# Structural Reliability of Existing City Bridges

*Analysis with Monte Carlo simulation including a load model based on weigh-in-motion measurements*

*Master Thesis – Final Draft*

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| **Student:** | Laura Hellebrandt |
| **Student number:** | 4247523 |
| **E-mail:** | l. hellebrandt@student.tudelft.nl |

# General Information

## Topic

Structural reliability of bridges with Monte Carlo simulation, including traffic loading data

## Student


***Laura Hellebrandt***
Student nr: 4247523
e-mail: l.hellebrandt@student.tudelft.nl / laura.hellebrandt@gmail.com
phone: 06-8380-9064


## Graduation Committee

Chairman: ***prof. ir. F.S.K. Bijlaard***
Technical University of Delft; Civil Engineering and Geo-sciences
Section: Structural and Building Engineering

Members: ***dr. ir. C.B.M Blom***
Ingenieursbureau Gemeente Rotterdam
Structural Engineering

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Technical University of Delft; Civil Engineering and Geo-sciences
Section: Structural Mechanics

***dr. ir. M.H. Kolstein***
Technical University of Delft; Civil Engineering and Geo-sciences
Section: Structural and Building Engineering

***prof. ir. R.D.J.M. Steenbergen***
TNO
Structural Reliability

***prof. ir. A.C.W.M. Vrouwenvelder***
TNO
Structural Reliability

# Preface

The current document has been prepared for the 5th meeting of the Graduation Committee (3rd September, 2014) and is the final draft of the Master's thesis with the title: *"Structural Reliability of Existing City Bridges - Analysis with Monte Carlo simulation including a load model based on weigh-in-motion measurements".*

The graduation project is conducted by Laura Hellebrandt as the final part of the Structural Engineering master curricula at the Technical University of Delft.

Ingenieursbureau Gemeente Rotterdam and TNO provide support for the graduation project and have delegated representatives to the graduation committee.

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# 1 Introduction

## 1.1 Background

Determining structural reliability is an increasingly important matter with regard to the aging infrastructure. Building codes allow for classifying a structure as adequate, if it is proven that the needed reliability index is reached, i.e. an excepted probability of failure is not exceeded. Calculations applying safety coefficients, loads and resistances prescribed by building codes may lead to conservative results as they are calibrated for “general” application. With more sophisticated calculations and stochastic input data, such as traffic loading, a deeper insight to the structural safety can be gained. This can lead to the proof of a higher structural reliability than determined by the analysis according to the building codes.

The Engineering Office of the Municipality of Rotterdam (Ingenieursbureau Gemeente Rotterdam - IGR) is concerned with the structural safety of several bridges. When, based on calculations according to current norms, namely the Eurocodes and relevant Dutch National Annexes are done and the outcome is that the structural reliability is insufficient, a possible step to gain a deeper insight and prove the structure safe would be a probabilistic calculation. It is therefore of interest to investigate the use of probabilistic methods that can be applied by practicing structural engineers to determine the reliability of a structure.

The most significant uncertainty in bridge analysis is related to traffic loading, as stated for example by (Caprani 2005). Traffic load measurements by weigh-in-motion technique have been carried out in the city of Rotterdam thus site-specific information is readily available. The data, which is being analysed by the Netherlands Institute for Applied Scientific Research (TNO), can serve as input for probabilistic calculations.

## 1.2 Problem description

### *1.2.1 State of infrastructure*

In the area of the city of Rotterdam, there are 325 traffic bridges (non-highway) according to inventories available. In *Figure 1* an example of the deck of a moveable bridge built in 1948, which has been re-analysed in 2009, can be seen.

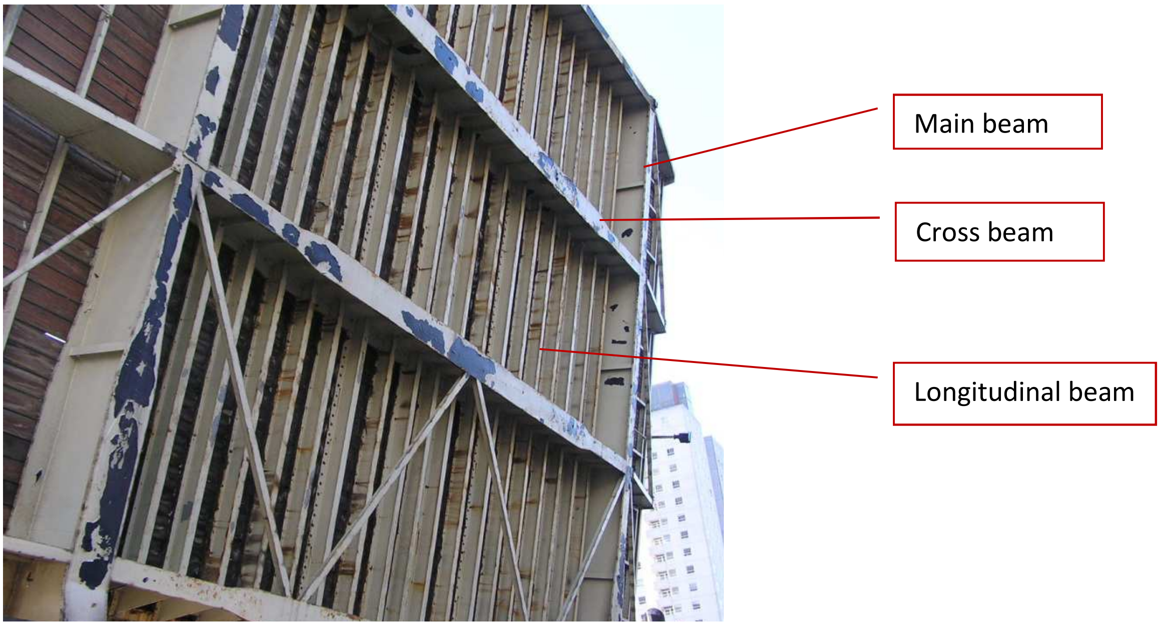


**Figure 1 - Deck of moveable bridge 'Rederijbrug'**

Below, in *Table 1* and *Figure* 2, the distribution of bridges with respect to material and age is shown. The dates are the original building dates; some of the bridges have been renovated since these mentioned times.

**Table 1 - Traffic bridges in Rotterdam by age and material**

| Built | | Age | Concrete | Steel | Steel & Concrete[1] | Other[2] | *Unknown*[3] | **Total** | |
|---|---|---|---|---|---|---|---|---|---|
| 1999 | 2014 | 0 - 15 | 68 | 8 | 0 | 1 | 11 | *88* | 27,1% |
| 1984 | 1999 | 15 - 30 | 28 | 1 | 3 | 1 | 11 | *44* | 13,5% |
| 1969 | 1984 | 30 - 45 | 40 | 2 | 2 | 1 | 14 | *59* | 18,2% |
| 1954 | 1969 | 45 - 60 | 40 | 0 | 5 | 0 | 9 | *54* | 16,6% |
| 1939 | 1954 | 60 - 75 | 10 | 0 | 1 | 2 | 6 | *19* | 5,8% |
| 1924 | 1939 | 75 - 90 | 29 | 3 | 4 | 0 | 2 | *38* | 11,7% |
| 1909 | 1924 | 90 - 105 | 8 | 1 | 0 | 0 | 0 | *9* | 2,8% |
| 1800 | 1909 | 105 - | 2 | 3 | 0 | 0 | 3 | *8* | 2,5% |
| *Unknown* | | | 4 | 0 | 0 | 0 | 2 | *6* | 1,8% |
| **Total** | | | 229 | 18 | 15 | 5 | 58 | **325** | |
| | | | 70,5% | 5,5% | 4,6% | 1,5% | 17,8% | | |

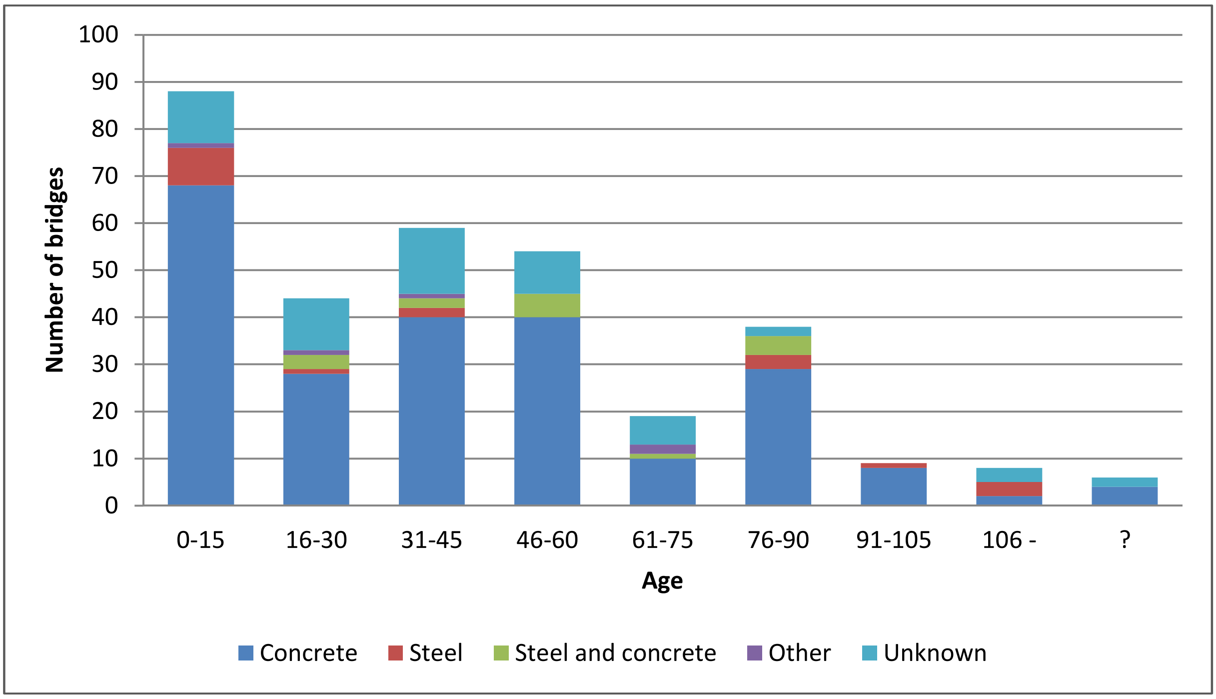


**Figure 2 - Traffic bridges in Rotterdam by age and material**

The state of the bridges is being monitored according to maintenance plans and the load bearing capacities are being checked according to a plan taking into account priorities, such as the material and age of the bridge or the type of road network it is a part of. Yearly 5-8 bridges are analysed by IGR, in consultation with the bridge administrator.

[1] *Bridge with both steel and concrete deck parts*
[2] Steel-concrete composite
[3] *Not known from inventory data available to date*

The main reason for re-evaluation of structures is the increase in traffic loading during the past decades, to which recent norms, namely the Eurocodes have been adapted. The change most relevant in the Netherlands is that the previously existing three loading categories for bridges with various expected traffic (traffic categories 30, 45 and 60, where the numbers refer to tons) has been replaced by a load models applicable for all traffic conditions in Eurocode 1.

The costs of demolishing and completely replacing a bridge structure are substantial. Therefore, in case a structure is proven unsafe, especially if with a low margin, economic consideration suggests that a deeper investigation to the safety of the structure is done. This can be carried out in several ways, such as more sophisticated structural analysis methods like taking into account load re-distribution or by performing measurements related to the strength of the structure, for example determining the concrete cover. In case the structure is not adequate, economic considerations motivate the decision for renovation, replacement or load restriction. The concept and methodology of re-analysis is described in various literature (JCSS 2000; Schneider 1997; Faber 2009). In practice, the methodology for IGR is elaborated in the relevant project description (Laarse 2012).

It can be concluded, that the problem from the "practical" point of view, expressed as need is the following:

> *Problem 1*
>
> In the coming years, several existing traffic bridges in the city of Rotterdam have to be evaluated for structural reliability due to increased traffic loading. A majority of structures is expected not to fulfill all requirements according to the basic calculations described in building codes.
>
> Based on past experience with evaluating structures, as well as on the knowledge that the requirements in the design codes are often conservative for specific cases it is expected that a portion of these structures has a sufficiently low failure probability. Probabilistic methods to prove this are currently not employed to a full extent.

*Implication*

It is of interest to investigate calculation methods for determining the failure probabilities of the structures. One option for a more sophisticated analysis is the application of probabilistic methods.

### *1.2.2 Traffic loading*

In order to know more about the actual traffic loading, weigh-in-motion measurements have been conducted at 2 locations (Matlingweg, Horvathweg) in Rotterdam. The measurement output is total weight, axle load, axle distance, vehicle distance and speed. The output also contains classification in categories

When converted to a load distribution, two strategies can be followed in order to take the output into account. Firstly, design traffic loads can be calibrated in a semi-probabilistic way. For example, research recommendations for national adaptations are made in Latvia (Paeglitis & Paeglitis 2002) and Slovenia (O'Brien et al. 2006). Secondly, site-specific loading may be determined and applied for the bridge where it was measured. Distribution of load effects can be used in probabilistic calculations.

> *Problem 2*
>
> Traffic load data is currently available for two specific locations in Rotterdam. Up to date it is not known what implication this data has for the loading condition of bridges in the city.

*Implication*

By analysing and extrapolating the data, it can be expected that more adequate loading conditions for a bridge in Rotterdam are determined than those described in Eurocode 1.

## 1.3 Work approach

### *1.3.1 Hypothesis*

Monte Carlo simulation with Excel can be used in certain situations to come to conclusions about bridge reliability which are more adequate than calculations performed using "traditional" (1st order) methods.

As the main uncertainty originates from traffic loading, the incorporation of traffic loading data in this analysis can significantly contribute to a refined outcome.

### *1.3.2 Problem statement*

> Monte Carlo analysis is a robust tool in the reliability analysis of existing bridges, however the applicability is not commonly known by structural engineers. Moreover it is not known how to incorporate in the analysis actual, site specific traffic loads derived from measurements.

### *1.3.3 Aim*

The aim of this thesis is to investigate the possibilities for using Monte Carlo analysis in bridge reliability calculations as well as the applicability of traffic load measurements as load data input. Building materials, structure types, sizes and failure modes of bridges in the city of Rotterdam are to be focused on.

It should be determined when Monte Carlo analysis or other probabilistic method can be used in the every-day practice in structural engineering, beyond the approach of Eurocodes. The need for specific software should be excluded, if possible, from the final applied methods and these shall mainly be used as comparison for finding limitations.

### *1.3.4 Objectives and research questions*

In order to reach the above mentioned aim, it is broken down to objectives and related research questions. These are the following:

I) Gain overview of methods in structural reliability analysis;
   a. What methods of structural reliability analysis are available?
   b. How are these methods applicable with respect to building codes and regulations?
   c. What are the advantages and disadvantages of each method and when are they applicable?
II) Determine relevant structure types and failure modes;
   a. What are "typical" structures among bridges in Rotterdam?
   b. Are there common failure modes and if yes, what are these?
   c. Is there potential for applying Monte Carlo analysis to investigate these failure modes?
III) Model the relevant (or otherwise chosen) structural failures;
   a. How can the resistance of these failures be modelled?
   b. What is the result of the analysis without input traffic loading data?
IV) Analyse and interpret traffic loading
   a. Convert weigh-in-motion data to traffic loading;
      i. What does WIM data represent and how is it related to standard load models?

ii. What is the best strategy for analysis of the data, with respect to data interpretation and extrapolation?

b. Convert traffic loading to load effects;

i. How can the loading be converted to load effects?

ii. What are the possibilities to use these load effects in simulation?

V) Determine structural reliability of relevant (or otherwise chosen) failure modes and a chosen specific case;

a. Incorporate load effects in limit state equations and carry out analysis;

VI) Evaluation of applied methods with respect to precision, usability and usefulness;

a. Are the methods applicable in practice?

b. What are the limitations?

c. What are costs and benefits in comparison to semi-probabilistic calculations?

d. What are costs and benefits in comparison to other, level II or III probabilistic assessment methods?

### 1.3.5 Outline

The current thesis is structured in three main parts. The 1st section *'Background'* consists of five chapters, it summarizes relevant literature and draws conclusions for application and for further strategy. Chapter 2 covers the main theoretical and practical background of structural reliability analysis, briefly introducing the various methods. Chapter 3 introduces the regulatory framework of both semi-probabilistic and probabilistic calculations. Chapter 4 investigates the bridges of Rotterdam as well as the currently applied process for re-evaluation of structures. In Chapter 5 a brief overview is given of statistical concepts which are applied in or have an influence on the further work.

The 2nd section, *'From WIM Measurements to Load Effect Distribution'* starts with a literature-review of traffic loading analysis in Chapter 6. Chapter 7 describes the strategy developed for traffic loading analysis, based on the interpretation of the previously summarized and evaluated research. Chapter 8 and 9 elaborate on the two main processes within the analysis: traffic simulation based on WIM data analysis and load effect analysis, respectively.

The 3rd section, *'Application and Evaluation'* consists of two chapters. Chapter 10 shows examples of probabilistic analysis of elementary structures with Monte Carlo simulation without and with traffic loading data input. Finally, Chapter 11 consists of an evaluation and recommendations.

# Section I

# Background

# 2 Structural reliability background

## 2.1 Introduction

"The safety of an existing structure is a matter of decision rather than of science. Reliability theory is a tool and a rational basis for preparing such decisions." (Schneider 1997)

### *Reliability*

Most problems in civil engineering consist of determining a capacity *R* and a load *S*, where the ultimate goal is to ensure that the first is larger than the latter.

$$R > S$$

On both sides of the inequality uncertainties are present. It is not possible to determine with absolute certainty either the strength or the load; therefore the concept of *absolute safety* is not applicable in practice. One can only be certain to some extent, that loads will not exceed the resistance of a structure in a given time period. Safety can therefore be expressed as the probability of non-failure $P_{nf}$. In practical applications, it is often simpler to determine the probability of failure, $P_f$, where the following relation holds:

$$P_f + P_{nf} = 1$$

The ***reliability*** of an element or a system can be defined as its probability of non-failure and can therefore be written as

$$reliabilty = 1 - P_f$$

### *Risk-based decision making*

Before introducing the basic concepts related to reliability, a short overview is given of the broader context of risk-based decision making.

The role of engineering is decision making or decision support, which is always done under some uncertainty. Risk-based decision making is a rational approach in several situations. In principle, the failure probability of any system is not the only information necessary for such a decision. Of real concern is the ***risk*** of the structure existing and being in use.

A basic definition of risk, as can be found in literature such as (CUR-publicatie 190 1997) is "probability multiplied by consequence". Thus risk-based decision making has two main aspects: probabilistic modelling (risk) and the modelling of consequences, as depicted in *Figure 3.*

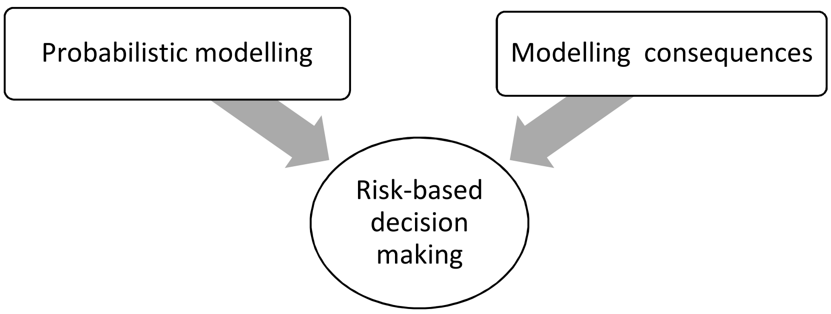


**Figure 3 - Risk based-decision model**

There are two approaches to include the aspect of safety as a probabilistic concept in the design or re-evaluation of a structure.

Firstly, design can be carried out based on acceptable failure probabilities. This criterion usually contains implicitly the consequences of failure in the form of consequence classes, such as in the Eurocodes. If the collapse of a structure is expected to have a relatively low consequence (for example a storage hall), a lower reliability will be determined from it than from a structure which would cause more damage by collapsing (for example a public building).

The second, more sophisticated approach, possibly used for example in the design process of flood protection systems (dikes) is risk-based design. Here the consequences are taken into account directly in a cost function, which includes investment costs and risk reduction also expressed in monetary terms, both as a function of design variables. The optimisation problem of design is tackled by minimising the cost function. This method is not or seldom used in structural engineering at the time of writing this work but has a high potential and is being developed for maintenance strategies.

In this thesis work the focus will be on probabilistic modelling while keeping in mind that in the overall result the consequence of possible failure also plays a role. In practice this is taken into account in building codes and norms by assigning consequence classes to objects and adapting the accepted probability of failure adequately, in order to have a standard acceptable risk. Thus, in practice a target failure probability to aim for is a relevant measure.

A possible situation when consequence might become of importance is in case bridge networks or a multitude of bridges with allowed higher failure probabilities are to be investigated.

