Concrete walls

The evaluation of the reinforcement design methods in D-regions around openings and overhangs

By

Jalal Fitoury

Graduation report presented to the faculty of Civil Engineering and Geosciences

In fulfillments of the requirements Master of Science in Engineering

By Jalal Fitoury

May 10, 2009

Members of the thesis committee:

Prof. dr. ir. J.C. Walraven

Section Structural and Building Engineering

Faculty Civil Engineering and Geosciences, Delft University of Technology

Prof. dipl. -ing. J.N.J.A. Vamberský

Section Structural and Building Engineering

Faculty Civil Engineering and Geosciences, Delft University of Technology

Dr. ir. P.C.J. Hoogenboom

Section Structural Mechanics

Faculty Civil Engineering and Geosciences, Delft University of Technology

Ir. F.H. Middelkoop

Directing manager, Corsmit Consulting Engineers

Ir. J.A. Bunkers

Department Building Control, The Municipality of Rotterdam

Ir. L.J.M. Houben

Section Road and Railway Engineering

Faculty Civil Engineering and Geosciences, Delft University of Technology

Acknowledgement

Special thanks to my supervisors, Prof. dr. ir. J.C. Walraven, Prof. dipl. -ing. J.N.J.A. Vamberský, Dr. ir. P.C.J. Hoogenboom, Ir. F.H. Middelkoop and Ir. J.A. Bunkers. I would like to show my gratitude for their guidance and for sharing their specialized knowledge with me. I would also like to thank Ir. J.A. Den Uijl for his important assistance.

Finally, I would like to thank my family for their support during this research.

Jalal Fitoury
May 10, 2009
Delft, The Netherlands

Abstract

Unlike for concrete beams and plates, relative little amount of theory is present for concrete walls. In combination with the underexposure of the reinforcement design of D-regions in the codes when concrete walls are considered, engineering companies struggle to find a clear reinforcement design procedure which is agreed upon by all relevant parties.

This report will therefore focus on the reinforcement design of D-regions around openings and overhangs for concrete walls loaded by a high distributed load. For this purpose 27 reinforcement designs are given in accordance with the Dutch Code NEN6720, using the Beam Method, and in accordance with the Eurocode EN 1992-1-1, using the Strut-and-Tie Method. The designs with the Dutch Code represent the way these cases are designed in practice, while the designs with the Eurocode show how these cases will be designed in the future. As a last step the designs are analyzed with ATENA using the Non-Linear Elastic Finite Element Method.

The goal of this report is to clarify whether both design methods are good methods for the reinforcement design of concrete walls with an opening or an overhang with the specifications as described in chapter 2. Furthermore the weaknesses of both methods are indicated and recommendations are given on how to improve the designs.

<u>Conclusions</u>

Both the Beam Method (Dutch Code) and the Strut-and-Tie Method (Eurocode) are conservative methods for the cases considered. This is caused by three phenomena:

1) Both methods do not take into account the contribution of the concrete tensile strength in the wall.
2) Both methods do not take into account that the reinforcement mesh will ensure a second load path for the tensile stresses in the wall when the concrete is cracked.
3) a) For the opening cases both methods do not take into account the positive effect of the large concrete mass which is present at both sides of the opening. This concrete mass will offer resistance in the increase of the opening, such that the tension tie will not be stressed as calculated by the codes. When less concrete mass is present at the sides of the opening, this effect will be smaller so that the stress in the tie will increase.
   b) For the overhang cases both methods do not take into account the positive effect of the concrete mass which is present over a large height above the tie. Due to the presence of this concrete mass, the tensile stresses in the concrete will be distributed over a large height so that the stresses in the tie will be less than calculated by the codes. When less concrete is present above the tie, this effect will be smaller so that the stress in the tie will increase.

In the cases considered the ULS condition is determined by crushing of the concrete at the corners of the openings. Increasing the amount of reinforcement in the ties for the cases considered will therefore not contribute to a higher load carrying capacity of the wall. In order to reduce the concentrated stress in these opening, the sharp edges in the corners should be round off so that the stress is distributed more equally. When a higher load carrying capacity of the wall is still desired, compression reinforcement can be added in order to decrease the compression stresses in the concrete.

The traditional design steps that start by defining the concentrated reinforcement ties and end with defining the minimum reinforcement mesh needed in the wall, might lead to a considerable amount of unnecessary reinforcement. The reinforcement mesh, which is not incorporated in the calculations for determining the reinforcement area needed in the ULS condition, will have a considerable contribution in transferring the forces in the wall. A better order in the design of concrete walls is to first determine the minimum reinforcement mesh needed in the wall. Then, if necessary, reinforcement can be added in order to meet the requirements in the SLS and ULS condition.

This research shows that different truss models, which lead to different reinforcements, will not result in a considerable change in the reaction of the wall. The designer should not make an effort in enhancing a truss model, which results in a complex system. It is better to choose a simple truss model that clarifies the force flow throughout the truss, as long as the truss model follows the overall image drawn by the flow of forces in the wall.

Detailing of the reinforcement is of big importance in order to transfer the loads trough the truss.

When a stress trajectory plot is used, the Eurocode calls upon the knowledge of the designer in order to translate the stress trajectory plot into a truss model that represents the flow of forces.

The ultimate load on a wall, according to the calculations used in practice, is limited by the τ_2-check in such a way, that it is only a function of the design concrete compressive strength, the thickness of the wall and the angle of the shear reinforcement.

Recommendations

The designer should realize that the crack width calculations presented in the codes are based on a bar loaded in tension. For walls these calculations do not provide solid ground for the determination of the crack widths.

In D-regions the maximum allowable stress in the struts is set to approx. 50% of the concrete design compressive strength according to the Strut-and-Tie Method of the Eurocode. Outside the D-region the maximum allowable stress is equal to the concrete design compressive strength. At the border of the D-region this causes a compatibility problem. Efforts should be made in order to understand the consequence of these compatibility problems.

In order to apply the Strut-and-Tie Method, it is important to have knowledge of the dimensions of the D-region for which the Strut-and-Tie Method is applicable. Research should be performed in order to achieve guidelines for the determination of this D-region in walls.

When high loading is applied, the struts in the Strut-and-Tie Method may become very large. In reality the compression stresses may not be distributed over such a large width. This might lead to a distorted image of the force flow through the D-region. Efforts should be made in order to determine the maximum width of the compression struts.

Table of contents

ACKNOWLEDGEMENT **I**

ABSTRACT **III**

TABLE OF CONTENTS **V**

1. INTRODUCTION **1**

2. SPECIFICATIONS **3**

2.1 THE CODES 3
2.2 THE LOADS 3
2.3 THE MATERIAL AND ENVIRONMENT 3
2.4 THE CASES 3

3. THE DESIGN METHODS **5**

3.1 THE DUTCH CODE 5
3.1.1 Backgrounds of the Beam Method for deep beams 5
3.1.2 Design steps 5
3.1.3 Comments on the design steps 9
3.2 THE EUROCODE 10
3.2.1 Backgrounds of the Strut-and-Tie Method 10
3.2.2 Design steps 11
3.2.3 Analytical investigation on optimizing different truss models 15
3.2.4 Summary 36

4 THE DESIGNS **45**

4.1 THE OVERHANG OF 1 M 45
4.1.1 The Dutch Code 45
4.1.2 The Eurocode, truss model 1 46
4.1.3 The Eurocode, truss model 2 46
4.1.4 The Eurocode, truss model 3 47
4.2 THE OVERHANG OF 5 M 48
4.2.1 The Dutch Code 48
4.2.2 The Eurocode, truss model 1 48
4.2.3 The Eurocode, truss model 2 49
4.2.4 The Eurocode, truss model 3 50
4.3 THE OVERHANG OF 10 M 51
4.3.1 The Dutch Code 51
4.3.2 The Eurocode, truss model 1 51
4.3.3 The Eurocode, truss model 2 52
4.3.4 The Eurocode, truss model 3 53
4.4 THE OPENING OF 1 M 54
4.4.1 The Dutch Code 54
4.4.2 The Eurocode, truss model 1 54
4.4.3 The Eurocode, truss model 2 55
4.4.4 The Eurocode, truss model 3 56
4.4.5 The Eurocode, truss model 4 56

4.5 THE OPENING OF 5 M 57
4.5.1 The Dutch Code 57
4.5.2 The Eurocode, truss model 1 58
4.5.3 The Eurocode, truss model 2 58
4.5.4 The Eurocode, truss model 3 59
4.5.5 The Eurocode, truss model 4 59
4.6 THE OPENING OF 10 M 60
4.6.1 The Dutch Code 60
4.6.2 The Eurocode, truss model 1 61
4.6.3 The Eurocode, truss model 2 61
4.6.4 The Eurocode, truss model 3 62
4.6.5 The Eurocode, truss model 4 62
4.7 COMPARING THE DIFFERENT DESIGNS 64

5. EVALUATION OF THE DESIGNS WITH ATENA 65

5.1 PRE-PROCESSING 65
5.1.1 Material properties 65
5.1.2 Mesh generation 66
5.1.3 The Loading 66
5.2 POST-PROCESSING 68
5.2.1 Simulation of the cases with unreinforced concrete 68
5.2.2 Simulation of the designs with a concrete tensile strength approaching zero 69
5.2.3 Investigating the importance of the reinforcement tie 72
5.2.4 Stress distribution 75

6. CONCLUSIONS AND RECOMMENDATIONS 81

6.1 CONCLUSIONS 81
6.2 RECOMMENDATIONS 82

REFERENCE 83

APPENDIX 85

1. Introduction

Unlike for concrete beams and plates, in practice no clear design procedure is present for the reinforcement design of D-regions in concrete walls. This is caused by the relative little amount of theory which is present for concrete walls in combination with the underexposure of this subject in the codes.

This report therefore focuses on the reinforcement design in D-regions around openings and overhangs for concrete walls loaded by a high distributed load. For this purpose the reinforcement design is performed multiple times in accordance with the Dutch Code NEN6720 and the Eurocode EN 1992-1-1. The designs with the Dutch Code using the Beam Method represent the way these cases are designed in practice. The designs with the Eurocode using the Strut-and-Tie Method are given in order to show how these cases will be designed in the future, as this code will be the European standard from March 2010. As a last step the designs will be analyzed with ATENA using the Non-Linear Elastic Finite Element Method (FNL-FEM).

In the first part of this report the design methods will be discussed. The design steps of each code will be presented after which comments on these design steps will be given. For the Eurocode an analytical investigation is performed in order to examine how the various parameters of different truss models can be optimized.

In the second part of this report a total of 27 designs will be given. 6 designs, representing 1 design per case, are made with the Dutch Code. 21 designs, representing several designs per cases, are made using the Eurocode.

In the last part of this report the designs are evaluated with ATENA. By means of ATENA it is investigated how the different elements of the design influence the performance of the wall. Based on this investigation conclusions are drawn.

The goal of this report is to clarify whether both design methods are good methods for the reinforcement design of concrete walls with an opening or an overhang with the specifications as described in the next chapter. Furthermore the weaknesses of both methods are indicated and recommendations are given on how to improve the designs.

2. Specifications

This report focuses on the evaluation of the reinforcement design methods. The starting points for this research are summed below.

2.1 The Codes

Two codes are considered in this investigation. The first code considered is the Dutch Code NEN6720, which is based on the Beam Theory. In practice engineering companies use this code for the design of concrete structures. The second code considered is the Eurocode EN 1992-1-1, which will be the European standard for concrete structures from March 2010. For the design of Discontinuity-regions, the Eurocode is based on the Strut-and-Tie Method Theory.

2.2 The loads

The evaluation of the reinforcement design methods will be investigated under high vertical distributed load on the wall equal to 1900 kN/m for the overhang cases and 2400 kN/m for the opening cases in the Ultimate Limit State. These high loads incorporate the total dead load and live load on the wall. The proportion of the dead load and live load is estimated to be 50%-50%. According to both NEN6720 and EN 1992-1-1, the safety factor for dead load and live load is respectively 1.2 and 1.5. Assuming the proportions as described above, the overall safety factor becomes: $1.2 \cdot 0.5 + 1.5 \cdot 0.5 = 1.35$. In this research other loads, such as horizontal wind load, in plane and perpendicular to the plane of the wall, are left out of consideration. Phenomena perpendicular to the plane of the wall, such as the lateral instability of the wall, are also left out of consideration. Furthermore the stiffness of the supports is assumed to be infinitely stiff.

2.3 The material and environment

The concrete used for this research is of class C35/45 and B45 for respectively the Eurocode and the Dutch code. The reinforcement used is of type FeB500. Because the design is based on walls in the inner side of the building, the environment class is chosen equal to XC1 and 1 for respectively the Eurocode and the Dutch code. Time effects, such as shrinkage and creep, are left out of consideration in this research as they are among other things influenced by the fabrication method.

2.4 The cases

In this research we focus on the reinforcement design in openings and overhangs in walls. In conferring with the committee members it is decided to consider walls with heights (h) and widths (b) much bigger than the dimensions of the opening or overhang (ℓ). For both types of geometrical discontinuities multiple dimensions are investigated equal to 1 m, 5 m and 10 m. The thickness of the wall is equal to 300 mm.

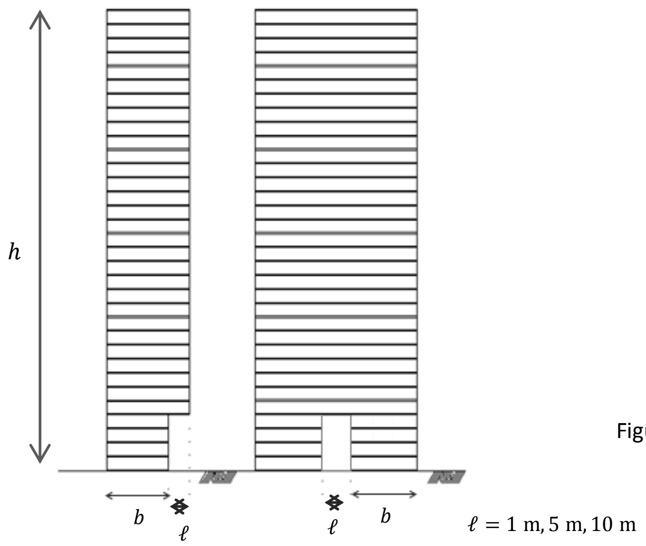


Figure 1: The cases considered

3. The design methods

3.1 The Dutch code

In practice a lot of engineering companies think in terms of the Dutch code. In order to give an impression of the calculations in practice, the calculations should be performed according to the Dutch code NEN6720 for concrete structures.

In the following chapters the background of the Beam Method for deep beams will be given in a concise manner. Then the design steps according to the Dutch code for concrete structures NEN6720 (Ref. 4) will be given. Finally comments will be given on the calculation method.

3.1.1 Backgrounds of the Beam Method for deep beams

The Dutch Code NEN6720 calculates deep beams by using the bending theory for thin beams on which a correction is applied due to the influence of the vertical shear stress. For determining this correction, which leads to a reduction for the accountable shear force, theories are used in accordance with the methods applied by Magnel, Paduart, Brendel, and others (Ref. 2). These theories are implemented in the GBV 1962, the VB 1974/1984 and NEN6720 (VBC 1990/1995).

3.1.2 Design steps

Step 1) Determining the material properties

Determining the material properties consist of the determination of the concrete properties according to art. 6.1 and the reinforcement properties according to art. 6.2 of the code.

The design concrete compressive strength is defined as:

$$f_b' = f_{brep} / \gamma_m$$

With:

$f_{brep}' = 0.72\, f_{ck}'$ with f_{ck}' is the cubic compressive strength according to table 3 of the code.

γ_m is the partial safety factor for concrete equal to 1.2 in the ULS and 1.0 in the SLS.

The design concrete tensile strength is defined as:

$$f_b = f_{brep} / \gamma_m$$

With:

$f_{brep} = 0.7\,(1.05 + 0.05\, f_{ck}')$

γ_m is equal to 1.4 in the ULS and 1.0 in the SLS.

The average tensile strength of the concrete is defined as:

$$f_{bm} = 1.4\, f_{brep}$$

The design average tensile strength of the concrete subjected to bending is defined as:

$$f_{br} = (1.6 - h)\, f_{bm} \nless f_{bm}$$

With:

h is equal to the height in m.

The design compressive strength of the reinforcement is equal to the design tensile strength of the reinforcement. The tensile strength of the reinforcement is defined as:

$$f_s = f_{srep} / \gamma_m$$

With:

f_{srep} is the representative value of the tensile strength according to table 12 of the code.

γ_m is the partial safety factor for the reinforcement equal to 1.15 in the ULS and 1.0 in the SLS.

Step 2) Specifications for the deep beam

Art. 8.1.4 states that a beam is considered to be a deep beam when it fulfills the requirement

$$l_{ov}/h \leq 2.0$$

With:

l_{ov} equal to the distance between to field points where the moment is zero.

h equal to the height of the beam.

The internal lever arm for a simply supported deep beam is equal to:

$$z = 0.2\,l + 0.4\,h \not> 0.6\,l$$

The internal lever arm for a console is equal to:

$$z = 0.2\,l + 0.4\,h \not> 0.8\,l$$

With:

l is equal to the span for the simply supported beam; for the corbel $l = 2\,a$.

a is the distance between the resultant of the loading and the point $L/4 \not> h/4$ of the inner side of the support.

L is the length of the corbel.

h is the maximum height of the corbel.

For the cases described in chapter 2, the height of the wall $h \gg l$, such that for the internal lever arm calculation it follows that $z = 0.6\,l$ for the simply supported beam and $z = 0.8\,l$ for the corbel, with l defined as described above.

Step 3) Determining the reinforcement to satisfy the bending moment condition

In art. 8.1.4. it is stated that the ultimate moment of the deep beam is calculated as:

$$M_u = A_s\, f_s\, z$$

By the requirement that the design moment has to be smaller or equal to the ultimate moment, the reinforcement can be determined. This reinforcement has to fulfill the minimum reinforcement requirement needed to carry the load when the concrete cracks. From art. 9.9.2.1. the cracking moment is determined according to:

$$M_r = 1.4\, f_{bm}\, W$$

With:

$W = \frac{1}{6}\, b\, h^2$ is the sectional modulus.

Because the height of the wall $h \gg l$, in practice it is not expect that the total height of the wall will be activated for to the cracking moment. As will be shown in step 4, art. 8.2 states that for the shear stress calculation of the deep beam $d = h$. Because $h \gg l$, in practice it is not expected that the total height of the wall will be activated due to the shear force at the discontinuity. Therefore the designer chooses to calculate d by using the equation $z = 0.8\,d$ (which is common for ordinary beams). To calculate the cracking moment, the height of the deep beam is determined as $h = d + c + \frac{1}{2}\,\emptyset_k + \emptyset_{st}$. The concrete cover follows from art. 9.1 of the code.

The anchorage length of the reinforcement is calculated according to art. 9.6.2. The basic anchorage length is formulated as:

$$l_{vo} = \alpha_1\, \emptyset_k\, f_s/\sqrt{f_b'}$$

With:

$\alpha_1 = 0.4\,(1 - 0.1\frac{c}{\emptyset_k}) \nless 0.24$	for ribbed steel, with c equal to the concrete cover.
$\emptyset_k$	is the diameter of the reinforcement bar.

When the reinforcing bar has a diameter smaller or equal to 25 mm, the anchorage length l_v is equal to the basis anchorage length l_{vo}, otherwise $l_v = 1.25\, l_{vo}$.

In art. 9.6.2. it is stated that the anchorage length may be reduced when the stress in the reinforcement bar is smaller than the representative tension strength of the bar. The reduced anchorage length is calculated according to:

$$l_{vr} = \frac{\sigma_{sd}}{f_s}\, l_v$$

Step 4) Determining the shear reinforcement

The shear stress in the critical cross-section is calculated according to:

$$\tau_d = \frac{V_d}{b\, d}$$

Art. 8.2 states that for deep beams $d = h$. As already mentioned in step 3, in practice designers choose $d = \frac{1}{0.8}\, z$, which is commonly the case for ordinary beams.

The ultimate shear stress of the concrete is calculated according to art. 8.2.3:

$$\tau_1 = 0.4\, f_b\, k_h\, k_\lambda\, \sqrt[3]{\omega_0} \nless 0.4\, f_b$$

With:

$k_\lambda = \frac{12}{g_\lambda} \sqrt[3]{\frac{A_0}{b\, d}} \nless 1$	for corbels and beams where between the loading and the support a compression diagonal can be formed. Otherwise $k_\lambda = 1$.
$g_\lambda = 1 + \lambda_v^2$	for $\lambda_v \geq 0.6$
$g_\lambda = 2.5 - 3\,\lambda_v \nless 1.36$	for $\lambda_v < 0.6$
$\lambda_v = \frac{M_{dmax}}{d\, V_{dmax}}$	
M_{dmax}	is the absolute maxim value of the moment.
V_{dmax}	is the absolute maxim value of the shear force.
A_0	is the smallest area where the loading has to go through.
$k_h = 1{,}6 - h \nless 1{,}0$	with h in meters.
$w_0 = 100\,\frac{A_s}{b\, d} \ngtr 2.0$ and $\nless 0.7 - 0.5\,\lambda_v$	is the present percentage of the reinforcement to carry the moment.

The shear stress that has to be carried by the shear reinforcement is calculated according to art. 8.2.1:

$$\tau_s = \tau_d - \tau_1$$

The amount of vertical shear reinforcement is then calculated according to art. 8.2.4:

$$A_{sv} = \frac{\tau_s\, b\, d}{z\, f_s \sin\alpha\ (\cot\theta + \cot\alpha)}$$

With:

$45 \ngtr \alpha \ngtr 90$	is the angle between the shear reinforcement and the longitudinal axis of the deep beam. In art. 8.2.4 it is stated that for deep beams $\alpha = 90°$.
$30 \ngtr \theta \ngtr 60$	is the angle between the compression diagonal and the axis of the deep beam. For deep beams θ is calculated according art. 8.1.4.

In art. 8.2.4 it is also stated that for deep beams horizontal shear reinforcement should be applied. If $\lambda_v \geq 0.4$, the amount of horizontal shear reinforcement should be equal to the vertical shear reinforcement. Otherwise the horizontal shear reinforcement should be twice the vertical shear reinforcement. This horizontal shear reinforcement should be distributed over the height of the internal leverarm z.

In art. 8.2.1 it is stated that the shear stress may not exceed the τ_2-value. This τ_2-value is formulated as:

$$\tau_2 = 0.2\, f_b^{'}\, k_n\, k_\theta$$

With:

$k_n = 1$	when no prestressing is applied.
$k_\theta = 1$	for $\alpha = 90$ or when no shear reinforcement is applied. Otherwise k_θ is a function of angles θ and α. However this not the case for deep beams as $\alpha = 90$.

Step 5) Determining the additional reinforcement requirements

If the concrete compressive stress is bigger than the design concrete compressive strength $f_b^{'}$, splitting reinforcement should be applied according to art. 9.13.1.

According to art. 9.9.3.1, 20% of the reinforcement needed to satisfy the ultimate moment should be applied as a minimum distributed horizontal reinforcement.

The reinforcement applied at the discontinuity must be able to carry the total shear force at the section according to 9.11.7

Step 6) Crack width control

The crack width control is performed in accordance with art. 8.7. In the SLS the stress in the uncracked concrete has to be determined. If the concrete stress is smaller than the average tensile stress of the concrete f_{bm}, the check has to be performed according to art 8.7.3. which states that the concrete is in the crack formation stage. Otherwise the check has to be performed according to art. 8.7.2 which states that the concrete is in the crack stabilizing stage.

These checks are related to the diameter of the reinforcing bar $\emptyset_k$ and the center to center distance of the bars. These are a function of the environment class and the stress in the reinforcing bars.

Step 7) Drawing the reinforcement

Before drawing the reinforcement the additional detailing requirements according to the code are first determined.

According to art. 9.11.4 the center to center distance of the reinforcement, needed to carry the ultimate moment, should not be bigger than 150 mm in the critical cross-section. In the other cross-sections the center to center distance may not be smaller than 50 mm and bigger than 250 mm according to art. 9.11.2. According to art. 9.11.3 the reinforcement may be distributed over an height equal to $0.2\, l \ngtr 0.2\, h$, with l being the span and h the height of the wall. When the reinforcement is bend, the radius may not be smaller than two and a half times the diameter of the bar according to art. 9.6.3. The diameter of the bar itself must fulfill the minimum requirements according to art. 9.9.1.

The center to center distance of the stirrups may not be bigger than 300 mm according to art. 9.11.4.4. The same condition holds for the minimum distributed horizontal reinforcement according to art. 9.9.3.1.

The concrete cover should fulfill the requirements according to table 44 of art 9.2. In addition to table 44 it is stated that the concrete cover must be bigger than one and a half times the diameter of the bar, when this diameter is bigger than 25 mm.

3.1.3 Comments on the design steps

A part of the shear force calculation in NEN6720, is the τ_2-check according to art. 8.2.1. This check consists of the verification $\tau_d \leq \tau_2$, with τ_d being the shear stress due to the loading and $\tau_2 = 0.2\, f_b^{'}\, k_n\, k_\theta$.

The τ_2 expression gives the maximum allowable shear stress based on the maximum allowable stress in the compression strut (App. 1). The τ_2 expression includes the reduction of the compressive strength of the strut due to the transverse tensile stresses caused by the reinforcement and the fact that the shear reinforcement gives rise to local stress concentrations. The maximum allowable stress for the compression strut is for this purpose determined by experiments. The following imperative relation was found for the compression strut (Ref. 1):

$$\sigma_b^{'} = \upsilon * f_b^{'} \qquad \text{with: } \upsilon = 0.7 - \frac{f_{ck}^{'}}{200} \geq 0.5$$

When the stress in the compression strut is equal to the maximum allowable stress according to the imperative relation, it can be shown that the shear stress in that cross-section is equal to the τ_2-value (App. 1).

When a wall, with an opening or an overhang, has to be checked according the shear stress calculations of NEN6720, the τ_2-check has to be satisfied. In order to use the τ_2-check, in practice the designers schematize the wall as a deep beam according to art. 8.1.4. However the deep beam schematization according to art. 8.1.4 was originally not meant for these types of problems. As a consequence it turns out that due to the schematization according to art. 8.1.4, the ultimate load defined by the τ_2-check is not influenced by the dimensions of the opening or overhang (App. 2). The ultimate load is than only influenced by the thickness of the wall, the concrete compressive strength and the angle of the shear reinforcement according to the equation:

$$q_{ult} = \beta * b * f_b^{'} \qquad \text{with: } \begin{array}{ll} \beta = 0.6 * \dfrac{\frac{5}{8} + \cot\alpha}{1+\left(\frac{5}{8}\right)^2} & \text{for the overhang} \\ \beta = 0.75 * \dfrac{\frac{5}{6} + \cot\alpha}{1+\left(\frac{5}{6}\right)^2} & \text{for the opening} \end{array}$$

In practice the shear reinforcement in a wall will be placed vertically, such that $\alpha = 90°$. The ultimate load will therefore be $q = 0.27 * b * f_b^{'}$ and $q = 0.37 * b * f_b^{'}$ for resp. the overhang and the opening.

The reason that the ultimate load is not influenced by the dimensions of the opening or overhang, is because the angle of the compression diagonal in art. 8.1.4 is fixed. This means that the factor k_θ is fixed, such that the value of τ_2 is only influenced by the concrete compressive strength.

Another point that catches the eye in the design steps presented, is that the shear reinforcement is not a function of the span due to the schematization of the wall according to art. 8.1.4. Art. 8.2 states that the shear reinforcement is expressed by:

$$A_{sv} = \frac{\tau_s\, b\, d}{z\, f_s \sin\alpha\, (\cot\theta + \cot\alpha)} \qquad \text{with:} \quad \tau_s = \tau_d - \tau_1$$

τ_1 is the ultimate shear stress of the concrete according to art. 8.2.2 and is not a function of the span. Art. 8.2 states that $\tau_d = \frac{V_d}{b\, d}$ and d is chosen equal to 1/0.8 $\cdot\, z$ according to the calculations in practice. Because the angle θ is fixed according to art. 8.1.4, the relation between the internal leverarm z and the span L is fixed. Therefore the value of d will increase exactly the same amount as the value of V_d, such that the value of τ_d stays unchanged. The shear stress that has to be transferred by the shear reinforcement τ_s is not a function of the span, because both τ_d and τ_1 are not a function of the span. In the expression of the shear reinforcement A_{sv}, the values of τ_s, b, f_s, α and θ are fixed as well as the relation of d/z is fixed. Therefore it can be concluded that due the schematization of the wall according to art. 8.1.4 the shear reinforcement A_{sv} $[\mathrm{mm^2/m}]$ is not a function of the span.

3.2 The Eurocode

3.2.1 Backgrounds of the Strut-and-Tie Method

According to art. 5.6.4 of the Eurocode (Ref. 7) the design of discontinuity regions should be performed according to the Strut-and-Tie method. The predecessor of this method is the truss method developed by Ritter in 1899 for the design of reinforced concrete beams loaded by shear. In 1920 Mörsch extended this truss method for beams loaded by torsion. The method idealizes cracked reinforced concrete beams by representing the main reinforcing steel as the tension chord, the stirrups as the vertical tension web members, and the concrete between the inclined cracks as the compressive diagonals inclined at an angle of $45°$ with respect to the longitudinal axis of the beam (Ref. 9, 10).

In the last century the truss method is improved by refining and expanding the truss analogy by Kupfer in 1964 and Leonhardt in 1965. It was observed that the angle of the concrete compression struts is generally not 45 degrees, but ranges between 25 and 65 degrees depending on the reinforcement arrangement. In 1971 and 1983 Lampert and Thürlimann introduced the truss method with compressive diagonals inclined at a variable angle. The truss method is subsequently expanded by considering deformations for the design of concrete beams in shear by Collins and Mitchell in 1980. However the design according to this method could only be used in the regions where the Bernoulli hypothesis is valid, which states that plane cross-sections remain planar after deformation. These regions are called the B-regions.

Due to discontinuities (caused by e.g. point loads or abrupt changes in the geometry) the stress distribution will be non-linear such that the Bernoulli hypothesis will not be valid. In order to design these regions, which are called the D-regions, the Strut-and-Tie method is developed by Marti in 1985. This Strut-and-Tie method is a generalization of the truss method and can be applied for the designs of D-regions. The determination of the ultimate strength by the Strut-and-Tie method is based on the lower bound plastic theory (Ref. 9).

In 1987 Schlaich introduced the concept that a structure can be subdivided into B- and D-regions. The B-regions can be designed by traditional sectional methods or by using the truss method, whereas for the design of the D-regions the Strut-and-Tie method should be used. The advantage of this subdivision is that the designer can focus on the D-regions, which are the potential weak spots, in order to ensure sufficient strength performance. In figure 9 a beam is shown which is subdivided into B- and D-regions.

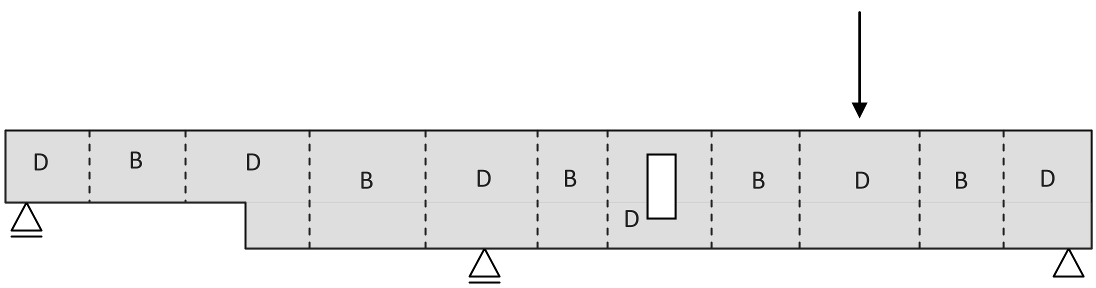


Figure 2: Subdivision of a beam into B- and D-regions

3.2.2 Design steps

Step 1) Determining the material properties

The first step is to determine the material properties of the structure. The material properties of the concrete can be determined according to art. 3.1, which are based on the characteristic cylinder strength (f_{ck}) determined at 28 days.

The mean compressive strength of the concrete at an age of 28 days is defined as:

$$f_{cm} = f_{ck} + 8 \text{ MPa}$$

The mean tensile strength of the concrete at an age of 28 days is defined as:

$$f_{ctm} = 0.30\, f_{ck}^{\frac{2}{3}} \qquad \text{for} \quad f_{ck} \leq 50 \text{ MPa}$$

$$f_{ctm} = 2.12\, \ln\left(1 + \frac{f_{cm}}{10}\right) \quad \text{for} \quad f_{ck} > 50 \text{ MPa}$$

The 5% fractile of the tensile strength of the concrete is defined as:

$$f_{ctk,0.05} = 0.7\, f_{ctm}$$

The design tensile strength and the design compressive strength of the concrete is resp. defined as:

$$f_{ctd} = \alpha_{ct}\, f_{ctk,0.05} / \gamma_c$$

$$f_{cd} = \alpha_{cc}\, f_{cm} / \gamma_c$$

With:

$\alpha_{cc} = 1{,}0$ taking into account the long term effects on the compression strength.

$\alpha_{ct} = 1{,}0$ taking into account the long term effects on the tensile strength.

γ_c is the partial safety factor for the concrete ($\text{ULS} = 1.5$ and $\text{SLS} = 1.0$).

The modulus of elasticity of the concrete is defined as:

$$E_{cm} = 22\left(\frac{f_{cm}}{10}\right)^{0.3}$$

The material properties of the reinforcing steel can be determined according to art. 3.2, which are based on the yield strength (f_{yk}). The design tensile strength of the reinforcing steel is defined as:

$$f_{yd} = f_{yk} / \gamma_s$$

With:

γ_s is the partial safety factor for the reinforcing steel ($\text{ULS} = 1.15$ and $\text{SLS} = 1.0$).

The modulus of elasticity for the reinforcing steel (E_s) is equal to 210000 MPa.

According to art. 6.5.2 the design strength for a concrete strut in a region with transverse compression stress or no transverse stress may be calculated according to $\sigma_{Rd,max} = f_{cd}$. The design strength of concrete struts with transverse tension is reduced to:

$$\sigma_{Rd,max} = 0.6\, v' f_{cd}$$

With:

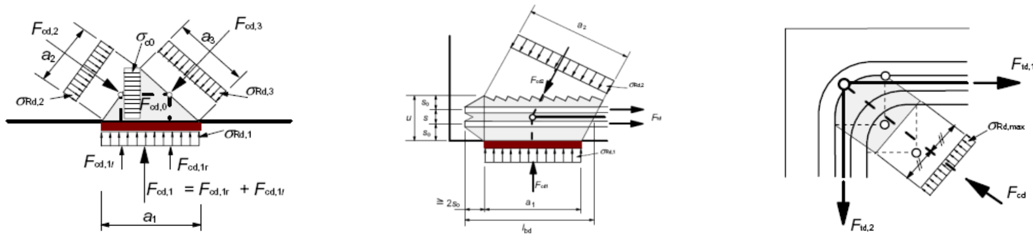
$v = 1 - \frac{f_{ck}}{250}$

According to art. 6.5.4 the design concrete compressive stress in the nodes is limited according to the loading on the nodes. A distinguishing is made between nodes loaded only by compression (C-C-C), nodes loaded in tension in one direction (C-C-T) and nodes loaded in tension in two or more directions (C-T-T). The maximum compressive strength of the nodes is calculated according to:

$$\sigma_{Rd,max,CCC} = v' f_{cd} \qquad \text{for nodes loaded by C-C-C}$$

$$\sigma_{Rd,max,CCT} = 0{,}85\, v' f_{cd} \quad \text{for nodes loaded by C-C-T}$$

$$\sigma_{Rd,max,CTT} = 0{,}75\, v' f_{cd} \quad \text{for nodes loaded by C-T-T}$$



a) C-C-C node b) C-C-T node c) C-T-T node

Figure 3: Nodes in Strut and Tie model

Step 2) Determining the minimum and maximum face reinforcement

According to art. 9.1 the minimum reinforcement is necessary in order to prevent brittle failure, wide cracks and to resist forces arising from restrained actions. According to art 9.7, deep beams should be provided with an orthogonal reinforcement mesh near each face in each orthogonal direction with a minimum equal to $A_{s,dbmin}$.

$$A_{s,dbmin} = 0.001\,A_c \not< 150\ \text{mm}^2/\text{m}$$

In addition to this, the centre to centre distance of the bars in the mesh should not exceed the minimum of two times the thickness of the wall and a distance of 300 mm.

