders the transfer of stresses via this area. Consequently, stresses σ_c in the strut are redirected and applied under an angle to the reinforcement bar.

Inclined stresses σ_c from the strut supporting on reinforcement bars can be decomposed in a horizontal and vertical stress component. Horizontal component $\sigma_{c,x}$ equilibrates bearing stress $\sigma_{p,x}$ in the reinforcement bar. In this direction, lateral bearing of reinforcement bars to concrete is possible. In vertical direction, however, equilibrium of compressive stresses is only possible via friction stresses in an evolving friction plane.

Thus, vertical stresses in the region close to reinforcement bars interact via shear stresses in a shear friction plane.

Draft – shear failure mechanism bent up bars

By reaching maximum friction capacity τ of the evolving shear friction plane, vertical equilibrium conditions are violated. With an increase of bearing stresses σ_p, the reinforcement bar (including confined region) cuts into the already existing splitting crack. Alternatively, the concrete strut loses its vertical support and slides over the reinforcement bar.

4.1.2 Validation mechanism with a finite element model

The development of this failure mechanism is validated with a finite element analysis. For the analysis a model is build with the program Diana.

Description finite element model

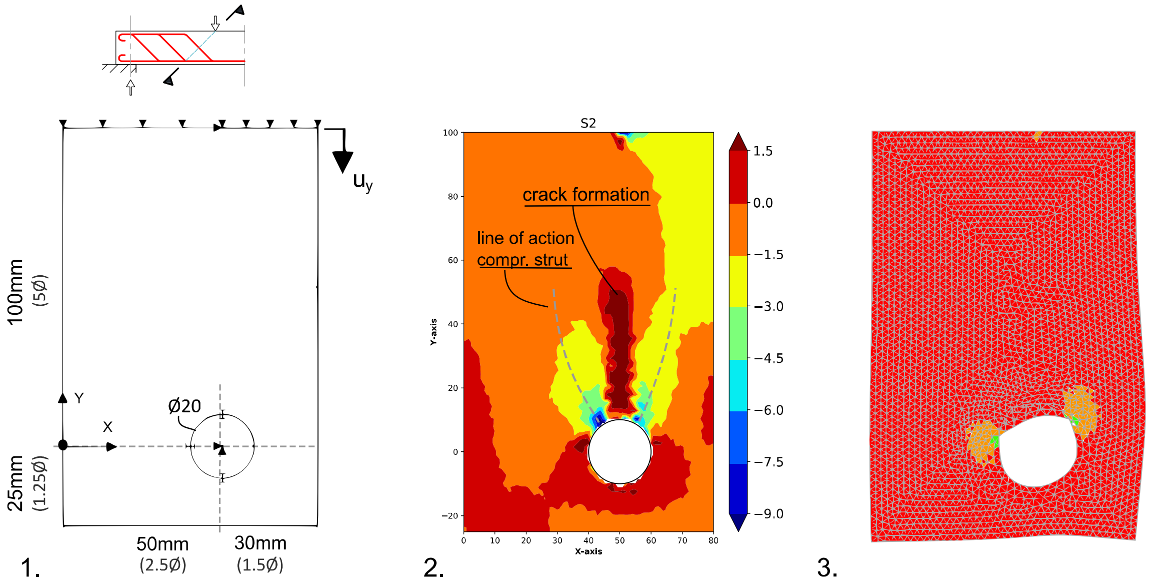
The mimic the effects of a concrete strut supported at curved sections of bent up bars, a rectangular two-dimensional plate (125*80) with an embedded circular section (ø20) are modelled. The plate and circular section represents the supporting concrete strut and crossing reinforcement bar along the strut axis respectively, see Figure 4-2a. The reinforcing bars is modelled asymmetric in the plate to analyse effects of concrete cover on the distribution of stresses over the plate. The reinforcement bar is fixed in x and y-direction in its centre point (50,0), while the plate is fixed in a y-direction along the top edge (y=125). In x-direction the plate is fixed in the point (50,100).

The material properties of concrete plate (C20), including smeared cracking model, are based on expressions found in the fib Modelcode2010 [35]. For the reinforcement bar linear elasticity is considered. Between the concrete plate and bar an interface is modelled to mimic friction and adhesion effects between reinforcement and concrete.

The mesh in model consists of triangular plane strain elements (T6EPS-elements) with an element size of approximately 3mm. As loading scheme, prescribed displacements are applied along the top edge of the plate.

More details of the finite element model can be found in Annex C.

Figure 4-2
Overview of examined finite element model and characteristic finite element results. 1) Geometric overview of finite element model, 2) distribution of principle stresses S2 over the model just before failure, and 3) screenshot of stresses S2 with the developed mechanism at failure.



<u>Results</u>
For readability of the figures in this section, the distributions of strains and stresses from the finite element model are drawn up via a small python script.

1. *Validation failure mechanism*

The distributions of principle stress $S2$ just before failure is shown in Figure 4-2b ($\sigma_{min} = 2.18Mpa$, $\sigma_{max} = -14.16Mpa$). From the shown stress distribution, the following observations are made:

- In front of the reinforcement bars, a region with tensile stresses equal to the tensile strength of concrete can be found. A closer look at the distribution of crack widths (not included in this report) confirms the development of splitting cracks in this region;
- The line of action of compressive stresses supporting at the reinforcement bar is shown in the figure explicitly and corresponds to the expected inclination of this line of action.
- After reaching the ultimate load in this model, equilibrium conditions around the reinforcement bar are violated. Figure 4-2c shows a screenshot of the developing sliding mechanism of the concrete strut over the reinforcement bar.

Since all ingredients of the introduced mechanism are included in the finite element model, the mechanism of previous section is confirmed.

2. *Mechanisms along the shear friction plane*

A closer look at the stresses destitution along the shear friction plane shows the development of three different mechanisms.