## 2.2 Basic Concepts [4]

Typical measures of reliability in civil engineering are the probability of failure and the reliability index. The relation between the two is defined as:

$$\beta = -\phi^{-1}(p_f)$$

The basic inequality R < S can be written as

$$\mathrm{Z} = \mathrm{R} - \mathrm{S}$$

Or:

$$\mathrm{Z(X)} = \mathrm{R(X)} - \mathrm{S(X)} \quad [5]$$

The previous equation is also called the ***limit state function*** and will be referred to in the current thesis work also as *reliability equation*. It is mentioned that in some cases there might be a limitation to this model. According to Diamantidis et al. (2012) "assumption of sharp boundary between desirable and undesirable state is a simplification that might not be suitable for all structural members and materials".

Both the resistance and load side consist of several stochastic parameters. Schneider (1997) gives the example of concrete strength to point out, that the number of these parameters is also a matter of judgement. Concrete strength depends on several aspects such as the water-cement ratio, the hardening process etc. However in the resistance model finally it will be reasonable to model the concrete strength with one single stochastic parameter - "at some point the branching off process has to be terminated". The variables which are finally considered in the reliability equation are termed ***basic variables***. Their choice depends on the problem.

Basic variable have three main types:

[4] Based on Schneider (1997)

[5] In EN 1990 expressed as $g(X) = R(X) - E(X)$

1. Environmental variables, which are not controllable by the designer.
   Seismic actions and wind can be relevant in structural engineering. From this wind can be expected relevant at opening of moveable bridges. Finally environmental variables are not considered in the current thesis.
2. Structural variables, which don't vary much during life except due to deterioration processes.
3. Utilisation variables, which can be controlled by supervision.
   It should be noted however, that there is also uncertainty also about "keeping agreements", thus full control might not be possible. In the current thesis work this can be the case of trucks loaded over the legal weight limit.

In the following, the elements of the limit state equation: the resistance *R* and the load *S* are described. For both, model uncertainties are present to which a separate section is dedicated.

### *2.2.1 Resistance*

A resistance model *R* can be expressed as:

$$\mathrm{R} = \mathrm{M} \times \mathrm{F} \times \mathrm{D}$$

| | | |
|---|---|---|
| Where: | M | Model uncertainty variable |
| | F | Material properties |
| | D | Dimensions and the derived quantities |

#### *Model uncertainty variable (M)*

Test results or the real behaviour of a structure deviates from the theoretical resistance model. The degree of this deviation is included in the model uncertainty of resistance. Its magnitude is different depending on failure mode considered. For example, more precise models exist for bending- than for shear failure of a concrete beam.

In calculations this uncertainty can be considered in two ways, as described by Diamantidis et al. (2012). Either it can be already included in safety factors of verifications according to building codes (*semi-probabilistic* methods, as will be described in *Section 2.3.1* and *3.4.1)*, or using ***probabilistic model factors*** in reliability analysis. The latter can be understood as including one or more additional stochastic parameters representing the model uncertainty.

#### *Material properties (F)*

When describing material properties, the concept of a ***transfer variable*** is relevant. This variable expresses that measurement results do not exactly represent reality. Reasons for this can be laboratory circumstances, scatter in lab versus scatter in structure or time dependence of material properties. With this in mind, a material property can be expressed as:

$$\mathrm{F} = \mathrm{P} \times \mathrm{T}$$

| | | |
|---|---|---|
| Where | P | Properties variable, i.e. the measured value |
| | T | Transfer variable<br>Typically has a mean $\mu < 1$ and a standard deviation $\sigma \sim 0.10\text{-}0.15$ |

#### *Dimensions and derived quantities (D)*

The mean value $\mu$ of dimensions and derived quantities is usually equal to or in the range of nominal value. The standard deviation is usually in the order of tolerances, thus the coefficient of variation, CoV = $\mu$ / $\sigma$, is larger for smaller dimensions.

### 2.2.2 Actions

An ***action*** can be defined as the cause of effects such as internal forces, deformations, material deterioration and other short- or long-term effects. (JCSS 2001)

***Load*** is an assembly of concentrated or distributed forces acting on the structure.

#### Action effect

The ultimate parameter of interest for a civil engineer is the effect that a certain action has on a structure, for example a bending moment *M*, shear *V* or normal force *N*. These effects are caused by the *action,* for example by wind pressure *w* [kN/m$^2$]. The action is caused by an *influence*, for example in the current case by the wind with the relevant parameter of wind speed *v* [m/s].

When modelling actions, there is usually a *leading action* and accompanying actions present. In a probabilistic approach, the first is typically described by an *extreme value distribution* while the second usually by a normal- or lognormal distribution. For further information about distribution types, refer to *Sections 5.2* and *5.4.*

The model of action effects, according to JCSS (2001) can be written as:

$$\mathrm{F} = \varphi(\mathrm{F_0}, \mathrm{W})$$

Where: $F_0$ Basic random variable which is often time- and space dependent. It is directly associated with the event causing the action and should be as independent from the structure as possible.

W Is a random or non-random field, which may depend on structural properties of the structure and transforms $F_0$ to F. Is often time independent.

It is noted that the model may include material properties as well, for example in the case of self-weight.

#### Model uncertainty

Similarly to the case of a resistance model, the action effect originating from a given action or influence contains uncertainty. This is taken into account by model variables on the load side of the limit state equation.

The degree of model uncertainty is often estimated subjectively and not measured. Serviceability limit state models contain a higher uncertainty and standard deviation σ 0.05 up to 0.3. For structural safety the model uncertainty is often taken with a mean of 1 and a standard deviation of 0, i.e. it is neglected due to effects “cancelling each other out”.

### 2.2.3 Models

According to the JCSS (2001): “it is understood that modelling is an art of reasonable simplification of reality such that the outcome is sufficiently explanatory and predictive in an engineering sense. (…) Models should generally be regarded as simplifications which take account of decisive factors and neglect the less important ones.”

When describing a limit state equation, both the resistance and load side are described by models. One can speak of *action models*, *structural models* that describe the action effects, *resistance models* which give resistance corresponding to action effects, *material*- and *geometry models*. These models often can’t be totally separated.

Structural or mechanical models can be further sub-divided to the following categories (Diamantidis et al. 2012):

a) Static response
Which is usually an elastic or a plastic model

b) Dynamic response
Where stiffness, damping and inertia are modelled

c) Fatigue
Which can be a so called "S-N model", based on experiments or a more sophisticated fracture mechanics model

In these models other things can be included, such as degradation or fire. In the current thesis work static response models are used.

### Model uncertainty

In most cases, the model describing relations between relevant variables is **incomplete** and **inexact.** Cause may be the lack of knowledge or simplification. (JCSS 2001) This has already been indicated in relation to resistance and load models.

In some cases, such as for example a steel bar in tension, the simplicity of the physical problem considered may allow for not including a model uncertainty in the calculation. An example can be found for this in Diamantidis et al.(2012)[6]

The model uncertainties are assumed to be partly correlated throughout the structure. (JCSS 2001) The correlation is estimated, according to JCSS (2001), however in the available version this information is not included. As the current thesis work focuses on failure within one cross section, this fact will not have an impact on the results.

### Determining model uncertainty

It is possible to determine model uncertainty in applied research. For example, when probabilistic models are set up for load effect calculations at TNO (Steenbergen et al. 2012), in some cases the coefficients are determined and the methodology is written in the reports. There is further reference to the specific case of traffic loading analysis in *Section 6.4.3.*

Each type of uncertainty has a distribution, usually assumed to be normal, which can be described by mean $\mu$ and variance V. If the overall model uncertainty is taken as the product of the specific model uncertainties, then the parameters (mean $\mu_M$ and variance $V_M$) can be described as:

$$\mathrm{V}_M = \sqrt{\mathrm{V}_1^2 + \cdots + \mathrm{V}_\mathrm{i}^2}$$

$$\mu_M = \mu_1 \cdot \ldots \cdot \mu_\mathrm{i}$$

For several common cases model uncertainties for both load and resistance models are recommended by the Probabilistic Model Code (JCSS 2001)and are visible in
*Table 2.*

[6] Based on the publication on JCSS (2000) and is also available in the lecture notes of Faber (2009)

**Table 2 - Recommended Probabilistic Models for Model Uncertainties – (JCSS, 2001)**

| | Model type | Distr | mean | CoV | corre lation |
|---|---|---|---|---|---|
| | **load effect calculation** | | | | |
| | moments in frames | LN | 1.0 | 0.1 | |
| | axial forces in frames | LN | 1.0 | 0.05 | |
| | shear forces in frames | LN | 1.0 | 0.1 | |
| | moments in plates | LN | 1.0 | 0.2 | |
| | forces in plates | LN | 1.0 | 0.1 | |
| | stresses in 2D solids | N | 0.0 | 0.05 | |
| | stresses in 3D solids | N | 0.0 | 0.05 | |
| | **resistance models steel (static)** | | | | |
| | bending moment capacity (1) | LN | 1.0 | 0.05 | |
| | shear capacity | LN | 1.0 | 0.05 | |
| | welded connectio capacity | LN | 1.15 | 0.15 | |
| | bolted connection capacity | LN | 1.25 | 0.15 | |
| | **resistance models concrete (static)** | | | | |
| | bending moment capacity (1) | LN | 1.2 | 0.15 | |
| | buckling | LN | 1.4 | 0.25 | |
| | shear capacity | LN | 1.0 | 0.1 | |
| | connection capacity | | | | |

(1) including the effects of normal and shear forces.

### *2.2.4 Elements and systems*

One can speak of the reliability of an element or that of a (sub-)system. When speaking of a bridge, an element can be for example one specific beam, while the system can be the whole bridge. It is worth to consider at least on a theoretical level, how reliability analysis should be approached in practice with regard to these two concepts. System reliability analysis is a complex task which includes knowledge of the reliability of all relevant elements and also their contribution to the functionality of the total system.

#### *Components*

***Component reliability*** "is the reliability of one single structural component which has one dominating failure mode."(JCSS 2001) Considering that a bridge has several components, how should an engineer approach reliability analysis? According to  Diamantidis et al. (2012) "limit state design is based on the consideration of local and not global failure, since design equations are usually defined and applied on a local level only. The global reliability (...) of the entire system is treated in the robustness requirements."

Therefore it is concluded that in practice it is allowed to consider reliability for one element, one failure mode. However, it is interesting to know what the limit to this approach is.

#### *Systems*

A ***system*** is a number of components or one component with multiple failure modes which are of nearly equal importance. (JCSS 2001)

Based on this, ***system reliability*** can be defined as "the reliability of a structural system composed of a number of components or the reliability of a single component which has several failure modes of nearly equal importance. " (JCSS 2001) Therefore if the failure probability of for example both shear and bending moment are close to each other and in the range of the allowed value, it may be necessary to consider system reliability.

Another case when system reliability is a relevant concept is in the case of statically indeterminate systems. In these structures usually only combinations of failing elements lead to failure of the system. (Schneider 1997) However, in practice it is allowed to assess only component reliability: "when it comes to analysing the probabilities of failure of statically indeterminate structural systems, it is appropriate to consider the element with the largest failure probability as the one dominating the problem."

In practice the main focus is on components, according to the JCSS (2001). "Probabilistic structural design is primarily concerned with component behaviour.( …) The requirements for the reliability of the components of a system should depend upon the system characteristics."

Attention should be paid to whether limits or targets are related to individual failure modes or the failure modes of a system. In the Eurocodes, reliability targets are related to components and the issue of system reliability is treated in robustness requirements. However, the JCSS (2001) suggests to carry out a probabilistic system analysis to establish redundancy (i.e. alternative load-carrying paths) and the state and complexity of the structure (multiple failure modes).

#### Implication

It is not completely clear what are the situations in probabilistic analysis when the target reliability may be considered as a requirement for a given failure mode, and when for a system.

There is no exact requirement for when probability of multiple failure modes in one cross section should be considered (one section as a system). There is also no exact requirement for when probability of failure in various cross sections should be considered "together" (i.e. as system failure).

The current thesis work focuses on component reliability, as in daily engineering practice the "cross-section checks" are typically done according to Eurocodes. Requirements are also given on the component level, therefore comparison will be possible.

### 2.2.5 Target reliability

As introduced in *Section2.1* , probabilistic design and assessment are concerned with probabilities of failure. The main requirement that a structure or element must comply with is therefore the target reliability or target failure probability. The reliability of an element $\beta_{el}$ should be higher than the target reliability and its failure probability $P_{f\,el}$ should be lower than the target failure probability.

$$\beta_{el} > \beta_{target}$$

$$P_{f\ el} < P_{f\ target}$$

But what are the target values and how are they defined? The concept of *risk based decision making* has already been introduced in *Section 2.1*. The question is therefore: *How safe is safe enough?*

#### What is "safe enough"?

Different requirements are valid for ultimate limit states and serviceability limit states. (Diamantidis et al. 2012)

Several criterions can be used to give indications of acceptable risk. Two of these which are typically applied in civil engineering (regulations) are the *individual risk criterion* and the *risk-consequence criterion*. The first refers to the acceptable probability of a fatal accident for one person. The second takes into account a "psychological" effect that a high number of casualties from a single accident is found less acceptable in society than the same amount of casualties caused by multiple "smaller" accidents.

*Individual risk criterion* is based on statistics of average safety. Average death rates per year from accidents are in the range of $10^{-4} – 5*10^{-4}$. Based on global statistics, the average death rate per year from structural failure is in the range of $10^{-6} – 10^{-7}$, which is a rough estimation. (JCSS 2000)

The acceptable, involuntary, individual death risk from structural failure $P_p$ is set as:

$$\mathrm{P_p} \leq \frac{\mathrm{B}}{\mathrm{P}_{(D|F)}}$$

Where B is a constant set at $10^{-6}$ (for the public)

$\mathrm{P}_{(D|F)}$ is the probability of a person being in or around the structure in case of collapse.

In codes applicable in the Netherlands for example, NEN 8700, the lowest bound is based on a maximum risk to human life of $4x10^{-4}$/year.

*Risk consequence criterion* considers the dependence of acceptable risk on the number of people affected. It is expressed as:

$$\mathrm{P_s} \leq \mathrm{A} \times \mathrm{N}^{-\mathrm{k}}$$

Where A is a constant set at $10^{-6}$ for structural failure

k represents the risk attitude (averse / neutral / prone)

The criterion is visualised in *Figure 4.*

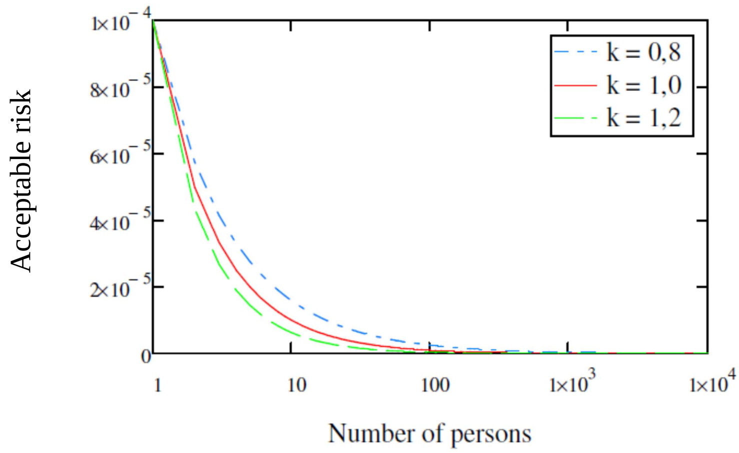


**Figure 4- Risk criterion for various risk attitudes**

According to the (JCSS 2000) „both limitations (…) are based on scarce observations with partially unknown or poorly defined reference populations. Both risk measures if used to set acceptability limits take account of all failure causes including non-structural causes and human error. They can serve at most as an orientation but not as a means to set up acceptability limits or targets for structures".

## Time dependence

It is necessary to define various measures of time in order to be able to discuss the concept of reliability. Failure probability is time-variant when the vector of basic variables is time-dependent. (Holický et al. 2005)

As an elementary example, we can think of throwing with a dice: with one roll the probability to get a 6 (1/6) is much lower than if we can roll three times. If we imagine that the value of the dice throw represents the load *S* and our resistance is for example 5.5, failure (6>5.5) will occur much more likely for three than for one roll.

Time-related concepts which are relevant for the current thesis work are: the *reference period*, the *design working life* and the *remaining working life*.

***Reference period*** is defined in the Eurocode (European Committee for Standardisation 2002) as "a chosen period of time that is used as a basis for assessing statistically variable actions, and possibly for accidental actions".

In practice this means that when speaking of reliability, the reference period is a relevant measure. According to Faber (2009) "in reliability analysis the main concern is to evaluate the probability of failure corresponding to a reference period." It doesn't make sense to speak of failure probabilities without attaching a certain time period.

When $P_d$ is the failure probability for $T_d$, the probability of failure for a reference period $T_n$=n*$T_d$ can be given as:

$$P_n = 1 - (1 - P_d)^n$$

For small failure probabilities this can be reduced to:

$$P_n = \frac{P_d \times T_n}{T_d}$$

As an example, a reference period of 1 year is taken, reliability indices and failure probabilities are plotted for *n* years. These are not related to targets elaborated in *Section 3.3.2*. The following graphs represent the time-dependence and are based on an example from Holický et al. (2005).

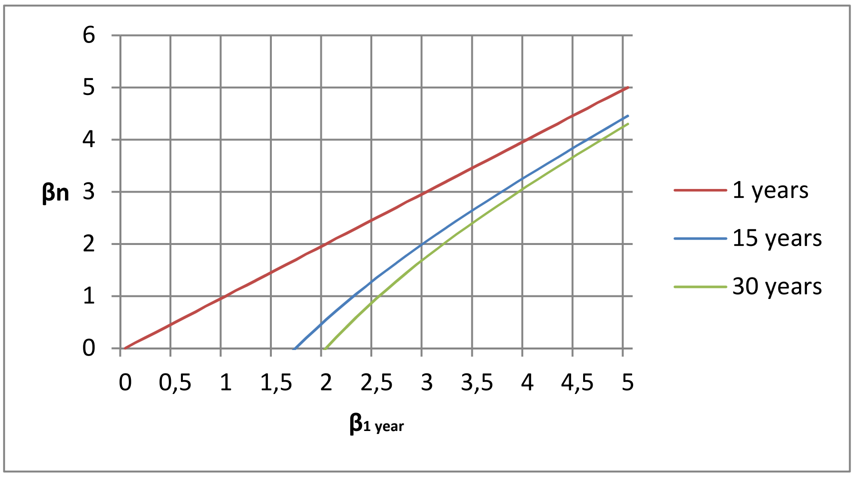


**Figure 5 - Reliability index for n years as function of reliability index of 1 year**

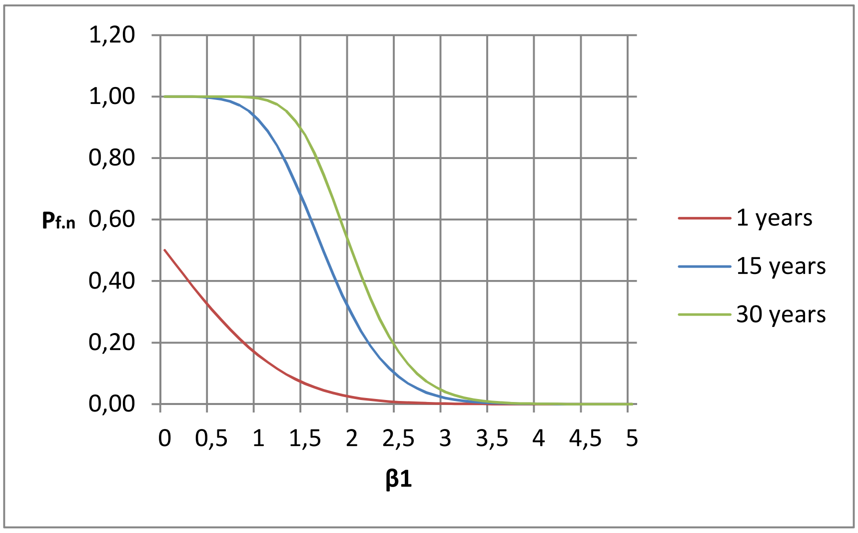


**Figure 6 - Failure probability for n years as function of reliability index of 1 year**

***Design working life*** is defined by Diamantidis et al. (2012) as "duration of the period during which a structure or a structural element, when designed, is assumed to perform for its intended purpose with expected maintenance but without major repair being necessary."

In the assessment of existing structures, which is the focus of the current thesis work, the ***remaining working life*** is of more concern. This can be defined as "the period for which an existing structure is intended/expected to operate with planned maintenance." (Diamantidis et al. 2012)

An example of various criteria is shown in *Table 3.* For application in codes as required target reliabilities, refer to *Section 3.3.2*.