According to art. 9.6 the maximum reinforcement mesh near each face in each orthogonal direction is equal to:

$$A_{s,dbmax} = 0.02\,A_c$$

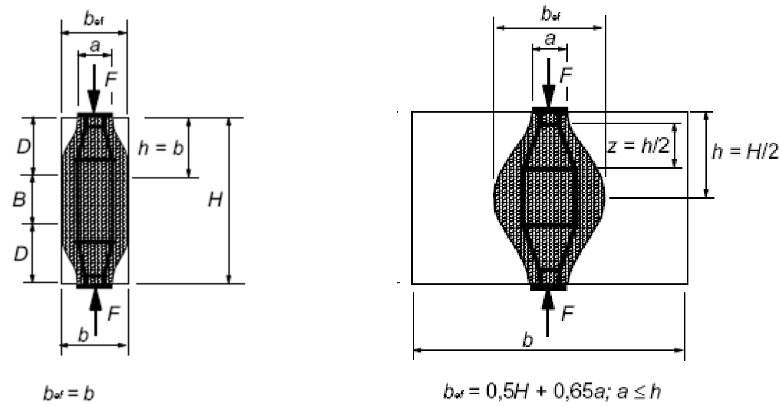
Step 3) Selecting a truss model

In this step an appropriate truss model must be chosen. In order to use this truss model for verification in the SLS, art. 5.6.4 states that compatibility of the Strut-and-Tie model must be ensured. In particular the position and direction of important struts should be oriented according to the linear elastic theory. After choosing a Strut-and-Tie model the space needed in the struts and ties due to the loading must be checked. Then the forces in the struts and ties is used in order to determine the reinforcement bars needed and to check the bearing capacity of the struts.

Due to curved trajectories in the struts, transverse tension stresses in the struts will occur. According to art. 6.5.3 the value of the tension force depends on the shape of the stress trajectories and the width of the member, or the available space around the struts. According to art. 6.5.3 the reinforcement required to resist the forces at the concentrated nodes may be smeared over a certain length. When the reinforcement in the node area extends over a considerable length of an element, the reinforcement should be distributed over the length where the compression trajectories are curved. The tensile force T may be obtained by:

$$T = \frac{1}{4}\frac{b-a}{b}\,F \qquad \text{for } b \le \frac{H}{2} \text{ (partial discontinuity regions)}$$

$$T = \frac{1}{4}\left(1 - 0.7\frac{a}{H/2}\right)F \qquad \text{for } b > \frac{H}{2} \text{ (full discontinuity regions)}$$



a) Partial discontinuity regions b) Full discontinuity regions

Figure 4: Parameters for determining the transverse tensile force

With this tension force T, one can calculate the reinforcement needed in the direction orthogonal to the strut direction. This reinforcement can be translated to a reinforcement mesh by using the strut angle θ such that it can be concluded whether the minimum reinforcement mesh calculated in designing step 2 is sufficient to carry this tension force T.

$$A_{s,db} = \frac{A_{s,\theta}}{2\,(\sin\theta + \cos\theta)} \not< A_{s,dbmin}$$

With:

$A_{s,dbmin}$	minimum face reinforcement calculated in designing step 2.
$A_{s,\theta}$	reinforcement needed in the direction orthogonal to the strut direction in order to carry the tension force T.
θ	the angle of the compression strut.

After calculating the reinforcement mesh needed for the struts, one has to calculate the reinforcement needed for the ties. The basic anchorage length needed for this reinforcement can be calculated according to art. 8.4.3.

$$l_{b,rqd} = \frac{\phi}{4}\frac{\sigma_{sd}}{f_{bd}}$$

With:

ϕ	is the diameter of the bar.
σ_{sd}	is the design stress in the bar.
$f_{bd} = 2{,}25\,\eta_1\,\eta_2\,f_{ctd}$	is the design value of the ultimate bond stress.
η_1	coefficient related to the bond condition (1.0 is for good bond condition).
η_2	coefficient related to the bar diameter (1.0 for $\phi \leq 32$ mm and $\frac{132-\phi}{100}$ for $\phi > 32$ mm).

According to art. 8.4.4 the design anchorage length is calculated according to:

$$l_{bd} = \alpha_1\,\alpha_2\,\alpha_3\,\alpha_4\,\alpha_5\,l_{b,rqd}$$

With:

α_1	coefficient related to the form of the bars.
α_2	coefficient related to the concrete minimum cover.
α_3	coefficient related to the confinement by transverse reinforcement.
α_4	coefficient related to influence of welded transverse bars.
α_5	coefficient related to the pressure transverse to the plane of splitting.

Step 4) Calculating the crack width

The crack width is calculated in the SLS according to art. 7.3.4.

$$w_k = s_{r,max}\ (\varepsilon_{sm} - \varepsilon_{cm})$$

With:

$$s_{r,max} = k_3\ c + k_1\ k_2\ k_4 \frac{\phi}{\rho_{p,eff}}$$

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 + \alpha_e\ \rho_{p,eff}\right)}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

ϕ	is the bar diameter. If a mixture of bar diameters is used $\phi = \frac{\Sigma\, n_i\, \phi_i^2}{\Sigma\, n_i\, \phi_i}$.
c	is the cover to the longitudinal reinforcement.
k_1	coefficient taking into account the bond properties.
k_2	coefficient taking into account the distribution of strain.
k_3	coefficient equal to 3.40 for the Netherlands.
k_4	coefficient equal to 0.425 for the Netherlands.
$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$	the effective height of the ties h_{eff}, in order to calculate $A_{c,eff}$, is according to art. 7.3.2 related to the concrete cover on the reinforcement by $h_{eff} = 2.5\ (h - d)$. This expression for h_{eff} can only be used when the tie lies along the edge of the member. If the tie lies inside the deep beam, an assumption is made for the effective height. If one layer of reinforcement is used $h_{eff} = 5\ \phi$, otherwise $h_{eff} = 2.5 *$ centre to centre distance of the reinforcing bars.
σ_s	is the stress in the tie reinforcement.
k_t	is a factor related to the duration of the load (0.6 for short term loading and 0.4 for long term loading).
α_e	is the ratio E_s / E_{cm}.
$f_{ct,eff} = f_{ctm}$	when cracks are expected after 28 days (the concrete is fully hardened).

Step 5) Drawing the reinforcement

Before drawing the reinforcement, the additional detailing requirements according to the code are first determined. This includes the minimum centre to centre distance of the bars according to art. 8.2 and 9.7. According to art. 8.3 the minimum mandrel diameter is determined such to avoid bending cracks in the bar and failure of the concrete inside the bend of the bar. If reinforcement bars are chosen with a diameter bigger than 32 mm, the additional requirements according to art. 8.8 need to be satisfied.

3.2.3 Analytical investigation on optimizing different truss models

It is observed that the Eurocode gives more freedom to the designer in comparison with the Dutch code, but at the same time requires from the designer to have detailed knowledge of the force flow within the system. This is especially the case when the designer has to choose an appropriate truss model for the Strut-and-Tie method. The code recommends choosing the angle of the compression diagonals in accordance with the stress trajectories of the linear elastic theory, when this model is used for calculations in the Serviceability Limit State. The code does not give more guidelines in this respect.

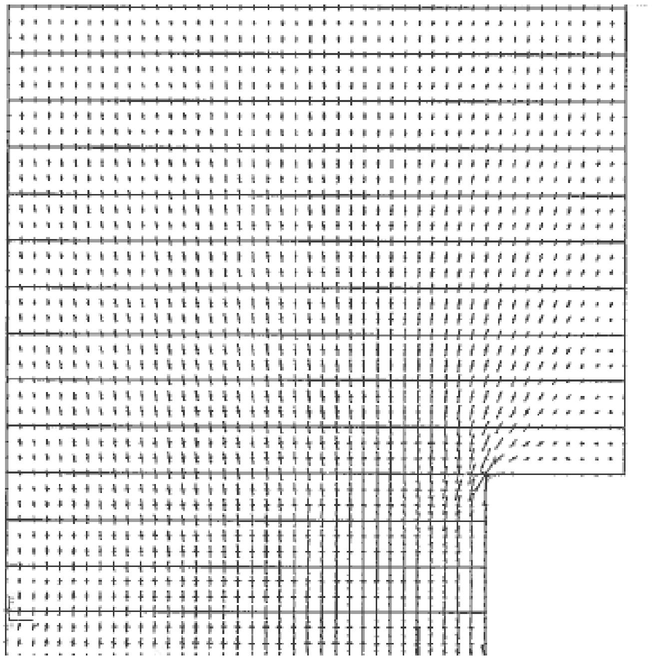


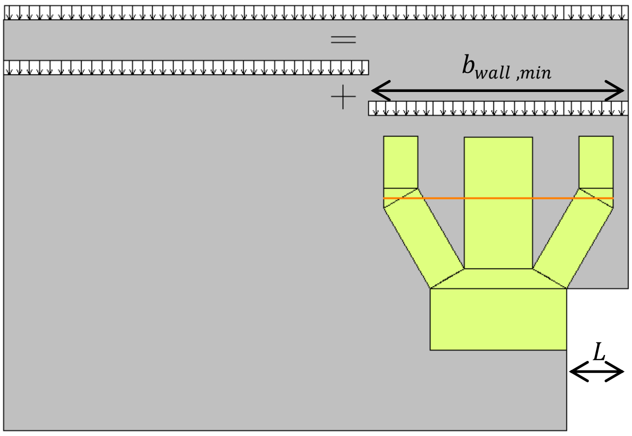
Figure 5: Plot of the stress trajectories for an overhang loaded by a distributed load

When the designer is confronted with this step, he has to make a decision on the amount of struts and ties that is to be used, where to place the connections between the struts and ties, and which value to choose for the angle, the height and the width of the struts and nodes. In most cases these decisions can not be obtained from a stress trajectory plot, as can be seen in the figure at the right side. The plot does give an indication for the orientation of the important struts, although it becomes more difficult to determine the angle of the compression diagonals when more diagonals are applied in the truss model. Because the loading on the truss results from a distributed load, these decisions influence the loading on the individual component of the truss which themselves influence the values of the thickness of the struts, the height of the struts, the thickness of the nodes, the amount of reinforcement needed in the ties, etc. This is a vicious circle which is solved by an iteration process. The iteration process can be described as following: first the designer chooses a truss model by determining the central axis of the components and the length of each component. Then the loading on the truss is calculated. After that, according to the maximum stresses in the struts, ties and nodes, the minimum dimensions of the struts and nodes are determined as well as the amount of reinforcement needed in the ties. If this does not fit in the geometry of the region, the positions of the struts, ties and nodes need to be changed. As a consequence, the loading on the truss will change. The dimensions of the individual elements are again calculated and related to the geometry of the region. This is repeated until the truss dimensions fit in the geometry. The designer can choose to stop and use this truss model, or continue in order to find dimensions of the truss that are more efficient in terms of the amount of reinforcement that is needed.

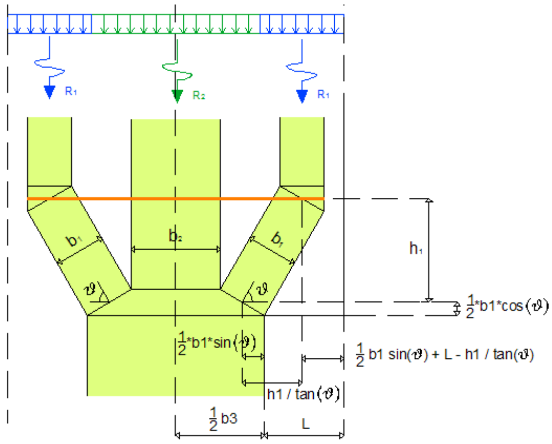
The main goal of this part of the investigation is to reduce the iterative procedure described above. This is done by considering different truss models for different cases. Guidelines are given in order to find the parameters of the truss models considered, which lead to an efficient design according to the Eurocode calculations.

For the overhang three truss models are considered, while for the opening four truss models are observed. For these truss models the iteration process as described above, will be reduced by expressing the parameters of the truss as a function of two variables which are determined in advance. Then with the help of the calculations in the Eurocode, the values of these variables are determined such that an efficient design is obtained. For one truss model these variables are determined using a stress trajectory plot of a linear elastic finite element program.

The overhang, truss model 1



a) Location truss model in the wall.



b) Parameters of the truss model

Figure 6: Truss model 1 for the overhang

The first truss model considered is shown in the figure above. It consists of one tie, two diagonal struts with both width b_1 and two vertical struts with widths b_2 and b_3. The variables of this truss model are the angle of the compression diagonals θ and the height of the compression diagonals h_1. The parameters of the truss and the forces on the truss can be expressed as a function of those two variables, the value of the distributed load and the dimensions of the overhang. This is done by considering the geometry requirements and the strength requirements of the individual elements, according to the Eurocode, as will be shown below.

It should be noted that the expressions derived in the next part relate the dimensions of the truss to the variables such that the stress in the struts is equal to the maximum allowable stress. This is a first step in order to achieve parameters for an efficient design. In order to achieve a lower stress in the struts, the widths of the struts have to be increased. When the width b_1 of the compression diagonals is increased, the amount of the distributed load that will go through the diagonals will increase, so that the force in the tie will increase. The amount of reinforcement needed will therefore also increase. The stress in the diagonal itself will decrease as $q_d < d_{wall}\ \sigma_{rd,max}$. This condition must hold according to the Eurocode, otherwise the thickness of the wall is not sufficient. When the width b_2 of the vertical strut is increased, the amount of reinforcement needed will not increase. However the length of the reinforcement will increase as the distance between the compression diagonals will increase.

First the minimum and maximum value of h_1 as a function of θ have to be observed. The minimum value of h_1 for a certain value of θ, is that value where the total load goes through the compression diagonals. The force on the compression diagonal results from a distributed load with a length of $(0{,}5\ b_3 + L)$ (see figure 14).

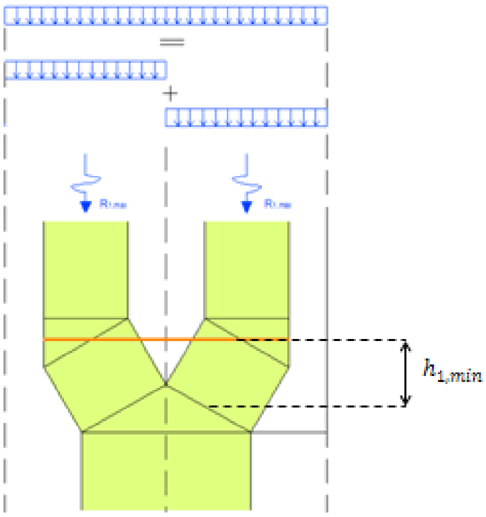
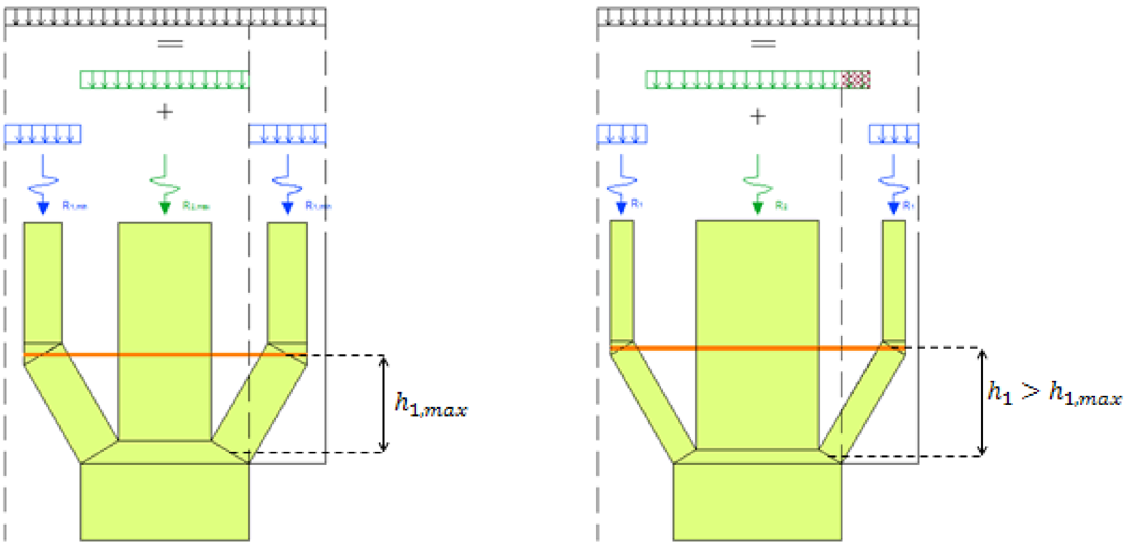


Figure 7: The minimum height of the compression diagonal

The maximum value of h_1 for a certain value of θ, is that value where the total load on the diagonal results from a distributed load with a length equal to the length of the overhang (see figure 15a). A bigger height of the compression diagonal is not possible, because the length of the distributed load belonging to the force on the diagonal will be smaller than the length of the overhang. The remaining part of the distributed load that is above the overhang will require a second tie in order to transfer this load to the middle strut. However in this truss model only one tie is available. In figure 15b the part that requires a second tie is cross-hatched.



a) Max. height of the compression diagonal b) Height bigger than max. height

Figure 8: The maximum height of the compression diagonal

Now for the situation where h_1 is equal to the minimum value as a function of θ, the maximum width of the compression diagonal and the maximum value of the force acting on the diagonal can derived by setting up the following equations:

$$R_{1,max} = q_d \left(L + b_{1,max} \sin\theta\right) \quad \text{eq. (1)}$$

$$N_{1,max} = R_{1,max} / \sin\theta \qquad \text{eq. (2)}$$

$$\sigma_{rd,max} = 0{,}6\, v' f_{cd} \qquad \text{eq. (3)}$$

$$\sigma_{rd,CCT} = 0{,}85\, v' f_{cd} \qquad \text{eq. (4)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (5)}$$

$$b_{1,max} = \frac{1}{d_{wall}} \cdot \left(\frac{N_{1,max}}{min\,(\sigma_{rd,max}\,;\sigma_{rd,CCC}\,;\sigma_{rd,CCT}\,)} \right) \qquad \text{eq. (6)}$$

Combining eq. (1) till eq. (6) gives:

$$R_{1,max} = \frac{q_d \cdot L}{1 - q_d / (d_{wall} \cdot \sigma_{rd,max}\,)} \qquad \text{eq. (7)}$$

$$b_{1,max} = \frac{R_{1,max}}{d_{wall} \cdot \sigma_{rd,max} \cdot \sin\,\theta} \qquad \text{eq. (8)}$$

Now the situation where h_1 is between the two limit values will be considered. The following equation for the compression diagonal can now be derived:

$$R_1 = 2\, q_d \left(\frac{1}{2}\, b_1 \sin\theta + L - \frac{h_1}{\tan\,\theta} \right) \qquad \text{eq. (9)}$$

$$N_1 = R_1 / \sin\theta \qquad \text{eq. (10)}$$

$$b_1 = \frac{1}{d_{wall}} \cdot \frac{N_1}{min\,(\sigma_{rd,max}\,;\sigma_{rd,CCC}\,;\sigma_{rd,CCT}\,)} \qquad \text{eq. (11)}$$

Combining eq. (9) till eq. (11) gives:

$$b_1 = \frac{2\, q_d}{\sin\,\theta\, (d_{wall} \cdot \sigma_{rd,max} - q_d)} \cdot \left(L - \frac{h_1}{\tan\,\theta} \right) \qquad \text{eq. (12)}$$

For the vertical strut above the node the following equations can be derived:

$$b_2 = 2\, (b_{1,max} \cdot \sin\theta - b_1 \cdot \sin\theta) \qquad \text{eq. (13)}$$

$$R_2 = N_2 = 2\, (q_d \cdot\, (b_{1,max} \cdot \sin\theta + L) - R_1) \qquad \text{eq. (14)}$$

Combining eq. (8), (9), (12), (13) and (14) gives:

$$b_2 = \frac{2\, q_d\, L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \cdot \left(\frac{2\, h_1}{L \cdot \tan\,\theta} - 1 \right) \qquad \text{eq. (15)}$$

$$R_2 = 2\, \left(q_d\, \left(\frac{1}{2} b_3 + L \right) - R_1 \right) \qquad \text{eq. (16)}$$

For the vertical strut below the node, the following equations are valid:

$$b_3 = b_2 + 2\, b_1 \sin\theta \qquad \text{eq. (17)}$$

Combining eq. (12), (15) and (17) gives:

$$b_3 = \frac{2\, q_d\, L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \qquad \text{eq. (18)}$$

The range of h_1 can now be determined according to the following equations:

$$\frac{1}{2}b_3 - \frac{1}{2}b_1 \sin\theta + \frac{h_1}{\tan\,\theta} \geq \frac{1}{2}b_1 \sin\theta + L - \frac{h_1}{\tan\,\theta} \qquad \text{eq. (19)}$$

$$\frac{1}{2}b_3 - \frac{1}{2}b_1 \sin\theta + \frac{h_1}{\tan\,\theta} \leq \frac{1}{2}\,b_3 + \frac{1}{2}\,L \qquad \text{eq. (20)}$$

Combining eq. (12) and (18) till (20) gives:

$$L\tan\theta \leq h_1 \leq \frac{1}{2}L\tan\theta\,\left(1 + \frac{q_d}{d_{wall}\cdot\sigma_{rd\,,max}}\right) \qquad \text{eq. (21)}$$

In theory the expressions above should be valid for widths of the wall bigger than the length of the distributed load that is transferred by the truss. Using the geometrical conditions this can be expressed as:

$$b_{wall\,,min} = b_2 + 2\,b_1 \sin\theta + 2\,L \qquad \text{eq. (22)}$$

Combining eq. (12), (15) and (22) gives:

$$b_{wall\,,min} = 2\,L\,\left(\frac{q_d}{d_{wall}\;\sigma_{rd\,,max} - q_d} + 1\right) \text{eq. (23)}$$

The relation between the angle and the height of the compression diagonals on the required reinforcement in the tie is investigated using the calculations provided by the Eurocode. It should be noted that the crack width calculations provided by the Eurocode concern cracks at the location of the tie, because the tie should be placed on those locations where cracks are expected. However in the case of the overhang, we cannot specify beforehand the exact location where cracks will develop. It is therefore not unthinkable that in the case of the overhang the dominant cracks might be located at a certain distance from the tie. However the Eurocode does not provide expressions in order to determine and control these cracks.

It turns out that the graph relating the variables of the truss model to the needed reinforcement in the ULS is qualitatively the same for the different dimensions of the overhang. The same holds true for the graph relating the variables of the truss model to the cracks at the location of the tie. From these graphs it can be concluded that as the angle θ increases, the reinforcement in the tie reduces and the crack width increases. Furthermore it seems that increasing the angle θ bigger than $60°$ will barely decrease the amount of reinforcement needed, while the crack width increases significantly. Choosing θ bigger than $60°$ is therefore not sufficient. The minimum reinforcement needed for a specific value of θ, is obtained when the height of the compression diagonal is chosen equal to the maximum height as a function of the angle (see the range of h_1 above).

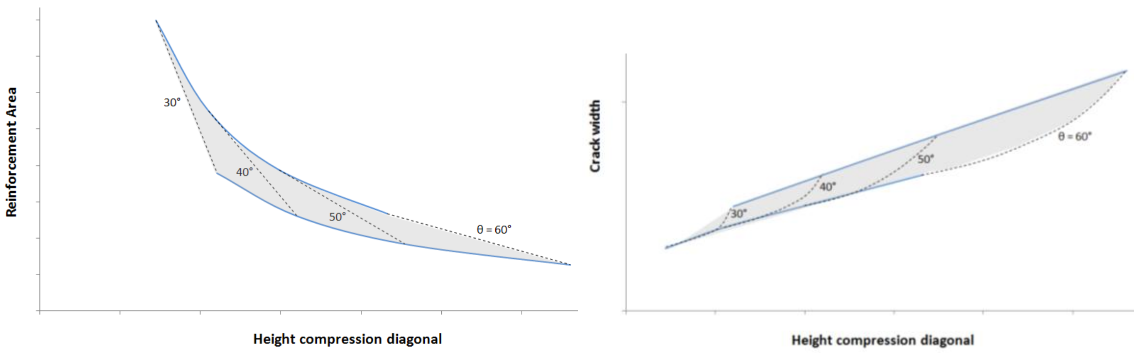
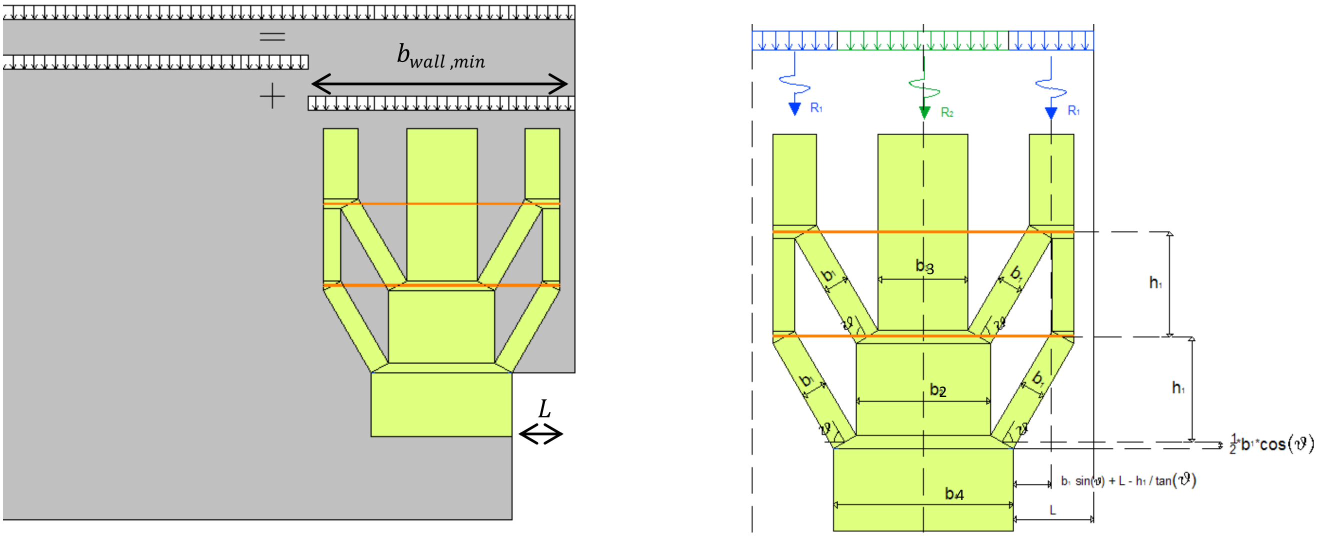


Figure 9: Graphs relating the angle and height of the compression diagonal to the reinforcement area and the crack width

The overhang, truss model 2



a) Location truss model in the wall.

b) Parameters of the truss model

Figure 10: Truss model 2 for the overhang

The second truss model considered is shown in the figure above. It consists of two ties, four diagonal struts with widths b_1 and three vertical struts with width b_2, b_3 and b_4. The variables of this truss model are the angle of the compression diagonals θ and the height of the compression diagonals h_1. In the same manner as done before, the dimensions of the parameters of this truss model can be expressed as a function of those two variables, the value of the distributed load and the dimensions of the overhang.

First the minimum and maximum values if h_1 as a function of θ have to be determined. The minimum value of h_1 for a certain value of θ occurs when the total load goes through the compression diagonals. The force on the compression diagonals results from a distributed load with a length of $(0{,}5\,b_4 + L)$ (see figure 19).

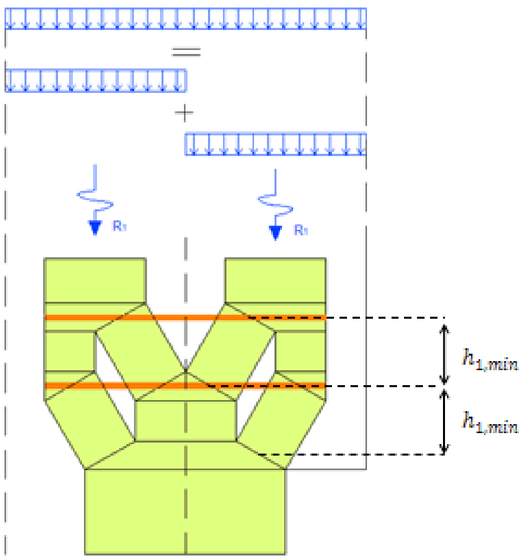
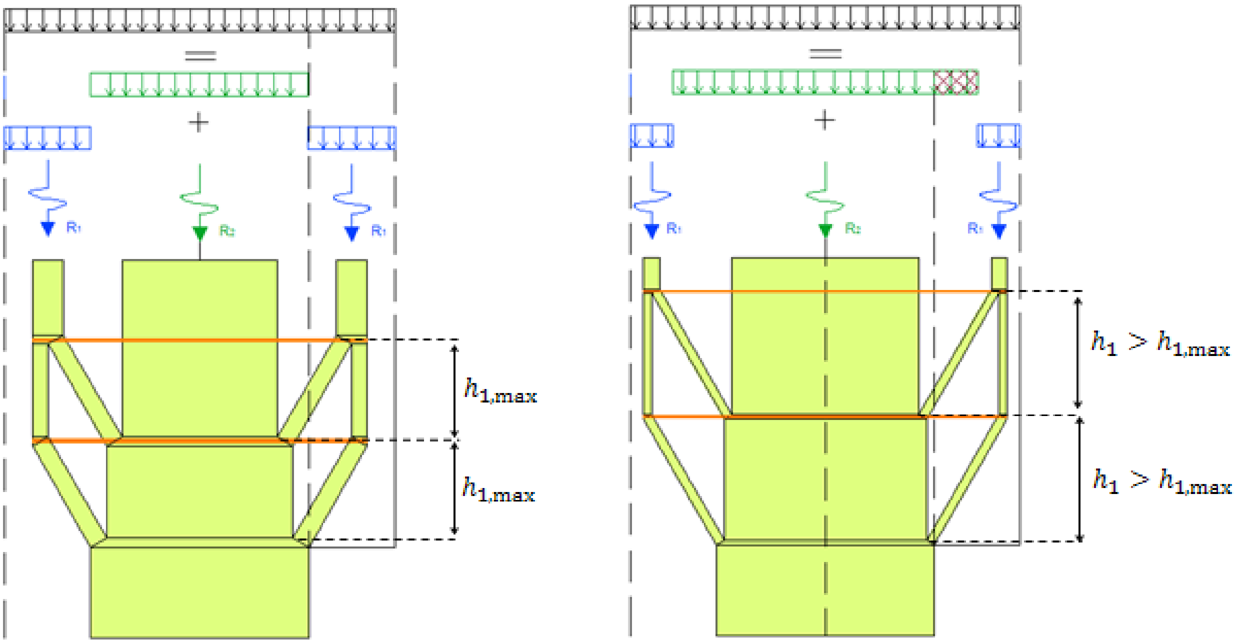


Figure 11: The minimum height of the compression diagonals

The maximum value of h_1 for a certain value of θ, is that value where the total load on the diagonals results from a distributed load with a length equal to the overhang (see figure 20a). A bigger height of the compression diagonals is not possible for this truss model, as the length of the distributed load belonging to the force on the diagonal will be smaller than the overhang. The remaining part of the distributed load that is above the overhang will require a third tie in order to transfer this load to the middle strut. However in this truss model only two ties are available. In figure 20b the part that requires a third tie is cross-hatched.



a) Max. height of the compression diagonal

b) Height bigger than max. height

Figure 12: The maximum height of the compression diagonal

Now for the situation where h_1 is equal to the minimum value as a function of θ, the maximum width of the compression diagonal and the maximum value of the force acting on the diagonal can be derived by setting up the following equations:

$$R_{1,max} = q_d \left(L + 2\, b_{1,max} \sin\theta\right) \quad \text{eq. (25)}$$

$$N_{1,max} = 1/2 \cdot R_{1,max} / \sin\theta \quad \text{eq. (26)}$$

$$\sigma_{rd,max} = 0{,}6\, v' f_{cd} \quad \text{eq. (27)}$$

$$\sigma_{rd,CCT} = 0{,}85\, v' f_{cd} \quad \text{eq. (28)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \quad \text{eq. (29)}$$

$$b_{1,max} = \frac{1}{d_{wall}} \cdot \left(\frac{N_{1,max}}{min\left(\sigma_{rd,max};\sigma_{rd,CCC};\sigma_{rd,CCT}\right)}\right) \quad \text{eq. (30)}$$

Combining eq. (25) till eq. (30) gives:

$$R_{1,max} = q_d \cdot \left(L + \frac{q_d \cdot L}{d_{wall} \cdot \sigma_{rd,max} - q_d}\right) \quad \text{eq. (31)}$$

$$b_{1,max} = \frac{q_d \cdot L}{2 \cdot \sin\theta \left(d_{wall} \cdot \sigma_{rd,max} - q_d\right)} \quad \text{eq. (32)}$$

Now the situation will be considered where h_1 is between the two limit values. The following equations for the compression diagonal can be derived:

$$R_1 = 2\, q_d \left(b_1 \sin\theta + L - \frac{h_1}{\tan\,\theta}\right) \qquad \text{eq. (33)}$$

$$N_1 = 1/2\, \cdot R_1/\sin\theta \qquad \text{eq. (34)}$$

$$b_1 = \frac{1}{d_{wall}} \cdot \frac{N_1}{min\,(\sigma_{rd\,,max}\,;\sigma_{rd\,,CCC}\,;\sigma_{rd\,,CCT}\,)} \qquad \text{eq. (35)}$$

Combining eq. (33) till eq. (35) gives:

$$b_1 = \frac{q_d}{\sin\,\theta\,(d_{wall}\cdot\sigma_{rd\,,max} - q_d)} \cdot \left(L - \frac{h_1}{\tan\,\theta}\right) \qquad \text{eq. (36)}$$

For the vertical strut above the lowest node, the following equations can be derived:

$$b_2 = 2\left(2\, b_{1,max}\, \cdot \sin\theta - b_1 \cdot \sin\theta\right) \qquad \text{eq. (37)}$$

$$R_2 = N_2 = 2\left(q_d \cdot\, \left(2\, b_{1,max}\, \cdot \sin\theta + L\right) - N_1 \cdot \sin\theta\right) \qquad \text{eq. (38)}$$

Combining eq. (32), (36) and (38) gives:

$$b_2 = \frac{2\, q_d}{d_{wall}\cdot\sigma_{rd\,,max} - q_d} \cdot \frac{h_1}{\tan\,\theta} \qquad \text{eq. (39)}$$

For the vertical strut above the highest middle node, the following equations can be derived:

$$b_3 = 2\left(2\, b_{1,max}\, \cdot \sin\theta - 2\, b_1 \cdot \sin\theta\right) \qquad \text{eq. (40)}$$

$$R_3 = N_3 = 2\left(q_d \cdot\, \left(2\, b_{1,max}\, \cdot \sin\theta + L\right) - R_1\right) \qquad \text{eq. (41)}$$

Combining eq. (32), (33), (36), (40) and (41) gives:

$$b_3 = \frac{2\, q_d\, L}{d_{wall}\cdot\sigma_{rd\,,max} - q_d} \cdot \left(\frac{2\, h_1}{L\,\tan\,\theta} - 1\right) \qquad \text{eq. (42)}$$

$$R_3 = 2\,\left(q_d\,\left(\frac{1}{2} b_4 +\, L\right) - R_1\right) \qquad \text{eq. (43)}$$

For the vertical strut below the lowest node, the following equations can be derived:

$$b_4 = b_2 + 2\, b_1 \sin\theta \qquad \text{eq. (44)}$$

Combining eq. (36), (39) and (44) gives:

$$b_4 = \frac{2\, q_d\, L}{d_{wall}\cdot\sigma_{rd\,,max} - q_d} \qquad \text{eq. (45)}$$

The range of h_1 is now determined according to the following equations:

$$\frac{1}{2} b_3 + b_1 \sin\theta + \frac{h_1}{\tan\,\theta} \geq \frac{1}{2}(\frac{1}{2} b_4 + L) \qquad \text{eq. (46)}$$

$$\frac{1}{2} b_3 + b_1 \sin\theta + \frac{h_1}{\tan\,\theta} \leq \frac{1}{2} b_4 + \frac{1}{2} L \qquad \text{eq. (47)}$$

Combining eq. (36), (42) and (45) till (47) gives:

$$\frac{1}{2} L \tan\theta \leq h_1 \leq \frac{1}{2}\, L \tan\theta\, \left(1 + \frac{q_d}{d_{wall}\cdot\sigma_{rd\,,max}}\right) \qquad \text{eq. (48)}$$

In theory the expressions above should be valid for widths of the wall bigger than the length of the distributed load that is transferred by the truss. Using the geometrical conditions this can be expressed as:

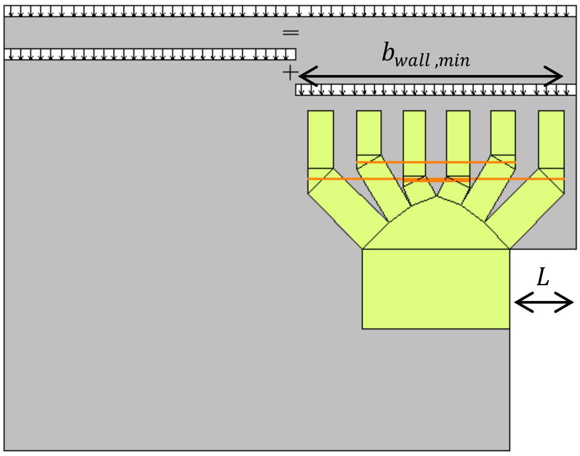
$$b_{wall,min} = b_3 + 4\, b_1 \sin\theta + 2\, L \qquad \text{eq. (49)}$$

Combining eq. (36), (42) and (49) gives:

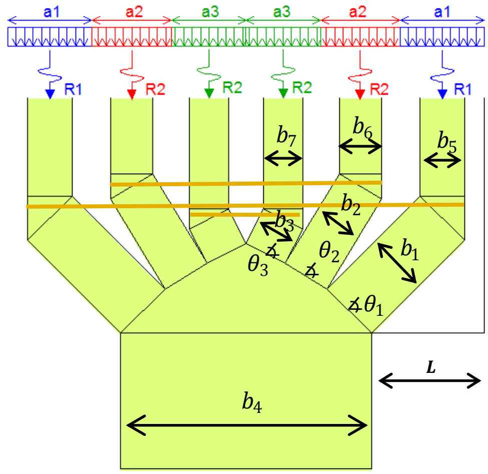
$$b_{wall,min} = 2\, L \left(\frac{q_d}{d_{wall}\ \sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (50)}$$

As for truss model 1, the calculations provided in the Eurocode are used in order to determine the optimum values for the variables of the truss model. Again it is denoted that the calculations in the Eurocode concerning the crack width control do not take into account the cracks elsewhere than the position of the tie. As for truss model 1, θ equal to $60°$ and h_1 equal to the maximum value as a function of θ gives an efficient design according to the calculations of the Eurocode.