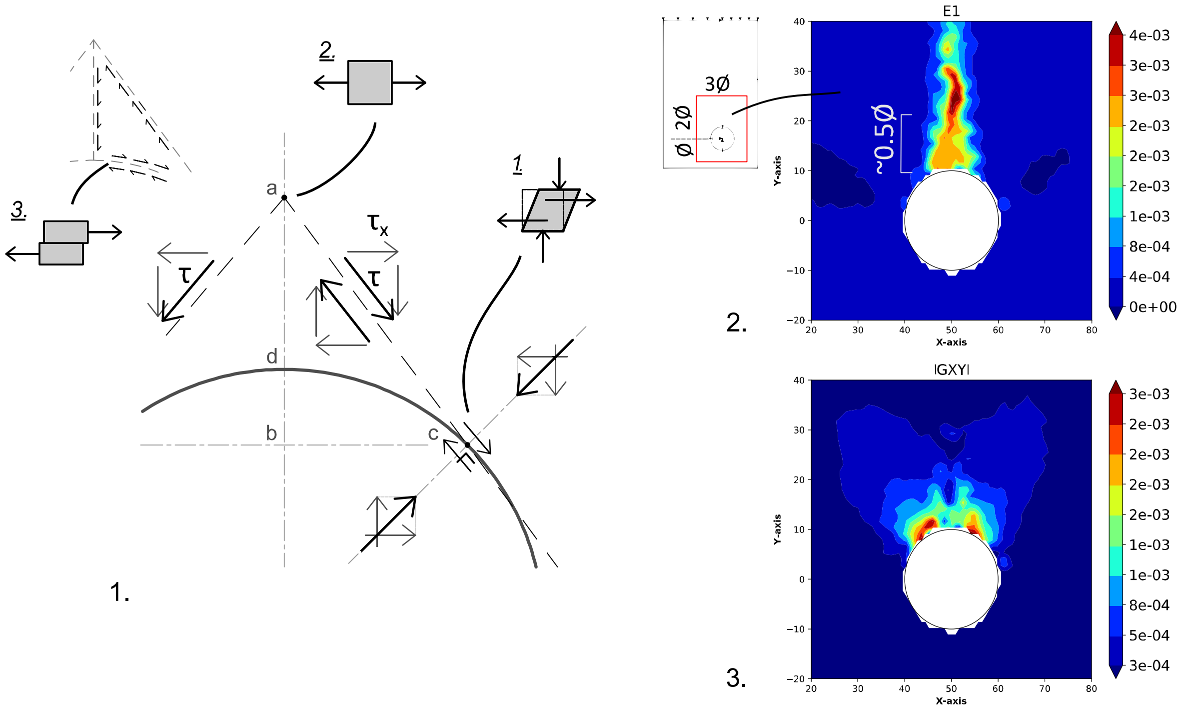
First mechanism is found at the intersection of the reinforcement bars and the shear friction plane (point c of Figure 4-3a). In this idealized situation, only compressive stresses and shear stresses are found at this point of the friction plane. Consequently, the resulting mechanism will influence the capacity to transmit shear stresses. The occurrence of shear strains at this location is found clearly in figure Figure 4-3c.

Second mechanism can be found around the tip of the confined region. Since shear stresses τ acts on an inclined plane, horizontal and vertical shear components will develop. In the tip of the confined region, tensile shear stresses τ_x from both sides of the confined region coincide and lead into a localized region with tension stresses. The concentration of tension at the tip of the confined region is presented by the distribution of principle strains $E1$ in figure Figure 4-3b.

A third mechanism can be found at the interface between the reinforcement bar and the confined region. The flow of shear stresses along the circumference of the confined concrete shows sliding mechanism between the reinforcement bar and confined concrete. The capacity to established equilibrium of latter sliding mechanism will depend on the friction strength between steel and concrete.

All of the described mechanisms can be decisive for the development of the failure mechanism. The decisive failure mechanism, however, is not being determined. A research towards the decisive mechanism is (far) beyond the scope of research. Although the exact cause of failure remain unknown, it is presumed in next section that the transfer of stresses from concrete strut to curved sections of bent up bars is limited by strength of this shear friction plane.

Figure 4-3
Overview of mechanisms along the shear friction plane and characteristic finite element results. 1) Schematization of developing mechanisms along the shear friction plane, 2) distribution of principle strains E1 over the model just before failure, and 3) distribution of shear strains GXY just before failure (in absolute values).



3. *Other observations*

Additionally, Figure 4-2b shows regions with tensile stresses around the reinforcement bar and near the edges of the plate. The developed tensile stresses near the plate edges indicates the increasing vulnerability for crack formation. Also, with an increase of loading developing cracks near the reinforcement bar propagates towards the edges of the plate. Effects and relations between curved sections of bent up bars and locations of crack formations have already been shown in the experiments by Saliger (Figure 3-8).

4.1.3 Reflection: what happens?

In previous sections, a concept of the developing failure mechanism have been presented. The question arises how this mechanism can be translates in consequences for engineering practice. Moreover, how this mechanism can be translated into an assessment method for bent up bars.

<u>After loss of vertical equilibrium</u>

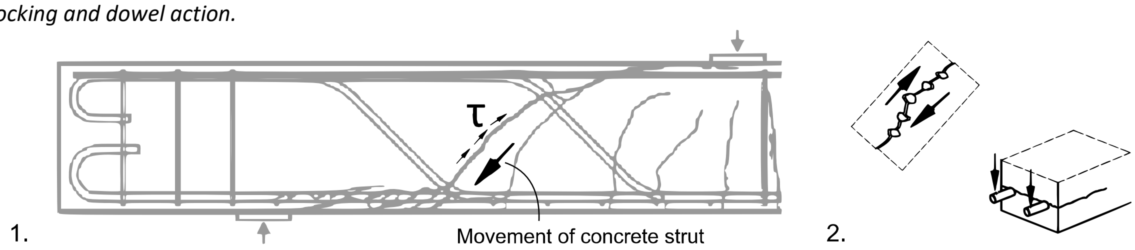
With exceeding of the shear strength of the shear friction plane, the concrete strut loses its vertical support. As result, the concrete strut will move in downward direction and generates the new (shear) cracks starting from the curved sections of bent up bar. This effect is exemplified in Figure 4-4.

Simultaneously, the structure searches for a new state of equilibrium. The capacity of a concrete member to resist shear stresses in this new state of equilibrium is primarily delivered by aggregate interlocking of plain concrete and dowel action [37][38]. Effects of aggregate action is found in developing shear crack as well as in the considered shear friction plane of the considered mechanical model. Dowel action effects are provided by flexural reinforcement bars, horizontal and/or inclined sections of bent up bars.

Thus, the capacity of concrete struts to transmit shear stresses and resist developing shear cracks, depends on the combined shear resistance of aggregate interlocking and dowel action. For the assessment method and maximum shear strength of bent up bars as shear reinforcement it is desirable to evaluate the combined shear strength of these mechanisms.

Figure 4-4

New equilibrium situation after the vertical equilibrium conditions of the supporting concrete strut are violated. 1) Developed mechanisms after loss of equilibrium, and (b) definitions of shear resisting mechanisms via aggregate interlocking and dowel action.



Note: Figure 1: Adapted from Huber (Beam PL26.0, fig.3, p.4) [38]. Figure 2: Adapted from Huber (fig.4,p.5)[38].

4.2 MAXIMUM SHEAR STRENGTH OF THE CONCRETE STRUT

From mechanics is known that the maximum shear strength τ_{max} of a shear plane is related to a shear friction coefficient μ and applied normal stresses. For a concrete member, the maximum normal stresses are equal to the compressive strength f_{ck} of concrete, resulting in expression Eq. 4-2.

$$\tau_{max} \leq \mu * f_{ck} \qquad \textbf{Eq. 4-2}$$

In latter expression friction coefficient μ is considered the critical parameter for design and describes the represents the ultimate shear strength of a concrete shear plane. With help of push-off tests, maximum shear strength of a concrete plane and the relation with exerted normal stresses can be determined.