**Table 3 - Example of various specified reliability / year. Based on (Diamantidis et al., 2012)**

| | Building | | Agricultural |
|---|---|---|---|
| | Specified lethal acc./ year | Specified Beta for 1 year | Specified Beta for 1 year |
| Fail / year | 1,00E-06 | 1,3E-06 | 1,335E-05 |
| Beta for 1 year | 4,75 | 4,7 | 4,2 |
| nr of years | 50 | 50 | 25 |
| Fail / n year | 5,00E-05 | 6,50E-05 | 3,34E-04 |
| Beta for n years | 3,89 | 3,83 | 3,40 |

*Implications*

- Consequence is taken into account in codes by consequence class, the appropriate class should be used when defining the target failure probability.
- When designing for target failure probabilities, the reference period for the prescribed Pf in the norms should be "matched" with the time periods of the resistance and load variables. Or, the obtained failure probability should be adjusted with the appropriate formula.

### *2.2.6 Other considerations*

"Every statement about the safety of an existing structure is person dependent and reflects the state of knowledge of the person that makes the statement. This is confirmed by the fact that expert opinions often differ considerably. However, as a rule in the course of discussions the views held by the experts tend to converge and experts can, eventually, even reach a full agreement. Experience shows that though views are subjective in a sense, there is rationalism in the final decision." (JCSS 2000)

Actual probabilities of failure are essentially governed by human error. Failure due to human error and unforeseeable random causes (dependent on quality assurance) is estimated to be in the order of 10. (JCSS 2000), also elaborated in (Faber 2009). When talking about probabilistic assessment, a structural engineering thesis is limited to structural matters, just one domain of the global issue of structural safety.

## 2.3 Methods of reliability analysis

Reliability assessment methods and described in several books and publications, such as CUR-publicatie 190 (1997); Faber (2009); Vrijling & Gelder (2002).

### *2.3.1 Overview*

An overview of reliability methods is given in *Table 4.*

**Table 4- Summary of probabilistic methods**

| | Method | | | |
|---|---|---|---|---|
| | **Level III - Numerical** | **Level III - Simulation** | **Level II** | **Level I** |
| **Method** | Fully probabilistic | Fully probabilistic | Fully probabilistic with approximations | Semi-probabilistic<br>from designer point of view deterministic |
| **Output** | Pf<br>Beta = $-\Phi$ (Pf) | - Failure frequency -> statistical analysis -> failure probability<br>- Failure frequency ~ failure probability most simple assumption | *Depends on method, see below*<br>Hasofer-Lind reliability index: distance of the origin to the transformed design point in the U space | OK / not OK;<br>Unity check |
| **Failure probability** | Unless speaking of very small failure probabilities, output can be used in extended context *(Schneider,1997)* | - Failure frequency -> statistical analysis as normally distributed variable -> failure probability<br>- Failure frequency ~ failure probability most simple assumption | "Statements about probability of failure are nominal and can only be used for comparison purposes. Such statements should not be used outside the context considered." *(Schneider, 1997)* | Statements about probability of failure not possible *(Schneider, 1997)*<br><br>But: it is linked to a concept of Beta (?) |
| **Application** | - Analytical<br>- Numerical integration | Monte Carlo simulation:<br>values generated from random variables and inserted into the probabilistic model | Linearize reliability in design point<br>Approximates probability distribution by standard normal distribution | Margin between characteristic values of R and S<br>Apply partial safety factors |
| | - | <u>Alternative:</u><br>Importance sampling - take more samples from / near the failure space | **1) FOSM**<br>reliability = derivative of function with respect to certain variable<br>**2) FORM**<br><u>a. Linear Z:</u> original distr. -> normal distr. -> normal space<br>$\beta$ = distance from origin to failure space<br>output: Beta; Alpha - influence coefficient<br><u>b. non-linear Z</u>: approximate by Taylor polynomial -> approx. Mu and sigma -> Beta=Mu/Sigma<br>point of approx. = in design point<br>iterative, optimisation problem!!<br>**3) SORM**<br>as FORM but considers second partial derivative (same curvature in design point) | $\alpha$-values (influence factors) are standardised and are considered independent of an arbitrary specific case |
| **Limitation** | - Problematic for complex limit state equations and / or several variables | - Computationally expensive<br>- No random number generator is "truly" random, thus always some imprecision left | In the example in prob. 2 notes, the Pf is smaller for each level 2 calculation than using the same data in MC!<br>-> in some case the more "complex" method is more conservative OR level II doesn't approximate on the safe side | - No conclusion about probability of failure<br>- Conservative due to generalised 'alpha' values |

### 2.3.2 Monte Carlo Simulation

In simulation methods, a large number of results are simulated through random sampling and the result is observed.

*Basics*

In structural reliability problems, even for a relatively simple model, several basic variables are usually necessary. These variables often have various distribution types, making the application of an analytical method cumbersome. “Stochastic simulation is an alternative approach: values are generated for the random variables and inserted into the model, thus mimicking outcomes for the whole system.” (Dekking et al. 2005)

The ***essence*** of Monte Carlo simulation in structural reliability analysis is that “exact or approximate calculation of probability density and of parameters of an arbitrary limit state function is replaced by statistically analysing a large number of evaluations using random realisations of the underlying distributions.” (Schneider 1997) We simulate several times all values which are necessary to construct the limit state function and then calculate the resistance *R*, load *S*, and *Z* values also several times.

The ***underlying idea*** of Monte Carlo simulation is drawing random numbers from a uniform probability distribution. If the distributions of the (original) input variables of the reliability function are known, values of these variables can be generated by making use of the inverse cumulative distribution function, as visualised in *Figure 7*. In practice, analytical formulas are available for some distribution types, while approximate formulas have been developed for others.

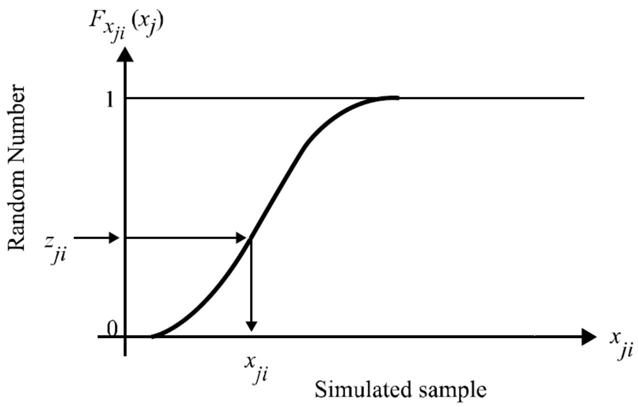


**Figure 7- Principle of simulation of a random variable** (Faber et al. 2007)

The ***outcome*** of a Monte Carlo analysis in this case are values for limit state function (Z(X)), with a statistical distribution. Resistance *R(X)* and load *S(X)* values can also be analysed and visualised. Failure frequency can be interpreted as failure probability, although the two concepts are not exactly equivalent. In some situations statistical analysis of the output is might be preferred, for example the distribution of the resulting *Z* values.

The main ***limitation*** of Monte Carlo analysis is the required number of simulations, which makes the method “computationally expensive”. This aspect is referred to several times in literature. To understand more this limitation, basic calculations for the needed number of simulations are presented in the *Appendix A*. It is concluded that in the range of failure probabilities which are defined by reliability requirements for existing structures (*Section 3.3.2*) the necessary number of simulations does not create a bottleneck for elementary structural reliability analysis. Here elementary refers to  the fact that probabilistic FEM is not applied.

### *Failure probability*

Failure probability can be determined in two ways when using Monte Carlo simulation. A simple frequency analysis can be carried out or the output can be analysed statistically. In the current thesis work frequency analysis of the results is applied.

In a simple *frequency analysis* the failure probability is determined as:

$$P_f \approx \frac{n_0}{n}$$

| | | |
|---|---|---|
| Where: | $n_0$ | Number of "failures" (Z<0) from the sample |
| | n | Number of realisations (i.e. total number of simulated Z values) |

More information about the spread of data is given by the approximate value of the variation coefficient $v_{Pf}$ as:

$$v_{Pf} \approx \frac{1}{\sqrt{n \times P_f}}$$

The relative error of the value has a mean of zero and is normally distributed. Consideration of maximum error with a required confidence level can be made. (Refer to methods described in *Section5.3*).

As mentioned previously, in some situations the *statistical analysis of output* may provide results of interest. As a first step, the mean value $\mu_Z$ and standard deviation $\sigma_Z$ of the limit state results *Z(X)* can be determined. From these, the reliability and failure probability are:

$$\beta \approx \frac{\mu_Z}{Z} \; ; P_f \approx \emptyset(u = -\beta)$$

The relations above assume that realisations are normally distributed, which is not always the case. More sophisticated statistical analysis can be performed. Further theoretical background is given in *Section5.3.*

### *Importance sampling*

The underlying idea of importance sampling is to take more samples from the failure space. The failure space is „increased" relative to the total space with the purpose to reduce the large number of simulations necessary (especially small failure probabilities). Importance sampling reduces the necessary simulations by a range of $10^2$ (CUR-publicatie 190 1997). As mentioned in the sub-section *'Basics'*, in the current case the reduction of the number of simulations was not necessary therefore the application of importance sampling was not considered.

### *Practicalities on random-number generation*

- The inverse CDF of commonly used distribution functions is available in (CUR-publicatie 190 1997) (5-14.) and are also given in *Appendix B*
- Programs (such as Excel, MathCad, MatLAB) have built-in inverse functions. Excel has more limitations in this aspect than MatLab.
- If no analytical form is available, the solution is to generate the original PDF function and use 'find' commands together with interpolation.

## 2.4 Reliability assessment of existing structures

### 2.4.1 Assessment versus design

There are several differences between designing new structures and assessing existing ones, both considering technical and economic aspects, for example a shorter remaining life, more expensive changes to the structural properties or aging of the material. (Diamantidis et al. 2012) When a structure is designed, the designer has influence on the overall strength of the structure. Thus it is not very "complicated" or un-economic to use more reinforcement, for example. In assessment, if the simplest models were used and all norms for new structures considered, several structures would not comply with them. Thus the engineer has to think "in depth" about the analysis.

### 2.4.2 Process of assessing structural reliability

The steps of safety assessment according to Schneider (1997) are:

1) Dimensioning the structure according to existing regulations. Then assessing this hypothetical structure with respect to $\beta_0$.
2) Calculating $\beta_0$ with obtained dimensions, using parameters in codes, such as Eurocodes or the Probabilistic Model Code (PMC) of the JCSS.
3) Determine $\beta$ using the actual dimensions of the structure in consideration, with up-to-date models and updated parameters.

If the structure doesn't comply with requirements, it is advisable to investigate certain variables further. FORM analysis can be used for example to gain insight to α-values and thus the relevance of the different variables. A flow-chart representing the procedure of assessing structures is given in *Figure 8.*

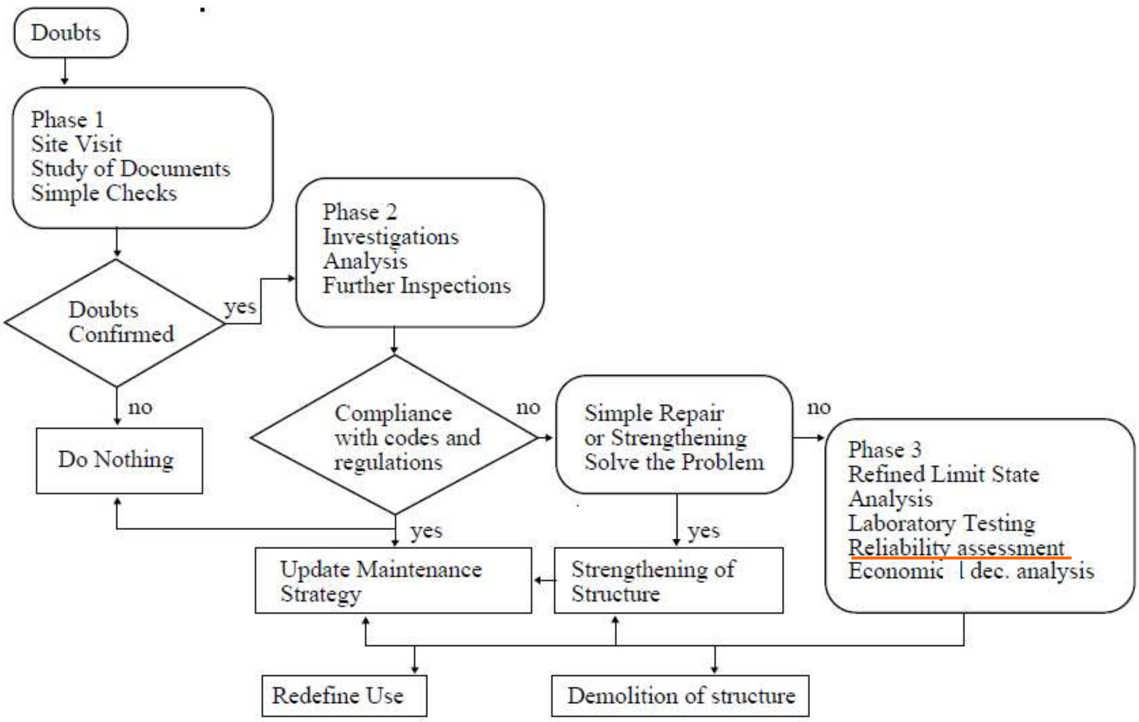


**Figure 8 - General adaptive approach for the assessment of structures** (Faber 2009)

*Similar flowchart in* Schneider (1997)*, added to PMC* (JCSS 2000)

The assessment procedure used in practice at Ingenieursbureau Gemeente Rotterdam (IGR) is described in *Section 4.2* of this study.

The Probabilistic Model Code (JCSS 2001)gives the following steps for component reliability assessment:

1) Select appropriate limit state function
2) Specify appropriate time reference
3) Identify basic variables and develop appropriate probabilistic models
4) Compute reliability index and failure probability
5) Perform sensitivity studies

# 3 Codes for structural safety and existing structures

## 3.1 Codes and their relations

In the current thesis work it is attempted to comply with all regulations (applicable building codes) in order to ensure that the results are directly applicable. The relevant building codes and other applicable norms have been overviewed in order to make sure that the applied and / or suggested methods of analysis fit into the philosophy of these norms (structural reliability according to international and European standards). Furthermore Eurocode load- and resistance models are studied, as these will serve as comparison for the suggested alternative probabilistic load model.

Codes and their relations are described in detail by Diamantidis et al. (2012). The main relevant standards are the ISO 13822 – *Basis for design of structures – Assessment of existing structures*, the Eurocodes with the Dutch National Annexes and the two additional codes within the Netherlands NEN 8700 and 8701. For description of the first two and their relations, refer to *Chapter 2* of Diamantidis et al. (2012). 'Annex C' of EN 1990 deals with 'Basis for partial factor design and reliability analysis'. (European Committee for Standardisation 2002)

## 3.2 Netherlands Normalisation – NEN Codes

Two documents treating existing structures have been published in recent years in the Netherlands. The **NEN 8700** is concerned with general principles of assessing existing structures, **NEN 8701** describes matters concerning loading. Both documents are meant to be used with the respective Eurocodes. This implies that on their "own" they are not useable and also that they are harmonised with the related Eurocode.

At the moment, besides NEN 8700 and 8701, there is no other regulation for existing structures in the Netherlands. NEN 8702 will concern concrete structures and is planned to be adapted in 2014. There are also Swiss, German, British and ISO regulations available. *(presentation of ir. Dieteren at NEN course, Dec. 2012.)*

For highway bridges, a specific guideline, the *Guidelines for assessment of existing structures (Richtlijn Beoordeling Kunstwerken – RBK)*(Rijkswaterstaat Technisch Document 2013)has been introduced. This document is specific for structures owned by the Ministry of Transport and Infrastructure (Rijkswaterstaat, RWS) and concerns main roads. Main roads have heavier loading than what can be expected in a city. Until NEN 8702 is introduced, these guidelines can serve as a possible alternative.

## 3.3 Applicability of probabilistic analysis

### *3.3.1 Methodology related specifications*

**JCSS PMC:** "The reliability method used should be capable of producing a sensitivity analysis including importance factors for uncertain parameters. The choice of the method should be in general justified. The justification can be for example based by another relevant computation method or by reference to appropriate literature. "(JCSS 2001)

Further requirements are given on accuracy as: "due to the computational complexity a method giving an approximation to the exact result is generally applied". The fundamental accuracy requirement is a maximum 5% overestimation of reliability with respect to the target level.

**ISO 2394** (Technical Commtitee ISO/TC 98 1998) lists acceptable methods for determining target failure probabilities. These are exact analytical methods, numerical integration, approximate analytical (FORM, SORM), simulation methods or a combination of these. Each of these is briefly described in *Section 16.*

**Eurocode:** 1.4 (5)It is permissible to use alternative design rules different from the Application Rules given in EN 1990 for works, provided that it is shown that the alternative rules accord with the relevant Principles and are at least equivalent with regard to the structural safety, serviceability and durability which would be expected when using the Eurocodes. (European Committee for Standardisation 2002)

### *3.3.2 Target reliabilities*

Ultimate limit states and serviceability limit states have different reliability requirements. In the following, ultimate limit states are considered.

#### *Reliability differentiation*

Target reliabilities are differentiated based on two aspects: consequences of failure and the relative cost of safety measures. (Diamantidis et al. 2012) The target reliability should be adjusted to the design life or remaining life, as the reference period defined in requirements of codes might not coincide with this. The adjustment should be done as described in *Section 2.2.5.*

#### *Required safety*

Each of the norms mentioned previously give values for target reliabilities. These slightly differ, the reason for this is the gradual development where the most basic regulation / code is the ISO 2394, while the most recent and locally applicable regulations are given in NEN 8700. The latter shall be the one used in practice in the current thesis work. Nevertheless a summary of reliability requirements in the different norms is given here.

**ISO 2394**
Target values for the reliability index are provided in ISO 2349, for a design working life. Differentiation for cost of safety measures and failure consequences is done.

**Table 5 - Target reliaility index βd for design working life Td, ISO 2394 (Diamantidis et al., 2012)**

| Relative costs of safety | Consequences of failure | | | |
|---|---|---|---|---|
| measures | small | some | Moderate | great |
| High | 0 | 1,5 | 2,3 | 3,1 |
| Moderate | 1,3 | 2,3 | 3,1 | 3,8 |
| Low | 2,3 | 3,1 | 3,8 | 4,3 |

**Probabilistic Model Code**
The probabilistic model code gives target failure probabilities for one year reference period and differentiates for costs and consequences, similarly to ISO 2394. Whether a consequence is determined as small or large is defined by the ratio of the total costs and construction costs.

**Table 6 - Target reliability indices β for 1 year reference period (ULS) PMC (JCSS, 2001)**

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| Relative cost of safety measure | Minor consequences of failure | Moderate consequences of failure | Large consequences of failure |
| Large (A) | $\beta$=3.1 ($p_F \approx 10^{-3}$) | $\beta$=3.3 ($p_F \approx 5\ 10^{-4}$) | $\beta$=3.7 ($p_F \approx 10^{-4}$) |
| Normal (B) | $\beta$=3.7 ($p_F \approx 10^{-4}$) | $\beta$=4.2 ($p_F \approx 10^{-5}$) | $\beta$=4.4 ($p_F \approx 5\ 10^{-6}$) |
| Small (C) | $\beta$=4.2 ($p_F \approx 10^{-5}$) | $\beta$=4.4 ($p_F \approx 5\ 10^{-6}$) | $\beta$=4.7 ($p_F \approx 10^{-6}$) |

**Eurocode**

The values in Eurocode reflect possible failure consequences by adapting the consequence class. They are given for the reference period of 1 and 50 years. The values are valid for component failures.

**Table 7 - Recommended min. values for reliability index β (ULS) EN 1990**
(European Committee for Standardisation, 2005)

| Reliability Class | Minimum values for $\beta$ | |
|---|---|---|
| | 1 year reference period | 50 years reference period |
| RC3 | 5,2 | 4,3 |
| RC2 | 4,7 | 3,8 |
| RC1 | 4,2 | 3,3 |

**NEN 8700**

A specific aspect of the norm is the different reliability requirements for existing structures. As additional safety measures for existing structures are usually more expensive than those for structures in the design phase, there is a relaxation in the safety requirement for such structures. For this purpose, the norm differentiates between levels:

- Rejection ('afkeur'): if an existing structure doesn't comply with the required reliability index, it should be rejected, in practice meaning refurbished / renovated.
- Reconstruction ('verbouw'): the level to which an existing structure should be renovated
  For structures being refurbished, a further differentiation in target reliability for individual elements is made:
  - Use ('gebruik'): concerns the newly built or strengthened element
  - Reconstruction ('verbouw'): concerns all parts of the structure which are not reconstructed.

The second specific aspect of NEN 8700 is that in case wind load is the dominant load, different reliability requirements are set. The reason for this is on one hand the high cost of safety measures for resistance to wind loading (for example concrete cores in high-rise buildings), on the other hand the high variance of wind loading which further increases the costs to reduce the failure probability.