The overhang, truss model 3



a) Location truss model in the wall.



b) Parameters of the truss model

Figure 13: Truss model 3 for the overhang

The third truss model is a fan model. It consists of three ties, six diagonal struts with widths b_1, b_2 and b_3 and seven vertical struts with widths b_4, b_5, b_6 and b_7. The parameters of this truss model will be achieved by using a stress trajectory plot and relating the parameters to the variables which are determined beforehand. The stress trajectory plot is obtained by using SCIA Engineer, a linear elastic finite element program. From the stress trajectory plot the angle of the compression diagonals θ_i and the relation between the width of the compression diagonals β_i will be obtained. These relations are coupled to the stress in the direction observed.

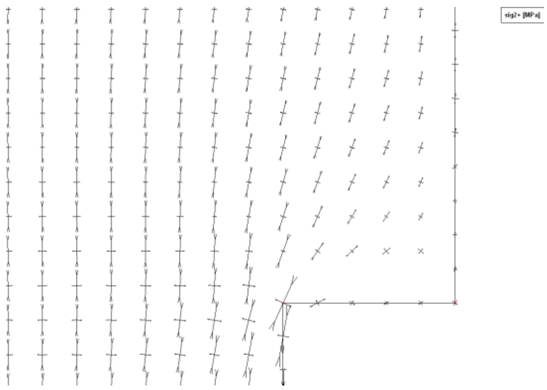


Figure 14: Stress trajectory plot with SCIA Engineer

Next the width of the strut b_4 is determined, so that the stress in this strut is equal to the maximum allowable stress according to the Code. This is a first step in order to achieve an efficient design, as explained in truss model 1.

The expression of b_4 can be determined as follows:

$$b_4 \; d_{wall} \; \sigma_{rd,max} = 2 \; q_d \left(L + \tfrac{1}{2} \; b_4\right) \qquad \rightarrow \qquad b_4 = \frac{2 \; q_d \; L}{d_{wall} \;\; \sigma_{rd,max} - q_d} \qquad \text{eq. (52)}$$

The width of each strut can now be determined by using the relation between the width of the diagonals (β_i), obtained from the stress trajectory plot, and the geometrical relations between the struts.

$$\left.\begin{array}{r} 2\;(b_1 \sin\theta_1 + b_2 \sin\theta_2 + b_3 \sin\theta_3) = b_4 \\ b_2 = \beta_2 \; b_1 \\ b_3 = \beta_3 \; b_1 \end{array}\right\} \quad \rightarrow \quad b_1 = \frac{1}{2} \; \frac{b_4}{\sin \theta_1 + \beta_2 \; \sin \theta_2 + \beta_3 \; \sin \theta_3} \qquad \text{eq. (53)}$$

$$b_2 = \beta_2 \; b_1 \qquad \text{eq. (54)}$$

$$b_3 = \beta_3 \; b_1 \qquad \text{eq. (55)}$$

$$b_5 = b_1 \sin\theta_1 \qquad \text{eq. (56)}$$

$$b_6 = b_2 \sin\theta_2 \qquad \text{eq. (57)}$$

$$b_7 = b_3 \sin\theta_3 \qquad \text{eq. (58)}$$

The height of the individual diagonal is geometrically determined by considering the available space of the distributed load that form the resultant force on the diagonals.

$$h_1 = \tan\theta_1 \left(L - \frac{1}{2}a_1 + \frac{1}{2}\, b_1 \sin\theta_1\right) \qquad \text{eq. (59)}$$

$$h_2 = \tan\theta_2 \left(L - a_1 - \frac{1}{2}a_2 + b_1 \sin\theta_1 + \frac{1}{2}\, b_2 \sin\theta_2\right) \qquad \text{eq. (60)}$$

$$h_3 = \tan\theta_3 \left(\frac{1}{2}a_3 - \frac{1}{2}\, b_3 \sin\theta_3\right) \qquad \text{eq. (61)}$$

With:

$$a_1 = \frac{\sin\theta_1\, d_{wall}\ \sigma_{rd,max}}{q_d}\ b_1 \qquad \text{eq. (62)}$$

$$a_2 = \frac{\sin\theta_2\, d_{wall}\ \sigma_{rd,max}}{q_d}\ b_2 \qquad \text{eq. (63)}$$

$$a_3 = \frac{1}{2}b_4 + L - a_1 - a_2 \qquad \text{eq. (64)}$$

The value of the resultant force acting on the individual diagonal is calculated according to:

$$R_1 = q_d\ a_1 \qquad \text{eq. (65)}$$

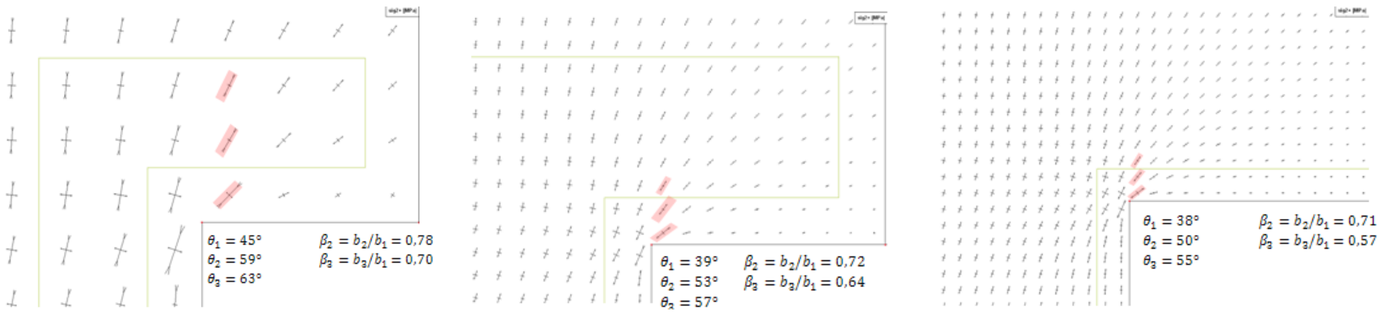
$$R_2 = q_d\ a_2 \qquad \text{eq. (66)}$$

$$R_3 = q_d\ a_3 \qquad \text{eq. (67)}$$

The minimum width of the wall for which these expressions are applicable can be expressed as:

$$b_{wall,min} = 2\,L\left(\frac{q_d}{d_{wall}\ \sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (68)}$$

Now the stress trajectory plot can be used in order to determine the angle of the individual compression diagonal and the relation between the widths of the compression diagonals. In figure 25 the trajectories that correspond with the diagonals are hatched for the dimensions of the overhang of 1m, 5m and 10m.



a) 1 m overhang b) 5 m overhang c) 10 m overhang

Figure 15: Stress trajectory plot for the upper left corner of the overhang

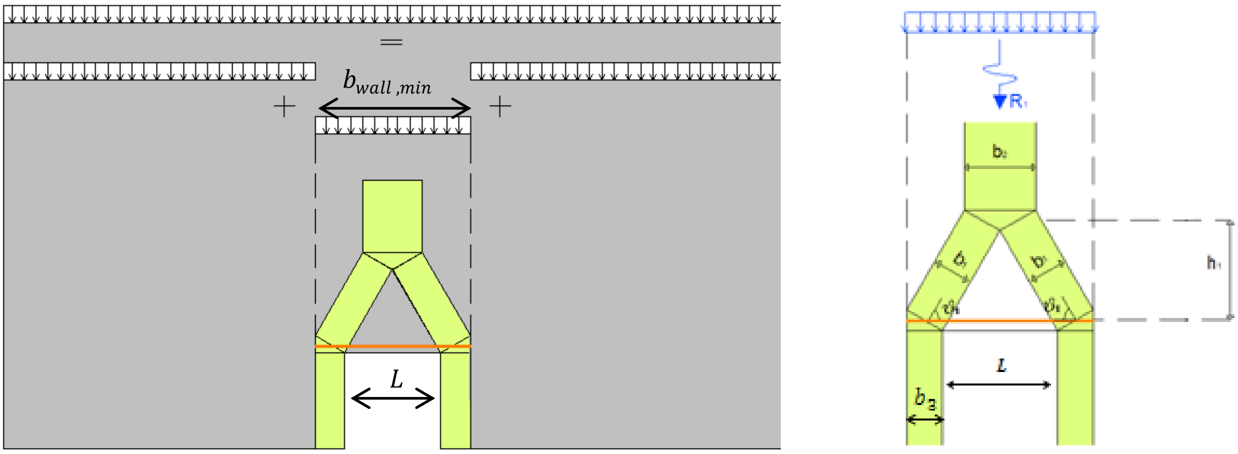
Now the stresses in the truss elements can be calculated. If nowhere the maximum stress is exceeded, the estimation of the ratio between the widths of the diagonals was correct. The iteration process is finished after one iteration step. If the stresses are exceeded somewhere in the truss, the correct relation between the width of the diagonals should be determined by using the forces in these diagonals. After checking the stresses in the struts, the reinforcement needed in the tie will be computed. Based on the crack width condition in the SLS the diameter of the bars and the centre to centre distance of the bars are determined. If necessary the reinforcement area is increased.

The properties of the truss model that fulfill all the requirements are given in a table below for the overhang of 1m, 5m and 10m. It should be noted that the Strut-and-Tie method as presented in the Eurocode takes for the struts only unreinforced concrete into account. The maximum stress in these struts is set to approximately 50% of the design compressive stress of the concrete. However when high loading is applied, the width of the struts will become very large. For this model this is the case for strut nr. 4, which has a width of approximately two times the dimensions of the overhang (see table). In reality the stresses will not distribute over such a large width. As a consequence, the concrete might locally fail in compression. However the Strut-and-Tie method does not indicate this. In a later part of this report, it will be shown whether in this case the concrete fails in compression by using a nonlinear elastic finite element program.

L [m]	1	5	10
$A_{s,tie,1}$ [mm^2]	10 Ø25	52 Ø25	112 Ø25
$A_{s,tie,2}$ [mm^2]	8 Ø20	30 Ø25	48 Ø25
$A_{s,tie,3}$ [mm^2]	2 Ø25	24 Ø25	45 Ø25
θ_1 [°]	45	39	38
θ_2 [°]	59	53	50
θ_3 [°]	63	57	55
β_2 [-]	0,78	0,72	0,71
β_3 [-]	0,70	0,64	0,57
b_1 [mm]	555	3186	6807
b_2 [mm]	434	2288	4837
b_3 [mm]	389	2049	3911
b_4 [mm]	2220	11099	22197
b_5 [mm]	392	2005	4191
b_6 [mm]	372	1827	3705
b_7 [mm]	347	1719	3204
h_1 [mm]	823	3318	6338
h_2 [mm]	798	3146	5429
h_3 [mm]	306	1192	2061
R_1 [kN]	1416	7240	15135
R_2 [kN]	1343	6598	13382
R_3 [kN]	1250	6206	11569

Table 1: Properties of the truss model for the overhang of 1m, 5m and 10m

The opening, truss model 1



a) Location truss model in the wall. b) Parameters of the truss model

Figure 16: Truss model 1 for the opening

The truss model considered is shown in the figure above. The variable of this truss model is the angle of the compression diagonals θ. De dimensions of the parameters of the truss and the force on the truss can be expressed as a function of this variable, the value of the distributed load and the dimensions of the overhang.

The following equations for the compression diagonal can be derived:

$$R_1 = q_d\ (L + 2\ b_1 \sin\theta) \qquad \text{eq. (69)}$$

$$N_1 = 1/2\ \cdot R_1 / \sin\theta \qquad \text{eq. (70)}$$

$$\sigma_{rd,max} = 0{,}6\ v' f_{cd} \qquad \text{eq. (71)}$$

$$\sigma_{rd,CCT} = 0{,}85\ v' f_{cd} \qquad \text{eq. (72)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (73)}$$

$$b_1 = \frac{1}{d_{wall}} \cdot \left(\frac{N_1}{min\ (\sigma_{rd,max}; \sigma_{rd,CCC}; \sigma_{rd,CCT})}\right) \qquad \text{eq. (74)}$$

Combining eq. (79) till eq. (74) gives:

$$R_1 = q_d\ \left(\frac{q_d\ L}{d_{wall} \cdot \sigma_{rd,max} - q_d} + L\right) \qquad \text{eq. (75)}$$

$$b_1 = \frac{q_d\ L}{2\ \sin\theta\ (d_{wall} \cdot \sigma_{rd,max} - q_d)} \qquad \text{eq. (76)}$$

For the vertical strut above the highest node the following equations can be derived:

$$b_2 = 2\ b_1 \cdot \sin\theta \qquad \text{eq. (77)}$$

Combining eq. (76) and (77) gives:

$$b_2 = \frac{q_d\ L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \qquad \text{eq. (78)}$$

For the vertical struts below the lowest node, the following equation can be derived:

$$b_3 = b_1 \cdot \sin\theta \qquad \text{eq. (79)}$$

Combining eq. (76) and (79) gives:

$$b_3 = \frac{q_d \, L}{2\,(d_{wall} \cdot \sigma_{rd\,,max} - q_d)} \qquad \text{eq. (80)}$$

In theory the expressions above should be valid for widths of the wall bigger than the length of the distributed load that is transferred by the truss. Using the geometrical conditions we can express this as:

$$b_{wall\,,min} = L \left(\frac{q_d}{d_{wall} \;\; \sigma_{rd\,,max} - q_d} + 1 \right) \qquad \text{eq. (81)}$$

The relation between the angle of the compression diagonal and the required reinforcement in the tie is investigated for the different dimensions of the overhang. It turns out that when θ increases the needed reinforcement in the ULS decreases, while the crack width at the location of the tie barely changes. This can be explained by the following; the crack width calculation is based on the concrete bar loaded in tension. The height of the bar is determined as a function of the concrete cover on the central axis of the reinforcement. This central axis is determined by the angle θ following from the geometry of the truss model (the central axis is located at $\{0.5\ b_1 \cos\theta\}$ from the edge). As θ increases the height of the bar in the calculation of the crack width will decrease. However the reinforcement needed in the ULS will also decrease. The decreasing height of the bar and the decreasing reinforcement area counteract each other in the calculation of the crack width, resulting in a constant crack width at the location of the tie. However it is not expected that this will be truly the case, because it would mean that the crack width at the location of the tie is not influenced by the reinforcement area in the tie.

By considering that an opening equals two overhangs connected to each other, we could assume that the best angle found in the overhang should be the same as for the opening. Therefore, for θ $60°$ degrees is chosen.

The opening, truss model 2

In the previous paragraph the parameters of truss model 1 have been determined. It can be argued that truss model 1 is not efficient, as it leads to very conservative solutions. This can be explained in the following manner. In truss model 1 the distributed load with a length of $(L + 2\, b_3)$ is schematized as a point load in the middle of the opening (see. Fig. 31). When we consider a simply supported beam loaded by a distributed load, the maximum moment in the beam will be equal to $\frac{1}{8} \cdot q \cdot l^2$. However a simply supported beam loaded by a point load in the middle equal to $q \cdot l$, will have a maximum moment equal to $\frac{1}{4}\; q\, l^2$. In order to obtain the same moment of $\frac{1}{8} \cdot q \cdot l^2$, the point load should be equal to $\frac{1}{2} \cdot q \cdot l$. When we translate this to the truss model for the opening, we obtain the truss model shown in figure 32. The expressions for this truss model are derived below.

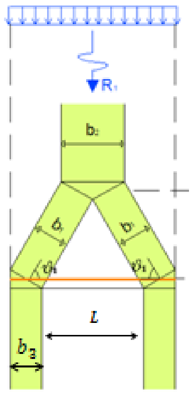
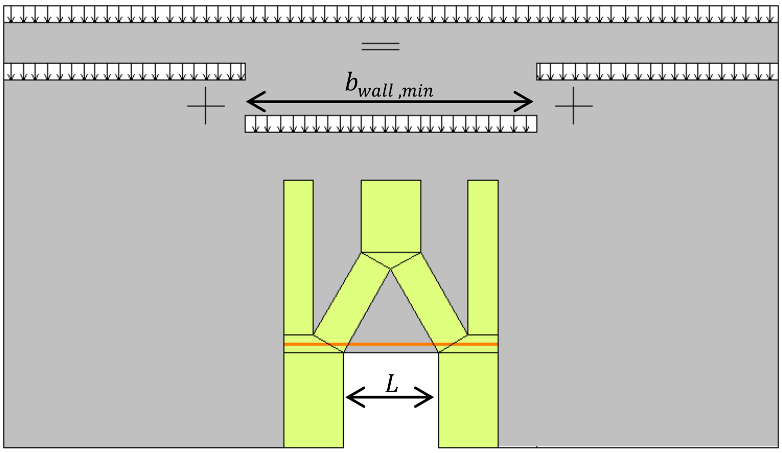
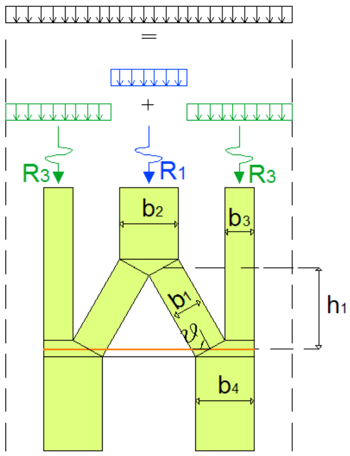


Figure 18: truss model 1 for the opening



a) Location truss model in the wall



b) Parameters of the truss model

Figure 17: Truss model 2 for the opening

The following equations for the compression diagonal are derived:

$$R_1 = q_d\; \frac{1}{2}(L + b_4) \qquad \text{eq. (82)}$$

$$N_1 = \frac{1}{2} \cdot R_1/ \sin\theta \qquad \text{eq. (83)}$$

$$\sigma_{rd,max} = 0{,}6\; v' f_{cd} \qquad \text{eq. (84)}$$

$$\sigma_{rd,CCT} = 0{,}85\; v' f_{cd} \qquad \text{eq. (85)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (86)}$$

$$b_1 = \frac{1}{d_{wall}} \cdot \left(\frac{N_1}{min\,(\sigma_{rd,max}\,; \sigma_{rd,CCC}\,; \sigma_{rd,CCT})} \right) \qquad \text{eq. (87)}$$

The vertical struts above the lowest nodes at the edge and the loading on these struts can be expressed as:

$$R_3 = q_d\; \left(\frac{1}{2} L + \frac{3}{2}\; b_1 \sin\theta + \frac{1}{2} b_3\right) \qquad \text{eq. (88)}$$

$$N_3 = R_3 \qquad \text{eq. (89)}$$

$$\sigma_{rd,max} = 0{,}6\, v' f_{cd} \qquad \text{eq. (90)}$$

$$\sigma_{rd,CCT} = 0{,}85\, v' f_{cd} \qquad \text{eq. (91)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (92)}$$

$$b_3 = \frac{1}{d_{wall}} \cdot \left(\frac{N_3}{min\,(\sigma_{rd,max};\sigma_{rd,CCC};\sigma_{rd,CCT})} \right) \qquad \text{eq. (93)}$$

The vertical struts below the lowest nodes at the edge can be expressed as:

$$b_4 = b_1 \sin\theta + b_3 \qquad \text{eq. (94)}$$

Combining eq. (82) till eq. (94) gives:

$$R_1 = 2\, q_d\, L \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (95)}$$

$$b_1 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}}{\sin\theta\,(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (96)}$$

$$R_3 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}\ (2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (97)}$$

$$b_3 = q_d\, L \frac{(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (98)}$$

$$b_4 = q_d\, L \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (99)}$$

The vertical strut above the highest node in the middle can be expressed as:

$$b_2 = 2\, b_1 \sin\theta \qquad \text{eq. (100)}$$

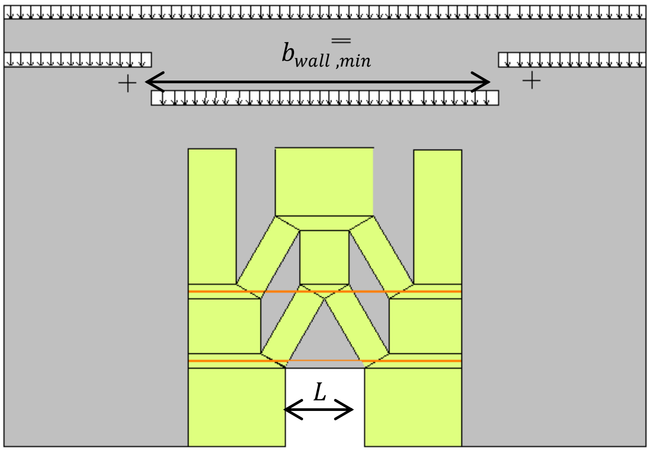
Combining eq. (96) and (100) gives:

$$b_2 = 2\, q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (101)}$$

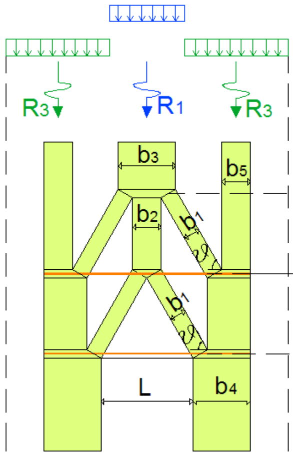
In theory the expressions above should be valid for widths of the wall bigger than the length of the distributed load that is transferred by the truss. Using the geometrical conditions this is expressed as:

$$b_{wall,min} = 2\, L \frac{d_{wall} \cdot \sigma_{rd,max}\ (3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (102)}$$

The opening, truss model 3



a) Location truss model in wall



b) Parameters of the truss model

Figure 19: Truss model 3 for the opening

The truss model considered is shown in the figure above. The variable of this truss model is the angle of the compression diagonals θ. The dimensions of other parameters of the truss and the resultant of the load on the truss can be expressed as a function of this variable, the value of the distributed load and the dimensions of the overhang.

The following equation for the compression diagonal can be derived:

$$R_1 = \frac{1}{2} q_d \ (L + b_4) \qquad \text{eq. (103)}$$

$$N_1 = 1/4 \cdot R_1 / \sin\theta \qquad \text{eq. (104)}$$

$$\sigma_{rd,max} = 0{,}6 \ v' f_{cd} \qquad \text{eq. (105)}$$

$$\sigma_{rd,CCT} = 0{,}85 \ v' f_{cd} \qquad \text{eq. (106)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (107)}$$

$$b_1 = \frac{1}{d_{wall}} \cdot \left(\frac{N_1}{min\,(\sigma_{rd,max} ; \sigma_{rd,CCC} ; \sigma_{rd,CCT})} \right) \qquad \text{eq. (108)}$$

For the vertical struts below the lowest nodes at the edge, the following equation can be derived:

$$b_4 = 2 \ b_1 \cdot \sin\theta + b_5 \qquad \text{eq. (109)}$$

For the vertical struts above the highest nodes at the edge of the truss, the following equations can be derived:

$$R_5 = 2 \ q_d \ \left(\frac{1}{2} L + b_4 - \frac{1}{2} b_5 - \frac{1}{2} \frac{R_1}{q_d} \right) \qquad \text{eq. (110)}$$

$$N_5 = R_5 \qquad \text{eq. (111)}$$

$$\sigma_{rd,max} = 0{,}6 \ v' f_{cd} \qquad \text{eq. (112)}$$

$$\sigma_{rd,CCT} = 0{,}85 \ v' f_{cd} \qquad \text{eq. (113)}$$

$$\sigma_{rd,CCC} = v' f_{cd} \qquad \text{eq. (114)}$$

$$b_5 = \frac{1}{d_{wall}} \cdot \left(\frac{N_5}{min\,(\sigma_{rd,max} ; \sigma_{rd,CCC} ; \sigma_{rd,CCT})} \right) \qquad \text{eq. (115)}$$

Combining eq. (103) and (115) gives:

$$R_1 = 2\, q_d\, L \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (116)}$$

$$b_1 = \frac{1}{2}\, q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}}{\sin\theta\,(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (117)}$$

$$b_4 = q_d\, L \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (118)}$$

$$R_5 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}\,(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (119)}$$

$$b_5 = q_d\, L \frac{(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (120)}$$

For the vertical strut above the lowest middle node, the following equation can be derived:

$$b_2 = 2\, b_1 \cdot \sin\theta \quad \text{eq. (121)}$$

Combining eq. (117) and (121) gives:

$$b_2 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (122)}$$

For the vertical strut above the highest middle node, the following equation can be derived:

$$b_3 = b_2 + 2\, b_1 \cdot \sin\theta \quad \text{eq. (123)}$$

Combining eq. (117), (122) and (123) gives:

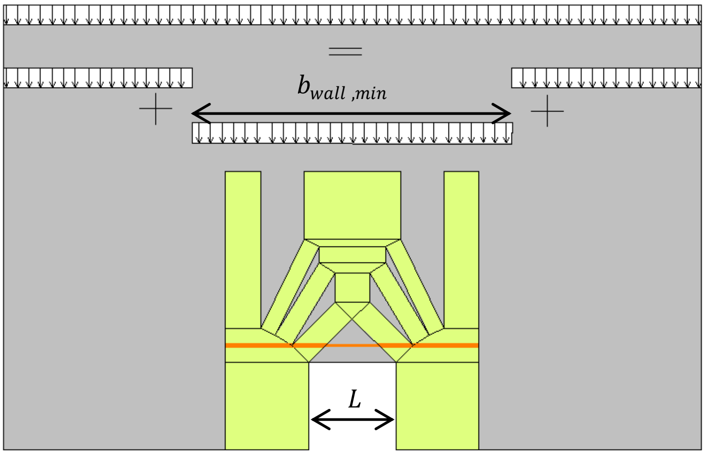
$$b_3 = 2\, q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (124)}$$

In theory the expressions above should be valid for widths of the wall bigger than the length of the distributed load that is transferred by the truss. Using the geometrical conditions this can be expressed as:

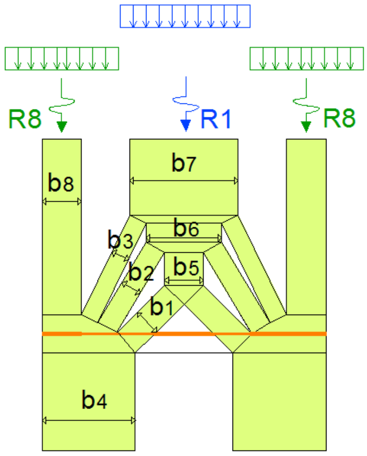
$$b_{wall,min} = 2\, L \frac{d_{wall} \cdot \sigma_{rd,max}\,(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \quad \text{eq. (125)}$$

In the same way as for truss model 1, the best value of θ cannot be based on the crack width calculation. Therefore the same procedure as truss model 1 is followed and θ is chosen equal to $60°$.

The opening, truss model 4



a) Location truss model in the wall.



b) Parameters of the truss model

Figure 20: Truss model 4 for the opening

The truss model considered is a fan model. The parameters of this truss model will be achieved by using a stress trajectory plot and relating the parameters to the variables which are determined beforehand. The stress trajectory plot is obtained by using SCIA Engineer, a linear elastic finite element program. From the stress trajectory plot the angle of the compression diagonals θ_i and the relation between the widths of the compression diagonals β_i will be obtained. These relations are coupled to the stress in the direction observed.

Next the width of the strut b_4 is determined such that the stress in this strut is equal to the maximum allowable stress according to the Code. This is a first step in order to achieve an efficient design, as explained before.

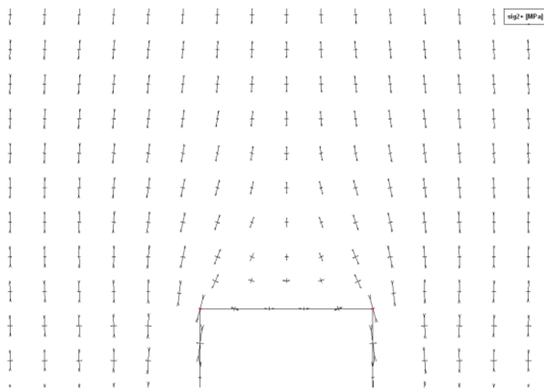


Figure 21: Stress trajectory plot with SCIA ESA Engineer

The following expressions for the determination of the parameters can be derived:

$$R_1 = \frac{1}{2} q_d \, (L + b_4) \qquad \text{eq. (126)}$$

$$R_8 = 2\, q_d \, \left(\frac{1}{2} L + b_4 - \frac{1}{2} b_8 - \frac{1}{2}\frac{R_1}{q_d}\right) \qquad \text{eq. (127)}$$

$$b_4 - b_8 = b_1 \sin\theta_1 + b_2 \sin\theta_2 + b_3 \sin\theta_3 \qquad \text{eq. (128)}$$

$$b_2 = \beta_2 \, b_1 \qquad \text{eq. (129)}$$

$$b_3 = \beta_3 \, b_1 \qquad \text{eq. (130)}$$

$$b_4 = \frac{1}{d_{wall}} \cdot \left(\frac{R_8 + \frac{1}{2} R_1}{min\,(\sigma_{rd,max}\,; \sigma_{rd,CCC}\,; \sigma_{rd,CCT}\,)}\right) \qquad \text{eq. (131)}$$

$$b_8 = \frac{1}{d_{wall}} \cdot \left(\frac{R_8}{min\,(\sigma_{rd,max}\,; \sigma_{rd,CCC}\,; \sigma_{rd,CCT}\,)}\right) \qquad \text{eq. (132)}$$

Combining eq. (126) till (132) gives:

$$R_1 = 2\, q_d\, L \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (133)}$$

$$R_8 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}\;(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (134)}$$

$$b_8 = q_d\, L \frac{(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (135)}$$

$$b_4 = q_d\, L \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (136)}$$

$$b_1 = \frac{b_4 - b_8}{\sin\,\theta_1 + \beta_2\,\sin\,\theta_2 + \beta_3\,\sin\,\theta_3} \qquad \text{eq. (137)}$$

The other parameters of the truss can be expressed as:

$$b_2 = \beta_2\, b_1 \qquad \text{eq. (138)}$$

$$b_3 = \beta_3\, b_1 \qquad \text{eq. (139)}$$

$$b_5 = 2\, b_1 \sin\theta_1 \qquad \text{eq. (140)}$$

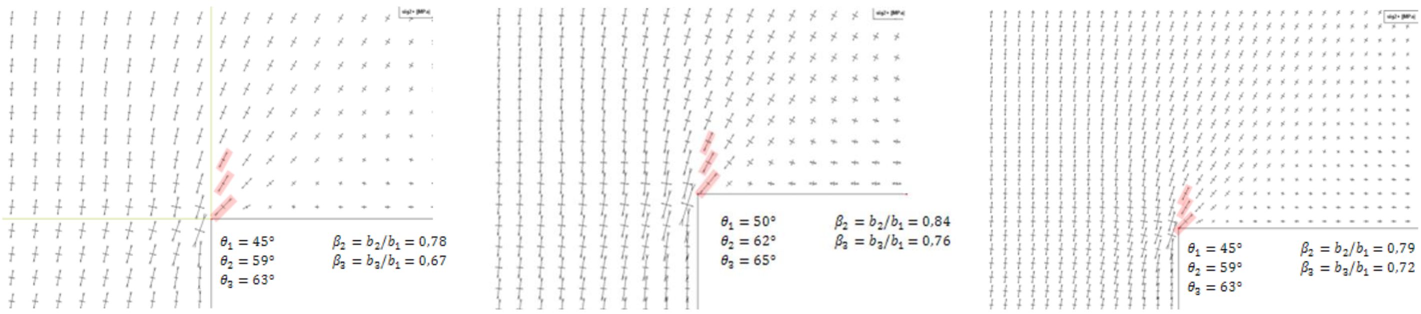
$$b_6 = b_5 + 2\, b_2 \sin\theta_2 \qquad \text{eq. (141)}$$

$$b_7 = b_6 + 2\, b_3 \sin\theta_3 \qquad \text{eq. (142)}$$

The minimum width of the wall for which these expressions are applicable can be expressed as:

$$b_{wall,min} = 2\, L \frac{d_{wall} \cdot \sigma_{rd,max}\,(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (144)}$$

Now the only unknowns in the expression above are the angle of each compression diagonal (θ_i) and the ratio between the width of the compression diagonals (β_i). We can use the stress trajectory plot in order to determine the angle of the individual compression diagonal and estimate the relation between the widths of the compression diagonals. In the figures 38 the trajectories that correspond with the diagonals are hatched for the dimensions of the opening of 1m, 5m and 10m.



a) 1 m opening b) 5 m opening c) 10 m opening

Figure 22: Stress trajectory plot for the upper left corner of the overhang

Now the stresses in the truss elements can be calculated. If nowhere the maximum stress is exceeded, the estimation of the relation between the widths of the compression diagonals was correct. The iteration process is finished after one iteration step. If the stresses are exceeded somewhere in the truss, the correct relation between the width of the diagonals should be determined by using the forces in these diagonals. Now the correct widths of the truss elements can be computed and the reinforcement needed in the ULS condition can be determined. Based on the crack width condition in the SLS, the diameter of the bars is determined. If necessary the reinforcement area can be increased. The centre to centre distance of the bars is determined such that the lowest layer of bars is located as near to the edge of the opening as possible, taking into account the concrete cover.

The properties of the truss model that fulfill all the requirements are given in a table below for the opening of 1m, 5m and 10m. As for truss model 3 of the overhang, the width of strut nr. 4 will become very large (see table below). As explained earlier, this is caused by the max. allowable compressive stress in the struts in combination with the high loading applied. In reality the stresses will not distribute over such a large width. As a consequence, the concrete might locally fail in compression. However the Strut-and-Tie method does not indicate this. In a later part of this report, it will be shown whether in this case the concrete fails in compression by using a nonlinear elastic finite element program.

L [m]	1	5	10
$A_{s,tie}$ [mm^2]	4 Ø25	16 Ø25	38 Ø25
θ_1 [°]	45	50	45
θ_2 [°]	59	62	59
θ_3 [°]	63	65	63
β_2 [-]	1,02	1,22	1,07
β_3 [-]	0,59	0,87	0,63
b_1 [mm]	117	467	1124
b_2 [mm]	119	568	1203
b_3 [mm]	69	406	704
b_4 [mm]	865	4324	8647
b_5 [mm]	165	715	1590
b_6 [mm]	368	1718	3651
b_7 [mm]	491	2453	4905
b_8 [mm]	620	3097	6194
R_1 [kN]	1771	8857	17714
R_8 [kN]	2237	11186	22373

Table 2: Properties of the truss model for the opening of 1m, 5m and 10m

3.2.4 Summary

The equations derived in the previous chapter can be translated in a number of steps in order to achieve an efficient design with these truss models. It must be noted that for this investigation the crack width calculations provided by the Eurocode have been used. However these crack width calculations only consider the cracks at the locations of the tie. Cracks elsewhere are not considered with this calculation, although those cracks might become dominant. In a later part of this report a nonlinear elastic finite element program will be used in order to verify whether the designs made with these equations indeed lead to satisfying results.