After a short description of a series of collected with push-off experiments, this section derives possible shear strength limitations for concrete struts supporting at curved sections of bent up bars. The section closes with a validation of the proposed expressions for specimen with bent up bars and reflects on the assessment methods for bent up bars treated in this report.

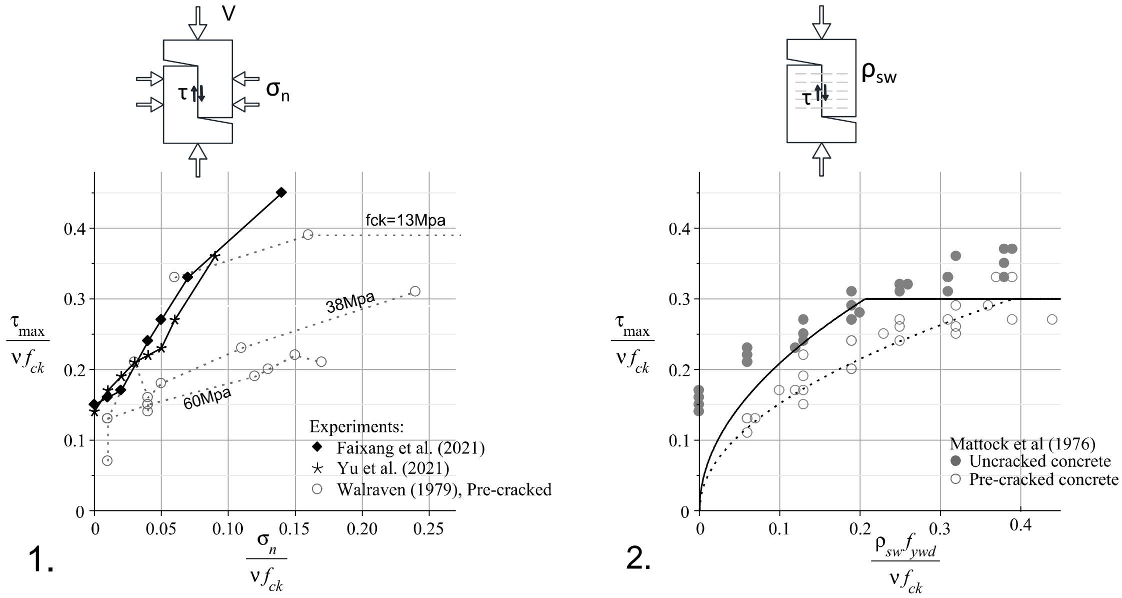
4.2.1 Push-off tests

Specimen with active confining stresses and uncracked friction planes have been found in Faxiang et al. [39] and Yu et al. [40]. Despite differences in concrete strength classes (fck=42.6Mpa and 26.9Mpa respectively), relative small deviation in maximum shear strengths can be observed. However, for the specimen with initially cracked friction planes examined by Walraven [41], significant influence of the concrete strength class is observed, see Figure 4-6a. Despite these differences, in all experiments the maximum shear strength is positively related to the amount of confining stresses.

Mattock et al. [42] examines the effects of initially cracked or uncracked shear planes for specimen with passive confinement. Outcomes of the experiments are included in Figure 4-6b (data from [43]). The empirically derived expressions are proposed by the Mattock et al. and taken from a later published discussion about their performed experiments [44].

Figure 4-5

Figures with outcomes of push-off tests. 1) Tests with active confinement, and 2) tests with passive confinement.



Note: Figure 1: Data points in the graph calculated based on data taken from Faixang et al. [39], Yu et al. [40] and Walraven [41]. Figure 2: Remake of figure 16 from Mattock et al. [44] (measurements by Mattock et al. taken from Krc et al. [43]). The included expressions are taken from Mattock et al. [44].

4.2.2 Shear strength limits concrete strut

From both figures in Figure 4-6, two design approaches for the shear strength of the concrete strut can be proposed.

<u>Lower bound approach</u>

Both series with passive and active confinement indicates the significance of the amount of confinement and concrete strength class on the maximum shear strength. Further, the disadvantageous effects of initially cracked shear planes on the maximum shear strength can be observed in both figures.

To overcome the variety of parameters on the maximum shear strength of concrete, a conservative lower bound solution can be considered. For this approach the maximum shear strength of the concrete strut should be limited to (Eq. 4-3):

$$\frac{\tau_{max}}{f_{ck}} = 0.10$$ **Eq. 4-3**

<u>Shear strength expression by Mattock et al.</u>

The combined shear mechanisms of aggregate interlocking and dowel action can be found in the experiments performed by Mattock et al [44], shown in Figure 4-6b. Mattock et al. proposes an empirical relation between the amount of shear reinforcement in the shear plane and corresponding shear strengths, presented in this report with Eq. 4-4. A corresponding lower bound solution for initially cracked concrete shear planes is indicated with dotted lines in the figure and shown by Eq. 4-5 [44].

$$\frac{\tau_{max}}{f_{ck}} = 0.66 * \sqrt{\omega} \leq 0.30$$ **Eq. 4-4**

Eq. 4-5

$$\frac{\tau_{max}}{f_{ck}} = 0.48 * \sqrt{\omega} \leq 0.30 \qquad (lower\ bound)$$

$Where$:

$$\omega = \rho_v * \frac{f_{ywd}}{f_{ck}}$$

$$\rho_v = \frac{A_{sw}}{b_w * s}$$

Mattock et al. mentioned yielding of reinforcement for relative small shear reinforcement ratios ρ_v crossing the shear plane during their experiments. With an increase of the number of reinforcement bars, premature failure of concrete occurs. An upper bound of $0.30f_{ck}$ is considered to avoid this brittle failure of concrete [44].

Interestingly, latter observation resembles the observed failure mechanisms in Figure 3-5 of specimen reinforced with bent up bars…

4.2.3 Validation design procedures for specimen with bent up bars

In this report different procedures have been threated for the assessment of bent up bars in concrete structures. The different procedures are based upon:

- Compressive strength f_{ck} of the concrete strut; section 0
- Yielding strength f_{ywd} of reinforcement bars; section 3.1.1

A summary of the design methodologies for bent up bars in shown Figure 4-6. In the figures data points of specimen which fail by shear or compressive strut failure mechanism are included. With plotting expression Eq. 3-4, it is presumed that the inclined section crosses the shear plane perpendicular ($\rho_v = \rho_{sw}$).