The reference period is minimum 15 years.  Similarly to Eurocode, consequence classes are taken into account. In the following tables, the required reliability indices are given for two different levels. The reliability levels which are of interest for most of the city bridges, thus structures belonging to consequence class 2, are indicated with the orange circles.

**Table 8 - Minimum reliability indices for reconstruction level (Normcommissie 351001, 2011a)**

| Minimumwaarden betrouwbaarheidsindices bij verbouw | | | |
|---|---|---|---|
| **Gevolgklasse** | **Minimum-referentieperiode** | ***β*** | |
| | | wn | wd |
| CC3 | 15 jaar [b] | 3,8 (3,6) | 3,3 [a] (2,6) |
| CC2 | 15 jaar [b] | 3,3 (3,1) | 2,5 [a] |
| CC1 | 15 jaar | 2,8 | 1,8 |

wn: wind niet dominant

wd: wind dominant

[a] Hierbij is de ondergrens voor persoonlijke veiligheid maatgevend.

[b] In het algemeen wordt aanbevolen een restlevensduur en derhalve een referentieperiode van 30 jaar.

De waarden tussen haakjes mogen alleen zijn toegepast bij bouwwerken waarvoor een omgevingsvergunning voor het bouwen is verleend onder het Bouwbesluit 2003 of daarvoor.

**Table 9 - Minimum reliability indices for rejection level** (Normcommissie 351001 2011a)

| Minimumwaarden betrouwbaarheidsindices bij afkeuren | | | |
|---|---|---|---|
| **Gevolgklasse** | **Minimum-referentieperiode** | ***β*** | |
| | | wn | wd |
| CC3 | 15 jaar | 3,3 [a] | 3,3 [a] |
| CC2 | 15 jaar | 2,5 [a] | 2,5 [a] |
| CC1b [b] | 15 jaar | 1,8 | 1,1 [a] |
| CC1a [b] | 1 jaar | 1,8 | 0,8 |

### *Implication*

When determining target reliabilities all norms take into account in some way consequences as well as the costs of increasing safety. However, the requirement levels are different in each relevant norm. Taking into account the moderate / normal cost target reliabilities from codes and calculating according to the method described in *Section 2.2.5*, target reliabilities from various codes are determined and shown in *Table 10*.

The current work focuses are bridges in cities, which are usually classified in Rotterdam as consequence class 2. Due to being in the context of the Netherlands, the reliability requirements from NEN 8700 are considered. The rejection level is of interest, thus in practice this means a required target reliability index of **β = 2.5** for a **15-year reference period**.

**Table 10 - Target reliabilities in norms for various consequence classes**

| 1 year | | | | | |
|---|---|---|---|---|---|
| **Consequence** | | small | low<br>some<br>minor | normal<br>moderate<br>moderate | high<br>great<br>large |
| EN | β | | 4,2 | 4,7 | 5,2 |
| | $P_f$ | | *1,33E-05* | *1,30E-06* | *9,96E-08* |
| ISO* | β | 2,9 | 3,5 | 4,1 | 4,7 |
| | $P_f$ | *1,87E-03* | *2,33E-04* | *2,07E-05* | *1,30E-06* |
| JCSS* | β | | 3,7 | 4,2 | 4,4 |
| | $P_f$ | | *1,08E-04* | *1,33E-05* | *5,41E-06* |
| **15 years** | | | | | |
| **Consequence** | | small | low<br>some<br>minor | normal<br>moderate<br>moderate | high<br>great<br>large |
| EN | β | | 3,54 | 4,11 | 4,67 |
| | $P_f$ | | *2,00E-04* | *1,95E-05* | *1,49E-06* |
| ISO | β | 1,92 | 2,70 | 3,42 | 4,11 |
| | $P_f$ | *2,76E-02* | *3,48E-03* | *3,10E-04* | *1,95E-05* |
| JCSS | β | | 2,94 | 3,54 | 3,77 |
| | $P_f$ | | *1,62E-03* | *2,00E-04* | *8,12E-05* |
| NEN<br>Verbouw, not wind dom. | β | | 2,8 | 3,3 | 3,8 |
| | $P_f$ | | *2,56E-03* | *4,83E-04* | *7,23E-05* |

## 3.4 Semi-probabilistic methods

### *3.4.1 Safety factors: relation of level I – II calculations*

*Safety factors* [7]

It is claimed that in the most recent guidelines a link has been sought between safety factors for load and strength parameters in codes and probabilistic design methods. The following formulas describe the relations between design values, distribution parameters $\mu$ and $\sigma$ of a normal distribution, sensitivity factors $\alpha$, required reliability $\beta$ and finally the partial factors $\gamma_R$ and $\gamma_S$.

$$R^* > S^* \qquad \text{where:} \qquad X^* = \mu_X + \alpha_X \beta \sigma_X$$

$$\frac{R_k}{\gamma_R} > \gamma_S S_k \qquad\qquad X_k = \mu_X + k\sigma_X$$

Thus the partial factors are determined by:

$$\gamma_R = \frac{R_k}{R^*} = \frac{1 + kV_R}{1 + \alpha_R \beta V_R}$$

$$\gamma_S = \frac{S^*}{S_k} = \frac{1 + \alpha_S \beta V_S}{1 + kV_S}$$

The influence coefficient α plays a role in determining the partial safety factor. Thus the spread of strength and loads influence the partial factor as well. As it is not an exact number, in behold of certain information, it may happen that a lower γ can be the outcome for target reliability.

[7] Based on CUR-publicatie 190 (1997) and *lecture of prof. Vrouwenvelder (2013, TU Delft)*

However in practice, two complications arise. Firstly, due to the dependence on too many random variables it is not practically feasible to calculate partial safety factors for each of these. Therefore variables are “bundled” and one safety factor is calculated for all of them._ Secondly, the safety factor is dependent on the reliability function and is therefore different for every case. Factors in regulations are thus calibrated as averages of a large number of reference cases

In code calibration procedures the method of determining the safety factors can be:

1. Large number of level II calculations are carried out for reference cases, as described by Vrouwenvelder & Siemes (1987) This is the method applied in old codes of the Netherlands (TGB).
2. Based on standardisation of $\alpha_x$ sensitivity factors, determining design point values based on probabilistic calculations.

„According to the Eurocode, the core of the level I design method is that the α-values are standardised and that they are considered independent of an arbitrary specific case.” (CUR-publicatie 190, 1997)

Within the interval $0.16 < \alpha_E/\alpha_R < 7.6$, the factors are $\alpha_E = -0.7$ and $\alpha_E = 0.8$. (European Committee for Standardisation 2002) These values are ”on the safe side”, as sum of the influence factors should be 1 (*here 1,13*). (Diamantidis et al. 2012)

For non-dominant loads the factor can be reduced according to a given formula. (Diamantidis et al. 2012; CUR-publicatie 190 1997) Influence coefficients to take into account are different for highly dominant load / resistance. Summary can be seen below.

**Table 11 - Standardised α values**

(Steenbergen, Morales-Nápoles, & Vrouwenvelder, 2012)

| | (1) | (2) | (3) |
|---|---|---|---|
| | volgens ISO2394 en NEN-EN 1990 | als de sterkte veruit dominant is | als de belasting veruit dominant is |
| $X_I$ | $\alpha_I$ | $\alpha_i$ | $\alpha_I$ |
| dominante sterkteparameter | 0.8 | 1.0 | 0.4 |
| overige sterkteparameter | 0.4 * 0.8 = 0.32 | 0.4 | 0.4 |
| dominante belastingparameter | - 0.7 | -0.4 | -1.0 |
| overige belastingparameters | - 0.4 * 0.7 = - 0.28 | -0.4 | -0.4 |

The partial safety factors cover various uncertainties, some of which have already been mentioned in *Section 2*. *(based on lecture of prof. Vrouwenvelder Level I. methods, 2013)* The partial factor of the load, $\gamma_S$, takes into account 1) deviations from the characteristic value; 2) uncertainties in the calculation model; 3) scatter in dimensioning; 4) difference between test results and the constructed object. The partial factor of the resistance, $\gamma_R$, takes into account deviations from the characteristic value and uncertainties in the calculation model.

### Changes to values in code

It is briefly investigated what applies for reliability requirements given by Eurocodes.

The highest level of requirements, a rule determines that:
“Partial factors (including those for model uncertainties) comparable to those used in EN 1991 to EN 1999 should be used”

The following further specifications are given:

- Annex B – B6 (1): A partial factor for a material or product property or a member resistance may be reduced, if an inspection class higher than that required according to Table B5 and/or more severe requirements are used.
- Annex C – C3 (2): In principle numerical values for partial factors and $\psi$ factors can be determined in either of two ways:

  a) On the basis of calibration to a long experience of building tradition.

  b) On the basis of statistical evaluation of experimental data and field observations. (This should be carried out within the framework of a probabilistic reliability theory.)

### *3.4.2 Specifics for structural re-analysis*

As mentioned in *Section 3.3.2*, in the Netherlands target reliabilities for existing bridges may be lower than for new ones. This results in lower partial factors allowed for both load- and resistance variables.

Furthermore, according to norms (Eurocode, NEN 8700 and 8701) the traffic loading for existing structures can be lowered by taking into account the real loading situation. This is done through the adjustment factors which are described in detail in *Section 6.3.5.*

# 4 Bridges in Rotterdam

## 4.1 Bridge inventory

In the area of Rotterdam, there are 325 traffic bridges (non-highway) according to inventories available. Based on inventory data analysis of age, material (deck) and partially, length of bridges in Rotterdam was analysed. Age- and material data can be found in the *Introduction.*

For approximately 50% of existing bridges the span was available in a form easy to handle. The distribution of the spans of these 192 bridges is shown in the *Figure 9*.

| Length [m] | Nr | % |
|---|---|---|
| 0 - 4,99 | 57 | 29,7% |
| 5 - 9,99 | 67 | 34,9% |
| 10 - 14,99 | 50 | 26,0% |
| 15 - 19,99 | 4 | 2,1% |
| 20 - 29,99 | 9 | 4,7% |
| 30 - | 5 | 2,6% |
| Total | 192 | |

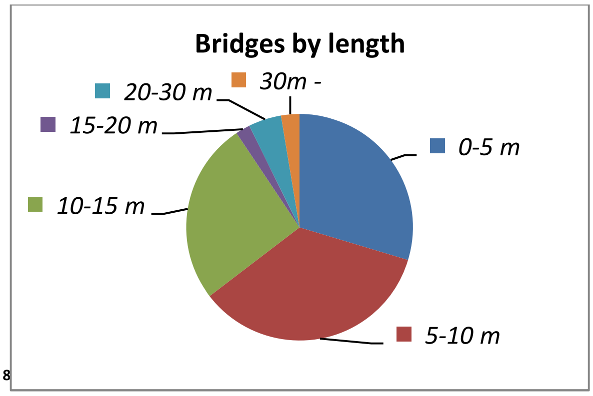


**Figure 9- Length of bridges in Rotterdam**[9]

It can be concluded that the majority of the bridges has a span below 15 meters. The scope of the current thesis work is bridges with spans up to 20 meters.

## 4.2 Methodology of analysing existing structures

### *4.2.1 When is a bridge re-calculated?*

As described in the Probabilistic Model Code (JCSS, 2000), it is the responsibility of the owner of the given infrastructure to initiate investigation / assessment of the object. In the current case, the bridge administrator *('beheerder')* takes this role.

### *4.2.2 Methodology in Gemeente Rotterdam*

#### Concept

Load bearing capacity and remaining lifetime are determined in cycles, which forms a part of planned maintenance. (Laarse 2012) A pilot project of analysis was carried out in 2008 with the aim to create cooperation between the bridge management and engineering departments and thus couple technical analysis and maintenance. The project started with three bridges and the long-term plan is to check the whole bridge park.

Later a similar project was developed for the municipality of Schiedam, which outsources the checks of existing civil infrastructure. In order to create tenders, IGR was asked to describe a process and a description of determining load bearing capacity and remaining lifetime was created.

[9] Based on inventory 'Lijst LdR Prioriteit TCV Verkeersbruggen DG Areaal' (20.11.2013.), partial inventory of all bridges

### *Loading*

Besides determining the adequacy of a structure for the remaining lifetime the aim is often to define the loads which can be withstood. Initially all objects are checked for rejection level using the maximum load. If the structure is not adequate, the maximum load bearing capacity is defined applying appropriate load reductions. *(Laarse, 2012)*

### *Required reliability*

For structures younger than 15 years the reliability requirements for new structures are taken into account. If the structure proves inadequate, the check is performed taking into account the real use situation (i.e. load reduction factors). Constructions older than 15 years are evaluated according to the rejection level requirements of NEN 8700.

### *Process*

The methodology applied at IGR is described in a flow chart, visible in *Figure 10.* (Laarse 2012)

A so called 'Risk analysis document' is prepared summarizing the first two steps. The report is sent to the bridge administrator, who decides what to do.

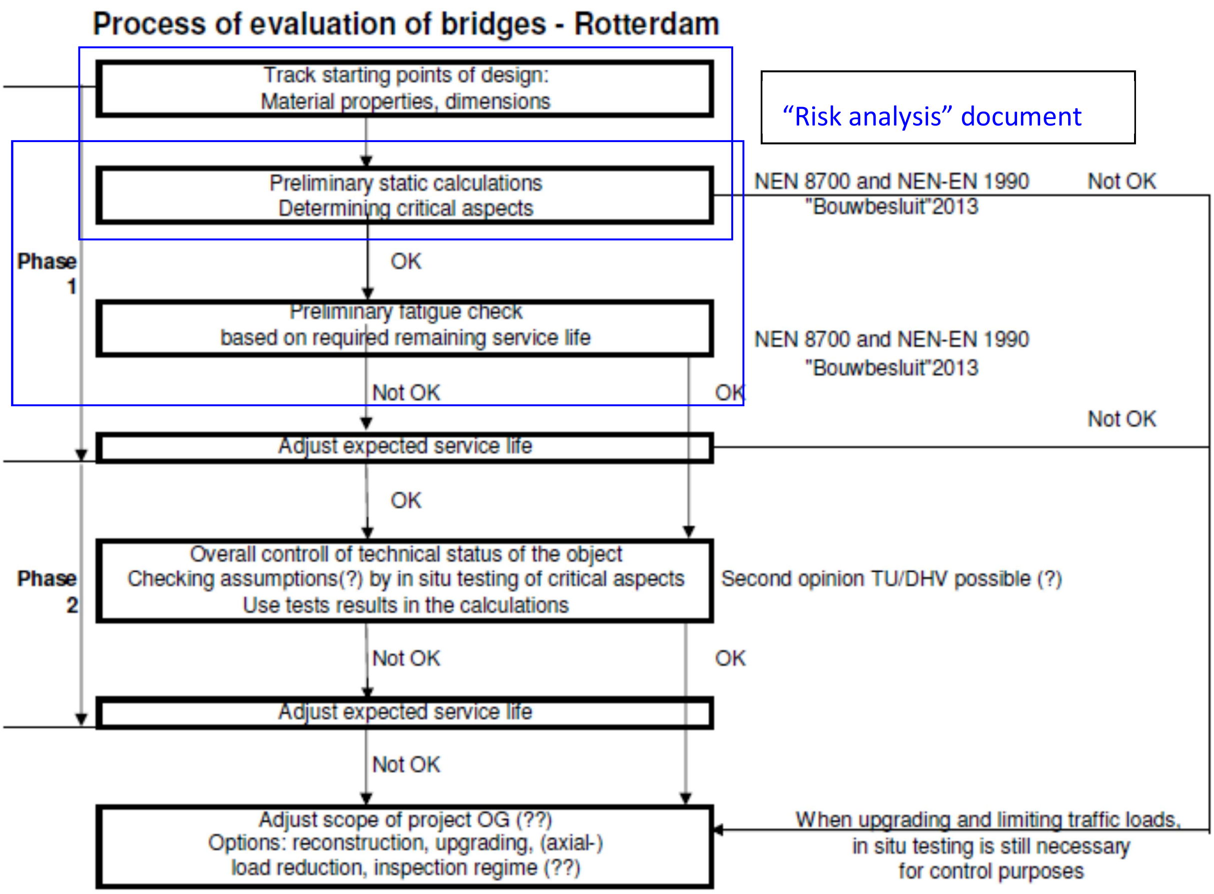


**Figure 10 - Process of analysing load bearing capacity** (Laarse 2012)

In the guideline created specifically for structural engineers (2nd and 3rd step in the flowchart), the following process of analysis is recommended:

1) Determining the load distribution
    a. Modelling
    b. Determining loads
    c. Determining dominant internal forces

2) Determining load bearing capacity of cross section
    a. With or without compressive reinforcement
    b. Concrete quality minimal (C30/37) or strength as determined by testing (max. C50/60)
    c. Shear capacity according to *Richtlijnen Beoordeling Kunstwerken (RBK)*
3) Checking
    a. If unity check proves non-sufficiency, check whether load-redistribution is possible.
4) Options if inadequate load bearing from initial check
   Re-calculate with
    a. Adapted Poisson – coefficient
    b. Orthotropic plate
    c. Truss-model
    d. Alternative load-path (TU Delft)

The engineers advise the bridge administrator regarding steps to take. In case of non-sufficiency the available options typically are to calculate further with more advanced methods or to impose load limitations.

### *4.2.3 Loading*

Load models NEN-EN1991+NB are applied to bridges, with the application of correction factors *(described in Section 3.4.2)*. An examples of is shown in *Section 4.3*.

These load models were calibrated to traffic loading on highways (bridges in A16, A15 and A12 highways). The frequency of heavy loads on bridges in the city areas is obviously lower. However, the maxima of loads is a complex matter and the simple fact of lower frequency does not directly justify the lowering of maximum loads.

The load models of Eurocodes are described in *Section 6.3.*

#### *Abnormal transport*

Loadings on bridges are used according to the codes. Exceptional transport exceeds size and / or weight limits prescribed by authorities. This is described in *Section6.3.3* . When a transporter wishes to use exceptional transport, a request for permission should be submitted to the authorities (RDW). This request in some cases should contain a route plan. In the case of Gemeente Rotterdam, if a request for the passing of exceptional transport is received, IGR is consulted for advice to support the decision of letting the vehicle pass in the city, on a certain route. A database is constantly being developed and upgraded in IGR with regard to the load bearing capacity of bridges for exceptional vehicles (various axle load combinations).

The decision to grant permit is under the pressure of safety on the one hand and economic profitability on the other hand. Though the decision is not directly in the hand of the engineer but of the owner / authority, there can be a pressure for getting outcomes supporting commercial transport. The ethical issues related to safety, which may appear also in this case, are out of the scope of this thesis. However this situation gives rise to certain implications. Firstly, the traffic loading data to be considered in calculations may not contain data of exceptional transport but later the bridge in consideration may be required to sustain such loads. The economic advantage of not upgrading a bridge may therefore be lower. Secondly, some bias can be expected towards the “unsafe” side in the cases of exceptional transport. At this point it is not known whether the quality assurance / checking system excludes such a bias to a reasonable extent thus in discussion about model uncertainty and human error this matter could be investigated.

## 4.3 Example of analysing existing structures

Examples of calculating existing structures were checked in order to understand the process as well as to get an indication about possible "typical" failure modes. It is to be noted that re-calculations before 2012 were done according to different standards than the currently applicable Eurocodes and NEN 8700 - 8701.

Among the overviewed bridge re-evaluations were the Rederijbrug, a moveable bridge with a steel beam grid deck, originally built in 1948; the Leuvebrug with a similar deck structure built in 1959 and renovated in 1981; and the local control of the concrete deck of the Jutphasebrug.

It is relevant to note the reduction factors to the loading applied in practice in the setting of Rotterdam. The background of these values is described in *Section 6.3.5.*

For the Rederijbrug these were:

- $\Psi_t$ – comes from NEN 6706, value of **0.944** in this case
  For reference period other than 100 years thus 5.6% reduction

- $\alpha_{trend}$ – from TNO report and NEN 8700
  Adjustment factor for traffic trends
  A factor 1.35 counts for uncertainty and traffic trend.
  Uncertainty factor 1.2
  Trend factor 1.35/1.2 = 1.125
  Thus correction factor 1/1.125 = **0.889**
  11% lowering. The used values were 15% for the UDL and 5% for the axle loads.

- $\alpha_q$ – from NEN 6706
  Accounts for less traffic
  For 1st lane: **0.91**

For the Leuvebrug the reduction factors were:

$$\left.\begin{array}{lll} - & \alpha \text{ based on intensity and reference period} & = 0.9 \\ - & \psi \text{ based on reference period} & = 0.98 \\ - & \alpha_{trend} \text{ based on reference period and span} & = 0.93 \end{array}\right\} \quad \mathbf{0.82}$$

Therefore in examples comparing semi-probabilistic and probabilistic calculations (*Chapter 10*) a reduction factor of 0.8 was considered, as what can be typically expected in Rotterdam.