The overhang, truss model 1

In order to achieve an efficient design with this truss model, θ is chosen equal to 60°. The other parameters of the truss are determined using eq. (145) till (148). The loading on the truss is calculated using eq. (149) and (150). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (151).

$$h_1 = \frac{1}{2} L \tan\theta \left(1 + \frac{q_d}{d_{wall} \cdot \sigma_{rd,max}}\right) \qquad \text{eq. (145)}$$

$$b_1 = \frac{2\, q_d}{\sin\theta\,(d_{wall} \cdot \sigma_{rd,max} - q_d)} \cdot \left(L - \frac{h_1}{\tan\theta}\right) \qquad \text{eq. (146)}$$

$$b_2 = \frac{2\, q_d\, L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \cdot \left(\frac{2\, h_1}{L \cdot \tan\theta} - 1\right) \qquad \text{eq. (147)}$$

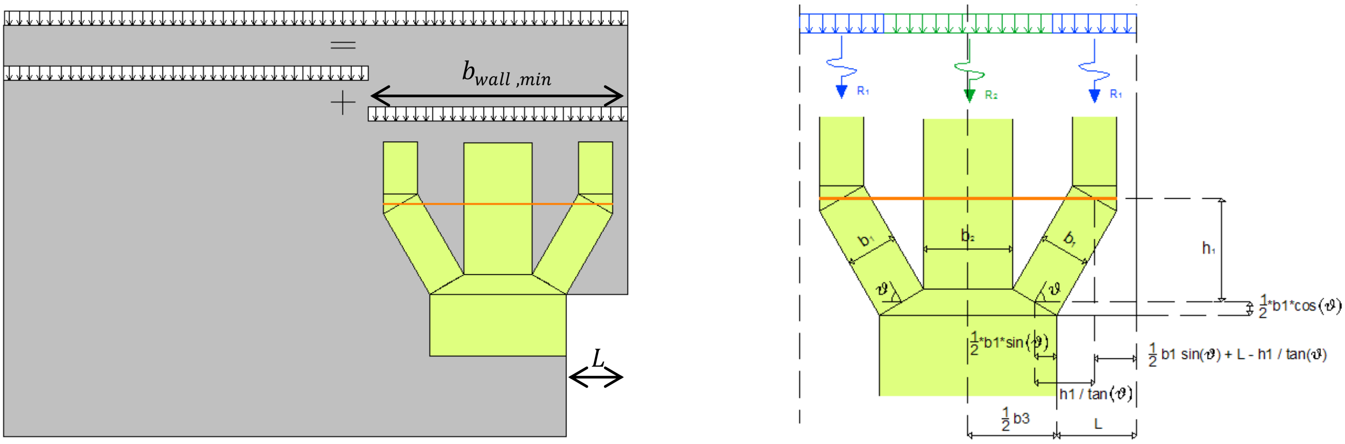
$$b_3 = \frac{2\, q_d\, L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \qquad \text{eq. (148)}$$

$$R_1 = 2\, q_d \left(\frac{1}{2}\, b_1 \sin\theta + L - \frac{h_1}{\tan\theta}\right) \qquad \text{eq. (149)}$$

$$R_2 = 2\left(q_d \left(\frac{1}{2} b_3 + L\right) - R_1\right) \qquad \text{eq. (150)}$$

$$b_{wall,min} = 2\, L \left(\frac{q_d}{d_{wall}\ \sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (151)}$$

After determining the parameters of the truss and the loading on the truss, the minimum reinforcement needed in the tie for the ULS conditions can be calculated. Based on the crack width condition at the locations of the tie in the SLS the diameter of the bars and the centre to centre distance of the bars can be determined. If necessary the reinforcement area can be increased.



a) Location truss model in the wall. b) Parameters of the truss model

Figure 23: Truss model 1 for the overhang

The overhang, truss model 2

In order to achieve an efficient design with this truss model, θ is chosen equal to 60°. The other parameters of the truss are determined using eq. (152) till (155). The loading on the truss is calculated using eq. (156) and (157). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (158).

$$b_1 = \frac{q_d}{\sin\,\theta\,(d_{wall}\cdot\sigma_{rd,max} - q_d)}\cdot\left(L - \frac{h_1}{\tan\,\theta}\right) \qquad \text{eq. (152)}$$

$$b_2 = \frac{2\,q_d}{d_{wall}\cdot\sigma_{rd,max} - q_d}\cdot\frac{h_1}{\tan\,\theta} \qquad \text{eq. (153)}$$

$$b_3 = \frac{2\,q_d\,L}{d_{wall}\cdot\sigma_{rd,max} - q_d}\cdot\left(\frac{2\,h_1}{L\tan\,\theta} - 1\right) \qquad \text{eq. (154)}$$

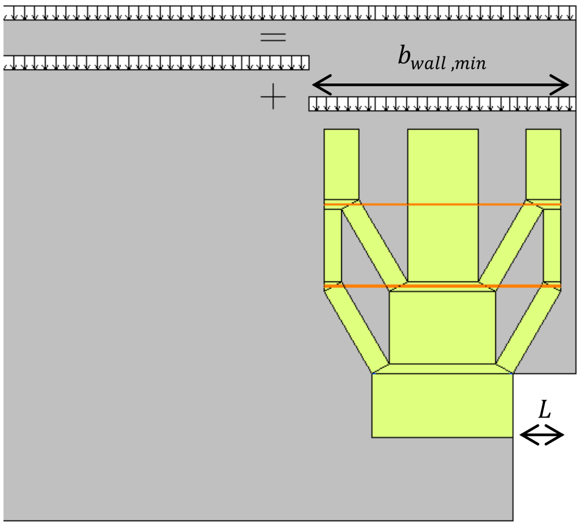
$$b_4 = \frac{2\,q_d\,L}{d_{wall}\cdot\sigma_{rd,max} - q_d} \qquad \text{eq. (155)}$$

$$R_1 = 2\,q_d\,\left(b_1\,\sin\theta + L - \frac{h_1}{\tan\,\theta}\right) \qquad \text{eq. (156)}$$

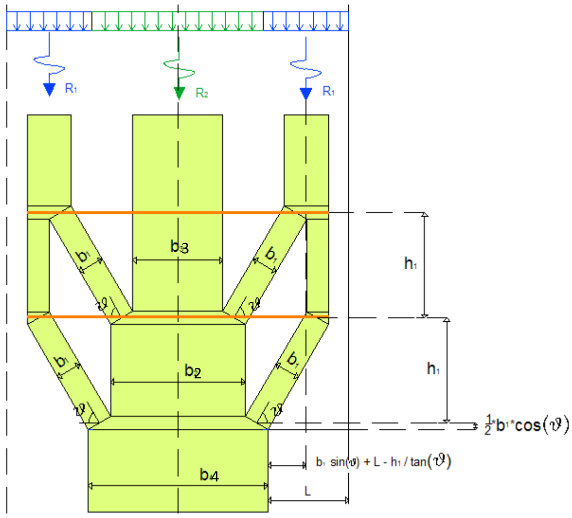
$$R_3 = 2\,\left(q_d\,\left(\frac{1}{2}b_4 + L\right) - R_1\right) \qquad \text{eq. (157)}$$

$$b_{wall,min} = 2\,L\,\left(\frac{q_d}{d_{wall}\;\sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (158)}$$

After determining the parameters of the truss and the loading on the truss, the minimum reinforcement needed in the tie for the ULS conditions is determined. Based on the crack width condition at the location of the ties in the SLS the diameter of the bars and the centre to centre distance of the bars are determined. If necessary the reinforcement area can be increased.



b) Location truss model in the wall.



b) Parameters of the truss model

Figure 24: Truss model 2 for the overhang

The overhang, truss model 3

In order to achieve an efficient design with this truss model, a stress trajectory plot is obtained from a Linear Elastic Finite Element program. From the stress trajectory plot the angle of each compression diagonal (θ_1 till θ_3) is determined and the ratio between the width of the compression diagonals (β_1 and β_2) is estimated. β_i are obtained by observing the stresses in the direction of each compression diagonal. The other parameters of the truss can be determined using eq. (159) till (171). The loading on the truss is calculated with eq. (172) till (174). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (175).

$$b_1 = \frac{1}{2} \frac{b_4}{\sin\theta_1 + \beta_2 \sin\theta_2 + \beta_3 \sin\theta_3} \qquad \text{eq. (159)}$$

$$b_2 = \beta_2\, b_1 \qquad \text{eq. (160)}$$

$$b_3 = \beta_3\, b_1 \qquad \text{eq. (161)}$$

$$b_4 = \frac{2\, q_d\, L}{d_{wall}\ \sigma_{rd,max} - q_d} \qquad \text{eq. (162)}$$

$$b_5 = b_1 \sin\theta_1 \qquad \text{eq. (163)}$$

$$b_6 = b_2 \sin\theta_2 \qquad \text{eq. (164)}$$

$$b_7 = b_3 \sin\theta_3 \qquad \text{eq. (165)}$$

$$a_1 = \frac{\sin\theta_1\, d_{wall}\ \sigma_{rd,max}}{q_d}\, b_1 \qquad \text{eq. (166)}$$

$$a_2 = \frac{\sin\theta_2\, d_{wall}\ \sigma_{rd,max}}{q_d}\, b_2 \qquad \text{eq. (167)}$$

$$a_3 = \frac{1}{2} b_4 + L - a_1 - a_2 \qquad \text{eq. (168)}$$

$$h_1 = \tan\theta_1 \left(L - \frac{1}{2} a_1 + \frac{1}{2}\, b_1 \sin\theta_1\right) \qquad \text{eq. (169)}$$

$$h_2 = \tan\theta_2 \left(L - a_1 - \frac{1}{2} a_2 + b_1 \sin\theta_1 + \frac{1}{2}\, b_2 \sin\theta_2\right) \qquad \text{eq. (170)}$$

$$h_3 = \tan\theta_3 \left(\frac{1}{2} a_3 - \frac{1}{2}\, b_3 \sin\theta_3\right) \qquad \text{eq. (171)}$$

$$R_1 = q_d\ a_1 \qquad \text{eq. (172)}$$

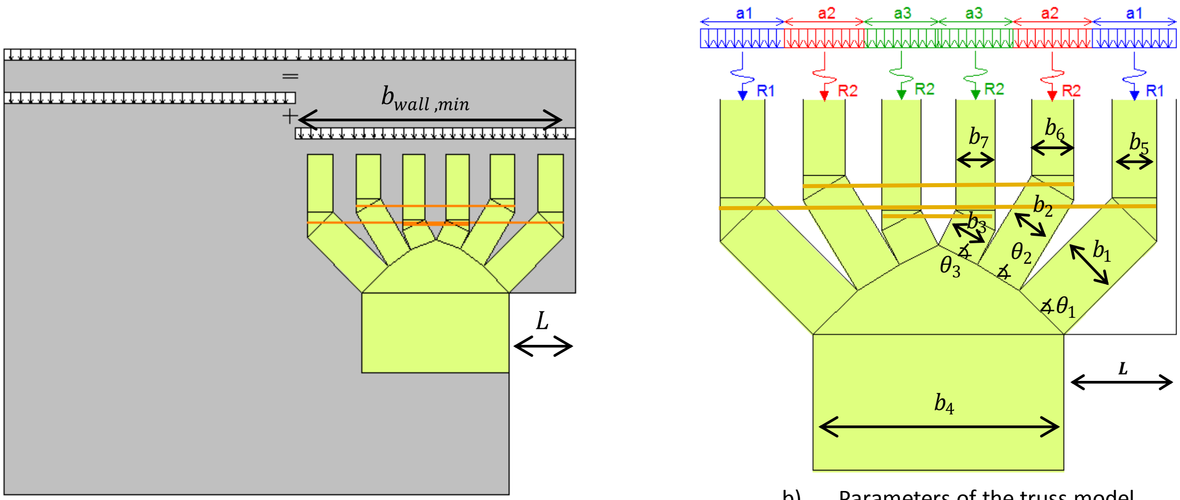
$$R_2 = q_d\ a_2 \qquad \text{eq. (173)}$$

$$R_3 = q_d\ a_3 \qquad \text{eq. (174)}$$

$$b_{wall,min} = 2\, L \left(\frac{q_d}{d_{wall}\ \sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (175)}$$

Now the stresses in the truss elements can be calculated. If nowhere the maximum stress is exceeded, the estimation of the ratio between the width of the compression diagonals (β_1 and β_2) is correct. The iteration process is than finished after one iteration step.

If the stresses are exceeded somewhere in the truss, the correct relation between the width of the diagonals have to be determined by using the forces in these diagonals. Now the new width of the truss elements can be computed and the reinforcement needed in the tie can be calculated. Based on the crack width condition at the location of the ties in the SLS, the diameter of the bars and the centre to centre distance of the bars are determined. If necessary the reinforcement area can be increased.



b) Location truss model in the wall.

b) Parameters of the truss model

Figure 25: Truss model 3 for the overhang

The opening, truss model 1

In order to achieve an efficient design with this truss model, θ is chosen equal to 60°. The other parameters of the truss are determined using eq. (178) till (180). The loading on the truss is calculated using eq. (181). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (182).

$$b_1 = \frac{q_d\ L}{2\ \sin\theta\ (d_{wall} \cdot \sigma_{rd,max} - q_d)} \qquad \text{eq. (178)}$$

$$b_2 = \frac{q_d\ L}{d_{wall} \cdot \sigma_{rd,max} - q_d} \qquad \text{eq. (179)}$$

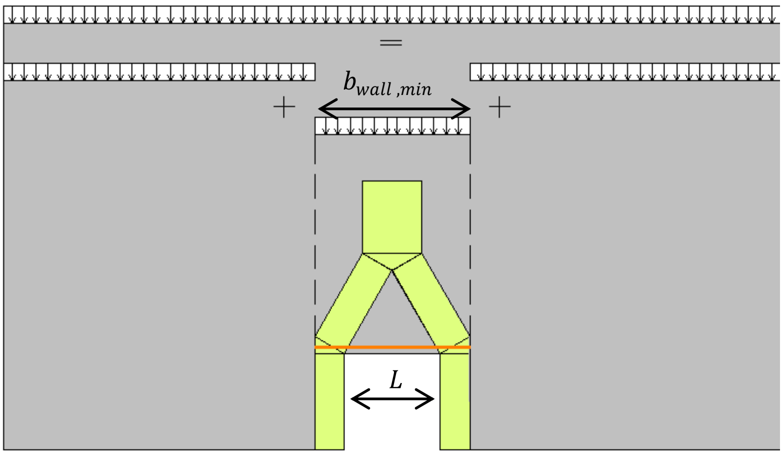
$$b_3 = \frac{q_d\ L}{2\ (d_{wall} \cdot \sigma_{rd,max} - q_d)} \qquad \text{eq. (180)}$$

$$R_1 = q_d \left(\frac{q_d\ L}{d_{wall} \cdot \sigma_{rd,max} - q_d} + L\right) \qquad \text{eq. (181)}$$

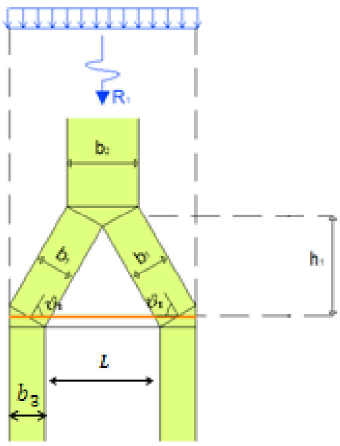
$$b_{wall,min} = L \left(\frac{q_d}{d_{wall}\ \sigma_{rd,max} - q_d} + 1\right) \qquad \text{eq. (182)}$$

After determining the parameters of the truss and the loading on the truss, the minimum reinforcement needed in the tie for the ULS condition is determined. Based on the crack width condition at the location of the tie in the SLS, the diameter of the bars is determined. If necessary the reinforcement area is increased. The centre to centre distance of the bars is determined such that the lowest layer of bars is located as near to the edge of the opening as possible, taking into account the concrete cover.

It should be noted that this truss model might lead to very conservative solutions. In truss model 1 the distributed load with a length of $(L + 2\ b_3)$ is schematized as a point load in the middle of the opening (see. Fig. 35). When we consider a simply supported beam loaded by a distributed load, the maximum moment in the beam will be equal to $\frac{1}{8} \cdot q \cdot l^2$. However a simply supported beam loaded by a point load in the middle equal to $q \cdot l$, will have a maximum moment equal to $\frac{1}{4}\ q\ l^2$. In order to obtain the same moment of $\frac{1}{8} \cdot q \cdot l^2$, the point load should be equal to $\frac{1}{2} \cdot q \cdot l$. When this is translated to the truss model for the opening, the truss model shown in the next paragraph is obtained.



a) Location truss model in the wall.



b) Parameters of the truss model

Figure 26: Truss model 1 for the opening

The opening, truss model 2

In order to achieve an efficient design with this truss model, θ is chosen equal to 60°. The other parameters of the truss are determined using eq. (183) till (186). The loading on the truss is calculated with eq. (187) and (188). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (189).

$$b_1 = q_d \, L \, \frac{d_{wall} \cdot \sigma_{rd,max}}{\sin\theta \, (d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (183)}$$

$$b_2 = 2\, q_d \, L \, \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (184)}$$

$$b_3 = q_d \, L \, \frac{(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (185)}$$

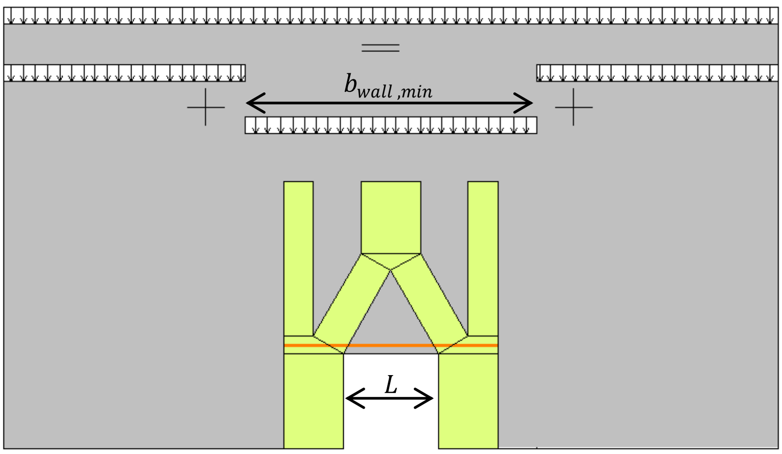
$$b_4 = q_d \, L \, \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (186)}$$

$$R_1 = 2\, q_d \, L \, \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (187)}$$

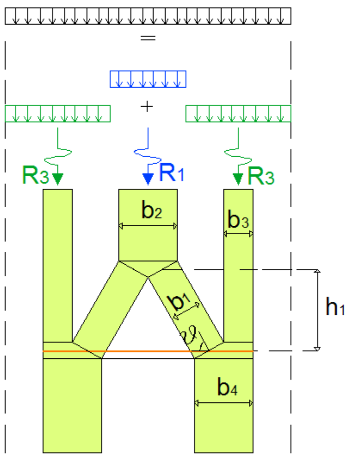
$$R_3 = q_d \, L \, \frac{d_{wall} \cdot \sigma_{rd,max} \; (2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (188)}$$

$$b_{wall,min} = 2\, L \, \frac{d_{wall} \cdot \sigma_{rd,max} \; (3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (189)}$$

After determining the parameters of the truss and the loading on the truss, the minimum reinforcement needed in the tie for the ULS condition is calculated. Based on the crack width condition in the SLS the diameter of the bars is determined. If necessary the reinforcement area is increased. The centre to centre distance of the bars of is determined such that the lowest layer of bars is located as near to the edge of the opening as possible, taking into account the concrete cover.



a) Location truss model in the wall



b) Parameters of the truss model

Figure 27: Truss model 2 for the opening

The opening, truss model 3

In order to achieve an efficient design with this truss model, θ is chosen equal to $60°$. The other parameters of the truss are determined using eq. (190) till (193). The loading on the truss is calculated will eq. (194) and (195). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (196).

$$b_1 = \frac{1}{2}\, q_d\, L\, \frac{d_{wall} \cdot \sigma_{rd,max}}{\sin\theta\, (d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (190)}$$

$$b_2 = q_d\, L\, \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (191)}$$

$$b_3 = 2\, q_d\, L\, \frac{d_{wall} \cdot \sigma_{rd,max}}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (192)}$$

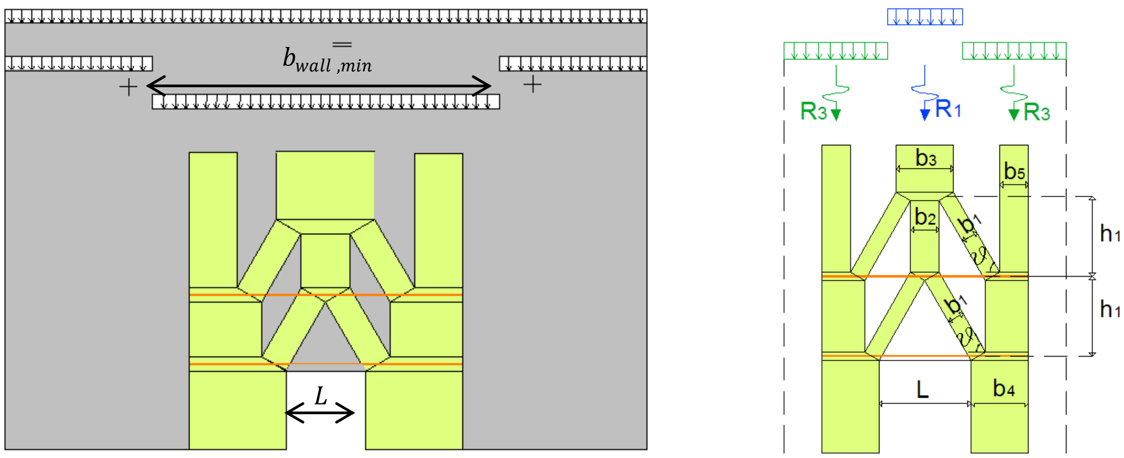
$$b_4 = q_d\, L\, \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (193)}$$

$$R_1 = 2\, q_d\, L\, \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (194)}$$

$$R_5 = q_d\, L\, \frac{d_{wall} \cdot \sigma_{rd,max}\,(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (195)}$$

$$b_{wall,min} = 2\, L\, \frac{d_{wall} \cdot \sigma_{rd,max}\,(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (196)}$$

After determining the parameters of the truss and the loading on the truss, the minimum reinforcement needed in the tie for the ULS condition is calculated. Based on the crack width condition in the SLS the diameter of the bars is determined. If necessary the reinforcement area can be increased. The centre to centre distance of the bars of the lowest tie is determined such that the lowest layer of bars is located as near to the edge of the opening as possible, taking into account the concrete cover.



a) Location truss model in wall b) Parameters of the truss model

Figure 28: Truss model 3 for the opening

The opening, truss model 4

In order to achieve an efficient design with this truss model, a stress trajectory plot is used from a Linear Elastic Finite Element program. From the stress trajectory plot the angle of each compression diagonal (θ_1 till θ_3) is determined and the ratio between the width of the compression diagonals (β_1 and β_2) is estimated. The value for β_i are obtained by observing the stresses in the direction of each compression diagonal. The other parameters of the truss can be determined using eq. (197) till (204). The loading on the truss is calculated with eq. (205) and (206). The minimum width of the wall for which these expressions are applicable, is calculated with eq. (207).

$$b_1 = \frac{b_4 - b_8}{\sin\theta_1 + \beta_2 \sin\theta_2 + \beta_3 \sin\theta_3} \qquad \text{eq. (197)}$$

$$b_2 = \beta_2\, b_1 \qquad \text{eq. (198)}$$

$$b_3 = \beta_3\, b_1 \qquad \text{eq. (199)}$$

$$b_4 = q_d\, L \frac{(3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (200)}$$

$$b_5 = 2\, b_1 \sin\theta_1 \qquad \text{eq. (201)}$$

$$b_6 = b_5 + 2\, b_2 \sin\theta_2 \qquad \text{eq. (202)}$$

$$b_7 = b_6 + 2\, b_3 \sin\theta_3 \qquad \text{eq. (203)}$$

$$b_8 = q_d\, L \frac{(2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (204)}$$

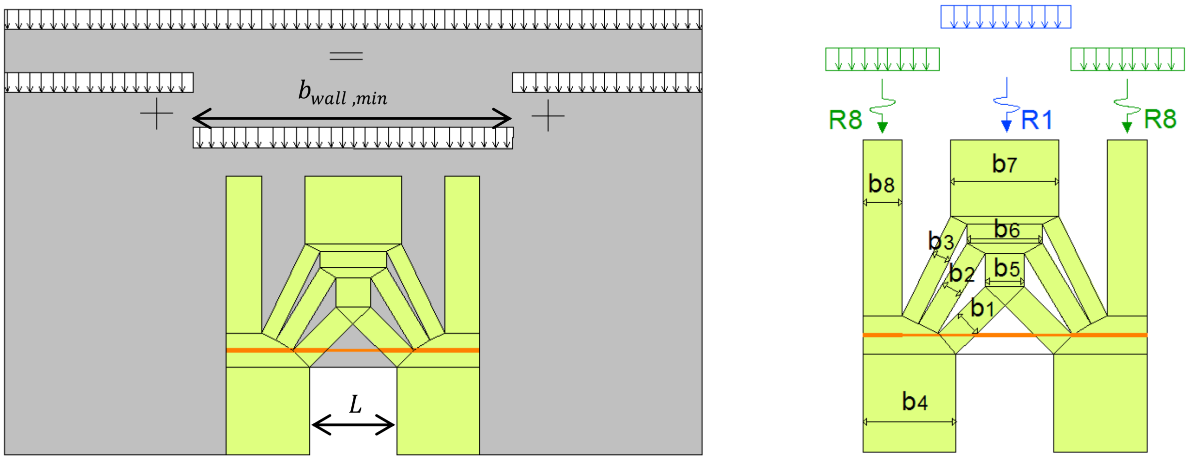
$$R_1 = 2\, q_d\, L \frac{(d_{wall} \cdot \sigma_{rd,max})^2}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (205)}$$

$$R_8 = q_d\, L \frac{d_{wall} \cdot \sigma_{rd,max}\ (2\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (206)}$$

$$b_{wall,min} = 2\, L \frac{d_{wall} \cdot \sigma_{rd,max}\ (3\, d_{wall} \cdot \sigma_{rd,max} + q_d)}{(d_{wall} \cdot \sigma_{rd,max} - q_d)\,(4\, d_{wall} \cdot \sigma_{rd,max} + q_d)} \qquad \text{eq. (207)}$$

Now the stresses in the truss elements can be calculated. If nowhere the maximum stress is exceeded, the estimation of the ratio between the width of the compression diagonals (β_1 and β_2) is correct. The iteration process is than finished after one iteration step.

If the stresses are exceeded somewhere in the truss, the correct relation between the width of the diagonals have to be determined by using the forces in these diagonals. Now the correct ratio between the widths of the diagonals can be computed. The minimum reinforcement needed is calculated from the ULS condition. Based on the crack width condition in the SLS, the diameter of the bars is determined. If necessary the reinforcement area is increased. The centre to centre distance of the bars is determined such that the lowest layer of bars is located as near to the edge of the opening as possible, taking into account the concrete cover.



a) Location truss model in the wall.

b) Parameters of the truss model

Figure 29: Truss model 4 for the opening

4 The Designs

In this chapter the detail drawing of the reinforcement for the discontinuity region is given, which is obtained by following the design steps of the codes and the results of the investigation on optimizing the different truss models.

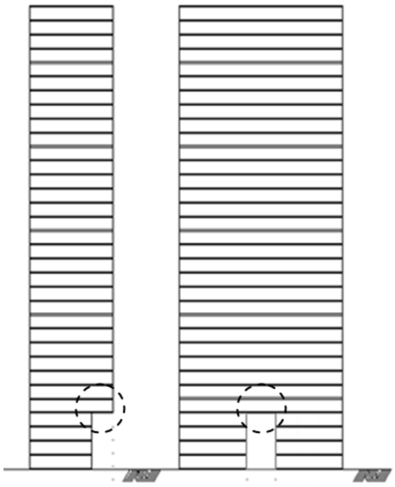


Figure 30: The dotted circle indicates the location for which the detail drawing is given.

4.1 The overhang of 1 m

4.1.1 The Dutch Code

The moment reinforcement consists of 6 $\emptyset 25$ subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. In accordance with art. 9.11.3 these 3 layers can be distributed over an height of $0.2\,l = 200$ mm. The centre to centre distance of the layers becomes 100 mm. The concrete is in the stabilized cracking stage ($f_{bm} < M/W$). The steel stress is equal to 311 N/mm^2. According to art. 8.7.2 the centre to centre distance of the bars for environments class 1 with a steel stress of 311 N/mm^2, should be smaller than 190 mm in order to fulfill the crack width requirements. This is satisfied. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 10 - 150$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction.

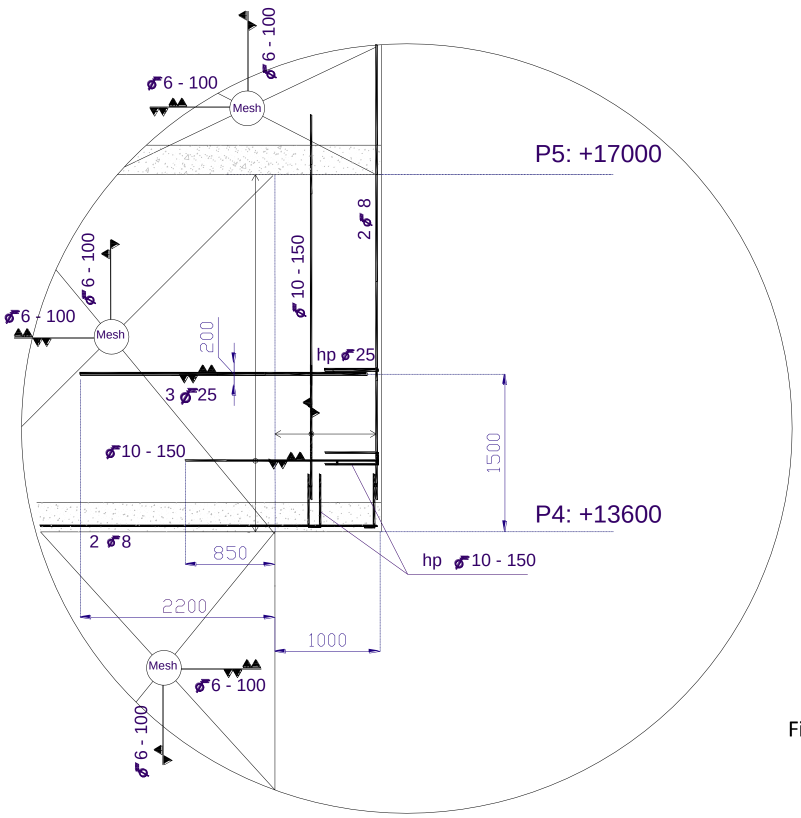


Figure 31: Reinforcement drawing for the design with the Dutch Code, overhang is 1 m

4.1.2 The Eurocode, truss model 1

The moment reinforcement consists of $8\ \emptyset 25$ subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 60 mm, such that the total height of the layers combined is 180 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the crack width will be equal to 0,32 mm, which fulfills the requirements of the maximum allowable crack width.

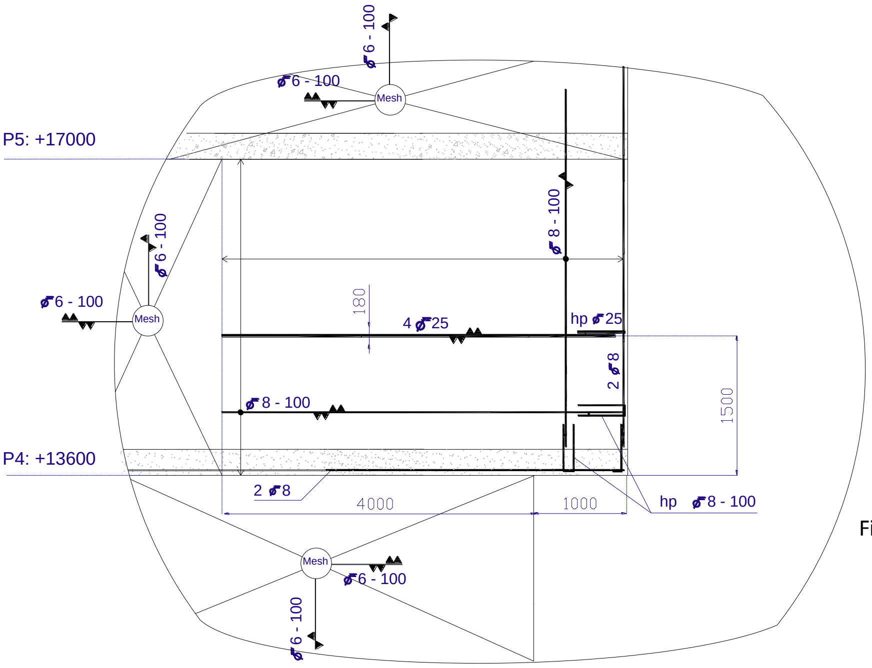


Figure 32: Reinforcement drawing for the design with the Eurocode with truss model 1, overhang is 1 m

4.1.3 The Eurocode, truss model 2

The moment reinforcement consists of two reinforcement ties positioned at a height of 1400 mm and 2750 mm from the overhang level. Each tie consists of 6 $\emptyset 20$, subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 60 mm, such that the total height of the layers combined is 120 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 125$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,35 mm, which fulfills the requirements of the maximum allowable crack width.