<u>Bent up bar design – compressive strength of concrete</u>

The design expressions proposed by Mattock et al. fits perfectly around the beams for which shear or compressive strut failure are observed. It confirms that in specimen with bent up bars the shear strength of the concrete strut is decisive. Moreover, it shows that the shear strength of bent up bars is based on the combined strengths of aggregate interlocking and dowel action for small percentages of bent up bars. For large number of bars, the shear strength is limited by the compressive strength of concrete with an upper bound equal to:

$$\tau_{max} \leq 0.30 f_{ck} \qquad \textbf{Eq. 4-6}$$

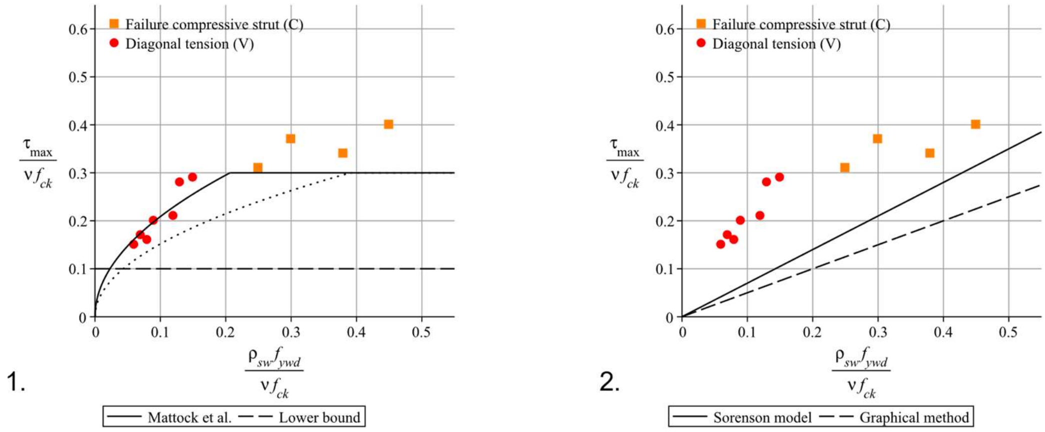
<u>Bent up bar design – strength reinforcement bar</u>

The graphical method and Sorenson method are shown to be conservative for the design of bent up bars. Despite, with the application of these expression no specific upper bounds are given to maximum stresses in the concrete strut.

Effects of a design procedure of inclined reinforcing members with truss models has been discussed earlier in the report. Consequently, this procedure is explicitly left from the figure.

Figure 4-6

Summary and validation of models proposed for the design of bent up bars. 1) Design procedures based on the strength of the compressive strut, and 2) based on the tensile strength of reinforcement bars.



Note: Data points in the graph are calculated from experimental results and specimen layout. The data is taken from Saliger[25][26] and Shoukry et al. [27].

4.3 ELABORATION ON EUROCODE RESTRICTIONS

The maximum shear strength of bent up bars prescribed in the Eurocode was presented in Eq. 4-1. Substitution of maximum shear strength $\tau_{Rdc,max}$ (eq.(6.5) of [7]) into Eq. 4-1 introduces a factor 0.50 in the expression. Factor ν encloses non-linear material effects and the effect of concrete cracking. Non-linear material effect are represented by a factor ν', the strength reduction of cracked concrete is introduced via a factor 0.60 [45].

Resulting expression Eq. 4-7 indicates that shear strength $\tau_{Rdc,max}$ should be limited to $0.30 * \nu' f_{cd}$ to avoid failure of the compressive strut. It corresponds to the value found in Figure 4-6.

$$|\tau_{Ed}| \leq 1/3 * \tau_{Rdc,max}$$

$$= 1/3 * [0.50 * \nu * f_{cd}]$$

$$= 1/3 * [0.50 * 0.60 * \nu' f_{cd}]$$

$$= 1/3 * [0.30 * \nu' f_{cd}] \qquad \textbf{Eq. 4-7}$$

Further elaboration of the $1/3$-term in latter expression shows that shear stresses τ_{Ed} exerted to the concrete member should be limited to $0.10 * \nu' f_{cd}$. This corresponds to the conservative lower bound expression proposed in section 0.

$$|\tau_{Ed}| \leq 0.10 * \nu' f_{cd} \qquad \textbf{Eq. 4-8}$$

4.4 FINDINGS

The development of an conceptual model, validation of developed failure mechanism and a reflection on its consequences for the shear strength of a concrete strut results into the findings:

- Close to reinforcement bars vertical stresses are transferred via shear stresses in a friction plane. By reaching maximum friction strength, the reinforcement bar cuts into the existing splitting crack;

- Aggregate interlocking and dowel action influences significantly the shear strength of the concrete strut;

- A conservative lower bound for shear strength of concrete struts is $0.10f_{ck}$, an upper bound $0.30f_{ck}$.

- Small shear reinforcement ratios ρ_{sw} crossing the shear plane leads to yielding of bent bars, large percentages to compressive failure of the concrete strut;

- Internal cracks develop in horizontal and vertical direction along the reinforcement bar and propagates towards the edges of the concrete member.

Most interesting observation of Part III is the resembles of the specimen with bent up bars and the shear friction expressions introduced by Mattock et al. It is surprising to find an almost perfectly fit of the design expressions with outcomes of the experiments. By the author's knowledge, this finding have not been observed neither mentioned in literature before.

5 DISCUSSION

Constraints (5.1) of the research project are discussed first, then the implications (5.2) are discussed. This chapter closes with recommendations for engineering practice (5.3) and further work (5.4).

5.1 REVIEW OF USED METHODOLOGY

5.1.1 Governing failure mechanism?

In this report, the specimen with bent up bars were classified per failure mechanism. These failure mechanisms were primary based on visual observations of crack formations by the authors. While descriptions and provided photographs can be checked visually or numerically, it is questionable if the observed failure mechanism corresponds to the actual mechanism(s) involved in failure process.

Especially bond failure mechanisms appears to be close related to other failure mechanisms. The horizontal cracks found in specimen 24 (failure in shear) and 26 (compressive strut failure) of Figure 3-8, indicates a delamination mechanism of reinforcement and concrete. Latter can be classified as a bond failure mechanism. While observations of the author are followed for these specimen, it is unknown if the observed delamination process (i) causes the failure mechanism, or (ii) results from the mentioned failure mechanism.

The importance of visual observations of cracks formation, however, is indicated in new researches towards structures designed with bent up bars [37][46]. Moreover, the visual observation of inclined shear cracks was one of the main reasons to prescribe bent up bars as shear reinforcement in the past (Section 2.1). Both aspects support the followed procedure in this report and confirms the importance of visual crack observation on the application of bent up bars as shear reinforcement.

Also, the shear strength of bent up bars is found dependent on the mechanisms of dowel action and aggregate interlocking. Both mechanisms contributes to the shear strength after cracks have been formed. Detailed insights in relations between observed crack formations and contributions of these and other shear mechanisms contributes to better understanding to the shear strength of bent up bars in concrete structures.