# 5 Statistics Background [10]

## 5.1 Basic concepts

### 5.1.1 Introduction and applied notations

In the current thesis work it is assumed that the reader is familiar with certain basic concepts. These are: *random variables* and their properties, such as the *probability density function* (probability mass function for a discrete random variable) $f_X(x)$, *cumulative distribution function* $F_x(X)$, and *quantile* $\xi_q$. The *mean* or *expected value* $\mu_X$ , the *standard deviation* $\sigma_X$ , the *variance* $Var[X]$ and the *coefficient of variation* $V_X$ or $CoV_X$ are also expected to be known concepts.

These properties and functions shall be denoted in the following sections as given above and as also summarized in *Table 12*.

**Table 12 - Basic concepts in statistics and applied notations**

| | |
|---|---|
| $f_X(x)$ | Probability density function |
| $F_x(X)$ | Cumulative distribution function |
| $\xi_q$ | Quantile |
| $\mu_X$ | Mean / expected value |
| $\sigma_X$ | Standard deviation |
| $Var[X]$ | Variance |
| $CoV_X$ | Coefficient of variation |

### 5.1.2 Uncertainty versus variability

The two terms refer to two different concepts. *Uncertainty* is associated with the lack of knowledge while *variability* corresponds to the spread of data or measurements.

*Model uncertainty*, which is described in *Section 2.2.3* is defined in the standard as "uncertainty related to the accuracy of models, physical or statistical". It is relevant for constructing the correct reliability equations and using model uncertainties related to both resistance- and load models. Note that these uncertainties contain variability – i.e. have a non-zero standard deviation.

*Statistical uncertainty* is described as "uncertainty related to the accuracy of the distribution and estimation of parameters". This concept is relevant when speaking of fitting distributions to data. The parameters of a distribution, for example the mean or standard deviation are random variables themselves. This is elaborated in *Section 5.3.2.*

For further information, ISO 2394 (Technical Commtitee ISO/TC 98 1998) categorizes uncertainties relevant for structural reliability (Appendix E).

### 5.1.3 Recommended literature

Definitions and examples can be found in any book on statistics, for example the *'Modern introduction to probability and statistics'* of Dekking et al. (2005). Some publications are specifically addressed for civil engineers such as the CUR-publicatie 190 (1997) or the *'Applied statistics for civil and environmental engineers'* (Kottegoda & Rosso 2008).

[10] Section based mainly on *Kottegoda & Rosso* (2008), *Dekking et al.* (2005) and *Vrijling & Gelder* (2002)

## 5.2 Applied distributions

### 5.2.1 Distribution types and references

It is shown for example in Diamantidis et al. (2012) that the distribution type of random variables strongly effect the probability of failure. For example, applying 3-parameter lognormal distribution, the same mean value and standard deviation but different skewness (the third parameter) gives failure probabilities in a large range.

In civil engineering the main interest is usually in small probabilities, thus in the so called *tail* of the distributions. Failure is expected to occur when the strength is relatively low (i.e. in the “left tail” of the distribution) while the load is high (i.e. in the “right tail” of the distribution). Therefore a distribution which approximates a sample well in the in the area of a mean value may be completely unsuitable in the area of the tails. (Schneider 1997)

For distributions of resistance and load parameters as well as model uncertainties the JCSS has collected or determined several relevant values and they can be accessed in the **Probabilistic Model Code** (JCSS 2001). The other sources to gain information about typical statistical parameters of interest in structural / bridge engineering are the **Background Documents of the EuroCodes**. When carrying out development and codification of resistance models, several tests were carried out to assess the reliability of the models. Thus, just as for material models, statistical properties have been assigned to the models of failure modes of for example joints in steel beams, the so called *model uncertainties*, which are described in detail in *Section 2.2.3*. When analysing a specific case and failure mode, information from these documents can therefore be applied.

When the task is the assessment of bridge structures, probabilistic traffic load models are unavailable or are currently used (to the knowledge of the author) only at a “scientific” level. In contrast, the JCSS PMC does give recommendation for the probabilistic model of wind loads and for certain live loads on buildings. These are supplemented by some basic examples such as in Vrouwenvelder et al. (2002).

Description of distributions which are used in the reliability functions or in the data analysis process are included in *Appendix B.* The analytical form of the *inverse cumulative distributions* is given (when available) as this is necessary for carrying out a Monte Carlo simulation, as described in *Section 2.3.2.*

### 5.2.2 Finite mixture models

When a dataset cannot be adequately represented by a single distribution, it is possible to construct a representation from a *mixture of distributions*. Each distribution in the mixture is described by its’ parameters and by its’ weight.

“A mixture model is able to model quite complex distributions through an appropriate choice of its components to represent accurately the local areas of support of the true distribution. It can thus handle situations when a single parametric family is unable to provide a satisfactory model for local variations in the observed data.” (McLachlan & Peel 2001)

Finite mixture models are used in analysis of vehicle weights in Steenbergen et al. (2012), where the final result is a composition of several normal distributions.

In the current thesis work uni-variate mixture distributions are used, these will describe the distribution of the gross vehicle weights (*Section 8.1*). For purpose of visualising data and investigating possible correlations in a simple way, a bi-variate multimodal mixture distribution is used in the traffic load analysis section (*Section 8.2.3*).

## 5.3 From data to probability distribution: statistical inference

### *5.3.1 Relevance and introduction*

In order to perform a probabilistic analysis the stochastic properties of input variables should be known, namely the *type* and the *parameters* of their probability distribution. For example, to evaluate the failure probability (determine the reliability index β) of a steel beam, the statistical distribution of its yield strength has to be known. As significant experience is available concerning well-known steel types, the distribution type and parameters can be taken from literature such as the JCSS PMC. In this case for example the yield strength is known to be lognormal distributed, with a certain relation between the nominal value, the mean and the standard deviation. In some cases however, this information (distribution parameters or even the distribution type) is not readily available. For example the concrete compressive strength of an old structure might not be known, or typically soil parameters, etc. In case of traffic loading, as stated above, probability distributions are not readily available. [11]

Statistics is useful for an engineer in order to gain relevant information from a sample of data. A dataset consists of observations of a phenomenon of interest, for example concrete compressive strength or the weight of a vehicle. A *population* is the aggregate of observations that might result from conducting an experiment. From a *data sample*, conclusions can be drawn about the whole *population* using *statistical inference*.

Whereas descriptive statistics describe a sample, inferential statistics infer predictions about a larger population that the sample represents.

In statistical inference the *type of distribution* and the *distribution parameters*, the latter denoted as $\theta$, are to be determined. For example, the distribution type can be lognormal, the parameters of interest the mean $\mu$ and the standard deviation $\sigma$. A second important area of statistical inference is the testing of the hypothesis, both concerning the type and the parameters of an estimated probability distribution. In the following, a brief overview of approaches in statistical inference is given.

### *5.3.2 Overview of statistical inference*

#### *Distribution type*

The **type of distribution** should be estimated based on known physical relations, if possible, and not based on data analysis. (Vrijling & Gelder 2002) When this is not possible, a distribution type is assumed.

It can happen that the random samples satisfy another type of distribution better, than the original distribution of the population. It should be proven that the selected distribution type is not improbable, with some form of *hypothesis testing* or in engineering practice possibly by *visual methods*. To check whether the data corresponds to the chosen distribution, *goodness-of-fit tests* can be used such as the so called $X$*-square test* or the *Kolmogorov-Smirnov test*. These will not be described here in detail, the reader is referred to literature. (Kottegoda & Rosso 2008; Vrijling & Gelder 2002)

[11] Even if probabilistic traffic load models are available, for example for a highway bridge, it would be reasonable to aim for utilizing measurement data of the city traffic. Based on data new and possibly more accurate probability distributions could be determined or existing ones could be *updated.*

### Practical approach to selecting the adequate distribution type

A practical approach to determine whether the theoretical distribution fits the empirical distribution is using visual methods. In a *probability plot* one axis corresponds to the empirical probability distribution (i.e. the cumulative frequency of the data sample) while the other to the probability distribution of the "hypothesised theoretical probability distribution". The grid on one axis (usually horizontal) is scaled to suit the cumulative distribution function of a certain probability distribution. If the data corresponds to the assumed distribution, a linear relationship is observed when plotting this data against the variate. "Such a graph is widely accepted by engineers as a form of presentation of data, usually for confirmation of an analysis."

Transformation formulas for the plot which can also be used for linear regression are summarized in *Table 13*.

**Table 13 - Transformation formulas**
(Vrijling & Gelder 2002)

| Distribution | X- axis | Y- axis |
|---|---|---|
| Uniform | Linear | Linear |
| Normal | Linear | Normally distributed |
| Log- normal | Logarithmic | Normally distributed |
| Exponential | Linear | Logarithmic |
| Gumbel | Linear | Double- logarithmic |
| Weibull | Logarithmic | Double- logarithmic |

However, according to Vrijling & Gelder (2002): "The 'position on a straight line' does not verify the correctness of the selected distribution. The position (…) on a straight line only leads to the conclusion that the observations present in the random sample can be modelled well by the selected distribution."

If the right-tail of the data is of interest, thus the maximum values, it is useful to adapt a plotting technique which shows deviation of the tail-data from the assumed distribution type. This will be the case in several steps of the traffic loading analysis as the maximum weights, heaviest axles, largest bending moments etc. will be of interest and have to be approximated most accurately when ultimate limit states (excluding fatigue) are of concern. A good representation is plotting the so called exceedance-frequency diagram of the data and the exceedance-probability diagram of the (assumed) theoretical probability distribution. The X-axis represents the values of the variable while on the Y-axis the probabilities of non-exceedance ($P_{NE} = 1-P_E$) are plotted on a logarithmic scale. Such figures are used in further sections of the thesis, for example *Figure 27* in *Section 75.*

In some cases it seems that more than one type of distribution fits the stochastic variables. More distribution types can be tried, for example Weibull- and Gamma distributions in Steenbergen et al. (2012). Plotting techniques can be applied similarly.

In practice, according to JCSS (2001), choice should be made for the "less safe" case, thus for the distribution which gives the higher failure probability in the reliability calculations.

### Distribution parameters

The **parameters** that describe the known or assumed distribution type can be estimated from a random sample in the process of *parameter estimation*. We speak of estimation because in statistical inference it is not possible to say with complete certainty that the parameter takes on exactly a specific value. In other words, the value of the parameter $\theta$ has *uncertainty* and as such is a random variable itself. In parameter estimation a value of $\theta$ is called an *estimate*, while the random variable is called an *estimator*. These are formally defined below.

> An *estimate* is a value *t* that only depends on the dataset $x_1, x_2, \dots, x_n$, i.e. *t* is some function of the dataset only:

$$t = h(x_1, x_2, \dots, x_n)$$

Let $t = h(x_1, x_2, \dots, x_n)$ be an estimate based on the dataset $x_1, x_2, \dots, x_n$. Then *t* is a realization of the random variable

$$T = h(X_1, X_2, \dots, X_n)$$

The random variable *T* is then called an *estimator*.

For example the sample mean $\overline{X}$ is an estimator of the population mean $\mu$. The value it takes on is the estimate.

Two main strategies are available for parameter estimation: *point estimates* and *interval estimates*.

A *point-estimate* describes an unknown parameter $\theta$ with a single value. There are several methods available to do this such as the method of moments, the method of maximum likelihood, its' more general form the Bayesian parameter estimation or the bootstrap. Knowing that the value $\theta$ is not certain, it is useful to define one or more measures which give an indication about whether the estimate is satisfactory, or how adequate it is. These measures are the properties of the estimator. An ideal estimator should have the properties of *unbiasedness*, *consistency, efficiency, sufficiency* and *robustness*. They are described in the following sub-section.

In contrast to a point estimate, which gives a single value for a parameter and information about the precision (property of estimate), an *interval estimate* accounts for uncertainty by determining a range in which $\theta$ falls with a certain probability. The bounds of this range are called *confidence limits*, while the interval is termed *confidence interval*.

### Properties of estimators

A point estimator $\hat{\theta}$ is an **unbiased estimator** of the population parameter $\theta$ if $E[\hat{\theta}] = \theta$. If the estimator is biased, the **bias** is $E[\hat{\theta}] - \theta$.
The mean and variance of a sample mean are for example unbiased estimators of $\mu$ and $\sigma^2$ (of the distribution).

An estimator $\hat{\theta}$, based on sample size *n*, is a **consistent estimator** of a parameter $\theta$, if for any positive number $\varepsilon$,

$$\lim_{n \to \infty} Pr\left[\left|\widehat{\theta_n} - \theta\right| \leq \varepsilon\right] = 1$$

**Efficiency** relates to variance of the sampling distribution: the smaller the variance the more efficient the estimator.

Next to minimising the variance, one can also speak of combining the criteria of efficiency and unbiasedness. The ***mean square error*** can be defined and one can aim to minimize this quantity instead of the variance.

$$mse = E\left[\left(\hat{\theta} - \theta\right)^2\right] = Var[\hat{\theta}] + bias$$

Related to the variation is the ***standard error***, which is the standard deviation of the sampling distribution of a statistic, thus in this case the standard deviation of $\hat{\theta}$.

A **sufficient estimator** gives as much information as possible about a sample of observations so that no additional information can be conveyed by any other estimator.

Finally, the term **robustness,** when applied to an estimate refers to "*insensitive to small deviations from the idealised assumption under which the estimate is optimised".*

In the following, some methods of statistical inference, which are relevant in the current thesis work, are described.

### 5.3.3 *Regression analysis*

Regression analysis estimates relationships between two variables, expressing the relation in the form of mathematical equations. According to Vrijling & Gelder (2002) "the essence of regression analysis is to minimise the deviations of the data from the (assumed) distribution model by an optimal selection of parameters." Regression analysis is a broad term and is used not just in the context of parameter estimation, but in analysis of relationship between two variables. Now however, we are curious about its application for parameter estimation.

The simplest form, termed *linear regression* assumes a "straight line relationship" between two variables, or in the current case between the data (sample) and the cumulative distribution function. Observations are organized in increasing order and are indexed ${}_N x_i$. If ${}_N x_i$ are interpreted as ordinates, a coordinate ${}_N y_i$ can be assigned to each. For some (assumed or known) distribution types axis transformation can be performed in order to have straight lines representing the cumulative distribution functions. Non-linear functions are thus "turned into" linear ones with the help of axis transformation.

For example in case of an exponential distribution:

$$\mathrm{Y} = \mathrm{A} \cdot \exp(\mathrm{BX})$$

$$\ln(\mathrm{Y}) = \ln(\mathrm{A}) + \mathrm{BX}$$

Meaning that X can be plotted on a linear and Y on a logarithmic scale.

The parameters can be estimated by fitting a straight line to the data. This can be done visually (the eye is expected to minimise the vertical distances from the estimated regression line) or by more precise methods such as the method of *least squares*. This is shortly described in the following sub-section.

#### *Method of least squares*

In this approach, the sum of the squared differences between the observations and the assumed model is minimised. This can be expressed as:

$$var(\varepsilon) = \chi^2 = \min_{\vec{\theta} \to \hat{\theta}} \sum_{i=1}^{N} \{y_i - (A + BX_i)\}^2$$

For a general case, where the relationship is not definitely linear this can also be expressed as:

$$var(\varepsilon) = \chi^2 = \min_{\vec{\theta} \to \hat{\theta}} \sum_{i=1}^{N} \{y_i - g(\vec{x}, \vec{\theta})\}$$

#### *Applicability*

According to Vrijling & Gelder (2002) it is "at least questionable" whether linear regression is suited for parameter estimation. A starting assumption for linear regression is that the deviations from the regression line are independent (and normally distributed). By organising observed data in increasing order, successive observations are not independent. Therefore doubts arise about adequacy of this method for parameter estimation.

### 5.3.4 *Method of maximum likelihood*

In order to find the distribution fit when the measured data points are "most likely" to appear, a possible method is the *method of maximum* likelihood.

Assume that the distribution of a population is known and has one parameter, $\theta$. The random sample likelihood function can then be defined as:

$$L(\theta|x_1, x_2, \dots x_N) = \prod_{i=1}^{N} f_{\underline{x}}(x_i|\theta)$$

This can be interpreted as:

- The (relative) probability of occurrence of a certain random sample, $x_1$, $x_2$, … $x_N$, as a function of a given parameter ($\theta$)
- The (relative) probability of a value occurring, given that certain random sample

The goal is to maximise this value. The maximum likelihood estimate of $\theta$ is the value for which the likelihood function *L(θ)* assumes a maximum.

In practice it is often useful to take the logarithm of the functions (then the multiplication turns into a sum). This can be termed *log-likelihood procedure*, which is in principle the same as the maximum likelihood method. The perquisite is that the likelihood function should be monotonous. The logarithm will then take on a maximum value at the same point as the original function. By derivation of both sides and equating the result to zero, the value of $\theta$ where *L(θ)* assumes a maximum can be determined.

#### *Bayesian method*

The Bayesian method is based on an a priori distribution of the parameter $\theta$, which indicates some "knowledge" of the parameter in advance, before the data is available. The method is based on the Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Applied to the maximum likelihood method it can be written as:

$$f_{\underline{\theta}}(t|x_1, x_2, \dots x_N) = C \prod_{i=1}^{N} f_{\underline{x}}(x_1, x_2, \dots x_N|t) f_{\underline{\theta}}(t)$$

#### *Applicability*

The method of Maximum Likelihood is used in literature on traffic loading analysis to determine adequate distributions of loads (Caprani 2005; Steenbergen et al. 2012). It is claimed to be "the method favoured by statisticians" by Kottegoda & Rosso (2008).

Contrary to (linear) regression, observations don't have to be sorted according to size therefore the dependence is excluded. It is *consistent* estimator, but a large sample of data is necessary before it becomes *unbiased*. In comparison to other estimates, it does not have a *low variance* and is therefore less *efficient*.

## 5.4 Extreme value analysis

### *5.4.1 Introduction*

Ultimate limit states are concerned with the "worst case scenario" loading. Therefore it is of interest to determine the statistical distributions of maxima – for example 15-year maxima values of the axle loads or of the bending moments.

For this extreme value analysis is necessary. Extreme value analysis is concerned with the probability of occurrence of events which are beyond an observed sample. (Kozikowski 2009, *based on Gumbel*) The largest or smallest value from a set of identically distributed independent random variables tends to an asymptotic distribution that only depends on the tail of the distribution of the basic variable. (Kottegoda & Rosso 2008)

If $X_1$,...$X_n$ is a sequence of independent random variables having the same distribution function *F(x)*

$$\mathrm{M_n} = \max(\mathrm{X_1}, \dots \mathrm{X_n})$$

Then the cumulative distribution function can be written as:

$$\mathrm{F_{M_n}}(\mathrm{x}) = \mathrm{F_X}(\mathrm{x})^{\mathrm{n}}$$

And the probability density function as:

$$\mathrm{f_n}(x) = \mathrm{nF}(\mathrm{x})^{\mathrm{n}-1}\mathrm{f}(\mathrm{x})$$

When the original distribution of a certain property or phenomena is known the extreme value distribution resulting for a certain time-span can be determined. For example, if the measured truck weights can be described with a normal distribution, this distribution represents the probability that one single truck at any moment has a weight *X.* If the information of interest is the heaviest truck among for example $10^6$, this will be described by an extreme value distribution with a mean significantly higher than that of the normal distribution describing the truck population.

The basic types of extreme value distributions and their properties are described in the following section.

### 5.4.2 Extreme value distributions

Three types of extreme value distributions can be defined (within each type one for maxima and one for minima as well). Exact definition of these distribution types can be found in literature, for example in Kottegoda & Rosso (2008). Here the analytical expressions that differentiate the three distribution types are not given. The main difference is the way the tail of the extreme value distribution behaves (right tail of maxima and left tail of minima distribution).

These probability density and cumulative distribution functions of all three distribution types can be written in a "generalised" format, described by three parameters: the location *u*, scale $\xi$ and shape *k.* When expressed in an analytical format, the distribution types are divided by the margin k = 0. This distribution is a *type I.* distribution and is called a *Gumbel distribution* for the case of maxima.

The parameter *k* corresponds to Type II distribution for positive and Type III distribution for negative values. [12] These distributions are also called "short-tail" and "heavy-tail" (or "fat-tail") distributions respectively. Short-tail distributions converge faster to the zero asymptote than a type I distribution, while fat-tail distributions converge more slowly.

An example of determining extreme value distribution based on measurements is given in *Section 6.4.3.*

[12] The expression varies in sources of literature therefore attention should be paid! MatLab® for example uses negative values for type II and positive for type III distributions

# Section II

# From WIM Measurements to Load Effect Distribution

# 6 Traffic Load Modelling - Review

## 6.1 Introduction

The current thesis work focuses on and limits itself to the main primary load on traffic bridges: traffic loading. Primary load refers to the load which expresses the purpose for which the bridge was built, as defined by O'Connor & A.Shaw (2000). Therefore the term ***loading*** will be used referring to ***traffic loading***, unless defined otherwise. Similarly, ***load*** effects will refer to ***load effects caused by traffic loading***, unless defined otherwise. Self-weight is to be considered in reliability calculations, while dynamic effects will be briefly discussed in *Section 6.5* in order to arrive to realistic and applicable conclusions.