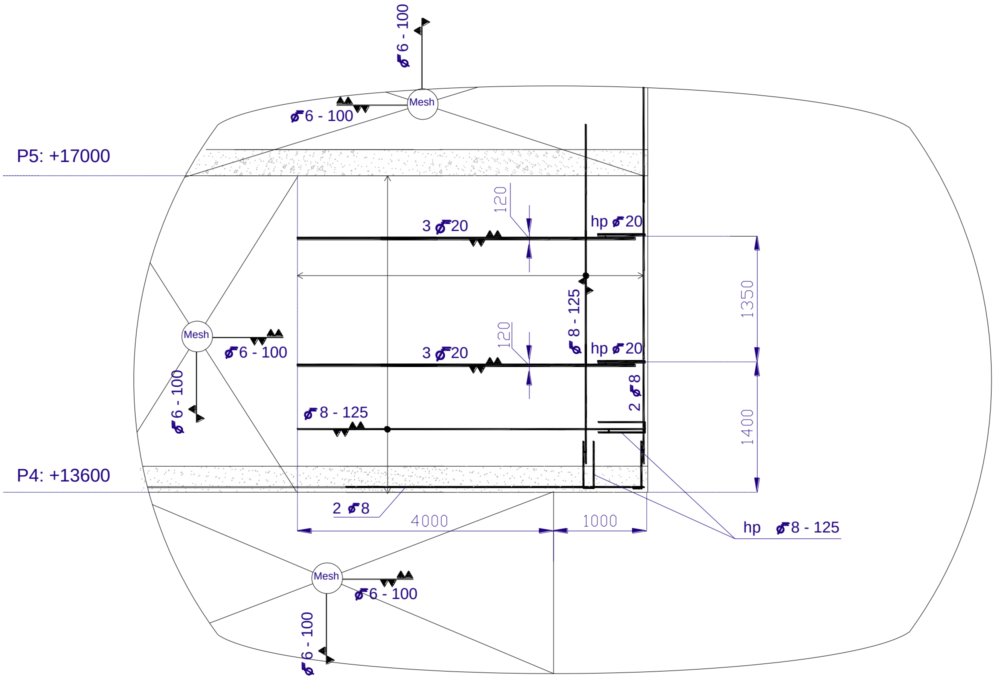


Figure 33: Reinforcement drawing for the design with the Eurocode with truss model 2, overhang is 1 m

4.1.4 The Eurocode, truss model 3

The moment reinforcement consists of three reinforcement ties of which two partially overlap. The ties are positioned at a height of 1020 mm and 1310 mm from the overhang level. The first tie consist of $10\ \emptyset 25$, the second of $8\ \emptyset 20$ and the third of $2\ \emptyset 25$. The third tie is located at the same height as the first tie, such that the total amount of reinforcement at the position of the third tie is equal to $12\ \emptyset 25$. In each tie the bars are subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 50 mm, such that the total height of the layers combined of tie 1, 2 and 3 are resp. 200 mm, 150 mm and 250 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 12 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

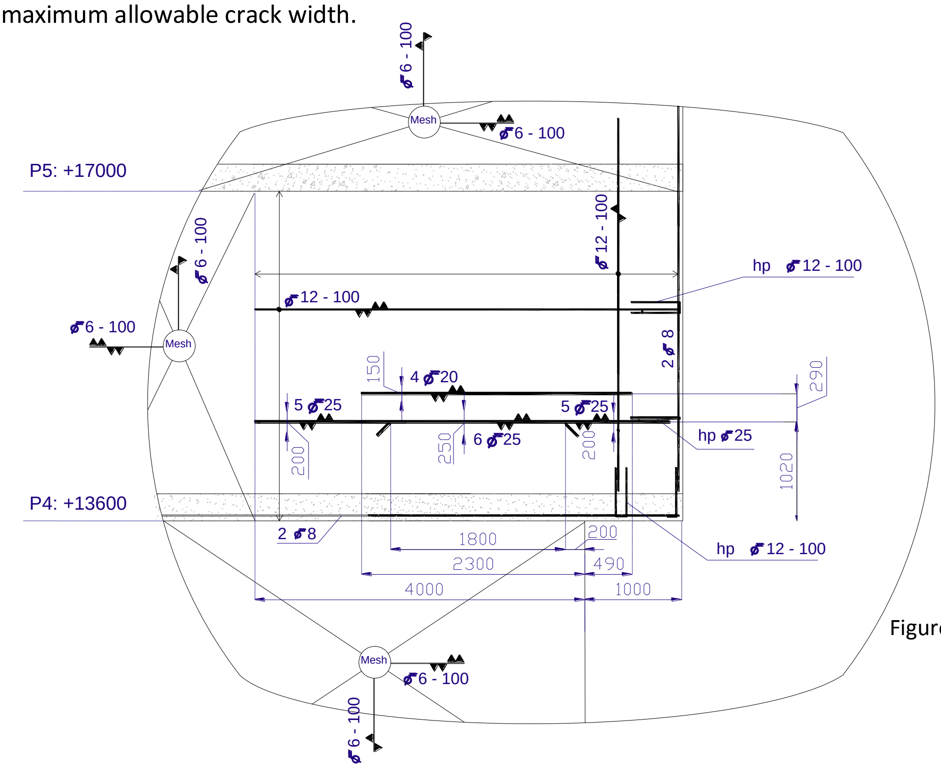


Figure 34: Reinforcement drawing for the design with the Eurocode with truss model 3, overhang is 1 m

4.2 The overhang of 5 m

4.2.1 The Dutch Code

The moment reinforcement consists of 30 Ø25 subdivided into 2 bars per layer connected by hairpins. The centre to centre distance of the bars equals $1000/14 = 70$ mm. The concrete is in the stabilized cracking stage. The steel stress equals 310 N/mm^2. For the crack width requirements, the centre to centre distance of the bars should be smaller than 190 mm. This is fulfilled. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 10 - 150$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction.

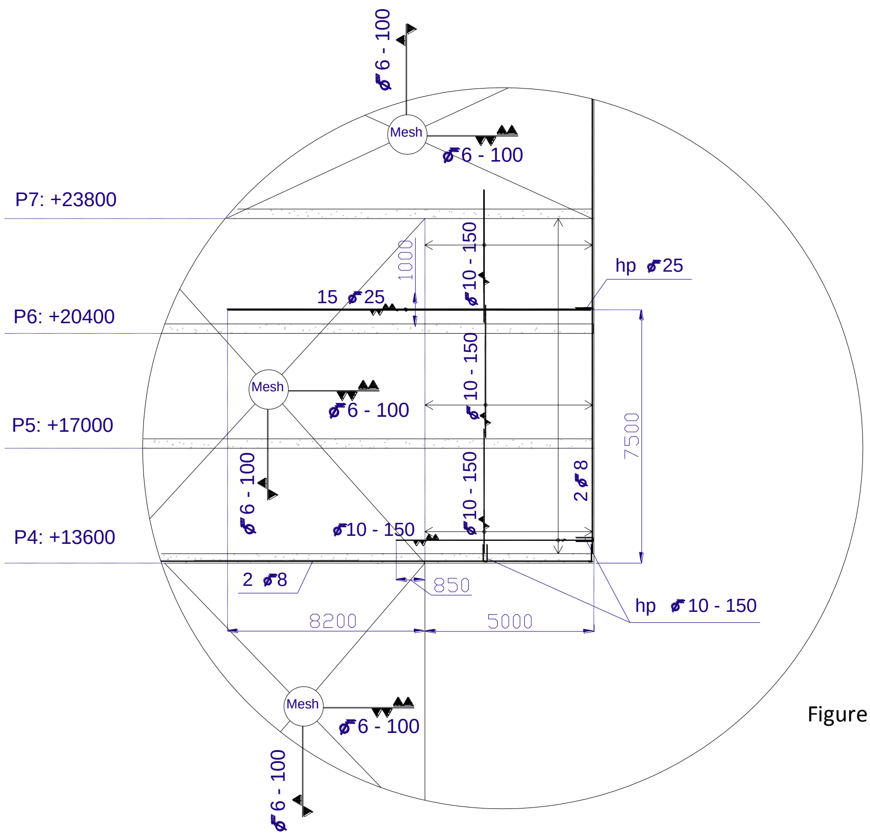


Figure 35: Reinforcement drawing for the design with the Dutch Code, overhang is 5 m

4.2.2 The Eurocode, truss model 1

The moment reinforcement consists of 32 Ø25 subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 50 mm, such that the total height of the layers combined is 750 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

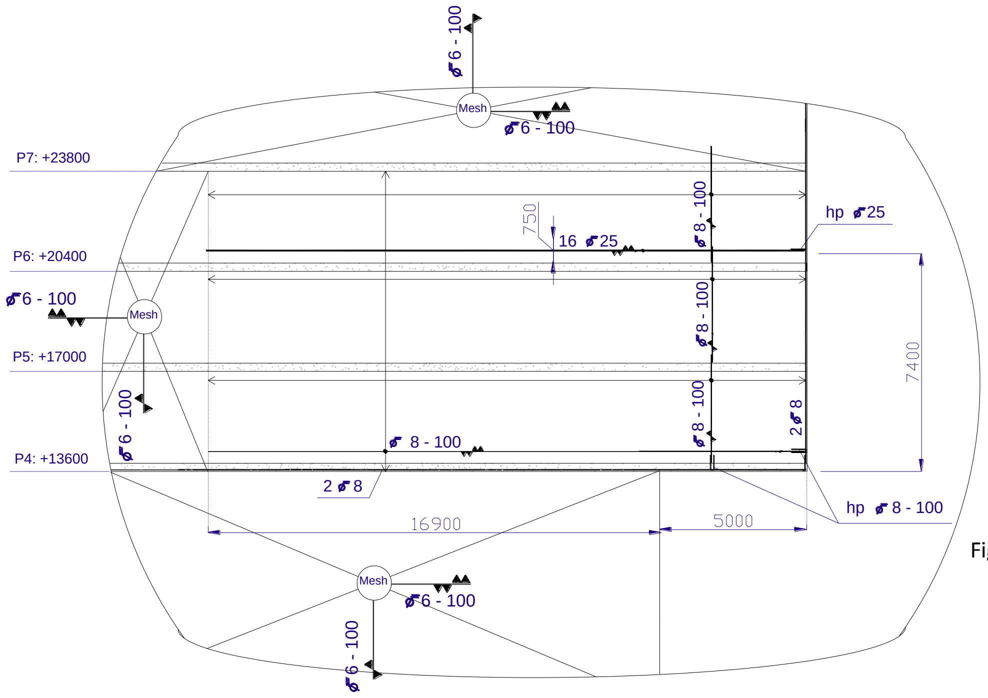


Figure 36: Reinforcement drawing for the design with the Eurocode with truss model 1, overhang is 5 m

4.2.3 The Eurocode, truss model 2

The moment reinforcement consists of two reinforcement ties positioned at a height of 7000 mm and 13600 mm from the overhang level. Each tie consists of $18\ \emptyset 20$, subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 60 mm, such that the total height of the layers combined is 480 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 125$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,37 mm, which fulfills the requirements of the maximum allowable crack width.

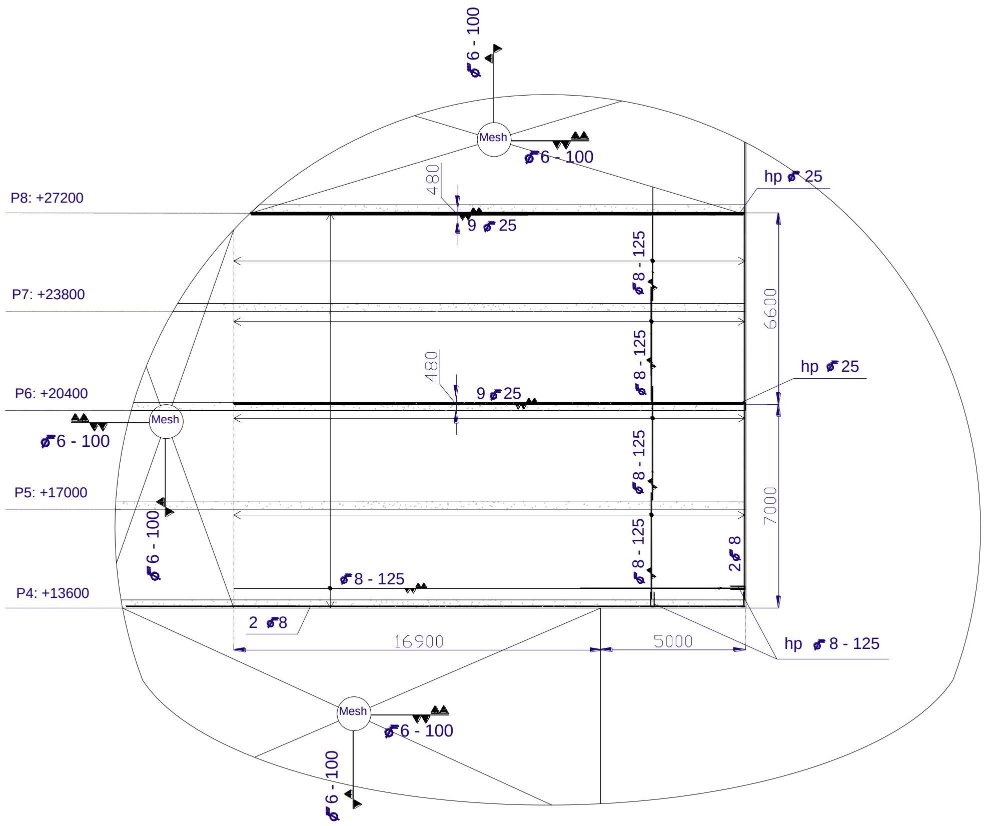


Figure 37: Reinforcement drawing for the design with the Eurocode with truss model 2, overhang is 5 m

4.2.4 The Eurocode, truss model 3

The moment reinforcement consists of three reinforcement ties positioned at a height of 4560 mm, 5610 mm and 6320 mm from the overhang level. The first tie consist of $52\ \emptyset 25$, the second of $30\ \emptyset 20$ and the third of $24\ \emptyset 25$. In the first tie the bars are subdivided into 2 bars per layer, with a centre to centre distance of the layers equal to 50 mm, such that the total height of the layers of tie 1 combined is equal to 1250 mm. In the second and third tie the bars are subdivided into 3 bars per layer, with a centre to centre distance of the layers equal to 50 mm. The reason that in the second and third tie 3 bars per layer are possible, is because these bars are anchored by their anchorage length such that the minimum mandrel diameter of the hairpin is not a requirement for the centre to centre distance of the bars in their layer. The total height of the layers of tie 2 and 3 are resp. 450 mm and 350 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 12 - 90$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,40 mm, which fulfills the requirements of the maximum allowable crack width.

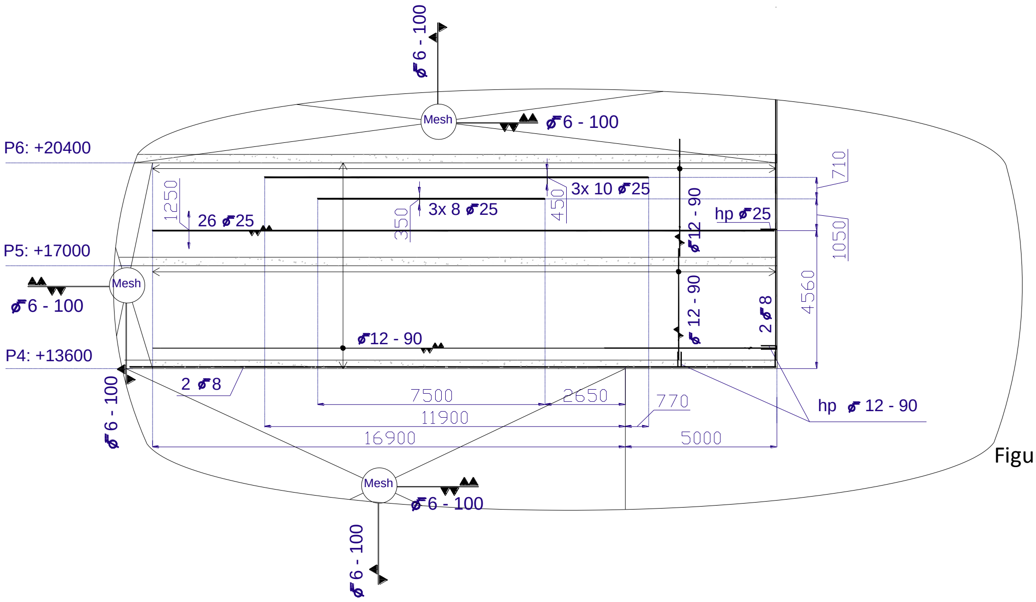


Figure 38: Reinforcement drawing for the design with the Eurocode with truss model 3, overhang is 5 m

4.3 The overhang of 10 m

4.3.1 The Dutch Code

The moment reinforcement consists of 60 Ø25 subdivided into 2 bars per layer connected by hairpins. The centre to centre distance of the bars equals $2000/29 = 70$ mm. The concrete is in the stabilized cracking stage. The steel stress equals 310 N/mm^2. For the crack with requirements, the centre to centre distance of the bars should be smaller than 190 mm. This is fulfilled. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 10 - 150$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction.

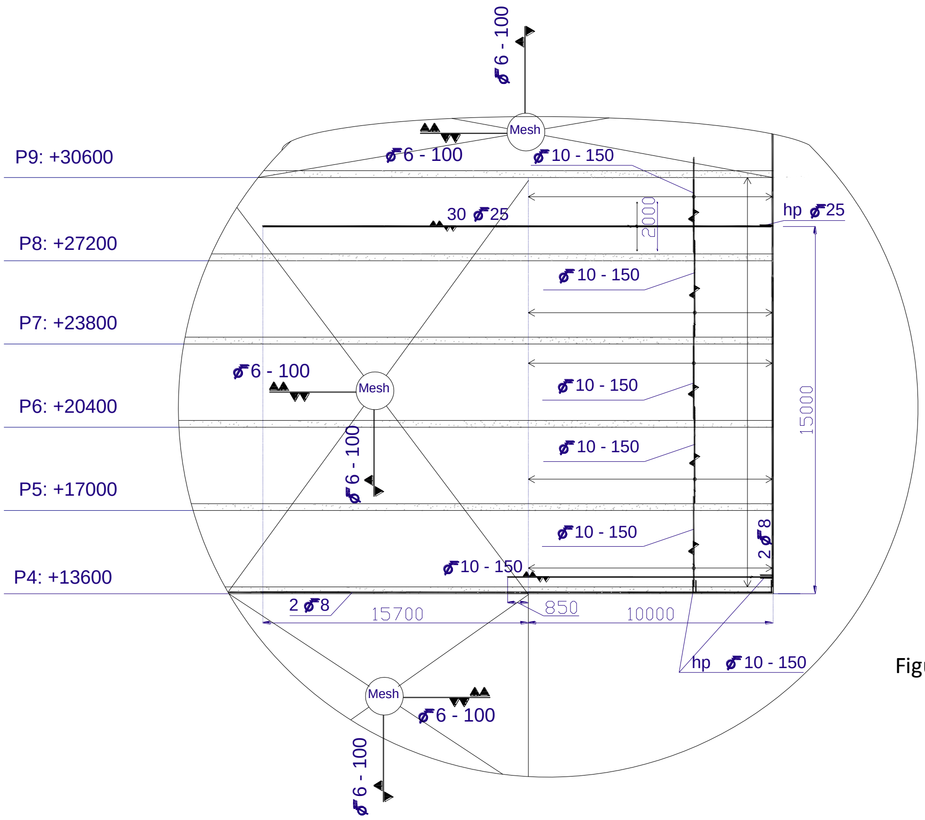


Figure 39: Reinforcement drawing for the design with the Dutch Code, overhang is 10 m

4.3.2 The Eurocode, truss model 1

The moment reinforcement consists of 64 Ø25 subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 50 mm, such that the total height of the layers combined is 1550 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

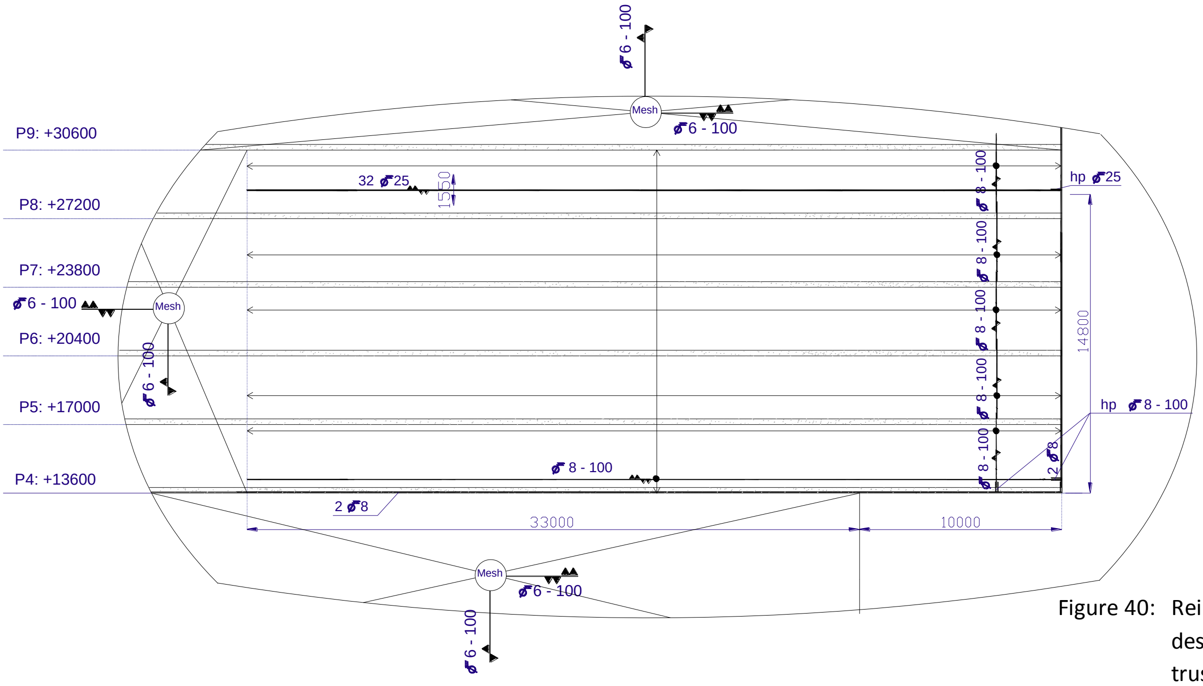


Figure 40: Reinforcement drawing for the design with the Eurocode with truss model 1, overhang is 10 m

4.3.3 The Eurocode, truss model 2

The moment reinforcement consists of two reinforcement ties positioned at a height of 14000 mm and 27200 mm from the overhang level. Each tie consists of $34\ \emptyset 20$, subdivided into 2 bars per layer which are connected by hairpins near the edge of the wall. The centre to centre distance of the layers is equal to 60 mm, such that the total height of the layers combined is 960 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 125$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,40 mm, which fulfills the requirements of the maximum allowable crack width.

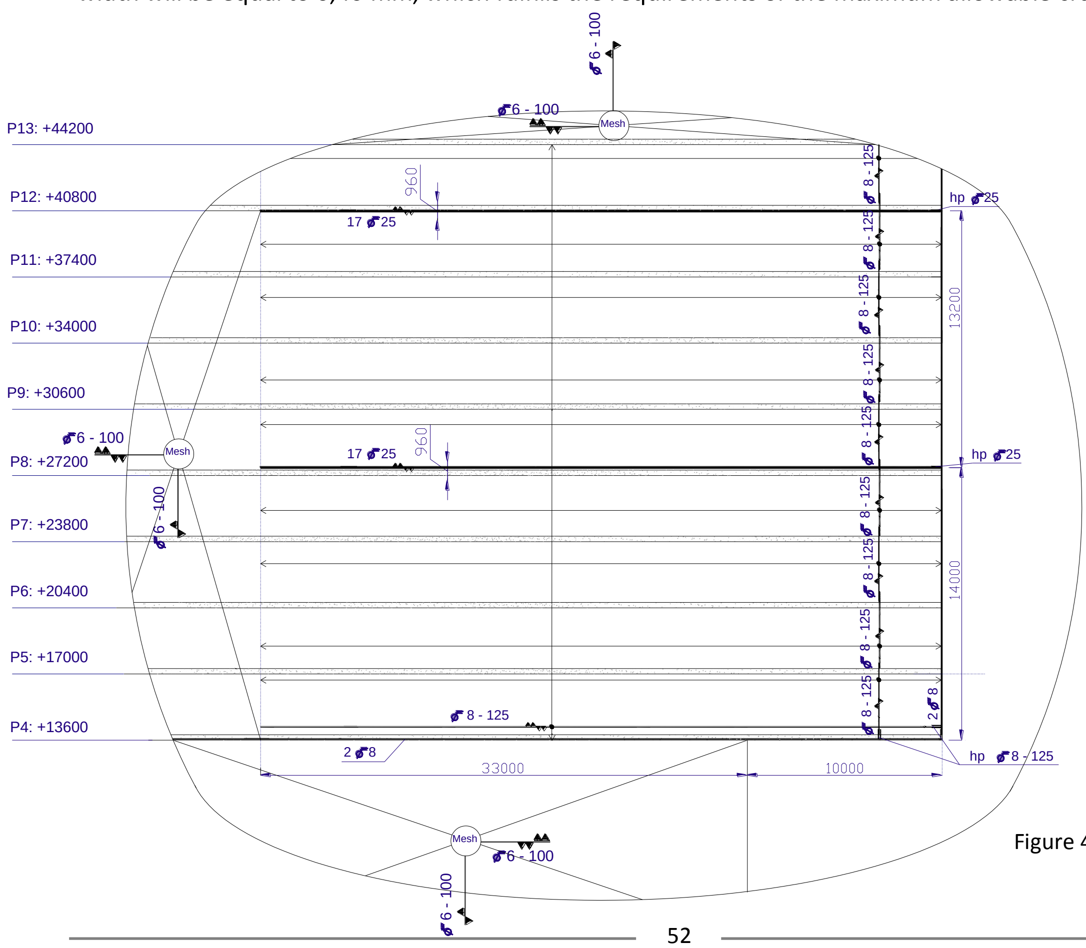


Figure 41: Reinforcement drawing for the design with the Eurocode with truss model 2, overhang is 10 m

4.3.4 The Eurocode, truss model 3

The moment reinforcement consists of three reinforcement ties positioned at a height of 9020 mm, 11620 mm and 12360 mm from the overhang level. The first tie consist of $112\ \emptyset 25$, the second of $48\ \emptyset 25$ and the third of $45\ \emptyset 25$. In the first tie the bars are subdivided into 2 bars per layer, with a centre to centre distance of the layers equal to 50 mm, such that the total height of the layers combined of tie 1 is equal to 2750 mm. In the second and third tie the bars are subdivided into 3 bars per layer, with a centre to centre distance of the layers equal to 50 mm. The reason that in the second and third tie 3 bars per layer are possible, is because these bars are anchored by their anchorage length such that the minimum mandrel diameter of the hairpin is not a requirement for the distance of the bars in their layer. The total height of the layers of tie 2 and 3 are resp. 750 mm and 700 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 12 - 80$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,40 mm, which fulfills the requirements of the maximum allowable crack width.

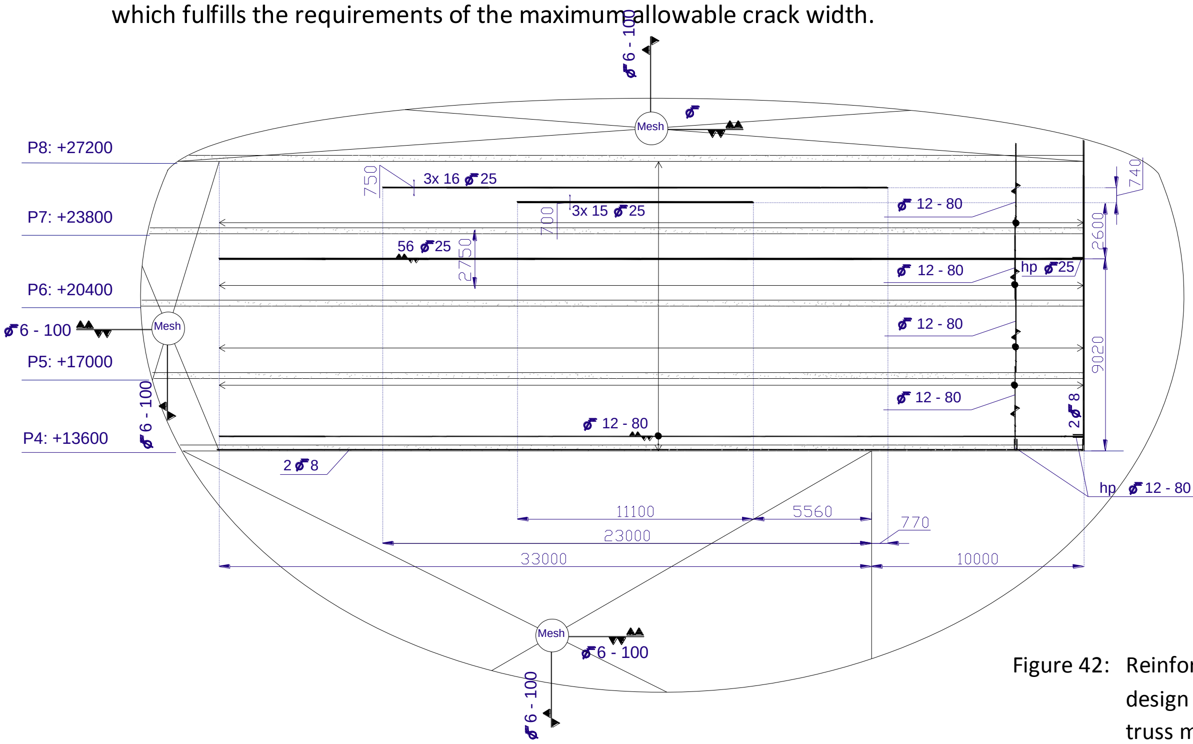


Figure 42: Reinforcement drawing for the design with the Eurocode with truss model 3, overhang is 10 m

4.4 The opening of 1 m

4.4.1 The Dutch Code

The moment reinforcement consists of $4\ Ø20 + 2\ Ø12$. The $2\ Ø12$ are chosen to be the primary reinforcement in the floor. The $4\ Ø20$ are subdivided into two layers with a centre to centre distance per layer equal to 150 mm. The concrete is in the stabilized cracking stage ($f_{bm} < M/W$). The steel stress is equal to 312 N/mm^2. According to art. 8.7.2 the centre to centre distance of the bars for environments class 1 with a steel stress of 312 N/mm^2, should be smaller than 190 mm in order to fulfill the crack width requirements. This is satisfied. The horizontal shear reinforcement near each face is equal to $Ø16 - 140$. The vertical shear reinforcement near each face is equal to $Ø10 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $Ø6 - 100$ near each face in each direction.

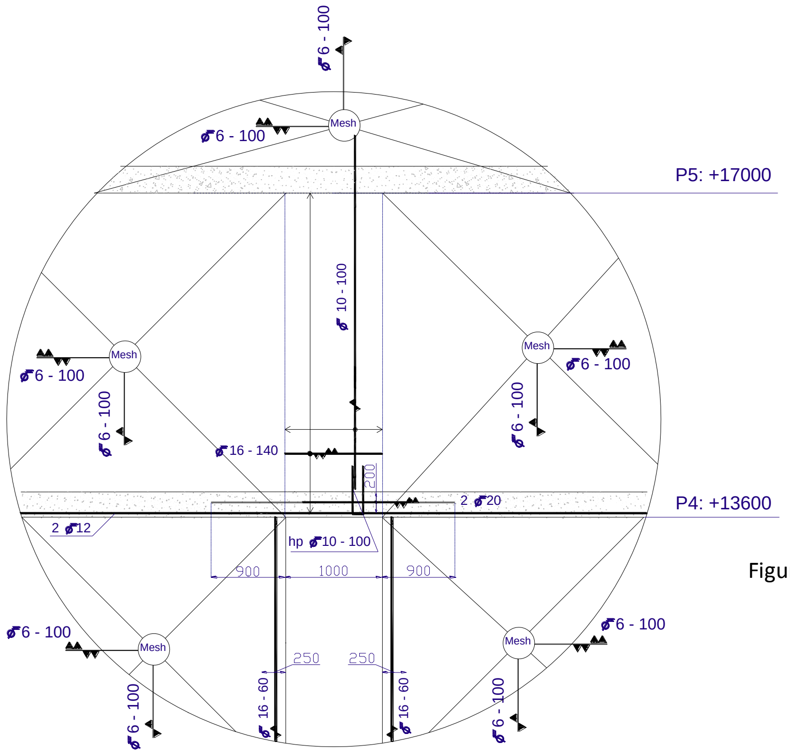


Figure 43: Reinforcement drawing for the design with the Dutch Code, opening is 1 m

4.4.2 The Eurocode, truss model 1

The moment reinforcement consists of $8\ Ø25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 70 mm, such that the total height of the layers combined is equal to 220 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $Ø8 - 90$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $Ø6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,31 mm, which fulfills the requirements of the maximum allowable crack width.

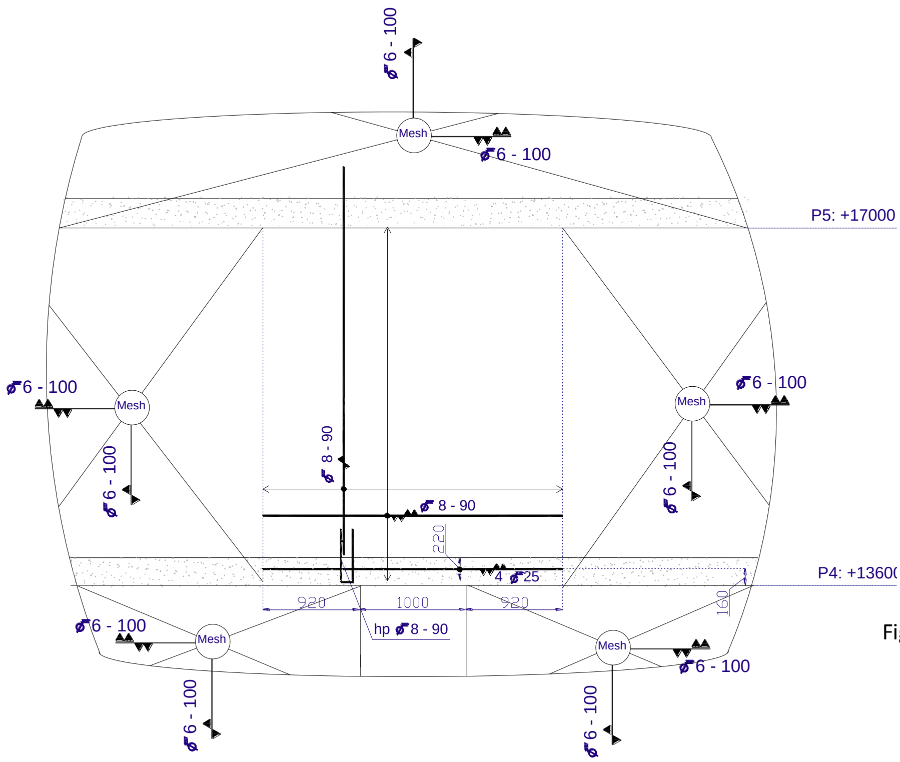


Figure 44: Reinforcement drawing for the design with the Eurocode with truss model 1, opening is 1 m

4.4.3 The Eurocode, truss model 2

The moment reinforcement consists of 2 Ø25 and 2 Ø20, both subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 50 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,30 mm, which fulfills the requirements of the maximum allowable crack width.

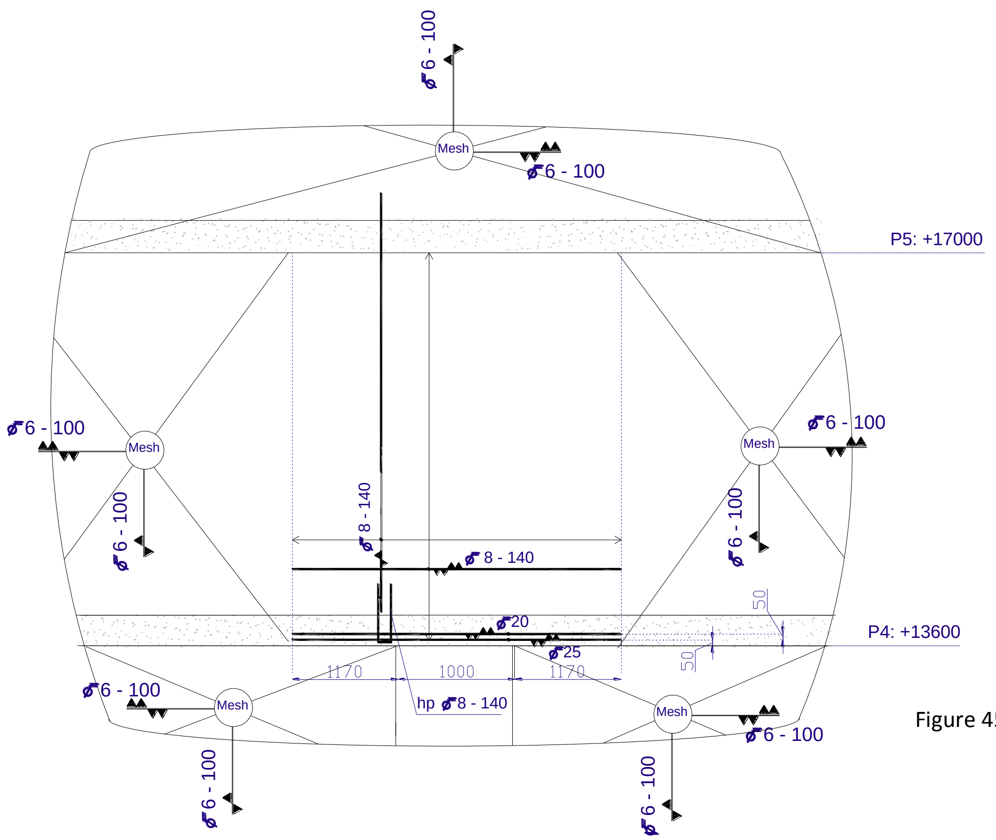


Figure 45: Reinforcement drawing for the design with the Eurocode with truss model 2, opening is 1 m

4.4.4 The Eurocode, truss model 3

The moment reinforcement consists of two reinforcement ties positioned at a height of 50 mm and 910 mm from the opening level. Each tie consists of $2\ \emptyset 25$, placed in one layer. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,27 mm, which fulfills the requirements of the maximum allowable crack width.