5.1.2 Critical design parameters?

During the research project many different parameters affecting the performance of bent up bars in reinforced concrete structures have been encountered. Despite, effects of bar diameters, effective depths, concrete covers, transverse bar spacing, continuous top sections, anchorage lengths, mandrel diameters, etc. were left. [25], [26][8]. Detailed insights towards effects of individual parameters on the performance of bent up bars can be subject for further research.

5.2 IMPLICATIONS

5.2.1 Shear strength restrictions in previous building codes

This report focusses on the maximum strength of bent up bars to resist *exerted* shear stresses. In previous building codes, however, the design of bent up bars was based on a limitation of *applicable* shear stresses. In this section, both design approaches will be compared.

Maximum shear stresses

The maximum allowable shear stresses found in Dutch building codes are recalculated based on the concrete strengths f_{cd}. Since these codes also prescribes limitations for compressive stresses, the compressive stress limitations $\sigma_{\mathrm{c,max}}$ is taken equal to compressive strength f_{cd}. Simultaneously, the definition of maximum shear stresses differs over the codes and is for comparison defined by expression Eq. 5-1. The resulting shear stress limits are presented in Table 5-1.

$$\tau_{Ed} = \frac{V}{b_w * d} \qquad \textbf{Eq. 5-1}$$

The maximum allowable shear stresses used in the design of concrete structures from the past corresponds well to the current shear strength restriction of bent up bars in the Eurocode. For a generally used concrete strength class C30 ($\nu' = 0.88$, Eq. 3-5), an almost identical result is obtained as in previous codes.

This comparison shows that similar strengths are assigned to bent up bars in previous building codes and current Eurocode. Thus, shear strength restrictions found in the current Eurocode for the design and (re)assessment of bent up bars makes sense.

Table 5-1
Comparison of shear strength restrictions for bent up bars in Dutch building codes.

Building code	Reference	$\tau_{Ed,max} \leq$	Remark
GBV1912	[47]	$0.07 * f_{cd}$	
GBV1918	[1]	$0.07 * f_{cd}$	
GBV1930[3]	[2]	$0.08 * f_{cd}$	
GBV1950[3] [4]	[3]	$0.08 * f_{cd}$	
GBV1962	[4]	$0.08 * f_{cd}$ $0.08 * f_{cd}$ $0.07 * f_{cd}$	*K160 (strength class)* *K225* *K300*
NEN-EN1992	[7]	$0.10 * \nu' f_{cd}$	

.

5.2.2 Truss models

While it was possible to study the distribution of stresses with truss models qualitatively, critical regions for bent up bars could not have been found. Further, larger stresses in horizontal sections, near end anchorages, and the development of splitting stresses in concrete remain unnoticed. Thus, aspects which limits the application of bent up bars in concrete structures cannot be modelled with trusses.

[3] Based on τ_{max} ≤5kg/cm2 and $\sigma_{c,max}$=60kg/cm2

[4] Consulted in Schrier (1951)[3]

The importance of this notion is related to the remark that the design of shear reinforcement is based on truss models in the Eurocode. Therefore, special care should be taken with the projection of Eurocode principles in design and reassessment of concrete structures reinforced with bent up bars.

5.2.3 Models for structural analysis of bent up bars

Figure 4-6 indicates that any model based on the tensile strength of inclined members is applicable for the analysis of bent up bars as long as it is applied in combination with the restriction $\tau_{Ed} \leq 0.10 f_{cd}$. It indicates that in this situation even truss models are applicable for analysis of structures with bent up bars.

When shear stresses exceed $\tau_{Ed} > 0.10 f_{cd}$, the graphical method or Sorenson method can be utilized effectively. However, since no limitations are given to avoid failure of the compressive strut, attention should be given to the possibility of this brittle mechanism at large shear reinforcement ratios. Alternatively, the lower procedure by Mattock et al. (Eq. 4-5) can be applied safely.

During the (re)assessment of reinforced concrete structures the original value of the design load is often asked. Since the graphical method is used for design of bent up bars in the past, this procedure can be employed to recover values of design loads for concrete structures.

5.2.4 Bent up bars and vertical stirrups

Additional stirrups can be applied to strengthen the concrete structure in transverse direction. As result, the possibility of concrete cover spalling mechanisms reduces. Also, the confining effect of stirrups is beneficial for the compressive strength of concrete and reduces widening of splitting cracks in curved sections of bent up bars. The beneficial effects of the combination of stirrups and bent up bars is already recognized in early experiments [14][21][31].

Important to notice is that the formation of cracks in the concrete strut within curved sections of bent up bars can not be avoided with the application of stirrups. The formations of splitting cracks depends namely on the amount of radial bearing stresses exerted to the concrete strut by curved sections of bent up bars. Latter stresses originates from tensile stresses in horizontal (flexural) sections of bent up bars. The application of additional stirrups does therefore not affect these stresses exerted to the concrete strut.

5.2.5 Beams versus plates

In the current Eurocode, bent up bars may be applied in beams only combination with additional stirrups [7]. Despite, Figure 3-5 indicates that beams reinforced with bent up bars only are capable to resist shear stresses. In latter situation, one should take care of:

- Sufficient shear *and* flexural reinforcement design;
- Effects of concrete cover spalling;
- Detailing of end anchorages (Figure 3-3b);
- To early or to much reduction of flexural reinforcement bars, as indicated in [31][48];

The confining effect of surrounded concrete in concrete plates reduces the possibility of concrete cover spalling [32]. Combined with the beneficial effects of larger web width on the shear resistance, bent up bars in plates are considered less critical for shear failure mechanism [49]. Nevertheless, measures to avoid premature tension, anchorage or compressive strut failure mechanisms should be taken.

5.3 PRACTICAL RECOMMENDATIONS

Minimum requirements of additional stirrups

The application of additional stirrups is a suitable solution to enlarge the strength of concrete structures reinforced with bent up bars only. Researches show that the replacement of $20 - 25\%$ of required shear reinforcement by closed stirrups is sufficient to avoid brittle failure mechanisms [50][25]. Latter solution is only valid in concrete beams if the shear reinforcement ratio in the form of stirrups is respected.

If a combination of stirrups and bent up bars as shear reinforcement is useful and effective by following this design procedure can be subject for debate.

End anchorage region

In contradiction with the findings from truss models (section 2.3.1), the large stresses exerted to concrete near end anchorages are important for the strength of concrete structures with bent up bars. Especially early structures with plain reinforcement bars are more vulnerable for bond and anchorage failure mechanism (Figure 3-3). Therefore, special attention should be given to these regions during the reassessment and design of structures with bent up bars.

Application of hooks (stirrups)

In section 2.4 the importance of physical anchorage for the structural integrity of concrete structures has been mentioned. Since disadvantages of the negligence of hooks can be found in experiments [21], it is recommended for engineering practice that hooks are added to stirrups and positioned around longitudinal reinforcement bars in the fictious top and bottom chord of concrete structures. Thus, stirrups should embrace the cross section properly by, for example, application in closed form.