Accidental-, thermal and earthquake loading are not considered. Based on recent analysis carried out by TNO for the Dutch Ministry of Transport, Water Management and Public Works (Rijkswaterstaat, RWS) (Steenbergen et al. 2012), this can be considered a typical approach in load modelling for bridges in the Netherlands.

This section will give an overview on the nature of traffic loading, measurements, codes, examples of updating design values as well as analysis of weight-in-motion (WIM) data by various authors. The purpose of the overview is to arrive to a strategy for WIM data analysis and traffic loading simulation.

It is noted that besides WIM data analysis there are other possibilities to determine load effects, for example by placing strain gauges on girders of a steel bridge. By measuring strain and with knowledge of the elastic parameters of steel the stresses can be determined and evaluated. The scope of the current thesis work however is the application of WIM data.

## 6.2 Measurements

### 6.2.1 Data of interest

In order to determine the effects on bridges originating from traffic, information about vehicles is gathered over a time period. The information of interest can be: gross vehicle weight (GVW), axle weight, vehicle distance, axle distance, time and speed (a derived quantity). The latter two, time and speed are typically useful for advanced traffic flow models that can optionally include congestion modelling (i.e. 'traffic jam').

In relevant literature it can be observed, that *axle load* and *GVW* is often the basis for creating load models. It is relatively straight-forward to apply descriptive and inferential statistics to these datasets: typically distribution functions, usually multi-modal, are fitted to the measured data. One of the main challenges in creating an adequate traffic load model is that knowing the distributions and / or design values of axle loads and GVW-s does not give direct information about the global load effects. Information about *axle- and vehicle distances* has to be used and / or assumed, resulting in a complex task.

### 6.2.2 Weigh-in-motion systems

Traditionally, until the 70's, static measurement systems were used. Selected vehicles, which appeared to be heavily loaded were measured at weighing stations. The statistical relevance of such data is questionable (Sedlacek et al. 2008), for example due to overloaded vehicles successfully avoiding the measurement stations.

Since the 70's the use of weigh-in-motion systems has spread. Initially so called weigh bridges were used, since the 80's piezoelectric equipment has been developed and applied. Piezoelectric materials convert mechanical stress or strain to proportionate electrical energy.

### 6.2.3 Europe

The traffic loading models in the Eurocodes are based on measurements from two measurement campaigns (1977-1982 and 1984-1988). Recorded daily traffic flows are 1000 – 8000 for slow lanes and 100-200 for fast lanes of motorways, 600 – 1500 for main roads and 100-200 for secondary roads.

Since these campaigns, several further measurements have been carried out. In their research Enright & O'Brien (2013) use traffic loading data from five EU-countries measured in the period 2005-2008, including approximately 2 700 000 trucks. The data corresponds to 0.5 to 1.5 years of measurements, depending on the location.

### 6.2.4 Netherlands, highways

In the Netherlands continuous measurements are being carried out on highways and are being used to update and re-evaluate the load effects provided in the codes. The measurements are used to monitor the actual loading on the infrastructure and in certain instances also to adapt design values of loads. The measurements and their use is described in detail in TNO-060-DTM-2011-03695-1814 (Steenbergen et al. 2012).

### 6.2.5 Data used in the thesis: Netherlands, urban bridges in Rotterdam, 2013 [13]

#### Reason for measurements

As elaborated in the *Introduction* of the current thesis work, it is expected that traffic loads on city bridges are lower than on highways, to which the currently applicable European and Dutch norms have been calibrated. Therefore an attempt is made to use local measurements to determine site-specific loads for bridges in the city and to compare them to the loads on highways in the Netherlands.

If the measurement program leads to reducing the design loads on city bridges, new bridges can be designed in a more economical way. The main gain however is expected for existing structures, where expensive refurbishment may be deemed unnecessary if the expected loads for the remaining lifetime of the structure were lowered.

#### Measurement project 2013

In 2013 a measurement program started involving besides the municipality a contractor for carrying out the measurements and TNO to analyse the data.

The measurements were carried out with the system *WIM Hestia*, which is a piezoelectric WIM system and had already been applied for measurements on highways. Data was collected at two locations over a period of five months. The system was then calibrated, as preliminary analyses suggested that the results contain a significant error. In one location measurements were carried out for two further months (30 September – 28$^{th}$ November). The data of these two months of calibrated measurements is used in the current thesis work.

The two months measurements correspond to 53 853 heavy vehicles, of which after a "data cleaning" process data of 48 586 is used. Heavy vehicle corresponds to a gross vehicle weight (GVW) of 3.5 tons. Lighter vehicles are expected not to contribute significantly to the extreme loading situations on short bridge.

[13] Based on the report of TNO to IGR (Huibregste et al. 2014)

### 6.2.6 North-America

In the USA design loads, namely the *'AASHTO LRFD HL93'* (comparable to Load Model 1 of Eurocode) were based on measurements carried out in Ontario, Canada in the 1970's with 9250 measured vehicles (Kozikowski 2009). These measurements were performed at weighing stations thus may likely not be statistically representative, as explained shortly in *Section 6.2.2.* At the same time, the values are quite accurate due to the precision of the method compared to a WIM system for example.

In the recent years, departments of transportation (DoT) of several states conducted extensive WIM measurement campaigns. In the study of Kozikowski (2009) use is made of 47 000 000 measurements conducted at 32 locations of six states. The data corresponds to half – one year of measurements, depending on the location.

### 6.2.7 Pre-processing data

#### Measurement errors

Analysis of data starts with filtering or cleaning measurement data. According to Enright & O'Brien (2011) *"the purpose of data cleaning is to identify gross measurement errors on individual vehicles and either attempt to correct these errors or eliminate the vehicle from the record so as to create a database of reliable readings".* Unrealistic data may distort the result of the analysis.

In the current thesis work pre-processing data is not carried out as the available measurements have already been processed by TNO. The current section attempts to give a brief overview of the necessity of dealing with two main types of measurement errors and summarizes how it has been done in different cases, including the analysis carried out by TNO. (Huibregste et al. 2014)

Two types of error are present in WIM measurements: *gross errors* and *random errors*. Cleaning data deals with *gross errors*: these are to be corrected, if possible or if not, then the data which is suspected to be a result of a gross error is eliminated. Various criteria are applied in research for the elimination procedure. Enright & O'Brien (2011) summarizes methods applied to European data (Netherlands , Slovakia, Slovenia, Poland, Czech Republic) as well to US data. When comparing the data gathering process of European countries, they conclude that the system used in The Netherlands is the most comprehensive and useful for data cleansing processes.[14]

*Random errors* are quantified by the accuracy of a WIM system. For example, the WIM Hestia station used in Rotterdam gives an accuracy for the gross vehicle weight of $\pm$ 10% with 95% confidence . Note that this is the value determined by the supplier of the system. Such errors can be treated by using an adequate model uncertainty. The current work does not elaborate on the relation of randomness in measurements and the applied values for model uncertainty.

#### Filtering measurements

Some criteria used in various publications for filtering gross errors are for example:

- Speed is below and above a certain speed (in measurements on highways)
- Sum of axle spacing is greater than the length of the truck
- If photos are available: vehicles not corresponding to measured axle numbers and distance
- Vehicles with low GVW
  *As they don't have significant influence on the extreme loading events*

[14] The measurement system referred to by Enright & O'Brien (2011) is that applied in TNO-060-DTM-2011-(…) (Steenbergen et al. 2012). The main difference between this system (used for highways) and the one applied in Rotterdam in 2013 is that in the first photos are made of the vehicles.

- Vehicles with un-realistically high axle load (for example 40t)
- (First) axle distance is below a certain value

## 6.3 Traffic load models in codes

### *6.3.1 Introduction*

Australian, Canadian, USA and European codes tend to model traffic loading with one or two major axle groups and a uniformly distributed load. The latter model the effect of a sequence of minor vehicles. The point loads are meant to represent local effects and, in combination with the distributed load, to create representative load effects. (O'Connor & A.Shaw 2000)

These values are determined in the process called "code calibration". The values to be used according to design codes are developed based on a combination of statistical analysis of measured vehicle- and / or axle weights and on the evaluation of load effects.

### *6.3.2 Loads - EN 1991*

As a result of the code calibration procedure, *Eurocode 1991 Part 2: Actions on Structures: Traffic Loads on Bridges*(European Committee for Standardisation 2003) describes the applicable traffic load models for Europe. A document to support application has been developed as well (Sanpaolesi et al. 2005).

The basic load models (LM) which are applicable for carriageways of up to 42 m and loaded lengths of up to 200 m are described below.

LM 1: two concentrated axle loads representing a tandem system and a uniformly distributed load (UDL)
*Refer to Appendix D for sketch of the load model.*

- LM 1 represents the loading creating maximum global load effects. A very heavy vehicle in one lane with several smaller vehicles in. Lighter vehicle models are used in on second and third lanes.
- Only one tandem system per notional lane
- Impact factor is included in $\alpha_{Qi} \times Q_{Qi}$ and $\alpha_{qi} \times q_{qi}$

  *Where* $Q_{Qi}$ is a concentrated load
  300 kN, 200 kN and 150 kN respectively
  $q_{qi}$ is a uniformly distributed load
  9 kN/m$^2$ for lane 1 and 2,5 kN/m$^2$ for the others
  $\alpha_Q$ accounts for the type of road and is given in the National Annex
  i is the number of the notional lane

LM2: one axle

- Represents the impact effect of a characteristic axle load from irregularities in the road surface.
- Impact factor is included in $\beta_Q \times Q_{ak}$

  *Where* $\beta_Q$ is same as $\alpha_Q$, unless specified otherwise

LM3: special vehicles

- Assumed to move at low speed (5m/s), therefore given axle load values include the dynamic effect.

LM4: crowd loading

- Is represented by a UDL of 5 kN/m$^2$

- Should be considered only when explicitly demanded, usually urban areas

The current thesis work focuses mainly on Load Model 1 and considers in some cases Load Model 2. LM 3, special vehicles, is neglected as it is expected that these need a permit to cross a city bridge. This is elaborated in *Section 6.3.3*. LM 4 is not expected to be dominant in comparison to the traffic load caused by vehicles and is therefore neglected.

### *6.3.3 Abnormal loads*

#### *Eurocode*

"The EU has laws on the maximum weights and dimensions allowed for road freight. Loads that surpass the limits – referred to as 'abnormal loads' – require special permits from regional or national authorities. Different countries have different rules and procedures for obtaining these permits – concerning vehicle escorts, allowed time frames, authorised speeds, etc. " (European Commission 2013)

Objects which may be affected by passing of such vehicles have to be checked for sufficient load bearing capacity. The procedure for dealing with these specific cases is described in guidelines in the engineering office of Rotterdam.

#### *Netherlands specific*

In the Netherlands the Dutch Road Administration, RDW is responsible for the authorisation of such special transport. The vehicles can gain one-time (temporary) or long-term permission for certain routes.

According to NEN 8701 (Normcommissie 351001 2011b) it is necessary to consider only exceptional loads which were considered in the original design as well. In practice, during the general re-evaluation of structures the exceptional vehicles are therefore usually not taken into account.

### *6.3.4 Adapting design values*

Design values have been set in codes and must be adapted according to law. However, it is permitted to adjust the value of the loads based on analysis of measurements. In the following, a brief overview of examples for such cases is given.

#### *Highway bridges*

In the Netherlands substantial effort is made to monitor traffic and to make use of the traffic load measurements, as described in *Section 6.2.4*. Specific of the traffic situation on highways in the Netherlands is the high traffic intensity and large vehicle weight, which is the result of extensive commercial transport. When compared to traffic load effects in some Central- / Eastern-European countries, a typical Dutch highway may have 20% higher "notional load model ratio" (O'Brien et al. 2006), both due to the presence of very heavy vehicles and to the more intense traffic .

It is of interest to the owner of the highways, the Dutch Ministry of Infrastructure (*Rijkswaterstaat, RWS*) to know the actual loading on the infrastructure in order to assess the safety of existing structures and to design new ones based on adequate and realistic loading criteria. Therefore the traffic measurements have been analysed in the past decades multiple times and were used for adapting design loads. The methodology adapted by TNO to do this is described in *Section 6.4.3*.

#### *Bridges in the urban areas*

As shown in *Section 6.3.2*, the measurements on which traffic load models are based were carried out on highways. The traffic composition on a bridge in an urban, for example in a residential area can be expected to differ from that on the highways.

It can be expected that the maximum load in the life-time of such a structure will be lower than that for a highway. This assumption should be made carefully though: for example in case the maximum vehicle load has an upper bound, a lower flow rate (number of vehicles per day) may not definitely guarantee a lower extreme load.

Following the recommendation of EN-1993, adjustments may be done. In the Netherlands, the possibility to adapt load models to the actual traffic loading situation has been worked out and is described in NEN 8700 and 8701, which are two norms dealing with the evaluation of existing structures (Normcommissie 351001, 2011a and b).

#### Adaption of fatigue loading based on measurements[15]

As an example for detailed loading analysis and the "gains" compared to loading described in norms is the research carried out by (Otte 2009). An existing model (Groendijk) was used to conclude stresses from the loading. Then WIM data was analysed and inserted to the model.

The main conclusions of the research, which may be relevant for the current investigations are the following (Otte 2009):

- Standard WIM measurements can't detect wheel types (VRSPTA can)
- Eurocode determines traffic on fast lane as 10% of that on the slow lane, while measurements indicate a ratio of up to 15-20%.
- Difference in volumetric density of cargo results in a large spread of axle loads
  First axles carrying the driver cabin have a similar axle load
- In the next 100 years axle loads are expected to grow by 20% while the number of trucks by 40% (TNO);

### *6.3.5 Netherlands: loading according to NEN*

As mentioned briefly in *Section 3.4.2*, Specifications in the Netherlands allow for the reduction of traffic loads. These factors consider that the loading situation is different in a specific location and structure than what the Eurocode load models have been calibrated for. These factors are described below.

#### *Service life and reference period*

This factor accounts for the fact that the remaining service life *t* is not equivalent to design life *T*. The time dimension is taken into account by allowing for a reduction factor $\Psi_t$(NEN-EN 1991-2:2003/NB:2009).

$$\psi_t = \left(\frac{\ln(\mathrm{vt})}{\ln(\mathrm{vT})}\right)^{0,45}$$

#### *Heavy vehicles and influence length*

This value is regulated in the national annex and takes into account the number of heavy vehicles per year. The reduction of short influence lengths is smaller than on long ones. The recommended value is minimum 0.8 if no traffic restriction signs are present. The values are summarized in *Table 14*.

[15] Based on 'Proposal for modified Fatigue Load Model related to EN 1991-2' (Otte 2009)

**Table 14 - Factor for shorter reference period**
(Normcommissie 351001 2011b)

| Referentieperiode | Ψ-factor [a] | | | |
|---|---|---|---|---|
| | Lengte van de overspanning of invloedslengte *L* | | | |
| | 20 m | 50 m | 100 m | ≥ 200 m |
| 100 jaar | 1,00 | 1,00 | 1,00 | 1,00 |
| 50 jaar | 0,99 | 0,99 | 0,99 | 0,99 |
| 30 jaar | 0,99 | 0,99 | 0,98 | 0,97 |
| 15 jaar | 0,98 | 0,98 | 0.96 | 0,96 |
| 1 jaar | 0,95 [b] | 0,94 [b] | 0,89 | 0,88 |
| 1 maand | 0,91 [b] | 0,91 [b] | 0,81 | 0,81 |

[a] Voor andere invloedslengten en referentieperioden mag lineair zijn geïnterpoleerd.
[b] Zie de opmerking onder de tabel.

### *Traffic trend*

The traffic trend factor $\alpha_{trend}$ takes into account that load models of the Eurocode are calculated with an extrapolation to 2050. If the life time of a structure is shorter, the characteristic value of the load can be reduced.

**Table 15 - Reduction factor for traffic trend compared to 2060**
(Normcommissie 351001 2011b)

| Invloedslengte *L* [m] | Reductiefactor $\alpha_{trend}$ [a] | | | | | |
|---|---|---|---|---|---|---|
| | 2010 | 2020 | 2030 | 2040 | 2050 | 2060 |
| 0 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| 20 | 0,89 | 0,91 | 0,93 | 0,96 | 0,98 | 1,00 |
| 50 | 0,82 | 0,86 | 0,89 | 0,93 | 0,96 | 1,00 |
| 75 | 0,78 | 0,83 | 0,87 | 0,91 | 0,96 | 1,00 |
| 100 | 0,76 | 0,81 | 0,85 | 0,90 | 0,95 | 1,00 |
| 150 | 0,75 | 0,80 | 0,85 | 0,90 | 0,95 | 1,00 |
| ≥ 200 | 0,75 | 0,80 | 0,85 | 0,90 | 0,95 | 1,00 |

### *Typical values*

Typical values when considering all three reduction factors are in the range of 0.7 – 0.9 on the city bridges of Rotterdam.

### *Netherlands specific weight limitations*

The maximum load legally allowed is 50 tons, passing special vehicles are allowed in the range of 50 to 100 tons. As a comparison, these values are 44 tons in Belgium and 40 tons in Germany (PPT Steenbergen NEN course).

## 6.4 Literature review of creating traffic load models

### *6.4.1 Introduction*

Methods of load analysis are described in detail among others in the works of (O'Brien et al. 2006; Zhou et al. 2012; Guo et al. 2012; Enright & O'Brien 2013; Caprani 2005; Paeglitis & Paeglitis 2002).

Two main “types” of output can result from the process of creating a traffic load model: design values (deterministic or for modern codes semi-probabilistic) or a probabilistic load model. In daily engineering practice it is typical to use deterministic / semi-probabilistic load models which are results of code calibration procedures. In some cases factors accounting for statistical effects may be used, for example the reduction factors allowed for by NEN 8700 and NEN 8701. If one wants to carry out a fully probabilistic analysis however, the loads or the load effects should also be expressed in a stochastic form, i.e. with distribution functions. Therefore the second type of output of traffic loading analysis process is stochastic load- or load effect models.

It is emphasized that both type of load models result from statistical procedures and creating them has several common steps. In some cases the semi-probabilistic (design) values result from fully probabilistic calculations as well, where on the level of (applied) research the distribution functions had been determined and used in several fully probabilistic calculations to calibrate a closely optimal design value.

Therefore the overview of literature concerned with determining design values and with creating fully probabilistic load- or load effect models is overviewed comprehensively. Research in the field of traffic loading analysis and (load effect) prediction is presented in the following sections. Some specific aspects are touched upon by others such as an evaluation of extrapolation methods by Zhou et al. (2012).

### *6.4.2 Determining traffic load models – Eurocodes*

The development of traffic load models used in EC is described in ‘Background Document to EN 1991: Part 2 – Traffic Loads on Bridges’ (Sedlacek et al. 2008). In the process of developing these load models measurements from several locations, as described in *Section 6.2.3,* were analysed.

Zhou, Schmidt, & Jacob (2012) give explanation and critique, mainly concerning the applied extrapolation techniques.

#### *Local effects - Axle loads*

Frequency distributions are approximated by bi-modal Rayleigh distributions, explained by the presence of loaded and unloaded vehicles. An example can be seen in *Figure 11.*

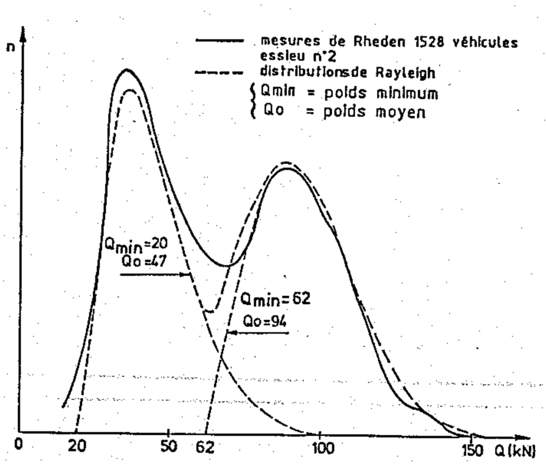


**Figure 11 - Axle load dsitributions for Eurocode calibration** (Sedlacek et al. 2008)

To determine extreme value distributions, axle load values $P_A \geqq 14$ kN are used and a half-normal distribution is fitted. Exceedance-frequency diagrams are used to derive representative values for the code. Daily extreme (mean), annual extreme, and 1000 years extreme (characteristic) values are defined and based on these the $Q_{Q1}$ = 300 kN axle load of LM 1 is determined. The values in LM 1 also include a dynamic amplification factor (DAF) resulting from un-evenness of the road surface. The $Q_{ak}$ =400 kN characteristic load of LM 2 includes a DAF of 1.3, which refers to a "bump" in the road. The axle distances of LM 1 are based on measured axle distances, considering the driver cabin.