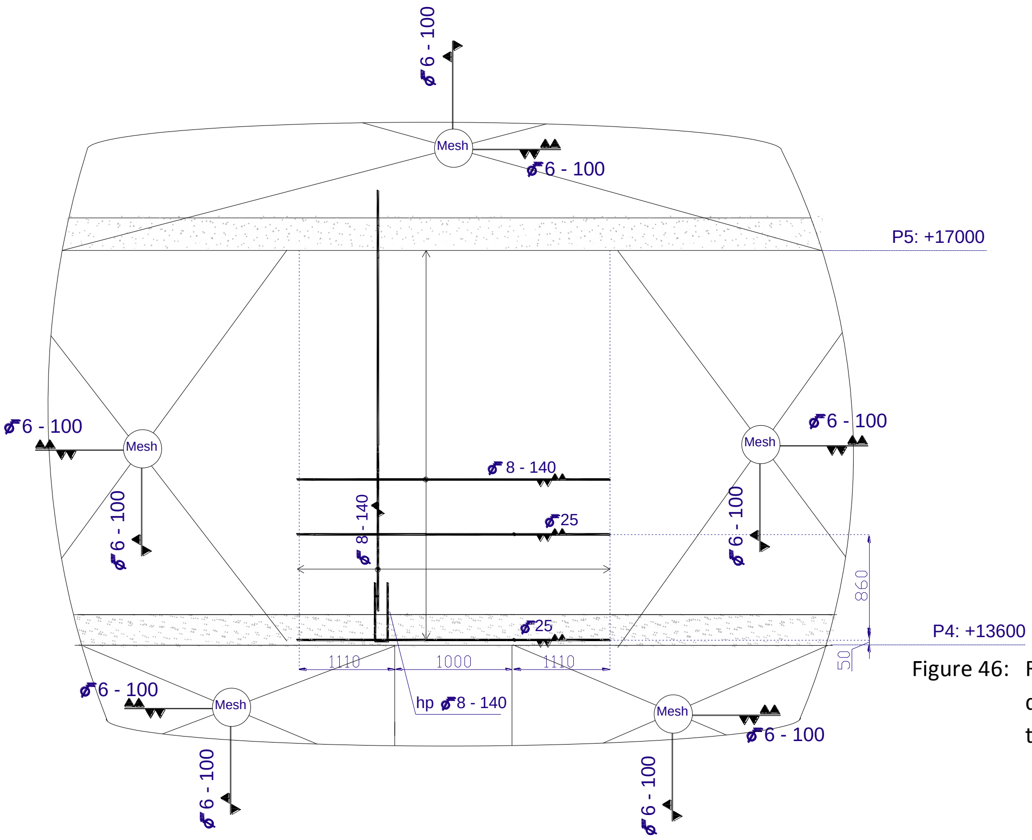


Figure 46: Reinforcement drawing for the design with the Eurocode with truss model 3, opening is 1 m

4.4.5 The Eurocode, truss model 4

The moment reinforcement consists of $4\ \emptyset 25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 74 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,35 mm, which fulfills the requirements of the maximum allowable crack width.

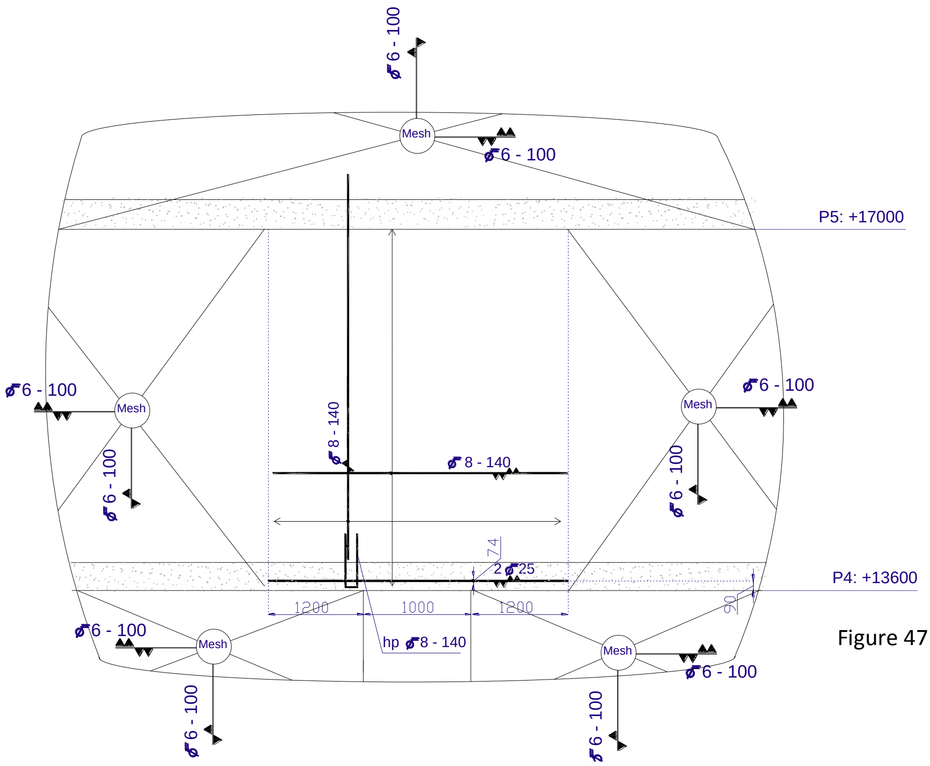


Figure 47: Reinforcement drawing for the design with the Eurocode with truss model 4, opening is 1 m

4.5 The opening of 5 m

4.5.1 The Dutch Code

The moment reinforcement consists of 16 Ø25. The 16 Ø25 are subdivided into eight layers with a centre to centre distance per layer equal to 140 mm. The concrete is in the stabilized cracking stage. The steel stress is equal to 295 N/mm^2. According to art. 8.7.2 the centre to centre distance of the bars for environments class 1 with a steel stress of 295 N/mm^2, should be smaller than 200 mm in order to fulfill the crack width requirements. This is satisfied. The horizontal shear reinforcement near each face is equal to $Ø16 - 140$. The vertical shear reinforcement near each face is equal to $Ø10 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $Ø6 - 100$ near each face in each direction.

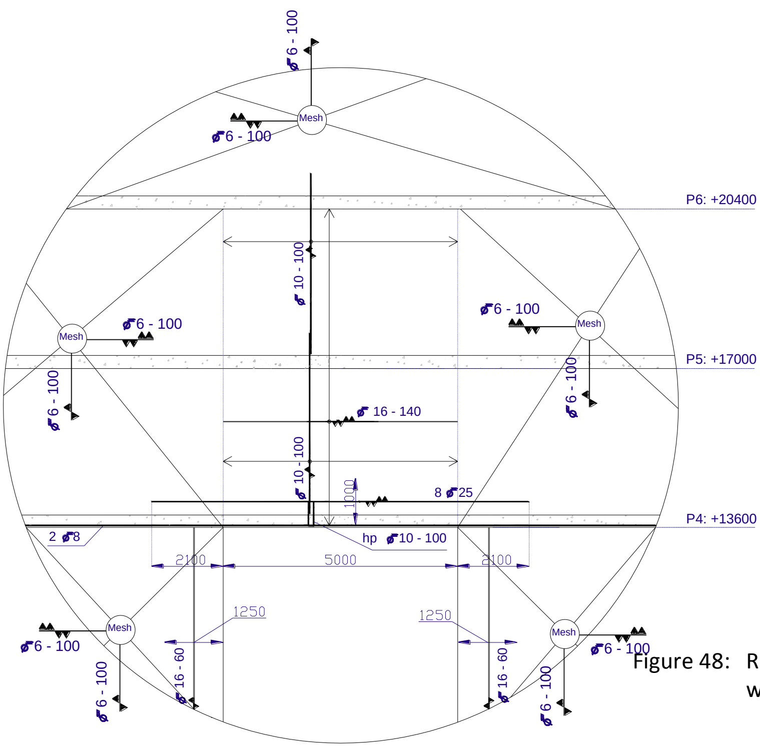


Figure 48: Reinforcement drawing for the design with the Dutch Code, opening is 5 m

4.5.2 The Eurocode, truss model 1

The moment reinforcement consists of 34 Ø25 subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 95 mm, such that the total height of the layers combined is equal to 1500 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 90$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

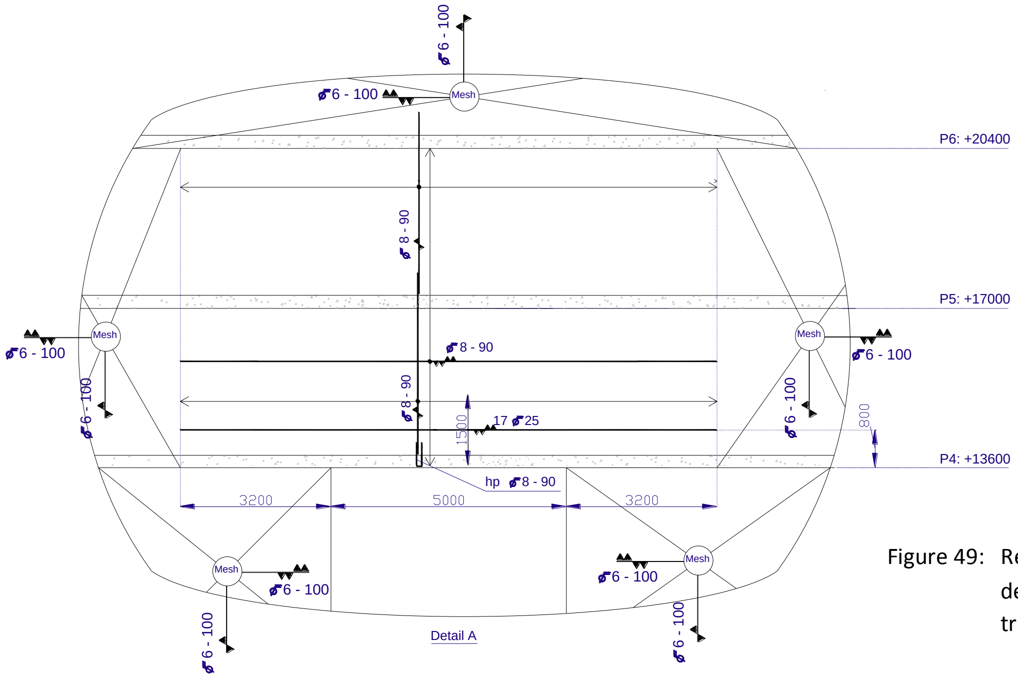


Figure 49: Reinforcement drawing for the design with the Eurocode with truss model 1, opening is 5 m

4.5.3 The Eurocode, truss model 2

The moment reinforcement consists of 16 Ø25 subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 85 mm, such that the total height of the layers combined is equal to 600 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,36 mm, which fulfills the requirements of the maximum allowable crack width.

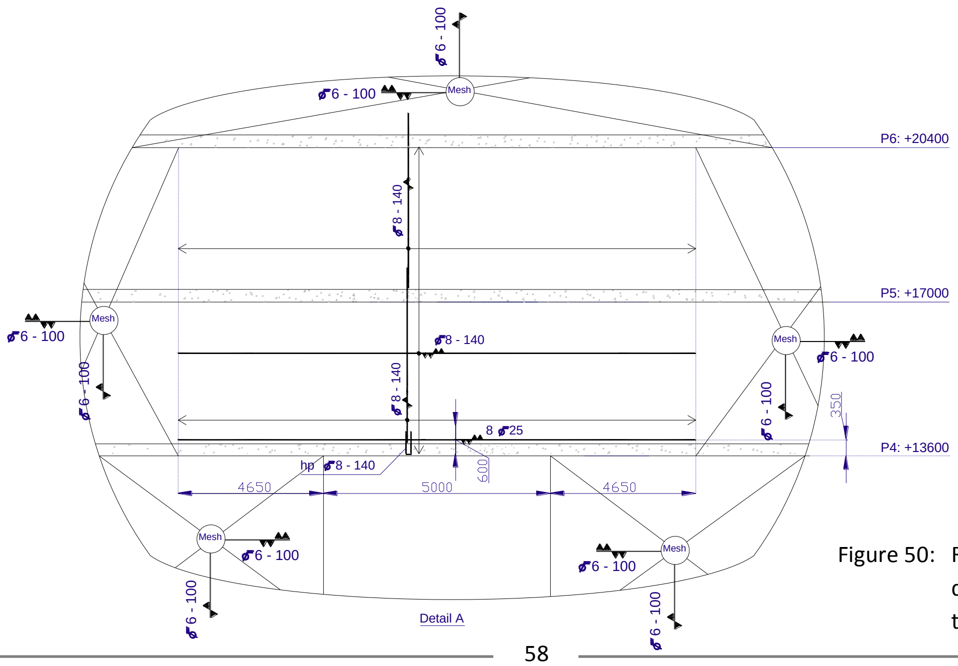


Figure 50: Reinforcement drawing for the design with the Eurocode with truss model 2, opening is 5 m

4.5.4 The Eurocode, truss model 3

The moment reinforcement consists of two reinforcement ties positioned at a height of 180 mm and 4500 mm from the opening level. Each tie consists of $8\,\emptyset 25$, subdivided into 2 bars per layer. The centre to centre distance of the layers of the lowest tie is equal to 85 mm, such that the total height of the layers combined is 250 mm. The centre to centre distance of the layers of the upper tie is equal to 60 mm, such that the total height of the layers is equal to 180 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

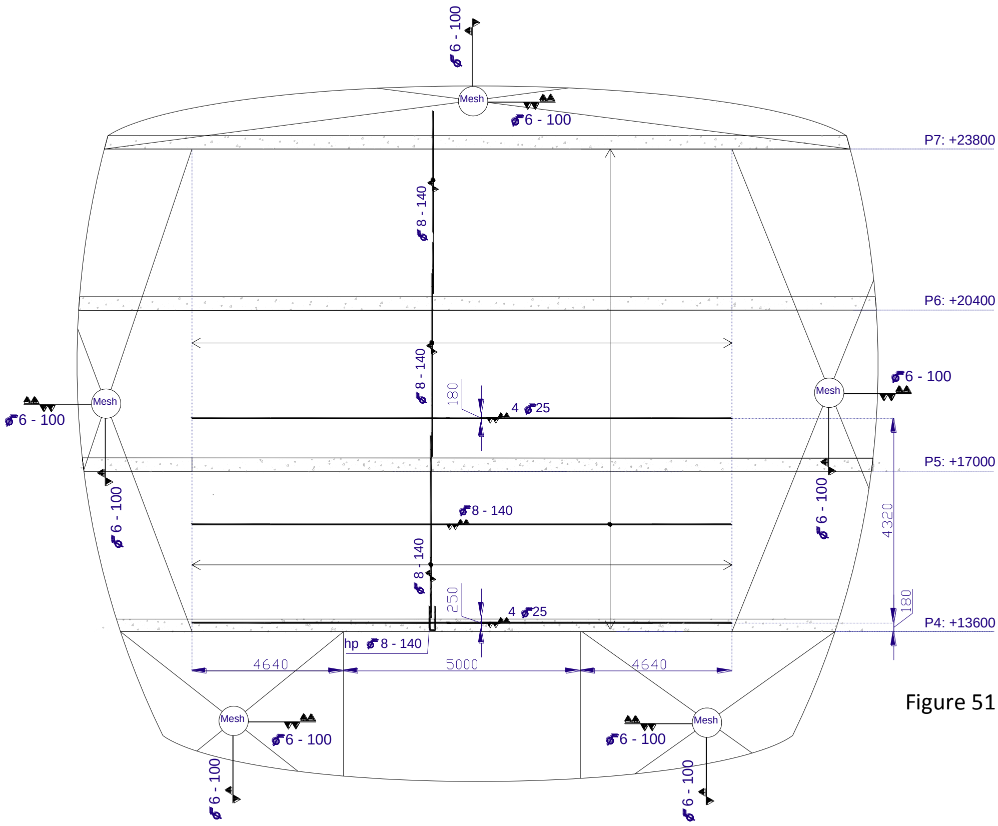


Figure 51: Reinforcement drawing for the design with the Eurocode with truss model 3, opening is 5 m

4.5.5 The Eurocode, truss model 4

The moment reinforcement consists of $16\,\emptyset 25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 90 mm, such that the total height of the layers combined is equal to 640 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,38 mm, which fulfills the requirements of the maximum allowable crack width.

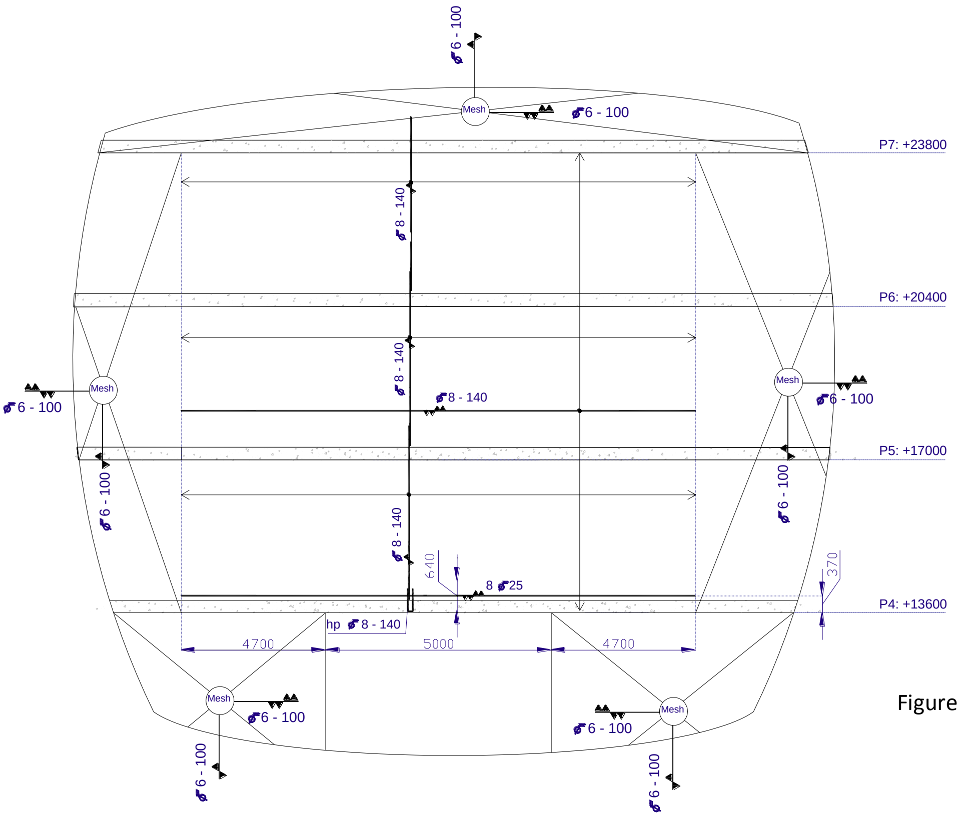


Figure 52: Reinforcement drawing for the design with the Eurocode with truss model 4, opening is 5 m

4.6 The opening of 10 m

4.6.1 The Dutch Code

The moment reinforcement consists of 30 Ø25. The 30 Ø25 are subdivided into fifteen layers with a centre to centre distance per layer equal to $2000/14 = 140$ mm. The concrete is in the stabilized cracking stage. The steel stress is equal to 314 N/mm^2. According to art. 8.7.2 the centre to centre distance of the bars for environments class 1 with a steel stress of 314 N/mm^2, should be smaller than 187 mm in order to fulfill the crack width requirements. This is satisfied. The horizontal shear reinforcement near each face is equal to $\emptyset 16 - 140$. The vertical shear reinforcement near each face is equal to $\emptyset 10 - 100$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction.

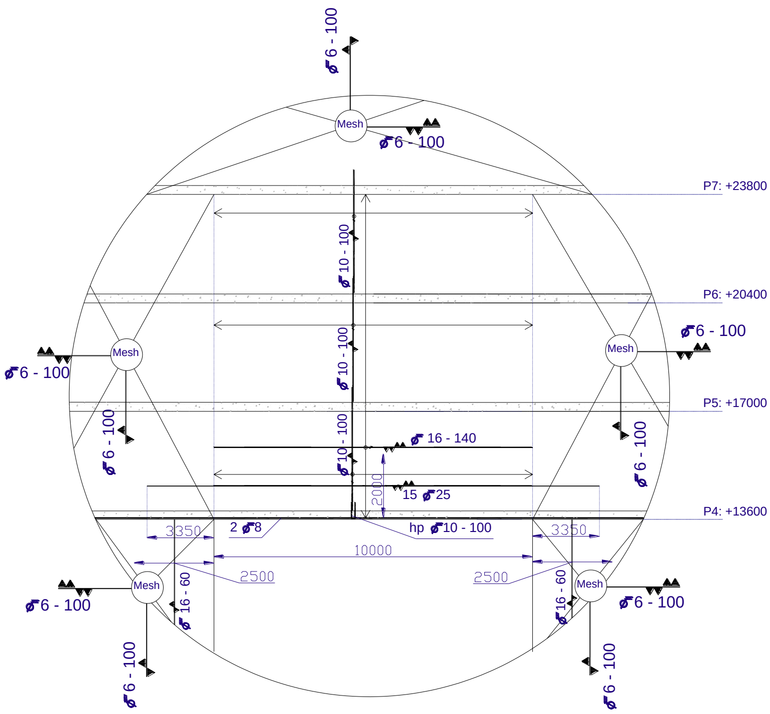


Figure 53: Reinforcement drawing for the design with the Dutch Code, opening is 10 m

4.6.2 The Eurocode, truss model 1

The moment reinforcement consists of 68 $\emptyset 25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 95 mm, such that the total height of the layers combined is equal to 3100 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8-90$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6-100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

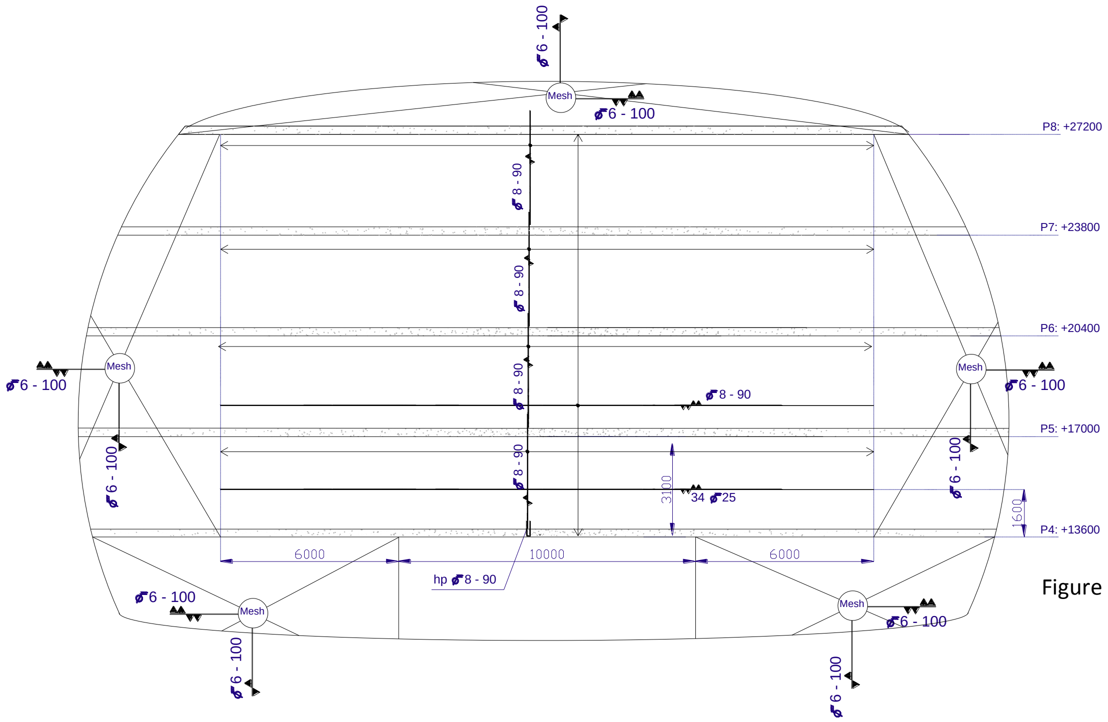


Figure 54: Reinforcement drawing for the design with the Eurocode with truss model 1, opening is 10 m

4.6.3 The Eurocode, truss model 2

The moment reinforcement consists of 30 $\emptyset 25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 95 mm, such that the total height of the layers combined is equal to 1320 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8-140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6-100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

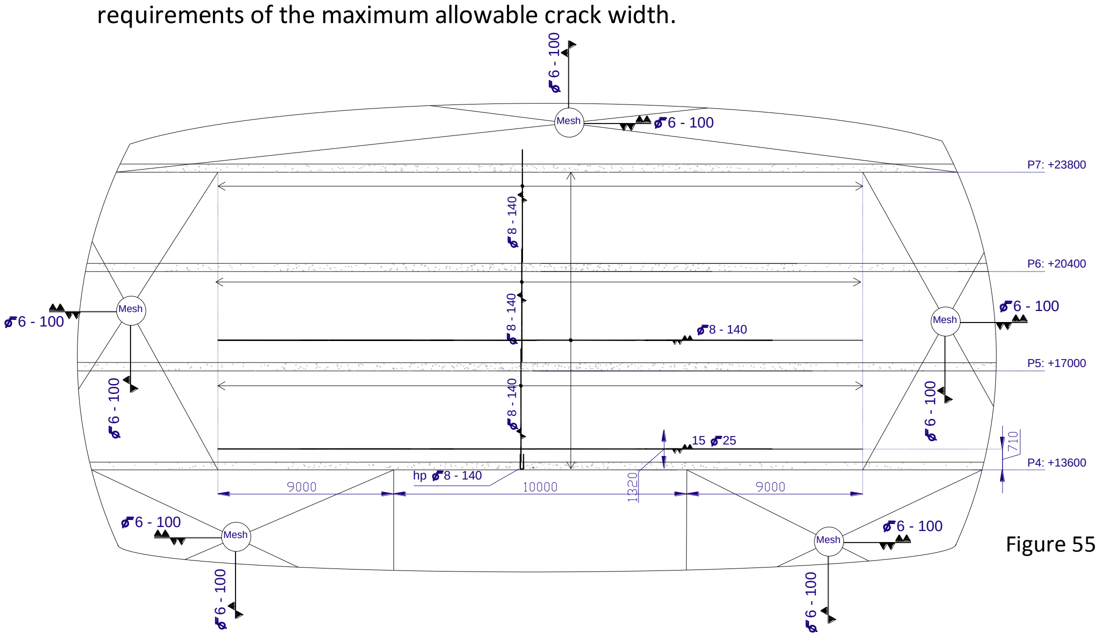


Figure 55: Reinforcement drawing for the design with the Eurocode with truss model 2, opening is 10 m

4.6.4 The Eurocode, truss model 3

The moment reinforcement consists of two reinforcement ties positioned at a height of 350 mm and 9020 mm from the opening level. Each tie consists of $16\,\emptyset 25$, subdivided into 2 bars per layer. The centre to centre distance of the layers of the lowest tie is equal to 85 mm, such that the total height of the layers combined is 600 mm. The centre to centre distance of the layers of the upper tie is equal to 60 mm, such that the total height of the layers is equal to 420 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,39 mm, which fulfills the requirements of the maximum allowable crack width.

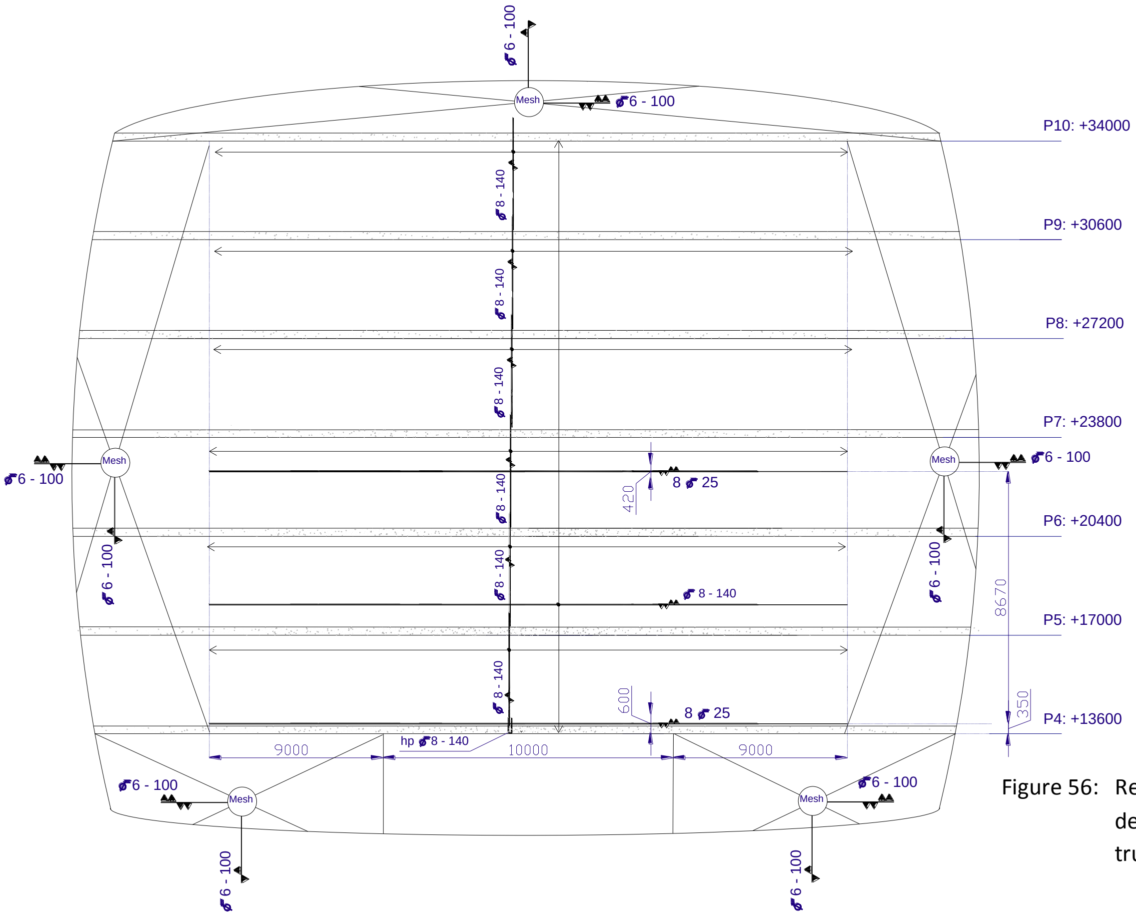


Figure 56: Reinforcement drawing for the design with the Eurocode with truss model 3, opening is 10 m

4.6.5 The Eurocode, truss model 4

The moment reinforcement consists of $38\ \emptyset 25$ subdivided into 2 bars per layer. The centre to centre distance of the layers is equal to 90 mm, such that the total height of the layers combined is equal to 1635 mm. The orthogonal reinforcement mesh near each face in each direction is equal to $\emptyset 8 - 140$. Outside this region a minimum orthogonal reinforcement mesh is applied equal to $\emptyset 6 - 100$ near each face in each direction. According to the crack width calculation, the maximum crack width will be equal to 0,38 mm, which fulfills the requirements of the maximum allowable crack width.

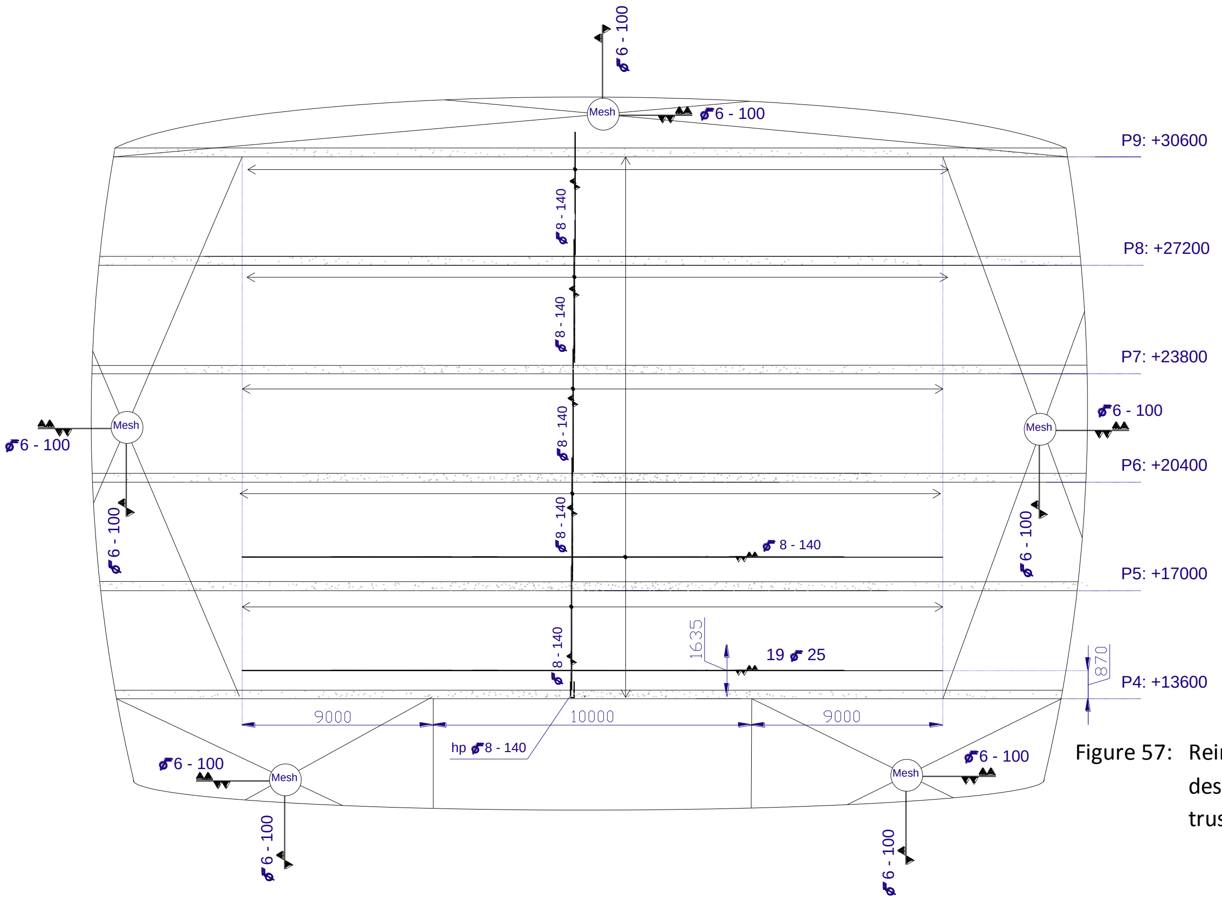


Figure 57: Reinforcement drawing for the design with the Eurocode with truss model 4, opening is 10 m

4.7 Comparing the different designs

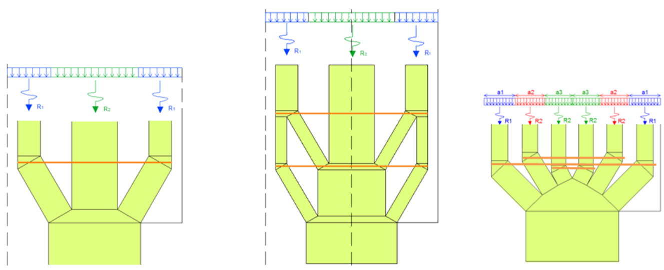
In the previous paragraphs the detailing of the reinforcement is given for the different dimensions of the opening and the overhang. According to the calculations of the Dutch Code and the Eurocode all designs presented in the previous chapter fulfill the requirements in the ULS and the SLS. In the tables below we compare the amount of reinforcement needed per design taking into account the reinforcement in the ties and mesh.

For the overhang it is observed that the designs made with the Dutch Code (DC) require the least amount of reinforcement. For the designs made with the Eurocode (EC) we observe that truss model 1 and 2 need a comparable amount of reinforcement, while the amount of reinforcement needed for truss model 3 is significantly more.

For the designs of the opening we observe that the reinforcement needed for the DC is the most. This is among other things caused by the large amount of reinforcement mesh needed above the opening and the compression reinforcement at the side of the opening. Furthermore it is observed that truss model 2, 3 and 4 of the EC need a comparable amount of reinforcement, while the reinforcement needed for truss model 1 is significantly more. This was expected as denoted in the chapter Analytical optimization of the different truss models.