5.4 FURTHER WORK

No engineering models have been found with which the combination of bent up bars and stirrups can be modelled. Design expression for the reassessment of concrete structures with combined interaction between bent up bars and stirrups are derived in [46], however these expressions are comprehensive and time consuming to apply in practice. The obtained insights in this report can be used as starting points for the development for a more manageable model.

Subject for further research can be the evaluation of the contribution of dowel action of bent up bars to the shear resisting capacity. In specimen of Figure 4-4, a crack form perfectly in the centre of the curved section. In this situation, effects of dowel action are expected to occur in the horizontal sections of bent up bars and flexural reinforcement bars. The individual contributions of these bars to the total shear strength is unknown. Alternatively, a crack can develop before or directly behind the point of bending upwards. The contributions and consequences of horizontal and inclined members on shear resistance to dowel action is questionable.

6 CONCLUSION

By conducting this research, background for the shear strength restrictions of bent up bars in the Eurocode have been provided. With analyses of the transfer and distribution of shear stresses in concrete structures, effects and consequences of bent up bars in concrete specimen could have been evaluated. From resulting assessment procedures and shear strength limits, it is possible to formulate an answer to the central research question:

What is the structural limitation of bent up bars as shear reinforcement?

Answer:
With the application of bent up bars in concrete structures, local stress concentrations are introduced in the concrete strut via the curved sections. With increase of tensile stresses in reinforcement bars, cracks are promoted into the supporting concrete strut by curved sections of bent up bars. The remaining shear strength of the concrete structure depends on the shear resistance of the cracked concrete strut.

6.1.1 What prefers the application of stirrups or bent up bars as shear reinforcement?

Throughout the twentieth century, changing design concepts changes the design procedure of shear reinforcement. In the early twentieth century, the general idea was to prevent brittle shear failure at any time. Together with the observation of inclined shear cracks around the neutral axis, the application of bent up bars in concrete structures was preferred.

After the sixties, this design concept changes to a strength-based approach in which crack formation was allowed. The behaviour of bent up bars in concrete structures after cracking, however, is less favourable compared to the application of stirrups. The confining effects of stirrups increases the strength concrete structures after cracking. Also, stirrups does not initiate cracks into concrete strut, can be designed accurately with simple truss models and are less precarious in design.

6.1.2 What is the capacity of members reinforced with bent up bars to resist shear stresses?

While inclined sections of bent up bars are positioned more effectively in concrete structures to resist inclined shear cracks, inclined section of bent up bars are unexpectedly not decisive for the limited shear strength of concrete members. Firstly, tensile stresses developed in these sections are relative small. Also, shear failure mechanisms can develop before full tensile strength of inclined sections of bent up bars is reached.

Instead, the interaction of stresses in curved sections of bent up bars with the concrete strut restricts the shear strength of the concrete members. The shear strength of uncracked concrete strut diminishes with the development of splitting cracks around curved sections of bent up bars. The remaining shear strength depends largely on the combined contributions of dowel action and aggregate interlocking. Comparison of the outcomes of shear friction test towards these mechanisms and maximum shear strengths of specimen reinforced with bent up bars shows large similarities.

The maximum capacity of members with bent up bars to resist shear stresses reaches a conservative maximum of $\tau_{Ed} \leq 0.3 f_{ck}$. With exceeding this limit, failure of the compressive strut can be observed.

6.1.3 How should structures reinforced with bent up bars be assessed?

Different design and assessment approaches have been examined in this report. Based on the yielding strength of reinforcement, truss models, the graphical method and Sorenson method are examined. All of these approaches are found applicable by respecting the shear strength restriction of $\tau_{Ed} \leq 0.1 f_{ck}$. In structures for which this limit is exceeded, the graphical method and Sorenson method are competent. However, with application of latter methods the maximum shear strength should being limited to $\tau_{Ed} \leq 0.3 f_{ck}$ to avoid failure of the compressive strut.

Design methods based on the compressive strength of reinforced concrete appears both to be competent for the assessment of bent up bars. The lower bound approach $\tau_{Ed} \leq 0.1 f_{ck}$ as found in the Eurocode gives conservative results. The more elaborated lower bound expressions proposed by Mattock et al. appears to give also conservative results for specimen reinforced with bent up bars.

Besides the assessment of the shear strength, additional attention should be given to these details in concrete structures reinforced with bent up bars: synchronization of amounts of shear and flexural reinforcement, effects of concrete cover spalling and the detailing of end anchorages (especially for plain reinforcement bars).

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ANNEX A: OVERVIEW DATASET

Throughout the report, different specimen were employed to analyse the shear strength of bent up bars. A description of specimen characteristics are given below, references can be found in the main report.

Description:
Specimen from Bach [51] are the oldest considered in the report. In contrast to the large amount of shear reinforcement in latter experiments Saliger [25], [26] performed experiments with a variable number of bent up bars and smaller web widths. While most experiments were performed with T-shaped cross sections, Richart [21] performed tests with rectangular beams. Uchimura [29] analysed effects of stresses in bent up bars subjected to point loads with small a/d-ratios. Similar tests are performed by Walther [30] in mid 50's with higher strength concrete classes. Modern researches by Huber[38] analysed beams with small amounts of shear reinforcement ratios. Shoukry et al. [27] analysed the effect of a/d-ratio on the shear strength.

A summary of the dataset and variation of parameters are shown in tabulated form below.

Table A1: Overview of collected specimen reinforced with bent up bars

Author	**Year**	**Shape**	**Nr. of tests**	A_{BU}/A_{sl}	a/d	f_{ck}	b_w
			[-]	[-]	[-]	[Mpa]	[mm]
Bach	1912	T	23	0.45-0.90	2.94	24.2	200
Richart	1927	R	59	0.40-0.86	3.60-4.80	20.9-27.3	203
Saliger	1913	T	16	0.00-0.80	3.25-3.42	24.3-29.4	160
Saliger	1915	T	16	0.00-0.75	3.29	24.3-29.4	75-160
Uchimura	1922	T	2	0.50	1.71	27.1-29.0	203
Walther	1956	R	1	0.40 *)	1.52	42.8-49.6	260
Shoukry et al.	2023	R	3	0.50	2.50**)	37.0-43.5	120
Huber	2023	R	6	0.00-0.75	3.50	38.6-41.7	500

*) specimen contains additional stirrups
**)specimen B5, B7,B12

ANNEX B: EXPRESSIONS TRUSS MODEL METHODOLOGY

This Annex presents the derivation of design expressions for inclined shear reinforcement based on the truss methodology. More detailed elaboration can be found in [28][52] or [53]