### *Global effects*

For influence areas over 10 meters, it is necessary to consider **vehicle weights and distances**. **Four vehicle types** are isolated, the weight distribution for each type is described by a bi-modal normal distribution. Dynamic effect is taken into account. Similarly to the procedure of determining axle loads maximum vehicle weight is extrapolated. The method of extrapolation is not elaborated in detail in the document. Congestion of lorries on one lane was not considered due to the low probability of occurrence - which is not quantified or elaborated further.

Vehicle distances are considered necessary for influence lines above 10 meters – the exact reason for this length not being elaborated. In **congestion** 1 m distance between vehicles is taken, while in free flow three options are suggested: a formula from Davenport, probability density functions of vehicle distances based on analysis from Germany or a simplified formula using the reaction time of drivers as input.

Simulations of traffic were carried out on two-lane box girder type bridge for spans of 3 – 200 m. A characteristic equivalent load *Q'* was determined, based on the relation between a point load and a load effect (moment *M* or shear force *V*).

$$Q' = \frac{M}{L}k \qquad\qquad Q' = Vk$$

where 'k' is a factor from the influence line. Distributing Q' over the length, q' could be obtained.

It was also concluded that for spans over 30 meters, congestion is always the relevant loading situation.

Four types of traffic are considered:

- Flowing: considered especially important in bridges up to 30-40 m for characteristic effect. Was used for longer bridges as well, to determine frequent values.
- Slowed down: extracted from the recorded traffic, but the distance between vehicles is reduced to approximately 20 m.
- Congested with cars: a reduced distance to 5 m
- Congested without cars: eliminating light vehicles which have the tendency to overtake in case the traffic in a lane slows down.

### 6.4.3 Adapting design loads based on measurements - TNO, Netherlands [16]

As described in *Section 0*, in The Netherlands public authorities[17] are continuously investigating the loading conditions on the bridge network. As a part of these efforts, TNO received the assignment to determine the implication of traffic loading measurements on the loading conditions for (highway) bridges. This project has been preceded by similar traffic loading analysis in 1992 and 1998 therefore a comparison not just with the Eurocode load models but also with these earlier results was to be done.

In the following a brief review is given of the methods applied in determining design load values. For further details the reader is referred to the report (in Dutch).

Firstly, two properties which are highly relevant when speaking of traffic loading and its effects are analysed: axle loads and the gross vehicle weight (GVW) of trucks. An attempt is made to analyse the existing data as well as to make predictions for the future, using statistical and practical tools.

Secondly, the implication of the given axle-load and GVW distributions for the load effects on bridges should be determined. This is done in two ways: using a simulation model and by an analytical solution. The expected output was a resulting equivalent uniformly distributed load (UDL).

#### Wheel loads and axle loads

Axle loads are measured directly by the WIM system. As the measurements correspond to a shorter time period than the remaining life of the structure (which is of course a typical situation), some form of statistical inference *(Section 5.3)* must be used to draw conclusions for the extreme values that are expected.

For the analysis of ***axle loads***, one single distribution cannot fit the measurement data adequately. However, as in the current case only the right-tail of the data is expected to be of interest, additional attention is paid (only to) the “left tail” of the data. The level above which measurements are considered is termed *cut-off load*.

Now the question arises: what is the value of the cut-off load? In other words, which measurements belong to the “left tail” and will thus have an influence on the parameters and type of the fitted extreme value distribution? Besides the parameters of the distribution (for example two for a Gumbel, three for a generalised extreme value) the cut-off value is also to be determined. [18] The distribution parameters are determined for several assumed cut-off loads using maximum likelihood estimation. The final choice is made using a Bayesian approach or a suggested more practical one.

A summary of the exact procedure: using maximum likelihood estimation to determine the distribution parameters as well as the two methods for choosing the optimal design load are described in *Appendix I*.

For cases where it was not reasonable to fit a single extreme value distribution function to the tail of the distributions (i.e. exceedance frequency diagrams), a mixture of normal distributions *(Section 5.2.2)*was chosen.

**Resulting design loads**

[16] Based on TNO-060-DTM-2011-03685-1814 ‘Algemene veiligheidsbeschouwing en modellering van verkeersbelasting voor brugconstructies’ (Steenbergen et al. 2012)

[17] Typically the Ministry of Transport, Public Works and Water Management, Rijkswaterstaat (RWS)

[18] It should be considered that while using a higher cut-off load gives better description of the tail of the distribution, but a lower cut-off load allows for more data to be included in the analysis, thus less uncertainty in the parameters of the fitted distribution.

**Wheel load – local effect**

- Dynamic factor taken as 1,4 both according to EC and from RWS / TNO research
- EC: LM 2:
  - 1,50*200 kN = 300 kN CC 3
  - 1,35*200 kN = 270 kN CC 2
- TNO:
  - Axle load from measurements * dynamic factor * trend factor =
    = 246*1,4*1,35 = 465 kN
    → Wheel: 465 / 2 = 232,5 kN
    → 10 % higher dynamic factor for one wheel than for axle: 235,5*1,1 = 256 kN

Thus the values given in Eurocode are accepted as adequate for both consequence classes.

**Axle load - local effect**

- EC: LM 2:
  - 1,50*400 kN = 600 kN CC 3
  - 1,35*400 kN = 540 kN CC 2
- TNO:
  The design axle loads determined by the process above, depending on location, are in the range of 236 – 248 kN. It is mentioned that measurements from some locations were completely excluded after being termed unreliable. The value of approximately 250 kN is suggested. Using 246 kN the following final design axle load is calculated:
  - 1,4*1,35*246 kN = 465 kN

Thus the values given in Eurocode are accepted as adequate.

### Gross vehicle weight

For gross vehicle weights, it was decided to fit a mixture of normal distributions *(Section 5.2.2)* to the observed data. Based on relations described in *Section 3.4* , the design value of the GVW (a value of the distribution function belonging to a certain non-exceedance probability) is determined for each case.

The sufficient number of normal distribution functions to use in the mixture model were determined by calculating a design value ($GVW_d$) from 2 to 16 mixture components. The number of mixture components where the design value started to converge was chosen. This number was usually around 10 components.

### Load model

Knowing the design axle load is directly applicable for local verifications, such as for example a part of a steel plate in an orthotropic deck. However when the aim is to determine global effects such as bending moment or shear in a critical cross section of a bridge, the design axle load and design GVW do not directly serve as useable information. Two methods are used in the report.

The first is based on a Monte Carlo simulation of traffic, while the second is an analytical model which makes use of the distribution functions fitted to the GVW-s. Simply supported beam models of various lengths were analysed in both cases.

In the ***Monte Carlo method*** a row of vehicles is simulated for two lanes, representing the average number of vehicles in a day. The vehicles are sampled randomly from the measured population (axle number, axle weight, axle distance is given). The “tricky” part of the simulation is the headway. A so called consistent auto-correlation is described and with a special technique this correlation between headways is taken into account when generating the traffic. An example of the simulation result is shown in *Figure 12*.

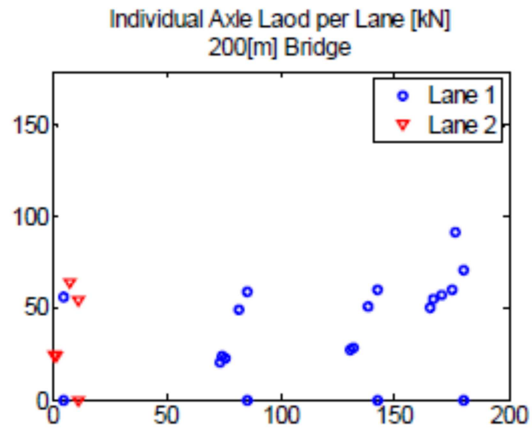


**Figure 12 – Axle loads per lane on, 200 m bridge** (Steenbergen et al. 2012)

The vehicles are run over the bridge (20, 50, 100 and 200 meter spans) in steps of 1m. Equivalent uniformly distributed load is calculated for each step and the maximum for a given time period (1, 10 and 30 days) is recorded.

The simulation model at the time of application was capable of simulating (multiple times) 30 days of traffic. This is not very high in comparison with other simulation models in this time period, which is due to the computational intensity of this method.

The ***analytical method*** considers free-flowing and congested traffic. It defines a "basic event" as a truck in the middle the span. The presence of a truck on the bridge is modelled with a uniformly distributed load over a *base length 'a'*. The beam is divided to sections 'd', the length of which is defined based on previous research (different values for free-flowing and congested traffic). Depending on the type of traffic, probabilities that a truck is present at a cross sections given a basic event are assigned (i.e. if a truck is present in the middle of the bridge, what are the probabilities that truck is present d, 2d, etc. distance behind / before it).

To each position *i* of a truck an equivalent UDL $q_i$ for the beam can be assigned, depending on the load effect of interest (bending moment or shear). Once all trucks on the bridge and their GVW are determined, the equivalent UDL, $q_{EUDL}$ for the full structure can be calculated. As the presence and the GVW are both stochastic variables, the resulting $q_{EUDL}$ will also be stochastic. The final result is determined using integration.

### Model uncertainty

In *Section* the need for including a quantified *model uncertainty* has been elaborated. The report of TNO gives an example of how this value is built up from various stochastic parameters.

The aspects considered in the research by TNO are summarized in *Table 16.*

**Table 16- Model uncertainty according to** Steenbergen et al. (2012)

| | Mean | St.dev. | Source |
|---|---|---|---|
| DAF vehicle | 1 | 0 | Within report |
| DAF bridge | 1,1 | 0,05 | Within report |
| Statistical uncertainty | 1 | 0,05 | Assumption |
| Spatial spread<br>(typical location in Nl less loaded than the location of measurements) | 0,86 | 0,07 | TNO 98-CON-R1813 |
| Trend factor 15 years | 1,1 | 0,1 | TNO 98-CON-R1813 |
| Load effect | 1 | 0,1 | JCSS PMC |
| **Total** [19] | 1,04 | 0,17 | |

### *Dynamic amplification*

The deterministic value of the applied dynamic amplification factor (DAF) is thoroughly considered and is chosen as 1.1 for global and 1.4 for local effects. The meaning of the DAF as well as a short summary of the considered literature is given in *Section 6.5 .*

## *6.4.4 University College Dublin – Caprani, O'Brien, O'Connor and Enright*

A research group at University College Dublin focuses extensively on interpreting and using traffic load measurements. Several publications in the topic present their coherent work carried out in this field. The dissertations of Caprani (2005) and Enright (2010) are major works dealing with analysis of WIM data and simulating traffic loading. They serve as a basis for several of the publications and for further developments of the research group.

The researchers generally aim to determine **life-time maximum load effects**. Thus their results cannot directly be used in a probabilistic analysis.

### *Caprani* [20]

Caprani developed a methodology and software programs in his PhD dissertation, using a complex approach to analyse and interpret traffic loading. The topic of **headway distributions** (i.e. the distance between the first axles of two consecutive trucks) is investigated in detail and is included in the model in a probabilistic way. Caprani simulates traffic loading for a maximum of 1250 days, based on distributions fitted to the relevant measured characteristics of truck traffic. Load effects are calculated from the simulation and are extrapolated to gain the life-time maximum load effects.

The main steps of the adapted strategy are described below.

1. **Statistical analysis of WIM data**
   Distributions are fitted to the following variables:
   - Gross vehicle weight (GVW)
   - Axle load - Correlated to the GVW for trucks with 4 or more axles
   - Speed - Normal distribution, independent of GVW
   - Headway - Special attention to headway modelling
     Most research takes a "minimum gap", here distribution is used based on measured data
   - Class (nr of axles) - Max. 6; ratios to full population are taken as relevant data; with ignoring 6+, extreme values may be neglected, but not enough data

[19] $\mu_{tot} = \sum \mu_i$ ; $\sigma_{tot} = \sqrt{\sum (\sigma_i)^2}$

[20] Based on *'Probabilistic Analysis of Highway Bridge Traffic Loading '* (Caprani 2005)

- Flow rate
- Axle spacing

2. **Traffic is generated based on the fitted distributions, using Monte Carlo simulation**, for a maximum of 1250 days (5 years).
   In order to minimise the needed computational capacity (to make the simulations feasible), only "**significant loading events**" are taken into account in the following steps. Caprani defines these events as single trucks with a GVW over 40t or more than 1 truck on a bridge. We will see later that Enright adapts more advanced criteria.

3. **Load effects are generated** as follows:
   - Input: significant loading event truck combinations & analytical influence lines (analytical, or fitted polynomial for complex structure such as beam grid)
   - Step: 0,1 sec – max. approximately 0,45 m
   - The maximum load effect for each crossing (moment and shear) are registered

4. **Analysis of load effects**
   The load effects generated in the previous step must be analysed. Separate 'Bridge Loading Events' are considered, based on the amount of trucks contributing to the maximum load effect.
   - 1- and 2 truck events often occur, thus only the first 50 000 values are considered
     3, 4 truck events rarely occur, thus all output load effects are considered
     5, 6, 7 trucks events - not enough data from the simulation to analyse it
     As a solution Caprani suggests some sort of importance sampling, but in the current research just excludes such events.
   - The loading events are separated and parent distributions are fitted to each set of results with *maximum likelihood* fitting
     Reason: if parent distributions are different, the population is not identically distributed.
     Result: distributions for load effect *j* from bridge loading event *n* on bridge length *m*, for example moment at mid span from 2-truck events on a 20 m long bridge.
     Most often generalised extreme value distributions result, with a non-zero shape parameter suggesting an "unbounded" distribution. Caprani considers this unrealistic and finally uses a *Gumbel* distribution.

5. **Extrapolation**
   - If the maxima are selected using a '*hybrid conventional approach'*, i.e. independently of the type of loading event, the extrapolation to extreme values may contain several errors.
   - On the contrary, adapting *'composite distribution statistics'* is an adequate method of extrapolation when the parent distributions are independent but not identically distributed.

6. **Prediction analysis** – estimating variability of extrapolation result is carried out with the help of the *method of predictive likelihood.*

7. **Accounting for dynamic interaction**
   When determining the total lifetime load effect, Caprani adapts a sophisticated approach for inclusion of dynamic effect in a probabilistic manner with the help of *multivariate extreme value analysis.*

**Conclusions**

- The means of modelling has a significant impact on the resulting characteristic effect.
- Predictive likelihood and composite distribution statistics should be used to evaluate distribution of 100-year load effect (when the load effect is extrapolated).
- A dynamic allowance of 6%, much lower than in most codes of the time, was proven using probabilistic methods.

- If the dynamic allowance is reduced, based on probabilistic analysis, free-flowing traffic will not definitely be the governing case, thus conjunction models should be used.

### *Enright* [21]

Enright used data of five countries (Netherlands, Slovakia, Czech Republic, Slovenia, and Poland) including approximately 3 million measured vehicles. His work is organized in 4 articles corresponding to 4 relevant topics[22]:

- Importance of modelling the upper tail of the gross vehicle weight distribution
- Monte Carlo simulation for traffic loading, with details of data "cleaning" and attention to modelling axle configuration
- Characteristic maximum load effects for bi-directional traffic, accounting for the effect of lateral load distribution
- Simulation of traffic in two parallel lanes and resulting load effects

Enright describes two main strategies, which can be adapted to determine life-time load effects:

1. Distribution fits to load effects: calculate load effects of measured traffic and fit distributions.
2. Monte Carlo simulation based on stochastic input, for longer time period than measured

Enright adapts the second approach: vehicles are simulated from parameters based on statistical distributions (GVW, axle load as ratio of GWV, axle distance, flow and gap) similarly to the work of Caprani, and load effects are calculated. The simulation is carried out for over 1000 years, which is the main difference in comparison to the previous work of Caprani. After having generated the traffic flow, similarly to Caprani Enright selects '*significant loading events'* in order to reduce the necessary computational capacity. However, he adapts a more advanced approach and selects scenarios where the combined GVW exceeds a certain threshold. This site-specific and length-dependent threshold is determined by observing daily as well as yearly maxima load effects for a short simulation period, based on input with no threshold. An example of such a plot can be seen below.

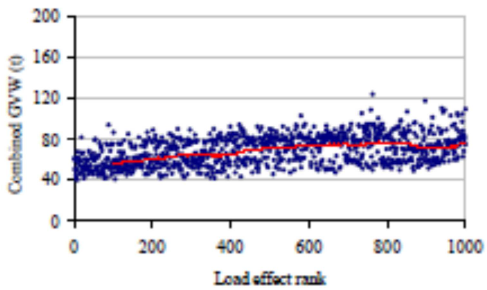


**Figure 13- Combined GVWs causing maximum load effects - Slovakia, 15 m** (Enright 2010)

The simulation model is calibrated by comparing load effects from the model to load effects from the measured WIM data.

As the simulation period is longer than the design life, the desired life-time mean maximum load effect can be obtained by interpolation. With this approach Enright excludes the need of extrapolation. He investigates multiple distribution types fitted to block maxima, concluding that a Gumbel distribution is overly conservative and deciding for a Weibull-distribution fitted to the upper 30% of the data.

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[21] Based on '*Simulation of Traffic Loading on Highway Bridges*' (Enright 2010)
[22] Articles: O'Brien et al. (2010); Enright & O'Brien (2013); Obrien & Enright (2011); Obrien et al. (2012)

Another advantage of this methodology is that insight is gained to the type of traffic causing a given load effect. The model is adapted to 2 lanes bi-directional and 2 lanes parallel traffic.

To the current knowledge of the author, and as claimed by Enright at the time of completing his work, this method is unique and is not described in other scientific works.

**Relevant conclusions**

- Both parametric and non-parametric methods for distribution fits to measured data are criticized. A semi-parametric approach is recommended and adapted in the dissertation.
- The result of the simulation and thus the obtained maximum lifetime load effect is sensitive to:
    - The extrapolation method of the GVW
    - Assumptions about vehicles with greater number of axles than measured
    - Very sensitive: modelling of axle spacing and wheel base
    - In case of loading events with multiple trucks, to the modelling of inter-vehicle gaps
- Variability of result is reduced by the long-run simulation
- Issues introduced by having to select distribution fits to load effects and by adapting appropriate extrapolation methods is overcome by long-run simulation

### 6.4.5 USA – Nowak, Czarniecki, Kozikowski

#### Nowak, Czarniecki [23]

Nowak was one of the main figures working on and influencing bridge codes development in Canada and the USA. In an article focusing on time variant reliability of steel girder bridges (Czarnecki & Nowak 2008) the researchers use as model for the loading input the load model developed for AASHTO LRFD code. The vehicle weight and the horizontal position of the vehicle on the deck are modelled as random variables, while the axle spacing and the ratio of load per axle is kept constant. The research focuses on system reliability analysis of steel girder bridges, therefore the lateral distribution becomes important.

#### Kozikowski [24]

In his PhD dissertation, Kozikowski aims to use WIM data collected on various sites of the USA to re-calibrate traffic loading for bridges. He uses recent data (2005-2007, 47'000'000 trucks) and compares the results also to those gained from measurements from Ontario (1970's, 9'250 trucks) which had served as a basis for the highway design code of the time. A specific focus of his work is the statistical description of *multiple presence events* (multiple heavy trucks on a bridge), including *correlation analysis*. This is however not made use of explicitly in the model but an assumption is made for which truck combination is relevant.

The research is focused on short- and medium span bridges: analysed spans were 9, 18, 27, 36, 67 m.

**Interpretation of original (1970's) measurements**: for each vehicle the maximum bending moment (positive and negative) and shear was determined. The ratio of these load effects to the load effect calculated from the design load of the codes was expressed and the cumulative probability distribution was plotted on a normal probability paper. The conclusion was drawn that the (relative) moments are not normally distributed and vary for different span lengths. Nevertheless the extrapolation for the design life was done assuming normal distribution. Values for 75 year maximum load effect were gained.

[23] Based on *'System reliability assessment of steel girder bridges'* (Czarnecki & Nowak 2006)
[24] Based on '*WIM Based Live Load Model for Bridge Reliability*' (Kozikowski 2009) – PhD Dissertation

**Application of new (2005-2007) WIM data**: the maximum load effects for a simple span (moment and shear) were recorded for <u>all trucks</u> per location. (Again, the values were divided by the load effects from the code loads.) Mean maxima for different return periods were tabulated. Results were also determined with light trucks removed (below a certain threshold). A sensitivity analysis was carried out to gain more information about the impact of very heavy trucks on the final result. It was shown that the distribution is highly sensitive to the removal of trucks – giving a practical visualisation of the tail sensitivity problem. Kozikowski checked configuration of extremely heavy trucks and concluded that they are most likely permit trucks (could be up to 5 times the legal weight limit). Finally extremely heavy trucks were included in determining the live load model, the exact reason for this decision is not elaborated however.

Three types of loading: low, medium and high were defined, based on the load effects resulting from measurements at various locations. The method of determining the thresholds is not described. For the following analysis, these cases were always treated separately.