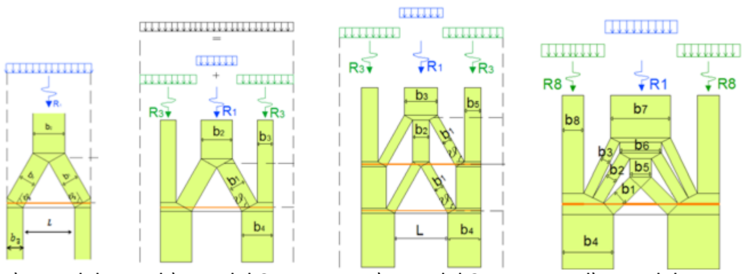
		Mass [Tons]	
		Overhang	Opening
L = 1 m	DC	0.237	0.337
	EC, Truss model 1	0.375	0.276
	EC, Truss model 2	0.360	0.175
	EC, Truss model 3	0.884	0.183
	EC, Truss model 4	-	0.187
L = 5 m	DC	4.950	3.764
	EC, Truss model 1	6.480	3.487
	EC, Truss model 2	5.859	2.436
	EC, Truss model 3	13.087	2.544
	EC, Truss model 4	-	2.546
L = 10 m	DC	19.860	12.634
	EC, Truss model 1	25.809	13.472
	EC, Truss model 2	24.289	9.020
	EC, Truss model 3	54.704	9.915
	EC, Truss model 4	-	10.351

Table 3: Reinforcement mass needed for the different designs



a) Model 1 b) Model 2 c) Model 3

Figure 58: The truss models considered for the overhang



a) Model 1 b) Model 2 c) Model 3 d) Model 4

Figure 59: The truss models considered for the opening

5. Evaluation of the designs with ATENA

In this chapter a nonlinear elastic finite element program is used in order to simulate and compare the behavior of the designed cases in reality. In consultation with the committee members, ATENA is chosen for this purpose.

ATENA is a Nonlinear Finite Element program written by Červenka Consulting group in order to analyze (un)reinforced and /or prestressed concrete structures. With this program it is possible to simulate the real behavior of concrete structures including concrete cracking, concrete crushing and reinforcement yielding. Several investigations show that analyses performed with ATENA give comparable results to the one found by real experiments, such as Leonhardt's shear beam test, prestressed concrete beam test, shear wall with an opening, bond failure test and more (Ref. 12). For the cases investigated in this report, ATENA 2D version 4.1.1 has been used.

5.1 Pre-processing

5.1.1 Material properties

In this research ATENA is used in order to simulate the designed cases in reality. In order to do so, the average values for the properties of the concrete should be incorporated in ATENA without the addition of any (material) safety factors. However, in order to obtain results which are verged to the safe side, the (5%) characteristic values are chosen for properties involving the strength conditions. The same holds for the reinforcement properties, which are modeled using a bilinear stress-strain diagram. The reinforcement bond, taking into account the transfer of forces between the concrete and the reinforcement, is generated using confined concrete, ribbed reinforcement and good bond quality.

The different parameters for the concrete, reinforcement and reinforcement bond are generated using settings in ATENA based on CEB-FIB Model Code 1990. The most important variables are listed below.

ATENA concrete properties	
$f_{ck,cube}$ [MPa]	45
f_c [MPa]	38.25
f_t [MPa]	3.036
μ [-]	0.2
E [MPa]	35570
G_f [N/m]	75.91

ATENA reinforcement properties	
Stress-strain law	Bilinear
σ_y [MPa]	500
E [MPa]	210000

ATENA reinforcement bond properties	
Bond stress [MPa]	Slip [mm]
5.5000	0.00
7.2215	0.25
10.423	0.50
13.750	1.00
13.750	3.00
5.5000	15.00
5.5000	1000.00

Table 4: Material properties in ATENA

5.1.2 Mesh generation

The finite element mesh is generated using eight noded quadratic elements. Mesh refinement is applied on locations where big stress gradients are observed in order to obtain accurate results. It should be noted that the crack width obtained in the post-processing part is related to the element size of the mesh. In ATENA the crack width in each element is based on the strain in the element. A small element size of the mesh will lead to many small cracks. However in reality these small cracks will be represented by one or more big cracks. In order to obtain a better estimation of the real crack width, all cracks over a distance equal to the maximum crack spacing $s_{r,max}$ should be summed up. The expression for $s_{r,max}$ can be found in both the Eurocode as the Dutch code. For the expression of $s_{r,max}$ you are referred to the design steps of the eurocode given in a previous chapter.

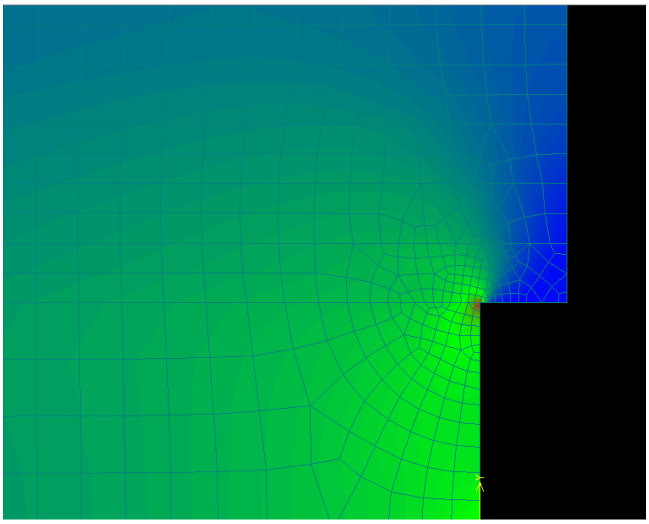


Figure 60: Mesh refinement on locations where big stress gradients are observed for the overhang of 10 m. The colors in the picture indicate the principle stress in the concrete. High compression stresses are indicated with in red.

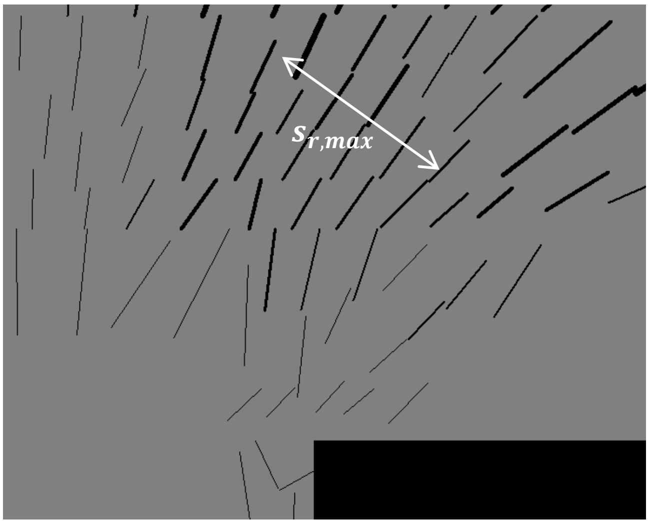


Figure 61: The picture indicates how the cracks can be summed up over a distance equal to the maximum crack spacing.

5.1.3 The Loading

In ATENA the load is build up by load steps each with a magnitude of one tenth of the load in the SLS, in order to obtain accurate results. At the end of the tenth step, the total load on the wall is equal to the load in the SLS. From load step eleven till seventeen the load will be increased with a factor of one twentieth. At the end of the seventeenth step, the total load on the wall is equal to the load in the ULS, as the safety factor is equal to 1.35. After the seventeenth step the load is increased with a factor of one tenth until the SLS and the ULS condition is normative. The SLS condition is defined to be normative when the max. allowable cracks width, defined by the codes, is exceeded. The ULS condition is defined to be normative when the stiffness of the structure as a hole tends to zero in the load-displacement graph.

Load step	Load coefficient	Total load on the wall
1	0.10	
:	:	
:	:	
10	0.10	q_{rep}
11	0.05	
:	:	
:	:	
17	0.05	$q_d = 1.35 * q_{rep}$
18	0.10	
:	:	
:	:	
n	0.10	$\{0.1 \cdot (n-7) + 0.35\} * q_{rep}$

Table 5: Load steps in ATENA

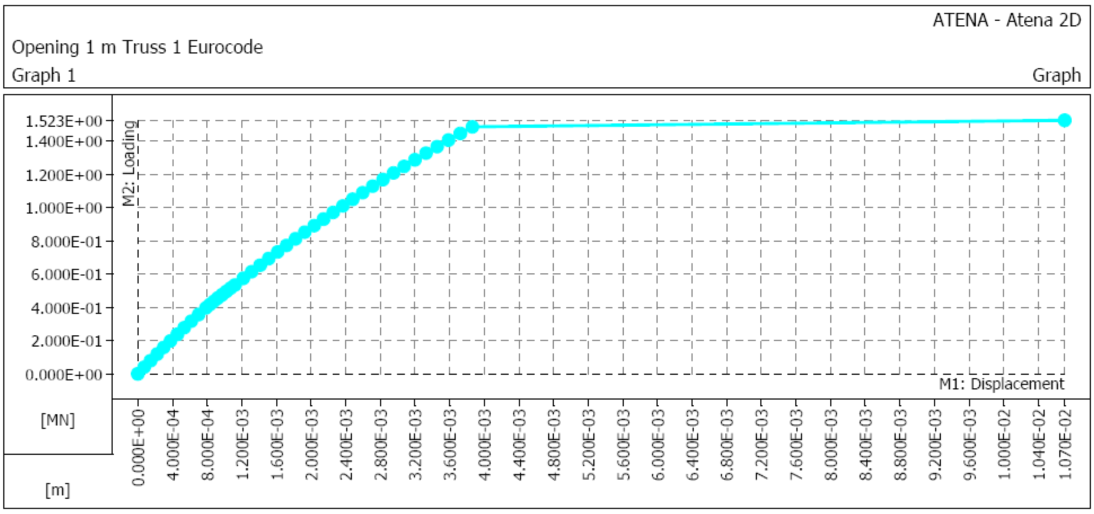


Figure 62: Load-displacement graph for the definition of the load step at which the ULS condition is normative. On the y-axis of this graph the load does not represent the total load on the wall, but the load on the element considered.

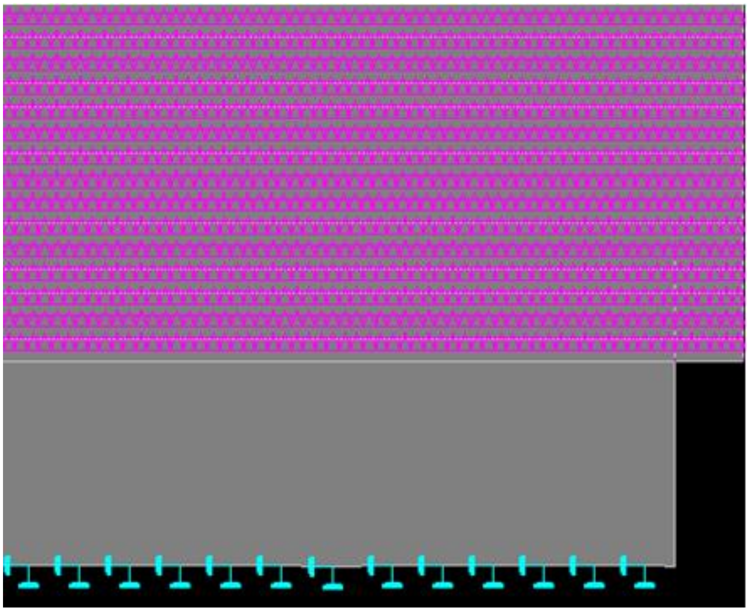


Figure 63: Loads and supports in ATENA for the overhang. The purple lines indicate the distributed load per floor on the wall. The blue lines indicate the supports.

5.2 Post-processing

After obtaining the results for the different designs it turned out that all designs satisfy the SLS and ULS criterion. It is even observed that the very first crack initiates at a load bigger than the load accounted for in the designs. From these results two conclusions could be made, videlicet:

- The design methods of the Dutch code and the Eurocode are conservative methods for the reinforcement design of the observed cases;
- The cases considered will fulfill the SLS and ULS conditions with unreinforced concrete, as the first cracks initiates at a load bigger than the one accounted for in the designs (SLS and ULS).

The second conclusion is very surprising. In order to verify this in ATENA, it is decided to simulate the cases considered with unreinforced concrete.

5.2.1 Simulation of the cases with unreinforced concrete

The results obtained with this simulation are represented in tables, like the one on this page. In these tables the crack width is given for the SLS condition of the designs (load step 10), the max. compressive and tensile stress in the concrete for the ULS condition of the design (load step 17), the factor between the loading in ATENA and q_{rep} of the designs for which the SLS condition is normative and finally the factor between the loading in ATENA and q_d of the designs for which the ULS condition is normative.

For the overhang it turns out that the crack width requirements in the SLS condition (load step 10) are met as well as the strength requirements in the ULS condition (load step 17). Furthermore the SLS and ULS condition will be satisfied up to a load equal to 2 times the load which is present in the designs, see table below.

Overhang	**1 m**	**5 m**	**10 m**
SLS condition design (load step 10)			
Crack width [mm]	-	-	-
ULS condition design (load step 17)			
max. compressive stress in concrete [MPa]	19	22	23
max. tensile stress in concrete [MPa]	0.83	0.87	1.23
SLS condition normative			
$q_{loading\ ATENA} / q_{rep}$	3.0	2.6	2.0
ULS condition normative			
$q_{loading\ ATENA} / q_d$	2.2	2.0	2.0

Table 6: Results of the unreinforced concrete wall with an overhang of 1 m, 5 m and 10 m.

For the opening it can also be verified that the crack width requirements in the SLS condition (load step 10) are met as well as the strength requirements in the ULS condition (load step 17). However we see a clear division in the results obtained for the opening of 1 m and the opening of 5 m and 10 m. For the opening of 1 m we see that the highest load at which all the requirements are satisfied, is equal to 2.7 times the load which is present in the designs. For the dimensions of 5 m and 10 m we see that the highest load at which all the requirements are satisfied, is equal to 1.1 times the load which is present in the designs. The reason for this division is the initiation of the first crack, which in the case of the opening of 5 m and 10 m occurs at al load equal to 1.1 times the load which is present in the design. It is observed that for these dimensions the first crack has a considerable width in the order of 0.5 mm.

Opening	**1 m**	**5 m**	**10 m**
SLS condition design (load step 10)			
Crack width [mm]	-	-	-
ULS condition design (load step 17)			
max. compressive stress in concrete [MPa]	16	18	15
max. tensile stress in concrete [MPa]	0.98	0.77	0.80
SLS condition normative			
$q_{loading\ \ ATENA} / q_{rep}$	3.7	1.1	1.1
ULS condition normative			
$q_{loading\ \ ATENA} / q_d$	2.7	2.4	2.7

Table 7: Results of the unreinforced concrete wall with an opening of 1 m, 5 m and 10 m.

It is verified that the cases considered with unreinforced concrete will fulfill the requirements of the SLS and ULS conditions of the designs. Because the concrete is not cracked in the SLS and ULS stage of the designs*, we cannot compare the different reinforcement designs made in the previous chapters, as they would give the same results. In order to compare these different designs it is decided to reduce the tensile strength of the concrete up to 10 % of it's initial value. In order to avoid having a viscous material, the fracture energy is also reduced with the same ratio. It should be noted that the results obtained in this manner should only be used in order to compare the different reinforcement designs. The values obtained for the crack widths should not be read as the real occurring crack widths.

5.2.2 Simulation of the designs with a concrete tensile strength approaching zero

On the next two pages the tables are shown which give the results obtained with ATENA for the different designs. These tables show that for all the designs in the ULS condition (load step 17) the stress in the reinforcement tie is very low in contrast to the stress in the reinforcement mesh. This might suggest that less reinforcement in the tie would be sufficient or; the reinforcement tie is of less importance in transferring the stresses in contrast to the reinforcement mesh or; the location of the reinforcement tie is such that is it not stressed efficiently. In the next paragraph the importance of the reinforcement tie in the ULS condition will also be determined.

* For the opening of 5 m and 10 m the concrete is cracked in the ULS stage of the design.

Overhang 1 m	**DC**	**EC**		
		Truss 1	**Truss 2**	**Truss 3**
SLS condition design (load step 10)				
Crack width [mm]	0.34	0.31	0.26	0.07
ULS condition design (load step 17)				
max. compressive stress in concrete [MPa]	23	26	25	27
max. tensile stress in reinforcement tie [MPa]	67	51	46	30
max. tensile stress in reinforcement mesh [MPa]	155	99	119	49
SLS condition normative				
$q_{loading\ \ ATENA} / q_{rep}$	1.1	1.3	1.3	1.7
ULS condition normative				
$q_{loading\ \ ATENA} / q_d$	1.7	2.1	2.1	2.1

Overhang 5 m	**DC**	**EC**		
		Truss 1	**Truss 2**	**Truss 3**
SLS condition design (load step 10)				
Crack width [mm]	0.58	0.52	0.54	0.31
ULS condition design (load step 17)				
max. compressive stress in concrete [MPa]	21	25	35	31
max. tensile stress in reinforcement tie [MPa]	46	43	66	37
max. tensile stress in reinforcement mesh [MPa]	104	111	178	51
SLS condition normative				
$q_{loading\ \ ATENA} / q_{rep}$	0.7	0.7	0.6	1.2
ULS condition normative				
$q_{loading\ \ ATENA} / q_d$	1.5	1.7	1.5	1.7

Overhang 10 m	**DC**	**EC**		
		Truss 1	**Truss 2**	**Truss 3**
SLS condition design (load step 10)				
Crack width [mm]	0.84	1.08	1.19	0.44
ULS condition design (load step 17)				
max. compressive stress in concrete [MPa]	34	34	34	34
max. tensile stress in reinforcement tie [MPa]	81	68	82	49
max. tensile stress in reinforcement mesh [MPa]	197	203	248	83
SLS condition normative				
$q_{loading\ \ ATENA} / q_{rep}$	0.5	0.5	0.5	0.9
ULS condition normative				
$q_{loading\ \ ATENA} / q_d$	1.3	1.3	1.3	1.4

Table 8: Comparing the results for the designs of the overhang of 1 m, 5 m and 10 m.

Opening 1 m	**DC**	**EC**			
		Truss 1	**Truss 2**	**Truss 3**	**Truss 4**
SLS condition design (load step 10)					
Crack width [mm]	0.12	0.09	0.11	0.13	0.11
ULS condition design (load step 17)					
max. compressive stress in concrete [MPa]	17	18	17	17	17
max. tensile stress in reinforcement tie [MPa]	22	24	23	25	23
max. tensile stress in reinforcement mesh [MPa]	35	93	61	65	63
SLS condition normative					
$q_{loading\ \ ATENA} / q_{rep}$	2.4	2.4	2.3	2.3	2.3
ULS condition normative					
$q_{loading\ \ ATENA} / q_d$	3.1	2.8	2.9	2.9	3.0

Opening 5 m	**DC**	**EC**			
		Truss 1	**Truss 2**	**Truss 3**	**Truss 4**
SLS condition design (load step 10)					
Crack width [mm]	0.23	0.24	0.32	0.25	0.24
ULS condition design (load step 17)					
max. compressive stress in concrete [MPa]	19	25	18	18	18
max. tensile stress in reinforcement tie [MPa]	25	30	23	22	27
max. tensile stress in reinforcement mesh [MPa]	59	99	103	97	97
SLS condition normative					
$q_{loading\ \ ATENA} / q_{rep}$	1.6	1.5	1.2	1.3	1.4
ULS condition normative					
$q_{loading\ \ ATENA} / q_d$	2.9	2.4	2.8	2.9	2.8

Opening 10 m	**DC**	**EC**			
		Truss 1	**Truss 2**	**Truss 3**	**Truss 4**
SLS condition design (load step 10)					
Crack width [mm]	0.32	0.30	0.32	0.34	0.33
ULS condition design (load step 17)					
max. compressive stress in concrete [MPa]	22	18	18	18	18
max. tensile stress in reinforcement tie [MPa]	27	25	24	22	24
max. tensile stress in reinforcement mesh [MPa]	57	45	62	66	59
SLS condition normative					
$q_{loading\ \ ATENA} / q_{rep}$	0.9	1.2	1.2	1.1	1.1
ULS condition normative					
$q_{loading\ \ ATENA} / q_d$	2.9	2.8	2.7	2.7	2.6

Table 9: Comparing the results for the designs of the opening of 1 m, 5 m and 10 m.

5.2.3 Investigating the importance of the reinforcement tie

In order to investigate the importance of the reinforcement tie, an overhang and an opening of 10 m both will be analyzed with and without a reinforcement tie. The reinforcement mesh and the concrete properties are unchanged in comparison to the previous simulations.

The tables show that the simulations of the designs with and without the reinforcement tie give similar results in ATENA. It is observed that the reinforcement tie does not contribute to a higher ultimate load, nor does it substantially contribute to the crack width reduction.

Overhang 10 m truss 1	**With reinforcement tie**	**Without reinforcement tie**
SLS condition design (load step 10)		
Crack width [mm]	1.08	1.10
ULS condition design (load step 17)		
max. compressive stress in concrete [MPa]	34	36
max. tensile stress in reinforcement tie [MPa]	81	-
max. tensile stress in reinforcement mesh [MPa]	197	188
SLS condition normative		
$q_{loading\ \ ATENA} / q_{rep}$	0.5	0.5
ULS condition normative		
$q_{loading\ \ ATENA} / q_d$	1.3	1.3

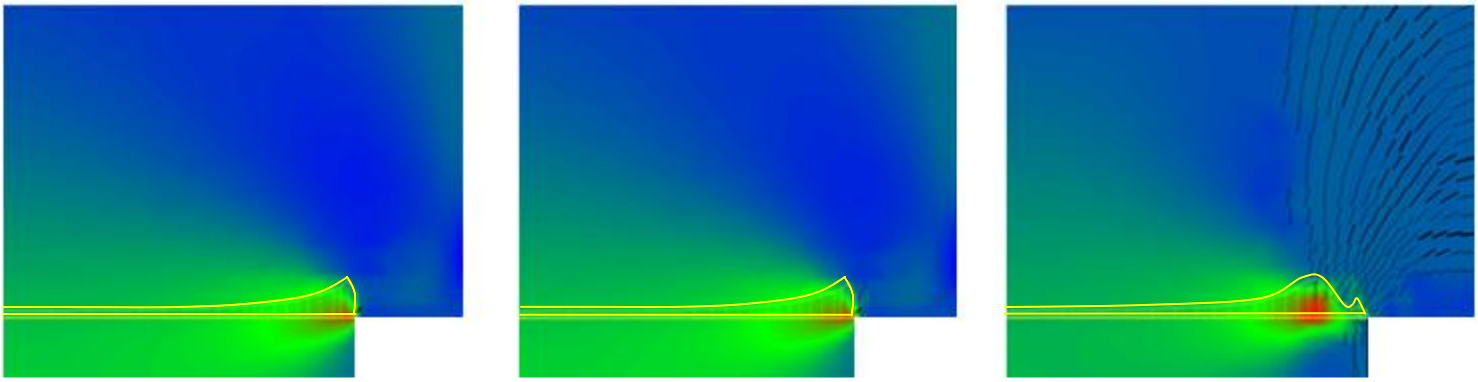
Table 10: Results of the concrete wall with an overhang of 1 m with and without a reinforcement tie.

Opening 10 m truss 1	**With reinforcement tie**	**Without reinforcement tie**
SLS condition design (load step 10)		
Crack width [mm]	0.30	0.28
ULS condition design (load step 17)		
max. compressive stress in concrete [MPa]	22	18
max. tensile stress in reinforcement tie [MPa]	27	-
max. tensile stress in reinforcement mesh [MPa]	57	94
SLS condition normative		
$q_{loading\ \ ATENA} / q_{rep}$	1.2	1.2
ULS condition normative		
$q_{loading\ \ ATENA} / q_d$	2.8	2.8

Table 11: Results of the concrete wall with an opening of 1 m with and without a reinforcement tie.

In order to explain why the reinforcement tie does not contribute to a higher ULS level, the concrete stresses are observed in ATENA at different stages.

For the overhang the first stage concerns the moment when the first crack starts to develop in the wall due to high compression stresses in the concrete. When the loading is increased, the crack width increases until the concrete in the corner fails under compression. At this instant the maximum compressive stress in the concrete displaces further into the wall while more cracks start to develop rapidly until the ULS condition is normative, see figures below. In the calculations with the codes it was assumed that the tensile stresses in the concrete would be exceeded which would determine the ULS condition. However ATENA shows that the compression stress in the concrete determine the ULS condition, because in the specifications (chapter 2) a wall is considered which has a large mass of concrete distributed over a large height above the overhang level. Therefore ATENA indicates that the tensile stress in the concrete will be distributed over this large height so that the tensile stress will remain very small. It should be noted that when a lower height is considered above the overhang level such that the ULS condition will be determined by exceeding the tensile stresses in the concrete, it is expected that the reinforcement tie will have a big contribution for the ULS level. The last mentioned case is however not considered as it is outside the borders of this report as laid down in the specifications (chapter 2).



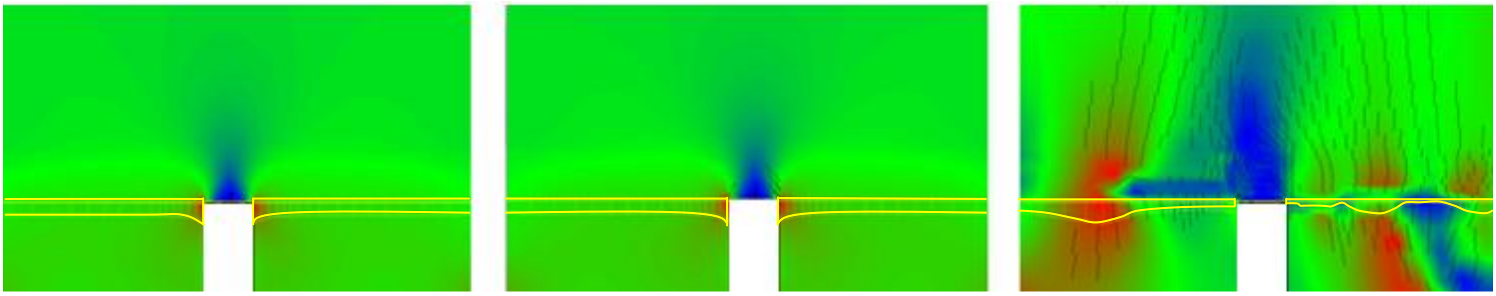
Analysis step 18
First crack appears in the direction of the schematized compression diagonal

Analysis step 27
Max. stress is equal to the concrete compressive strength

Analysis step 30
ULS condition is normative

Figure 64: Unreinforced concrete wall with an overhang of 10 m. The red areas show the location where high compressive stresses are found in the concrete. The lines indicate the distribution of the compressive stress over the width of the wall.

For the opening it is observed that the first crack starts at the location where the tension tie is schematized in the designs. When the loading is increased, cracks start to develop at the locations where the compression diagonals were schematized, until the concrete in the corner fails in compression. At this instant the maximum compressive stress in the concrete displaces further into the wall while more cracks start to develop rapidly until the ultimate load is reached, see figures below. In the calculations with the codes it was assumed that the tensile stresses in the concrete would be exceeded which would determine the ULS condition. However ATENA shows that also for the opening the compression stresses in the concrete determine the ULS condition, because in the specifications (chapter 2) a wall is considered which has a large concrete mass at both sides of the opening. This concrete mass will offer resistance in the increase of the opening, until the concrete starts to fail in compression. It should be noted that when the width of the wall near the opening becomes smaller, the ULS condition may be determined by exceeding the tensile strength of the concrete above the opening. When this is the case, the reinforcement tie is expected to have a big contribution in the ULS level. The last mentioned case is however not considered as it is outside the borders of this report as laid down in the specifications (chapter 2).



Analysis step 12
First crack appears on the location where the tie was schematized in the designs

Analysis step 31
Cracks appear in the direction of the schematized compression diagonal

Analysis step 40
ULS condition is normative

Figure 65: Unreinforced concrete wall with an opening of 10 m. The red areas show the location where high compressive stresses are found in the concrete. The lines indicate the distribution of the compressive stress over the width of the wall.

5.2.4 Stress distribution

In the calculations of the Beam Method and the Strut-and-Tie Method, the load is assumed to go vertically through the wall until it reaches the reinforcement tie, which transfers the load over the D-region. However it is not unthinkable that the concrete which is placed above the tie will perform preliminary work, such that the stress in the D-region will decrease. In order to confirm this, the stress distribution along the height of the wall should be analyzed. It is also of importance to analyze how this stress distribution changes when cracks propagate through the wall.

Unlike plates, walls transfer stresses mainly by shear action. In order to understand how the load is transferred through the wall, it is necessary to observe the shear stress distribution along the height of the wall.

For the unreinforced overhang of 10 m the shear stress distribution is given for several stages. For this purpose the real properties of the materials are simulated in ATENA.

In figure 66 the shear stress distribution is shown along the height of the wall in the elastic stage when the concrete is still uncracked. In figure 67 the shear stress distribution is given when the concrete compressive stress is equal to the concrete compressive strength. In spite of the cracks that are present above the overhang, it is observed that the shear stress distribution is unchanged. Figure 68 shows the shear stress distribution just before the ultimate load is reached. At the corner of the overhang the concrete is failed in compression, so that at this point less shear stress is transferred. This difference is however negligible small when the overall shear stress distribution of the wall is observed.

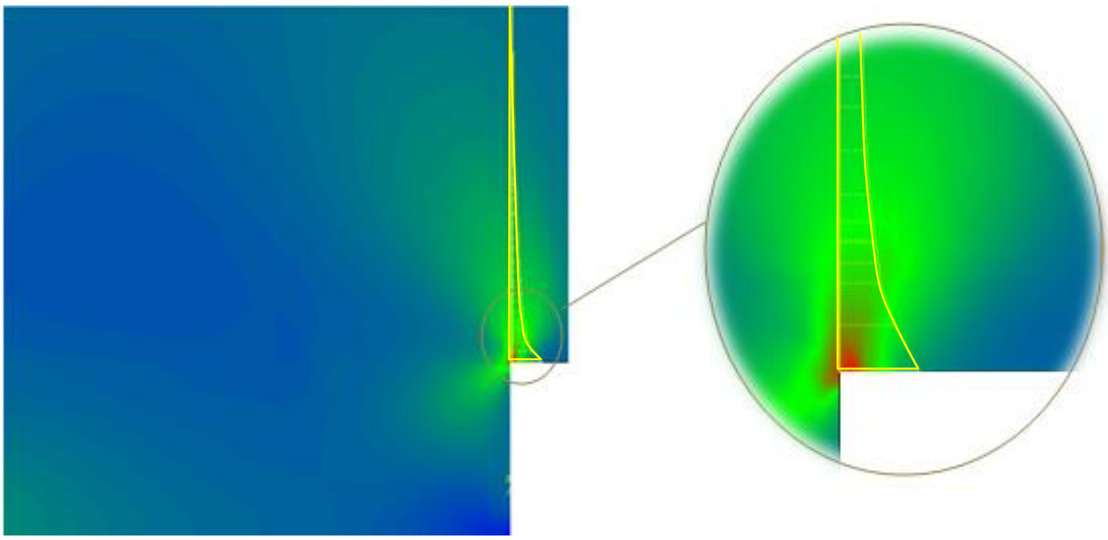


Figure 66: Shear stress distribution along the height of the wall when the concrete is uncracked.

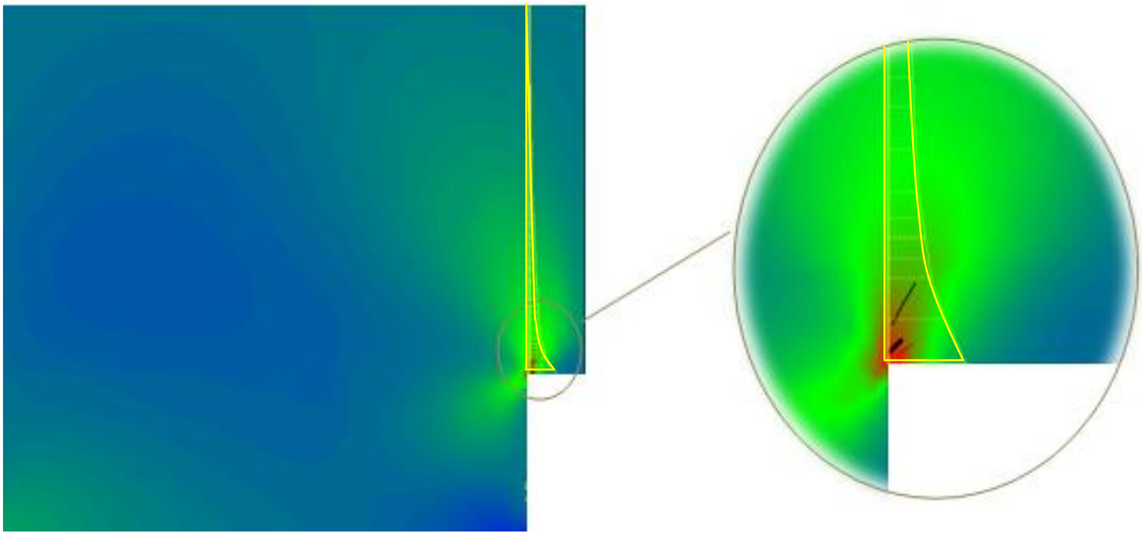


Figure 67: Shear stress distribution for the cracked concrete when the concrete compressive stress is equal to the concrete compressive strength.

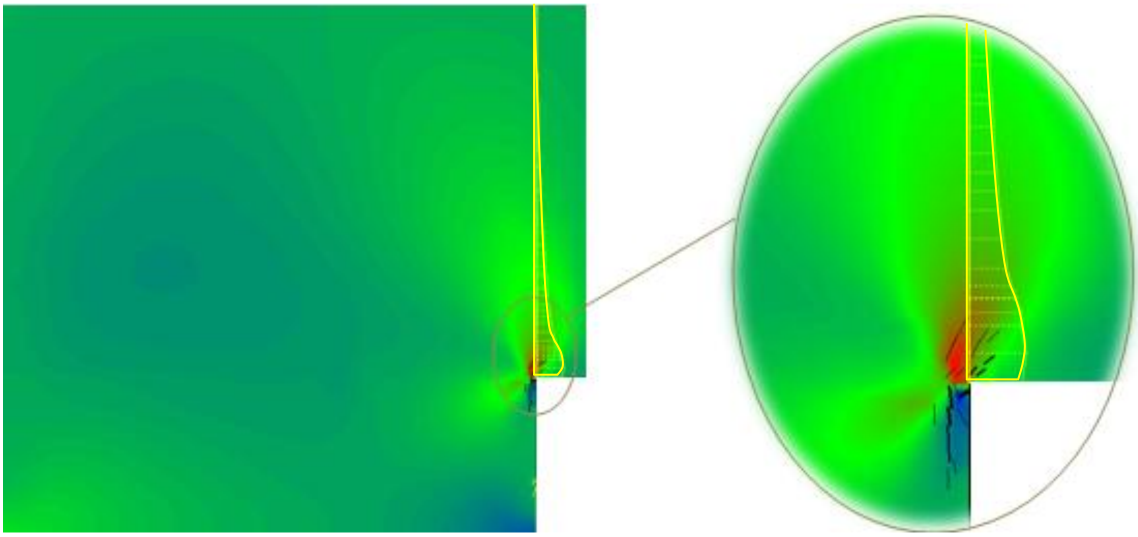


Figure 68: Shear stress distribution just before the ultimate load is reached.

The figures above show that, a certain amount of load is transferred by shear above the location where the reinforcement tie is placed in the designs. Because of this, the load which has to be transferred by the tie will be less, such that the stress in the tie will be less than accounted for in the calculations of the codes. It is interesting to determine the percentage of the load which is transferred by shear above the tie, because in this way the total load on the wall could be reduced with this percentage for the determination of the reinforcement area in the tie.

In the table on the next page, the percentage of the load is indicated which is transferred by shear above the reinforcement tie. Based on this percentage the relative stress in the reinforcement tie is given, which indicates the stress in the reinforcement tie according to the codes when the load on the tie is reduced with the above mentioned percentage. The stress in the tie according to ATENA is also presented in the table. This is done in order to compare this stress to the relative stress described above.

Overhang 10 m	**% load transferred by shear above tie**	$\sigma_{s,rep}$ **Calculations DC / EC** [MPa]	$\sigma_{s,tie,rel} = \frac{100-\%}{100} \cdot \sigma_{s,rep}$ [MPa]	$\sigma_{s,tie}$* **ATENA** [MPa]
Design DC	53 %	311	146	81
Design EC, Truss Model 1	53 %	243	114	68
Design EC, Truss Model 2	27 %	243	177	82
Design EC, Truss Model 3	73 %	261	70	49

* *Stress in the tie according to ATENA when the concrete tensile strength is reduces to 10%.*

Table 12: Effect of shear stress distribution on the stress in the tie.