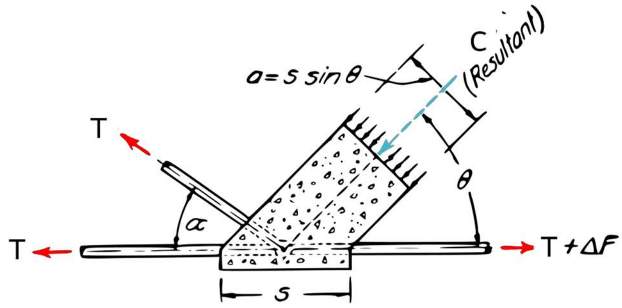

Figure B1: Generalized truss node loaded by tensile ties and compressive strut. After [28] *.*

Step 1:
Horizontal and vertical equilibrium:

$\Sigma F_x = 0: \quad T * \cos(\alpha) + C * \cos(\theta) = \Delta F$ $\Sigma F_y = 0: \quad T * \sin(\alpha) - C * \sin(\theta) = 0$ $Where$: $T = f_{ywd} * A_{sw}$ $C = f_{cd} * b_w * s * \sin(\theta)$ $\Delta F = V_{Ed} * \cot(\alpha) = V_{Ed} * \frac{s}{z}$	eq.B1 eq.B2

Step 2:
Rewriting eq2 and substitution into eq1:

$\Delta F = C * \left(\frac{\sin(\theta)}{\sin(\alpha)} * \cos(\alpha) + \cos(\theta)\right) \quad \rightarrow$ $\tau_{Rdc} = f_{cd} * \sin^2(\theta) * (\cot(\alpha) + \cot(\theta))$ $\Delta F = T * \left(\cos(\alpha) + \frac{\sin(\alpha)}{\sin(\theta)} * \cos(\theta)\right) \quad \rightarrow$ $\tau_{Rds} = \rho_{sw} * f_{ywd} * \sin^2(\alpha) * (\cot(\alpha) + \cot(\theta))$ $Where$: $\tau_{Rdc} = \frac{V_{Rdc}}{b_w * z}$ $\tau_{Rds} = \frac{V_{Rds}}{b_w * z}$ $\rho_{sw} = \frac{A_{sw}}{s * b_w * \sin(\alpha)}$	Eq.B3 Eq.B4

Step 3:
A ductile failure mechanism is guaranteed as reinforcement starts to yield after failure of the concrete strut. Result, shear strength of reinforcement should at least exceed shear strength of concrete. Finding the condition at which shear strength of concrete and reinforcement are equal.

$\tau_{Rdc} = \tau_{Rds} \quad \leftrightarrow \quad \frac{\sin^2(\theta)}{\sin^2(\alpha)} = \frac{\rho_{sw} * f_{ywd}}{f_{cd}} \ (= \omega)$	Eq.B5

Step 4:
From geometry follows eq.B6. Comparison with eq.B5 and working out results in eq.B7.

$\sin^2(\theta) = \frac{1}{1+\cot^2(\theta)} = \omega * \sin^2(\alpha)$	EqB.6
$\cot(\theta) = \sqrt{\frac{1-\omega * \sin^2(\alpha)}{\omega * \sin^2(\alpha)}}$	EqB.7

Step 5:
Validation of expression Eq.B4 for vertical stirrups (α=90°), figure B.2a. Effect of applications of expressions Eq.B4 and Eq.B7 for vertical stirrups (α=90°)and bent up bars (α=45°), figure B2b.

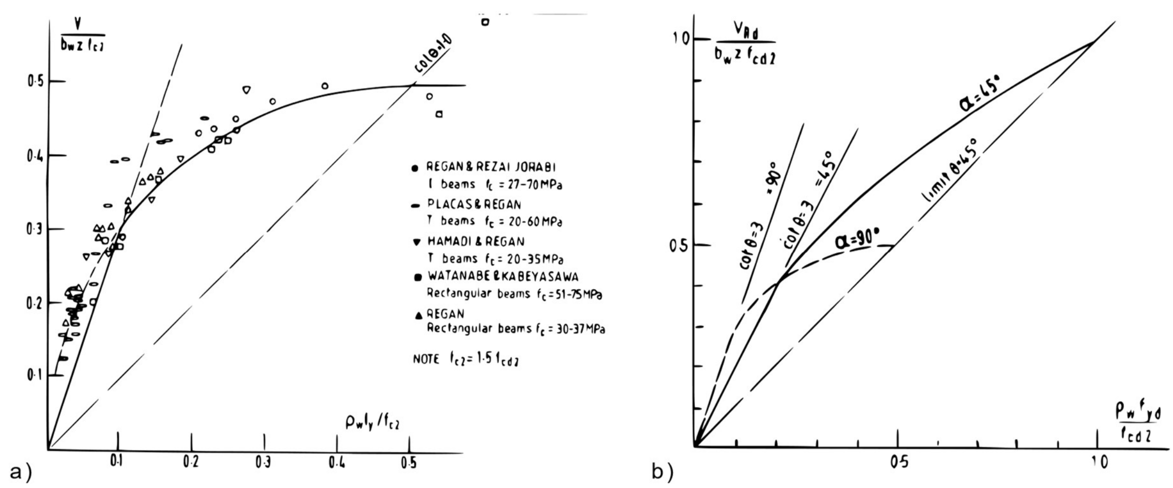

Figure B2: Validation of shear strength expressions for (a) vertical stirrups, and (b) shear strength compared to inclined stirrups. From [53] *.*

ANNEX C: DETAILS FINITE ELEMENT MODEL

With the software package Diana fea (release 10.5) a finite element model is build to validate the mechanism described in section 4.1 of the main report. This annex describes the characteristics of the modelling procedure in more detail. This annex closes with figures of distributions principle strains and stresses obtained from the Diana.

Model description:
A 2D rectangular plate (125*80) with an embedded circular section (ø20) are modelled representing a supporting concrete strut and reinforcement bar along the strut axis respectively. The reinforcing bars is modelled asymmetric in the plate to analyse effects of concrete cover on the distribution of stresses over the plate. The reinforcement bar is fixed in x and y-direction in its centre point (50,25), while the plate is fixed in a y-direction along the top edge (y=125) and in x-direction in (50,125), see figure C1a

Material properties of concrete (C20, normal weight), including smeared cracking, are based on fib Modelcode2010 [35]. For reinforcement linear elasticity is considered (E=210000, v=0.3).

Between the concrete plate and bar an interface is modelled to mimic friction and adhesion effects between reinforcement and concrete. The two-dimensional line interface has linear material properties with a large normal stiffness modulus (1E+13N/mm3) to neglect effects of deformations between concrete and steel in normal compressive direction. The shear stiffness modulus is taken conservatively to 1.6N/mm3. This value is based on the cohesion value of pull-out tests with plain bars obtained from (fig.2, p.668)[36](tests from Abrams,1913), see figure C1b. Also, the tensile strength for tensile stresses perpendicular to the reinforcement bar is set equal to 1.6Mpa. For Mode-1 brittle cracking behaviour is considered, for Mode II zero shear traction.