From the table it is observed that the relative stress in the tie according to the codes, is still more than the stress in the tie according to ATENA. This is caused by the fact that the codes do not incorporate the contribution of the reinforcement mesh in the reduction of the stress in the tie. However the reinforcement mesh will ensure that the tensile stresses are transferred although the concrete tensile strength is reduces to 10%. However the codes assume that these tensile stresses will only be transferred by the ties.

For the opening of 10 m the same procedure is followed as described above for the overhang. Below, the stress distribution is given for the unreinforced case in several stages. In figure 69 the shear stress distribution is shown along the height of the wall in the elastic stage when the concrete is still uncracked. In figure 70 the shear stress distribution is given when big cracks have developed in the concrete. In spite of the cracks, it is observed that the shear stress distribution is unchanged. Figure 71 shows the shear stress distribution just before the ultimate load is reached. At the corners of the opening the concrete is failed in compression, so that at this point less shear stress is transferred. As in the case of the overhang, this difference is negligible small when the overall shear stress distribution of the wall is observed

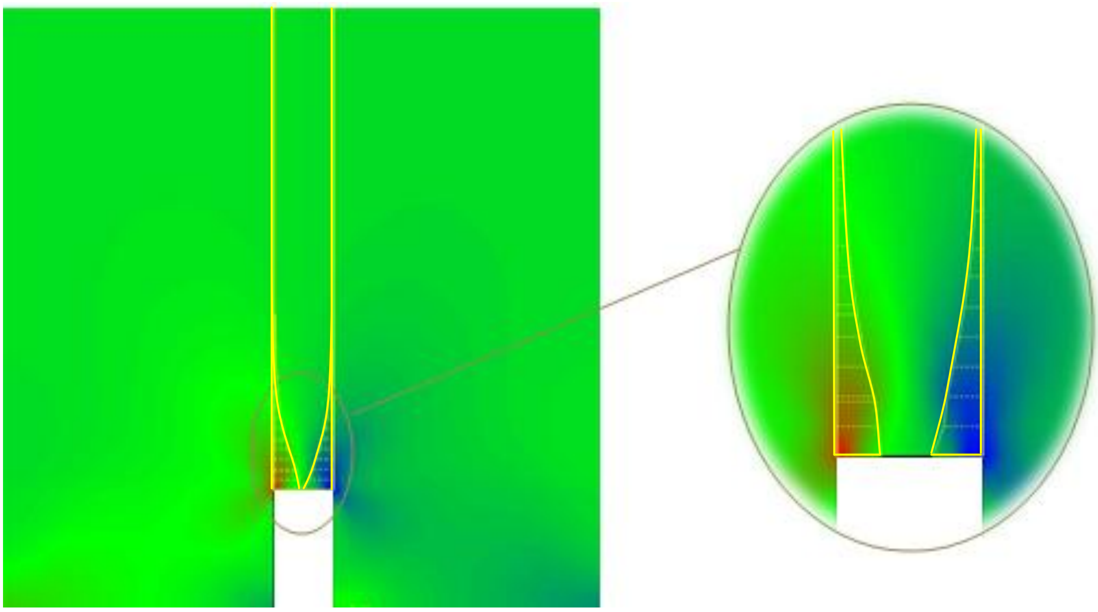


Figure 69: Shear stress distribution along the height of the wall when the concrete is uncracked.

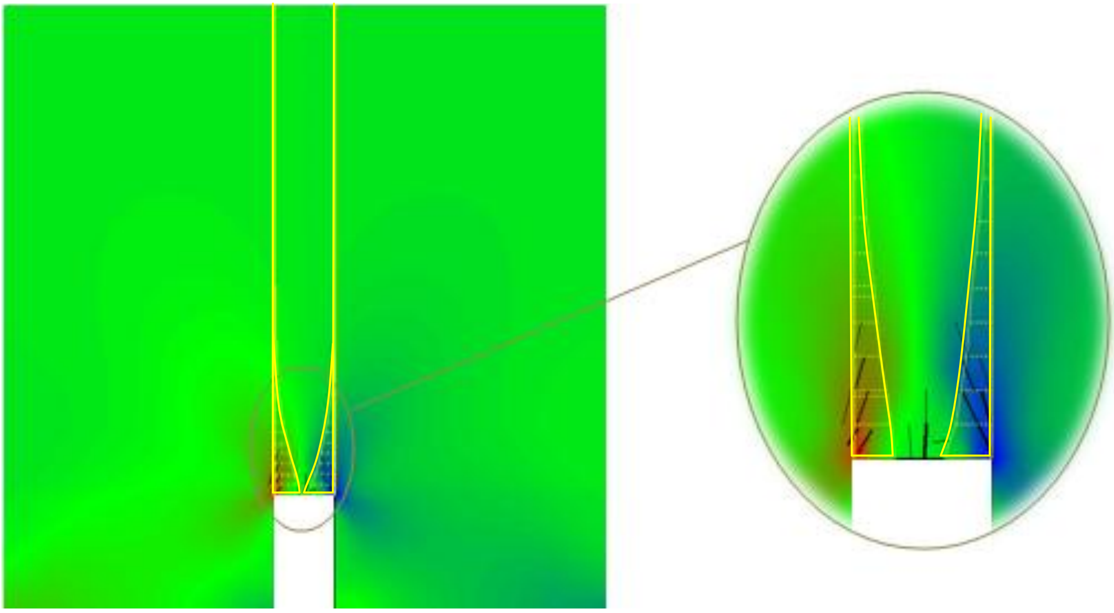


Figure 70: Shear stress distribution for the cracked concrete when the concrete compressive stress is equal to the concrete compressive strength.

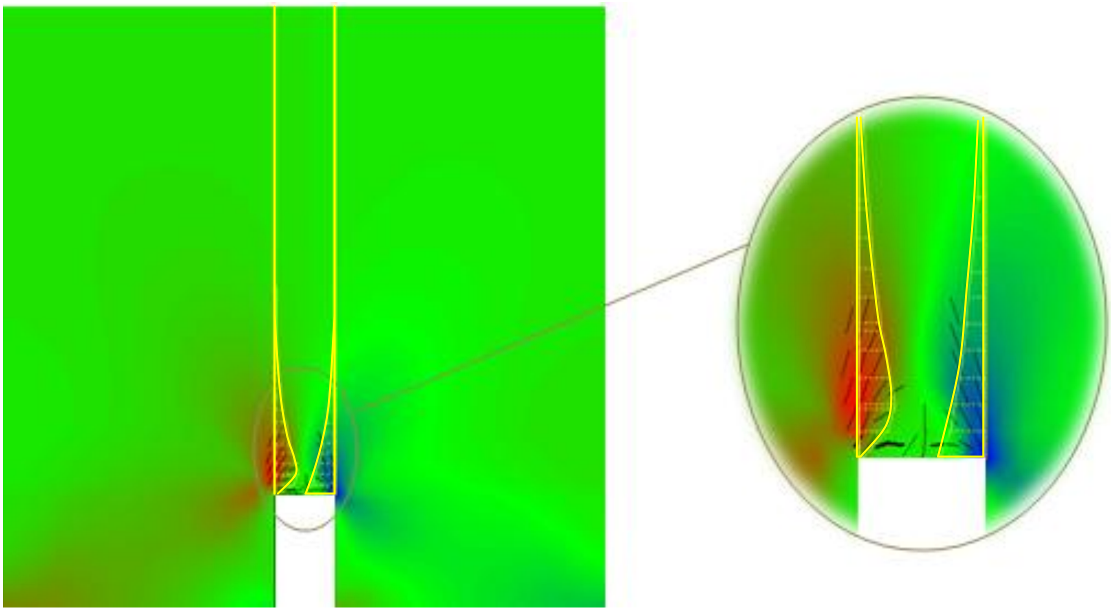


Figure 71: Shear stress distribution just before the ultimate load is reached.

As for the overhang, the table on the next page indicates the percentage of the load which is transferred by shear above the location where the reinforcement tie in the designs is present. Based on this percentage the relative stress in the reinforcement tie according to the codes is calculated. The stress in the tie according to ATENA is also presented in the table.

Opening 10 m	**% load transferred by shear above tie**	**$\sigma_{s,rep}$ Calculations DC / EC** [MPa]	**$\sigma_{s,tie,rel} = \frac{100-\%}{100} \cdot \sigma_{s,tie}$** [MPa]	**$\sigma_{s,tie}$* ATENA** [MPa]
Design DC	88 %	314	38	27
Design EC, Truss Model 1	80 %	257	51	25
Design EC, Truss Model 2	89 %	257	28	24
Design EC, Truss Model 3	90 %	241	24	22
Design EC, Truss Model 4	87 %	251	33	24

* *Stress in the tie according to ATENA when the concrete tensile strength is reduces to 10%.*

Table 13: Effect of shear stress distribution on the stress in the tie.

As for the overhang, the relative stress in the tie according to the codes is still more than the stress in the tie according to ATENA. As already mentioned, the reinforcement mesh will ensure that the tensile stresses are transferred although the concrete tensile strength is reduces to 10%. However the codes assume that these tensile stresses will only be transferred by the ties.

As the table indicates, the difference between the relative stress and the stress indicated by ATENA is for the opening much smaller than in the case of the overhang. This can be explained by observing the horizontal tensile stress distribution over the height of the unreinforced wall.

As can be seen in the figures below, for the case of the overhang the tensile stresses in the concrete are very low as they are distributed over a very large height. In the schematization of the codes, the tensile stresses are assumed to be transferred by a concentrated reinforcement tie. However, because the horizontal stress is distributed over such a large height, the tie will not be stressed efficiently.

For the case of the opening the horizontal stress in the concrete is relatively big, because this horizontal stress is distributed over a much smaller height. For this case the schematization of the codes that the horizontal stress is transferred by a horizontal reinforcement tie is therefore much better. That is why the relative stresses indicated in the tables are for the case of the opening much closer to the values obtained by the codes.

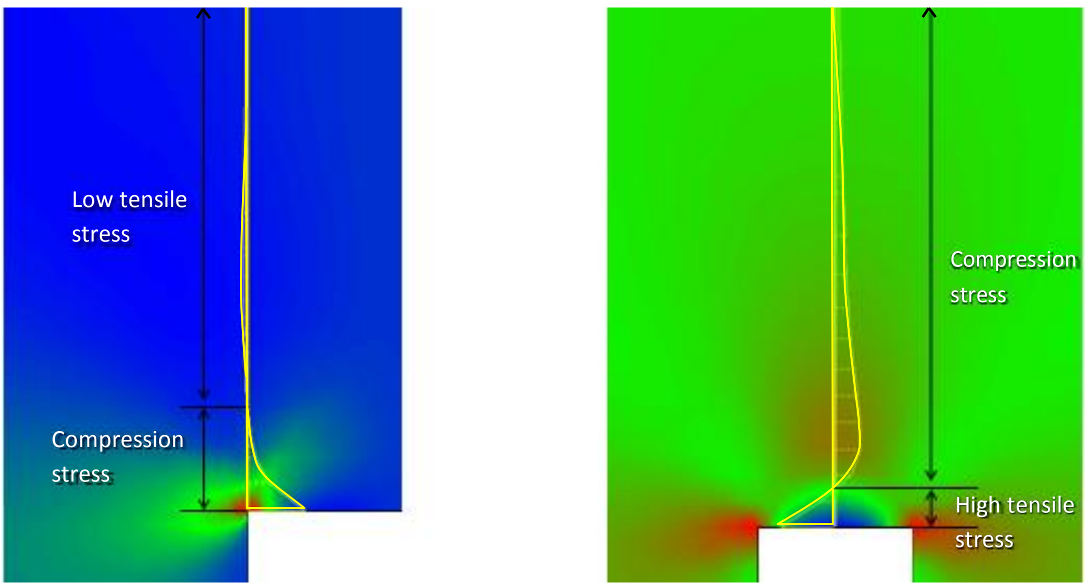


Figure 72: Horizontal stress distribution over the height of the wall. Left this is indicated for the overhang, while right this is done for the opening.

6. Conclusions and recommendations

Throughout the investigation several conclusions and recommendations were drawn. The most important ones are listed below.

6.1 Conclusions

Both the Beam Method (Dutch Code) and the Strut-and-Tie Method (Eurocode) are conservative methods for the cases considered. This is caused by three phenomena:

1) Both methods do not take into account the contribution of the concrete tensile strength in the wall.
2) Both methods do not take into account that the reinforcement mesh will ensure a second load path for the tensile stresses in the wall when the concrete is cracked.
3) a) For the opening cases both methods do not take into account the positive effect of the large concrete mass which is present at both sides of the opening. This concrete mass will offer resistance in the increase of the opening, such that the tension tie will not be stressed as calculated by the codes. When less concrete mass is present at the sides of the opening, this effect will be smaller so that the stress in the tie will increase.
   b) For the overhang cases both methods do not take into account the positive effect of the concrete mass which is present over a large height above the tie. Due to the presence of this concrete mass, the tensile stresses in the concrete will be distributed over a large height so that the stresses in the tie will be less than calculated by the codes. When less concrete is present above the tie, this effect will be smaller so that the stress in the tie will increase.

In the cases considered the ULS condition is determined by crushing of the concrete at the corners of the openings. Increasing the amount of reinforcement in the ties for the cases considered will therefore not contribute to a higher load carrying capacity of the wall. In order to reduce the concentrated stress in these opening, the sharp edges in the corners should be round off so that the stress is distributed more equally. When a higher load carrying capacity of the wall is still desired, compression reinforcement can be added in order to decrease the compression stresses in the concrete.

The traditional design steps that start by defining the concentrated reinforcement ties and end with defining the minimum reinforcement mesh needed in the wall, might lead to a considerable amount of unnecessary reinforcement. The reinforcement mesh, which is not incorporated in the calculations for determining the reinforcement area needed in the ULS condition, will have a considerable contribution in transferring the forces in the wall. A better order in the design of concrete walls is to first determine the minimum reinforcement mesh needed in the wall. Then, if necessary, reinforcement can be added in order to meet the requirements in the SLS and ULS condition.

This research shows that different truss models, which lead to different reinforcements, will not result in a considerable change in the reaction of the wall. The designer should not make an effort in enhancing a truss model, which results in a complex system. It is better to choose a simple truss model that clarifies the force flow throughout the truss, as long as the truss model follows the overall image drawn by the flow of forces in the wall.

Detailing of the reinforcement is of big importance in order to transfer the loads trough the truss.

When a stress trajectory plot is used, the Eurocode calls upon the knowledge of the designer in order to translate the stress trajectory plot into a truss model that represents the flow of forces.

The ultimate load on a wall, according to the calculations used in practice, is limited by the τ_2-check in such a way, that it is only a function of the design concrete compressive strength, the thickness of the wall and the angle of the shear reinforcement.

6.2 Recommendations

The designer should realize that the crack width calculations presented in the codes are based on a bar loaded in tension. For walls these calculations do not provide solid ground for the determination of the crack widths.

In D-regions the maximum allowable stress in the struts is set to approx. 50% of the concrete design compressive strength according to the Strut-and-Tie Method of the Eurocode. Outside the D-region the maximum allowable stress is equal to the concrete design compressive strength. At the border of the D-region this causes a compatibility problem. Efforts should be made in order to understand the consequence of these compatibility problems.

In order to apply the Strut-and-Tie Method, it is important to have knowledge of the dimensions of the D-region for which the Strut-and-Tie Method is applicable. Research should be performed in order to achieve guidelines for the determination of this D-region in walls.

When high loading is applied, the struts in the Strut-and-Tie Method may become very large. In reality the compression stresses may not be distributed over such a large width. This might lead to a distorted image of the force flow through the D-region. Efforts should be made in order to determine the maximum width of the compression struts.

Reference

Ref. 1 Lecture notes CT3051B Reinforced Concrete, 2006
Prof. dr. ir. J.C. Walraven

Ref. 2 CUR-Report 47 Gedrongen balken en korte consoles, 1971

Ref. 3 CUR-Report 94-13 Achtergronden bij de VBC, 1990

Ref. 4 NEN6720, 1995

Ref. 5 GTB 1990 part II

Ref. 6 Cement nr. 6, 2004

Ref. 7 NEN-EN 1992-1-1

Ref. 8 Design of concrete structures with stress fields, 1997
Dr. A. Muttoni, Dr. J. Schwarts, Prof. dr. B. Thürlimann

Ref. 9 Performance-based optimization of structures, 2005
Qing Quan Liang

Ref. 10 Design of concrete structures, 1997
Arthur H. Nilson

Ref. 11 Design and numerical analysis of reinforced concrete deep beams, 2007
Moufaq Noman Mahmoud

Ref. 12 ATENA program documentation part III, 2005
Petr Kabele, Vladimír Červenka, Jan Červenka

Appendix

App. 1 and App. 2 are added to this report in the following pages. Due to the size of App. 3, App. 4 and App. 5 it is chosen to exclude those in a different set of papers.

Appendix included in this report:

App. 1 The τ_2-expression
App. 2 Ultimate load according to the τ_2-expression

Appendix excluded from this report:

App. 3: Calculations Dutch Code NEN6720
App. 3.1: The overhang of 1 m
App. 3.2: The overhang of 5 m
App. 3.3: The overhang of 10 m
App. 3.4: The opening of 1 m
App. 3.5: The opening of 5 m
App. 3.6: The opening of 10 m

App. 4: Calculations Eurocode EN1992-1-1
App. 4.1: The overhang of 1 m, truss model 1
App. 4.2: The overhang of 1 m, truss model 2
App. 4.3: The overhang of 1 m, truss model 3
App. 4.4: The overhang of 5 m, truss model 1
App. 4.5: The overhang of 5 m, truss model 2
App. 4.6: The overhang of 5 m, truss model 3
App. 4.7: The overhang of 10 m, truss model 1
App. 4.8: The overhang of 10 m, truss model 2
App. 4.9: The overhang of 10 m, truss model 3
App. 4.10: The opening of 1 m, truss model 1
App. 4.11: The opening of 1 m, truss model 2
App. 4.12: The opening of 1 m, truss model 3
App. 4.13: The opening of 1 m, truss model 4
App. 4.14: The opening of 5 m, truss model 1
App. 4.15: The opening of 5 m, truss model 2
App. 4.16: The opening of 5 m, truss model 3
App. 4.17: The opening of 5 m, truss model 4
App. 4.18: The opening of 10 m, truss model 1
App. 4.19: The opening of 10 m, truss model 2
App. 4.20: The opening of 10 m, truss model 3
App. 4.21: The opening of 10 m, truss model 4

App. 5: Calculations ATENA (FNL-FEM)
App. 5.1: Simulation of the cases with unreinforced concrete
App. 5.1.1: The overhang of 1 m
App. 5.1.2: The overhang of 5 m
App. 5.1.3: The overhang of 10 m
App. 5.1.4: The opening of 1 m
App. 5.1.5: The opening of 5 m
App. 5.1.6: The opening of 10 m

App. 5.2: Simulation of the design with a concrete tensile strength approaching zero
App. 5.2.1: The overhang of 1 m, Dutch Code
App. 5.2.2: The overhang of 1 m, Truss model 1 Eurocode
App. 5.2.3: The overhang of 1 m, Truss model 2 Eurocode
App. 5.2.4: The overhang of 1 m, Truss model 3 Eurocode
App. 5.2.5: The overhang of 5 m, Dutch Code
App. 5.2.6: The overhang of 5 m, Truss model 1 Eurocode
App. 5.2.7: The overhang of 5 m, Truss model 2 Eurocode
App. 5.2.8: The overhang of 5 m, Truss model 3 Eurocode
App. 5.2.9: The overhang of 10 m, Dutch Code
App. 5.2.10: The overhang of 10 m, Truss model 1 Eurocode
App. 5.2.11: The overhang of 10 m, Truss model 2 Eurocode
App. 5.2.12: The overhang of 10 m, Truss model 3 Eurocode
App. 5.2.13: The opening of 1 m, Dutch Code
App. 5.2.14: The opening of 1 m, Truss model 1 Eurocode
App. 5.2.15: The opening of 1 m, Truss model 2 Eurocode
App. 5.2.16: The opening of 1 m, Truss model 3 Eurocode
App. 5.2.17: The opening of 1 m, Truss model 4 Eurocode
App. 5.2.18: The opening of 5 m, Dutch Code
App. 5.2.19: The opening of 5 m, Truss model 1 Eurocode
App. 5.2.20: The opening of 5 m, Truss model 2 Eurocode
App. 5.2.21: The opening of 5 m, Truss model 3 Eurocode
App. 5.2.22: The opening of 5 m, Truss model 4 Eurocode
App. 5.2.23: The opening of 10 m, Dutch Code
App. 5.2.24: The opening of 10 m, Truss model 1 Eurocode
App. 5.2.25: The opening of 10 m, Truss model 2 Eurocode
App. 5.2.26: The opening of 10 m, Truss model 3 Eurocode
App. 5.2.27: The opening of 10 m, Truss model 4 Eurocode
App. 5.3: Investigating the importance of the reinforcement tie
App. 5.3.1: The overhang of 10 m, Design truss model 1 with Reinforcement tie, Eurocode
App. 5.3.2: The overhang of 10 m, Design truss model 1 without Reinforcement tie, Eurocode
App. 5.3.3: The opening of 10 m, Design truss model 1 with Reinforcement tie, Eurocode
App. 5.3.4: The opening of 10 m, Design truss model 1 without Reinforcement tie, Eurocode

App.1: The τ_2-expression

The τ_2-norm is a part of the check to shear force failure. In this part a verification of the τ_2-norm will be given, in order to understand this check.

When shear force in the structure is present, according to art. 8.2 of NEN6720 one has to satisfy the condition:

$$\tau_d \leq \tau_u \qquad \text{with:} \qquad \tau_d = \frac{V_d}{b*d}$$
$$\tau_u = \tau_1 + \tau_s \leq \tau_2$$

τ_d represents the shear stress due to the shear force V_d, τ_1 represents the allowable shear stress when no shear reinforcement is applied, τ_s represents the shear stress carried by the shear reinforcement and τ_2 represents the maximum stress in the compression strut.

When we analyze the formula above, it can be concluded that the following condition must hold:

$$\tau_d \leq \tau_2 \qquad \text{with:} \qquad \tau_2 = 0.2\, f_b^{'}\, k_n\, k_\theta$$

k_n is a factor taking prestressing into account
k_θ is a factor as a function of the angle θ

In order to explain the expression for τ_2, we have to analyze the stresses in the structure. Due to a discontinuity in the structure distortion of the stress path will occur. In figure App. 56 this discontinuity is caused by the support. Due to this discontinuity a compression strut will develop. This compression strut has a width in the plane of the drawing equal to:

$$b_D = z * (\cot\theta + \cot\alpha) * \sin\theta$$

On basis of equilibrium it is found that $N_D^{'} * \sin\theta = V_d$, such that for the concrete stress in the compression diagonal is found:

$$\sigma_b^{'} = \frac{V_d}{b*z*\sin^2\theta*(\cot\theta+\cot\alpha)} \qquad \text{(eq. 1)}$$

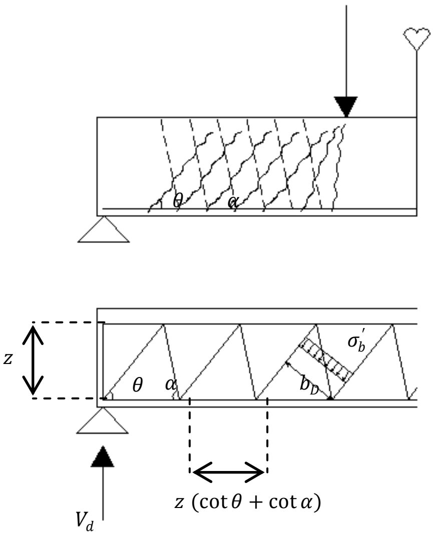


Figure App. 73: Geometry of the compression strut

It might be expected that the maximum stress in the compression strut is equal to the uniaxial strength of the concrete. However the shear reinforcement, loaded in tension, introduces by bond transverse tensile stresses. This causes a reduction of the strength as can be seen from the failure envelope for concrete loaded in two directions. See figure App. 73. Due to the occurrence of compressive stress in the direction of the diagonal and tensile stresses in the transverse direction, a reduction of the ultimate compressive stress occurs. Furthermore the shear reinforcement can give rise to local stress concentrations. From experiments it was shown that the following relation exists between the maximum stress and the concrete strength (Ref. 1):

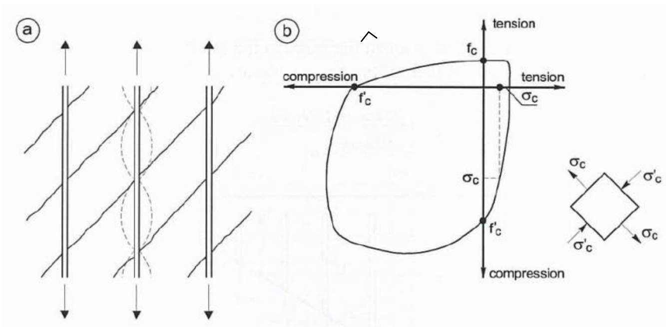


Figure App. 74: Reduction of the concrete compressive strength due to transverse tension

$$\sigma_b^{'} = \upsilon * f_b^{'} \qquad \text{(eq. 2)}$$

With:

$$\upsilon = 0.7 - \frac{f_{ck}^{'}}{200} \geq 0.5$$

Substituting eq. 1 and eq. 2 and assuming $\upsilon = 0.5$ will result in:

$$\frac{V_d}{b*z*\sin^2\theta*(\cot\theta+\cot\alpha)} \leq 0.5 * f_b' \qquad \text{(eq. 3)}$$

For ordinary beams it can be assumed that $z \cong 0.8 * d$ such that $d \cong 1.25 * z$. Substituting this into eq. 3 and knowing that $\tau_d = \frac{V_d}{b*d}$ will result in:

$$\tau_d \leq 0.4 * f_b' * \sin^2\theta * (\cot\theta + \cot\alpha) \qquad \text{(eq. 4)}$$

Eq. 4 agrees with the requirement formulated in art. 8.2.1 of NEN6720, when the wall is not prestressed in both directions:

$$\tau_d \leq \tau_2 \qquad \text{with:} \qquad \tau_2 = 0.2 * f_b' * 2 * \frac{\cot\theta+\cot\alpha}{1+\cot^2\theta} \qquad \text{(eq. 5)}$$

Choosing vertical stirrups as shear reinforcement we get $\alpha = 90°$ and choosing the angle of the compression diagonal equal to $\theta = 45°$ we get:

For eq. 4: $\tau_d \leq 0.20 * f_b'$

For eq. 5: $\tau_d \leq 0.20 * f_b'$

It can be seen that this gives exactly the same answer.

App. 2: Ultimate load according to the τ_2-expression

In practice it is claimed that, according to NEN6720, the dimensions of the opening doesn't influence the ultimate load according to the τ_2-norm. In this part it will be shown that the internal lever arm z, according to NEN6720, is a function of the dimensions of the openings in such a way, that the angle of the compression diagonal θ is constant. When the wall is not prestressed in both directions and because the angle θ is a constant, the value τ_2 can only be increased by increasing the concrete class of the stability wall, as will be shown.

The ultimate load q, according to only the τ_2-norm, will be derived for three different systems. First this load is determined for the cantilever system (fig. App. 74a). Then the ultimate load is determined for a system with an opening in the middle of the wall. This is schematized as a simply supported deep beam, as often done in practice (fig. App. 74b). The third system is equal to the second system with the difference that the wall is schematized as a deep beam that is fully clinched (fig. App. 74c).

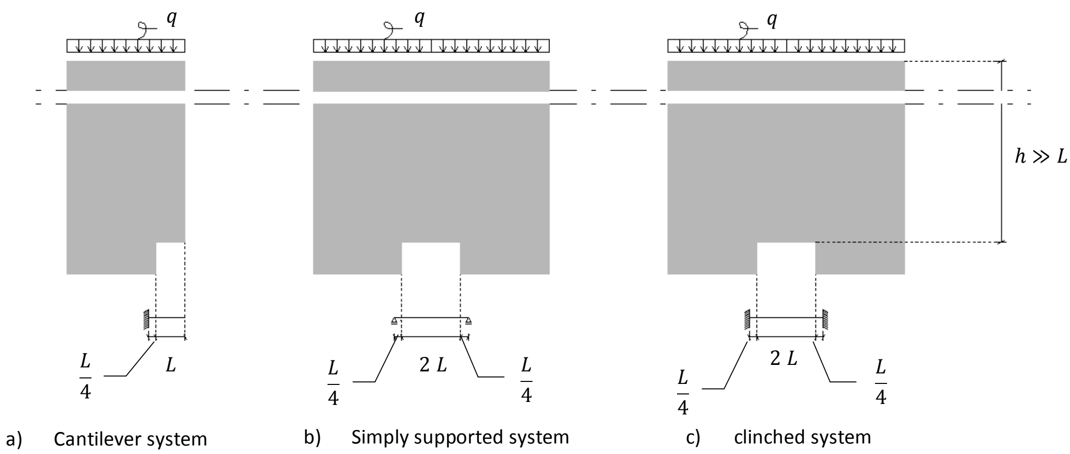


Figure App 75: Schematization of the system

Cantilever system

According to article 8.1.4 of NEN6720 the internal lever arm is equal to:

$$z = 0.4 * a + 0.4 * h \leq 1.6 * a \quad \text{(eq. 1)}$$

With:

- h is equal to the height above the opening as given in figure App. 3
- a is equal to the distance from the clinch till the resultant of the distributed load q above the opening

With the definition of a given above it becomes:

$$a = \frac{L}{2} + \frac{L}{4} = \frac{3}{4} L \quad \text{(eq. 2)}$$

Because $h \gg L$, the internal lever arm z is equal to:

$$z = 1.6 * a = 1.2 * L \quad \text{(eq. 3)}$$

The determined shear force, at the end of the opening, is equal to $V_d = q * L$. The shear stress according to art. 8.2.2 becomes:

$$\tau_d = \frac{V_d}{b*d} = \frac{q*L}{b*d} \quad \text{(eq. 4)}$$

In art. 8.2.2 it is mentioned that for deep beams $d = h$. Because in this case h is considered infinite compared to the dimension of the opening L, according to practice this statement cannot be true as it will lead to an infinite small shear stress. Because of this it is assumed that $z \cong 0.8 * d$, wich holds true for an ordinary beam. Substituting this and eq. 3 into eq. 4 we get:

$$\tau_d = \frac{q}{b*1.5} \quad \text{(eq.5)}$$

According to article 8.2.1 τ_2 is equal to:

$$\tau_2 = 0.2 * f_b^{'} * k_n * k_\theta \quad \text{(eq. 6)}$$

With:

- $k_n = \frac{5}{3}\left(1 - \frac{\sigma_{bmd}^{'}}{f_b^{'}}\right) \leq 1$
- $k_\theta = 1$ for $\alpha = 90°$
- $k_\theta = 2 * \frac{\cot\theta + \cot\alpha}{1+\cot^2\theta}$ for $45° \leq \alpha < 90°$

Because no prestressing is applied ($\sigma_{bmd}^{'} = 0$), k_n becomes equal to 1. The angle θ can be calculated according to art. 8.1.4 with $\theta = \arctan\frac{z}{a} = \frac{1.2*L}{\frac{3}{4}*L} = 58°$. Substituting this into k_θ we get:

$$k_\theta = 2 * \frac{\cot\theta + \cot\alpha}{1+\cot^2\theta} = 2 * \frac{\frac{5}{8}+\cot\alpha}{1+\left(\frac{5}{8}\right)^2} \quad \text{(eq. 7)}$$

Substituting eq. 7 into eq. 6, while knowing $k_n = 1$ we get:

$$\tau_2 = 0.4 * \frac{\frac{5}{8}+\cot\alpha}{1+\left(\frac{5}{8}\right)^2} * f_b^{'} \quad \text{(eq. 8)}$$

The τ_2-norm according art. 8.2.1 is equal to $\tau_d \leq \tau_2$. Substituting eq. 5 and eq. 8 into the τ_2-norm gives:

$$q \leq 0.6 * \frac{\frac{5}{8}+\cot\alpha}{1+\left(\frac{5}{8}\right)^2} * b * f_b^{'} \quad \text{(eq. 9)}$$

As can be seen from eq. 9 the distributed load q is according to NEN6720 dependent of the angle of the stirrups α, the width of the wall b and the concrete compressive strength $f_b^{'}$. It can therefore be concluded that the dimensions of the opening doesn't influence the ultimate load. This means that if one has an opening of for example 10 m in a wall, this will lead to exactly the same ultimate load as an opening of 10 cm in the same wall. It is furthermore stated that the highest distributed load, according to eq. 9, is obtained when $\alpha = 45°$. The ultimate load becomes $q = 0.70 * b * f_b^{'}$. The lowest load is obtained when $\alpha = 90°$ and is equal to $q = 0.27 * b * f_b^{'}$.

Simply supported system

The same calculation can be done for the simply supports system. Therefore the calculation will be done in a concise manner.

$$l_{ligger} = 2\,L + 2 * \frac{1}{4}L = 2.5\,L$$

Art. 8.1.4: $z = 0.6 * l_{ligger} = 1.5 * L$

Art. 8.2.2: $\tau_d = \frac{V_d}{b*d} = \frac{q*L}{b*\frac{15}{8}*L} = \frac{\frac{8}{15}*q}{b}$

Art. 8.2.1: $\theta = \arctan\frac{z}{\frac{1}{2}*l_{ligger}} = 50°$

$$k_\theta = 2*\frac{\cot\theta+\cot\alpha}{1+\cot^2\theta} = 2*\frac{\frac{5}{6}+\cot\alpha}{1+\left(\frac{5}{6}\right)^2}$$

$$k_n = 1$$

$$\tau_2 = 0.2*f_b^{'}*k_n*k_\theta = 0.4*\frac{\frac{5}{6}+\cot\alpha}{1+\left(\frac{5}{6}\right)^2}*f_b^{'}$$

$\tau_d \le \tau_2$ gives than:

$$q \le 0.75*\frac{\frac{5}{6}+\cot\alpha}{1+\left(\frac{5}{6}\right)^2}*b*f_b^{'}$$

For $\alpha = 45°$ we get the highest ultimate load equal to:

$$q \le 0.81*b*f_b^{'}$$

For $\alpha = 90°$ we get the lowest ultimate load equal to:

$$q \le 0.37*b*f_b^{'}$$

<u>Clinched system</u>

The concise calculation for the clinched system is as follows:

$$l_{ligger} = 2\,L + 2*\frac{1}{4}L = 2.5\,L$$

$$\frac{1}{8}*q*l_{ov}^2 = \frac{1}{24}*q*l_{ligger}^2$$

$$l_{ov} = 1.44\,L$$

$$\frac{1}{2}\,l_{os} = \frac{l_{ligger}-l_{ov}}{2} = 0.53\,L$$

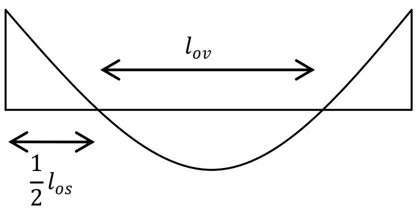


Figure App. 76: Moment distribution

The determining section due to the shear force is at a distance $\frac{L}{4}$ from the clinch. Because this section is within $\frac{1}{2}\,l_{os}$, the internal lever arm z becomes:

Art. 8.1.4 $z = 0.75*1.5*l_{os} = 1.19\,L$

Art. 8.2.2: $\tau_d = \frac{V_d}{b*d} = \frac{q*L}{b*1.49*L} = \frac{q}{1.49*b}$

Art. 8.2.1: $\theta = \arctan\frac{z}{\frac{1}{2}*l_{ligger}} = 44°$

$$k_\theta = 2*\frac{\cot\theta+\cot\alpha}{1+\cot^2\theta} = 2*\frac{1.05+\cot\alpha}{1+(1.05)^2}$$

$$k_n = 1$$

$$\tau_2 = 0.2 * f_b^{'} * k_n * k_\theta = 0.4 * \frac{1.05 + \cot\alpha}{1+(1.05)^2} * f_b^{'}$$

$\tau_d \leq \tau_2$ gives than:

$$q \leq 0.60 * \frac{1.05 + \cot\alpha}{1+(1.05)^2} * b * f_b^{'}$$

For $\alpha = 45°$ we get the highest ultimate load equal to:

$$q \leq 0.59 * b * f_b^{'}$$

For $\alpha = 90°$ we get the lowest ultimate load equal to:

$$q \leq 0.30 * b * f_b^{'}$$