The mesh consists of triangular element plain strain elements (T6EPS) with an element size of approximately 3mm. A step-wise application of prescribed displacements along the top edge gives with trial and error approach a suitable value for the maximum applicable displacement at failure (here: uy,max=0.044mm).

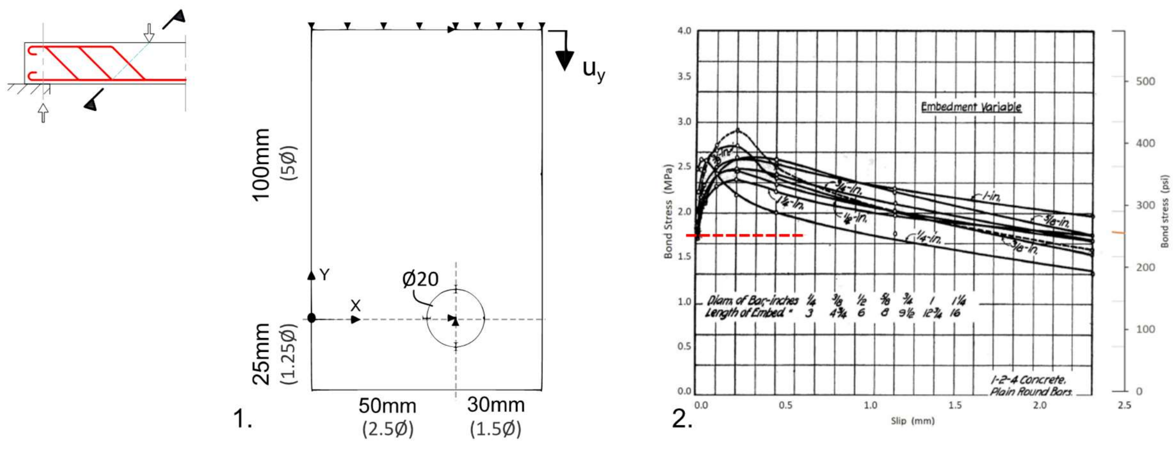

Figure C1: (1) Overview of geometry finite element model, and (2) diagram with bond-slip plots plain reinforcement used for the definition of interface adhesion. Figure taken from (fig.2, p.668)[36](tests from Abrams,1913)

Outcomes of finite element analysis:
The eccentricity of the reinforcement bar in the concrete seems to have no influence on the transfer of shear stresses from the strut to the reinforcement bar. Also, the adhesion effect modelled with the interface reveals no large effect on the experimental outcomes.

The shear stiffness modulus of the interface appears to have large effects on the total load carrying capacity of the system. Despite, since the research focusses on the qualitative distribution of stresses instead of the quantitative assessment of the maximum load carrying capacity, further attention to value for shear friction modulus have not been given.

Resulting distributions of principle stresses are shown in figure C2. Detailed distributions of principle strains E1, E2 and absolute values of shear strains Gxy around the reinforcement bar can be found in figure C3.

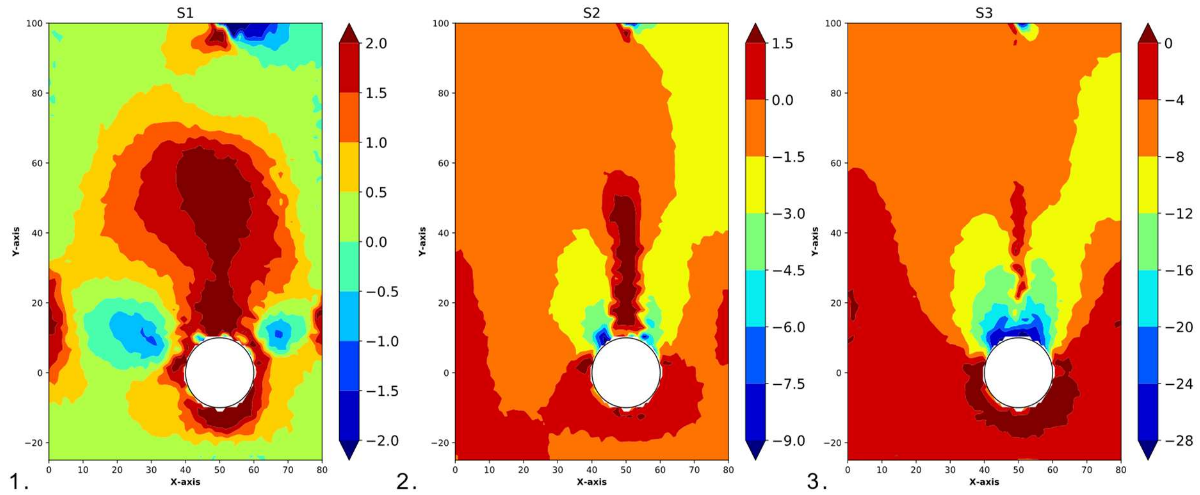

Figure C2: Principle stresses S1, principle stresses S2, and principle stresses S3 (in out-of-plane direction).

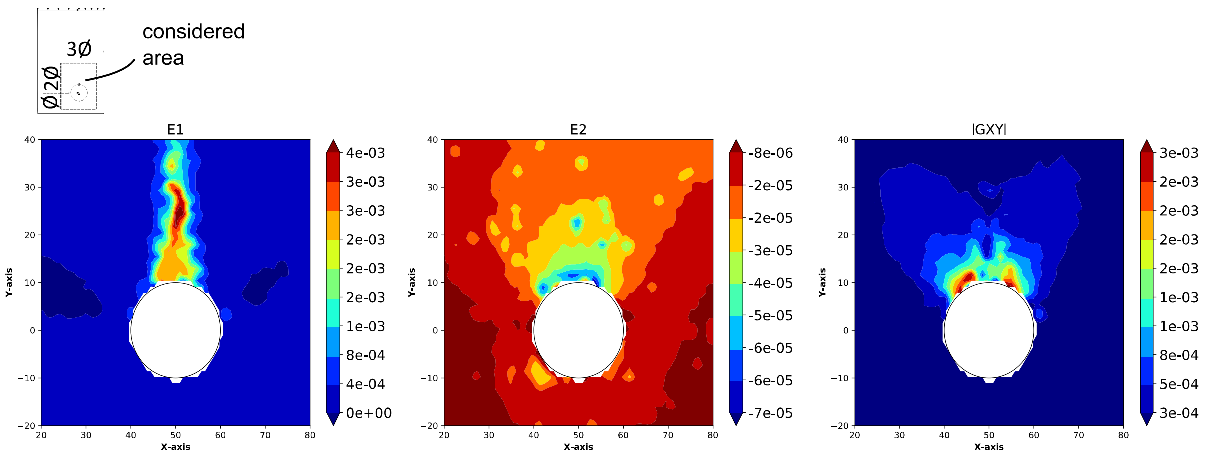

Figure C3: Principle strains E1, principle strains E2, and absolute values of shear strains Gxy.