Kozikowski claims that in order to fit a distribution to the load effects "extensive engineering judgement" would have been necessary, which he aimed to avoid. Thus he adapted a **non-**parametric method, namely the kernel density estimator, using normal distribution as kernel function and determining the bandwidth based on the best fit to the dataset. These fits were extrapolated from one year (measured data corresponded to one year) to 75 years using extreme value theory.

An example of the result can be seen in *Figure 14*. The red line represents the non-exceedance probability for the maximum load in 75 years ($P_{Non\text{-}Ex} = 1\text{-}1/(n_{trucks\,/\,year} * n_{years})$). The mean maximum value is the value of the instantaneous distribution belonging to this required level and the CoV can be determined based on the 75-year maximum distribution. The coefficients of variation are in the range of approximately 0,1 – 0,13.

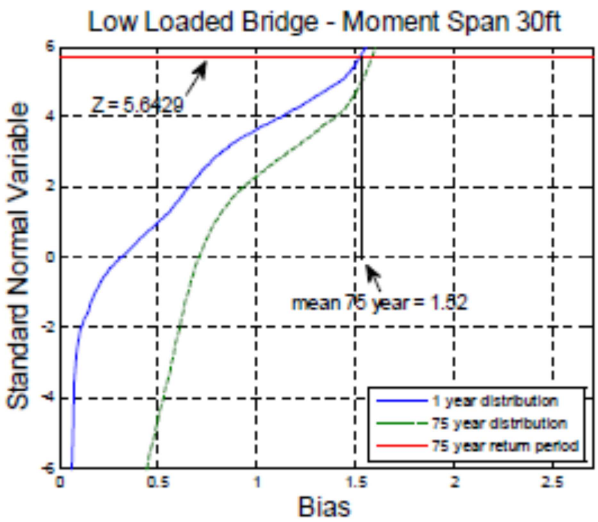


**Figure 14 – Load effect extrapolation** (Kozikowski 2009)

The values are all determined from one vehicle. (Thus it is not surprising to get ratios of load effect from WIM to load effect from design load above 1.) A crucial element of the approach of Kozikowski (based on Nowak) is therefore creating a model for multiple trucks on a bridge, denoted as *multiple presence events*.

The first step was the analysis of correlation, for which multiple presence events were filtered based on time of record and speed – resulting in events with headways smaller or equal to the bridge length. The correlation between "very similar" trucks was checked (same number of axles, GVW within 5%, axle spacing within 10% difference). In the original code, maximum load effect was based on two heavy side-by-side trucks. The correlation analysis proved that the assumption is conservative and that two fully correlated trucks are negligible. Thus, the multiple presence events analysed were reduced to two cases: 75 year maximum truck on the bridge, 1 year maximum truck and an average truck on the adjacent lane.

Dynamic load is based on previous research (NCHRP Report 368) with a mean value of 0,1 and CoV of 0,08. This is integrated with other uncertainties to arrive to new CoV-s for the load effects. It is noted that the CoV is larger for shorter periods.

Kozikowski performed analysis on steel girder bridges with various length and girder spacing. He used FEM calculations with a deterministic approach, moving two adjacent trucks transversally and finding the ratio of maximum load effect on the most heavily loaded girder to the load effect value obtained from analysis as a simple supported beam.

*"For the ultimate limit states, calculated reliability indices represent component reliability rather than system reliability. The reliability indices calculated for structural system are larger than for individual components by about 2. Therefore, selection of the target reliability level should be based on consideration of the system."*

As a result, Kozkowsky obtained a live load model for low-, medium- and high loaded bridges for various spans and design lives, in the form of a **multiplier** for load effect determined from loading according to code and a belonging CoV.

**Conclusions**:

- Probabilistic analysis based on use of FEM modelling and including loading data based on WIM measurements gave sufficient reliability indices for the tested structures, higher than those required, despite the increased load effects compared to the ones according to code. This is due to analysis in the form of a system.
- Unrealistic trucks may be removed, but no heavy vehicles can be discarded as they have a major impact on the extrapolation results. Removal of 0,03% of the heaviest trucks led to a 32% difference in the maximum load effect.
- Old truck data (Ontario, 1970's) cause more severe load effects than what was based on new WIM measurements, except for two very heavily loaded sights.
    - Kozikowski: concludes that quality of data is more important than quantity and that 1 year of measurements is not enough for such extrapolations
    - Comment of the author: the Ontario data was extrapolated in a very simplified way (assuming normal distribution). Thus, just because the mean values of the heavy trucks in Ontario are higher (also because only selected trucks were measured) should not definitely mean that the maximum live time load effect will also be higher or comparable.
- 1 year measurements are not enough to determine 75 year mean maximum, more yearly maximum values are needed.
- Time of record must have 0,01 sec accuracy for adequate headway distance

### 6.4.6 *Yoshida* [25]

Yoshida adapts a simplified method, compared to the researchers whose work is described above. His aim is, just as in the current work, to arrive to load probability distributions which can be used in the probabilistic analysis of a bridge. Yoshida considers only GVW and instead of WIM measurements, uses data from the Japanese bridge code. This code defines five traffic flow classes (A-E) and gives ratios of vehicle types (such as passenger car, small truck, …) in each class. More interestingly, the weight distribution of each type of vehicle is given in the code, approximated by log-normal distributions and truncated. This is a simplification but the impact of it is not quantified.

### 6.4.7 *Discussion*

The most advanced method currently seems to be that of **Enright** (2010).
Advantages - general:

- There is no need for extrapolation of load effects.
- The traffic scenario which causes a given (maximum) load effect can be shown.

Advantages for the case of short urban bridges would be:

- On short bridges, large axle weights are expected to have a higher impact than the overall GWV. It seems reasonable to opt for a strategy that avoids constructing an equivalent UDL..

Disadvantages - general:

- Extensive effort has to be made to model axle- and inter-vehicle distances
- It is expected that large computational capacity is necessary

Disadvantages for the case of short urban bridges would be:

- It is expected that a high number of measurements is necessary to be able to calibrate the Monte Carlo model. However, it is not definitely true that with the given number of measurements this MC model will be less precise than by simply extrapolating the traffic load effects.

The method of **Caprani** (2005) for the data analysis appears to be identical, while the simulation is carried out for a shorter time period. Thus, in case a maxima distribution of load effects is searched for, extrapolation will be necessary.

It seems reasonable to aim for adapting the methodology of these researchers, and when the necessary computational capacity is known (i.e. the time limit of the simulation is found), a decision can be made for adapting extrapolation or interpolation techniques for the load effects.

The method of **Kozikowski** is suited directly for probabilistic analysis, because the output is an extreme value distribution.

Advantages:

- Maxima distributions are gained, thus can be used directly in probabilistic analysis.
- There is no need to fit distributions to several vehicle properties and to combine them in relatively complicated software for simulation.

Disadvantages:

[25] Based on two articles: Yoshida & Akiyama (2011); Yoshida (2011)

- The multiple presence events are not modelled. It can reasonably be expected that such events would be dominant on a bridge, however only an assumption is made in the given research.
- The extrapolation  doesn't give any information about the type of loading event causing the maximum load effect, thus the result cannot be "understood" in a practical way.

Considering that the analysed urban bridges are of a span up to 20 m, the assumption that multiple presence events are not dominant may actually be reasonable.

## 6.5 Dynamic amplification factor [26]

### *6.5.1 Introduction*

The dynamic amplification factor (DAF) represents the ratio of the load effect on a bridge considering its dynamic behaviour to the load effect caused by an equivalent static loading. A distinction should be made between the dynamics of vehicles and of the bridge structure. (Steenbergen et al. 2012)

Eurocode includes this effect in the characteristic values of axle- and equivalent uniformly distributed loads. Therefore when comparing load effects on a structure resulting from a certain traffic load model based on WIM data (what the ultimate goal of the traffic loading analysis will be) with those caused by loads in the norm, this factor should be included.

### *6.5.2 Value of the dynamic amplification factor – summary of literature*

The research of TNO for Rijkswaterstaat (Steenbergen et al. 2012) includes a broad overview of literature on the dynamic amplification factor, consisting of a summary of articles and their evaluation. The starting point is that in previous research for RWS a DAF of 1.2 and 1.4 for very short spans had been used.

The main conclusions of the literature study are:

- DAF is a non-deterministic quantity and should be treated as a stochastic parameter
- The most relevant parameter is the road surface
- DAF is lower for higher static loads
- For combination of vehicles the DAF is lower than for individual vehicles
- All consulted research considers the DAF values in codes overly conservative. Applied values are, for example:
    - 1.05 in the European project ARCHES
    - 1.1 from simulations using stochastic traffic load- and road surface models

Furthermore, a comparison of static (weight bridge) and WIM measurements is carried out. For 3 different locations the ratio of WIM (dynamic) to static loads are:  0.99 / 1.0;  1.03 / 1.03; 1.044 / 1.052. (The first values represent the full population, the second correspond to data of trucks above 50 tons.)

The final decision is made for a DAF of 1.1 for global and 1.4 for local effects.

### *6.5.3 Discussion and chosen values*

The current work does not analyse in depth the dynamic effects, the decision for the DAF is therefore based on the cited TNO report.

[26] Based on TNO-060-DTM-2011-03685-1814 'Algemene veiligheidsbeschouwing en modellering van verkeersbelasting voor brugconstructies' (Steenbergen et al. 2012)

$\mu_{DAF} = 1{,}1$

$\sigma_{DAF} = 0{,}05 – 0{,}01$

The literature study points out that for the combination of vehicles the DAF is lower than for individual vehicles. Therefore the values, which in the TNO report were used for spans from up to 20 meters might be somewhat non-conservative. It is recommended to further investigate this aspect.

# 7 Traffic Loading Analysis and Simulation - Strategy

## 7.1 Considered strategies

### 7.1.1 Goal of traffic loading analysis

The goal of the traffic loading analysis is to gain stochastic input for probabilistic structural reliability analysis. The input may be in the form of a **load** or a **load effect** described by an extreme value distribution.

### 7.1.2 Load effects

The nature of load effects strongly influences the approach to be taken in the use of WIM data. Firstly, static and dynamic behaviour of the bridge can be separated. The focus of the current graduation project is static loading, but dynamic amplification of the traffic loading is to be taken into account, as described in *Section 6.5*.

Considering only static loading, the following categories of load effects caused by traffic load can be considered:

- Maximum life-time effects
    - Global: ex. bending moment on a structure caused by several trucks on a bridge;
    - Locals: ex. bending moment of a steel plate of a bridge, caused by one wheel load;
- Cumulative effects: fatigue, which is caused by several trucks passing over the bridge over a long (i.e. months, years) period of time creating a cumulative damage

These situations are to be treated differently when setting up a load model, whether semi-probabilistic or fully probabilistic.

For maximum lifetime load effects, such as for example a simple bending-moment or shear capacity, the ***life-time maxima distribution function*** will be relevant in probabilistic analysis. For fatigue, all passing trucks are of interest as failure is a result of a cumulative effect. From this point on the focus is on life-time maxima load effects.

### 7.1.3 Life-time reliability

One of the main challenges in the application of load measurements is that the maximum load / load effect for the life time should be considered. For the reliability analysis the input for the elements of the $Z = R - S$ reliability function should be the distributions referring to the design life or remaining life.

### 7.1.4 Maxima distribution of load effect

#### Effect caused by an individual axle

In this case, the relationship between the load and load effect is relatively straight forward. It can be expected, that the life time maximum load effect will be caused by a life-time maximum load (i.e. axle weight). The most simple case is the heaviest axle causing the largest local moment / shear in a part of the structure.

However, taking the axle load as a point load is a simplification, thus here the following factors, which also have a stochastic nature should be taken into account:

- Wheel contact surface
- Dynamic amplification

Thus, it may be overly simplified to use the life-time maxima distribution of the axle weights without considering the influence of the stochastic nature of the factors mentioned above. Nevertheless, for such a case it seems reasonable to adapt a strategy of the following steps:

1. Fit a statistical distribution to axle weights with attention to the tail modelling.
   Possibly adapt a peak-over threshold or block-maxima method to arrive to extreme value distributions. For details of a possible approach refer to *Appendix I.*
2. From the maxima of load, calculate directly the maxima of load effect.
   In this step, load effect uncertainty and dynamic amplification should be taken into account.

### *Effect caused by one or more vehicles*

When effects caused by one or more vehicles on the bridge are of concern, the relationship between the load and load effect is less straight-forward. In this context load is understood as the vehicles, each described by several characteristics (axle distance, axle weight, gross vehicle weight, etc.) and their relation (distance) also described by stochastic parameters. It is not possible to define the maximum load directly: would this be one very heavy truck on the mid-span, or two "medium" heavy trucks on a bridge? A truck with short axle distance may cause higher bending moment than a heavier truck with a larger axle distance, etc.

As described in detail in *Chapter 6, t*he values to be used according to design codes are developed based on a combination of statistical analysis of measured vehicle- and / or axle weights and on the evaluation of load effects, during the process of "code calibration". Australian, Canadian, American and European codes tend to model traffic loading with one or two major axle groups and a uniformly distributed load. The latter model represents a sequence of minor vehicles . The point loads are meant to represent local effects and, in combination with the distributed load to create representative load effects. (O'Connor & A.Shaw 2000)

In the case of a probabilistic analysis, the main point of interest is the life time maxima distribution of the load effect. Thus, it is not definitely necessary to come up with an intermediate step of equivalent loading. When searching for maximum life-time load effect, whether as input for semi-probabilistic or fully probabilistic calculation (in the 1st case the main question is a value with a given exceedance probability, in the 2nd case a maxima distribution), usually[27] traffic loading simulations are applied for various spans.

Focusing on the approach using traffic simulation, the main differences in the methods applied in literature are whether the input traffic consists of observed vehicles, such as in Steenbergen et al. (2012) and Kozikowski (2009), or whether also non-observed vehicles are generated, based on statistical properties of the (relevant) parameters describing a vehicle, such as Caprani (2005); Enright (2010). The two approaches are represented in *Figure 15*.

[27] In Steenbergen et al. (2012) one of the main strategies is to use statistical parameters of gross vehicle weights (GVW) to come to an equivalent uniformly distributed load. An analytical model is used which is less likely to be suitable for a short bridge where the axle distance distribution and the exact probabilities of 2- or more truck crossings have a larger influence.

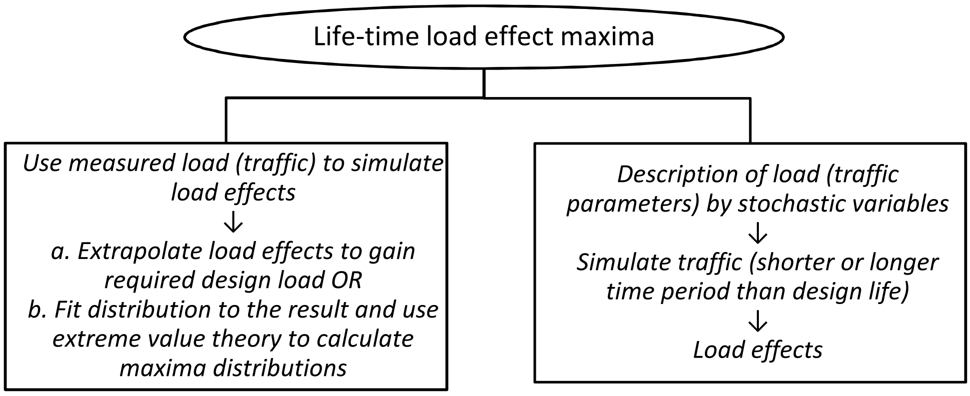


**Figure 15 - Approaches for traffic loading simulation**

## 7.2 Adapted strategy

### *7.2.1 Framework*

Based on review of literature and considering that the application is to be used for short-span bridges, an approach is selected which consists of the simulation of several vehicles based on analysed WIM measurements, determining the load effects of the several years traffic and finally analysing the block-maxima values. The approach is schematised in *Figure 16*.

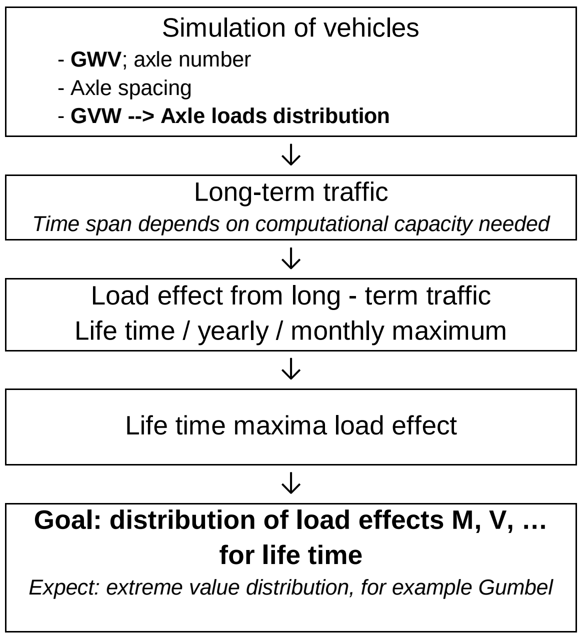


**Figure 16 - Approach for global life time load effects (caused by one or more vehicles)**

Once the general approach has been selected, the details are worked out. Two options are considered and schematised in *Figure 17*, from which the 'Strategy A' will be finally selected. 'Strategy A' is based on fitting statistical distributions to gross vehicle weights within one vehicle category. Vehicle categories can be defined in multiple ways: by axle number, by the 'statistical category' (available directly from WIM measurements), by sub-categories from the WIM data or by for example *NL-WIM* categories based on an algorithm of Rijkswaterstaat. The other relevant parameters of a vehicle are the axle weight and axle distance. In order to take into account the relation between the distance and axle weight (on closer spaced axles the GVW is distributed more evenly) in a simple way, a full correlation is assumed and a "vehicle parameter" is defined as a vector consisting of the ratio of the GVW per axle, and the axle distances.

If sufficient amount of data is available, the population of vehicle parameters is assumed to adequately represent the traffic. The sampling for creating the traffic is suggested to be done separately from the GVW distributions and the population of vehicle parameters, per category and then to be coupled for determining the exact values of axle loads.

**Strategy A**
**1. Fit GVW per category**

→ *What is a category?*
**a. per axle number**
**b. vehicle class**:
**b.a.** "statistical" category *(in R'dam WIM: EUR13)*
**b.b.** *NL WIM (RWS – highways; "precise algorithm")*
**b.c.**"make" categories

↓

Sampling:
- Axle number or category empirical (%of total)
- Draw values from GVW belonging to the given category
→ Result: GVW and axle number or GVW & vehicle category

**Strategy B**
**1. Multivariate GVW**

Fit a bivariate, multi-modal distribution to GVW - category variables
→ *What category?*
**a.** GVW - axle number *(as Enright)*
***a.a.*** Only to tail, rest is empirical (*as Enright)*
***a.b.*** Fitted for full range
**b.** GVW - vehicle class

↓

Sampling:
- Sample from bi-variate distribution
→ Result:
GVW and axle number or
GVW & vehicle category

↓ ↓

**2. Create sample space for axle weights & distances**

Inside one category, assume no correlation between GVW and rest of the parameters *

**2.1.** Axle distances - Empirical distribution, per category

**2.2.** Axle weights

↓ ↓

**2.2 a.** Empirical distribution of %GVW / axle

**2.2 b.** Some axles are maybe independent of / somewhat correlated to GVW (e.g. driver cabin) → separate distribution

↓

→ *How are 2.1 and 2.2 related?*

**a.** Not correlated at all
→ *This is an incorrect assumption, close axles are highly correlated* (e.g. Enright)
**b.** Fully correlated
→ *Simple assumption, makes it easy to draw from distribution*
**c.** Correlation depends on distance
→ *as Enright* → *too complicated for now, likely not enough data*

↓

Sampling:
- Per category, sample from vehicle properties
- 2.2.a % of weight
- 2.2.b for some axles sample from statistical distribution, for the rest from % of GVW

↓

Output: GVW, Axle number, Axle load

**Figure 17 - Considered approaches for traffic simulation**

Strategy B, similarly to the method of Enright (2010) would be to fit a multi-modal bivariate (or multivariate) distribution to the measured GVW-s and the relevant vehicle parameters, typically for example the number of axles. The sampling of the GVW and the correlated properties could be done from this joint distribution function.

### *7.2.2 Steps*

The proposed traffic load analysis and simulation therefore consists of the following main steps, which are described in detail in the respective sections of *Chapter 8 and 9*. The steps and their relation is also visualised in the flow chart of *Figure 18.*

1) WIM data is analysed and a sample space is created for the simulation
   Described in detail in *Section 8.1*
2) Traffic is simulated, where each truck is described by its:
   a. Gross vehicle weight (GVW)
   b. Category (by axle number or statistical category)
   c. Property P (axle distances and %of GVW / axle)

   Described in detail in *Section 8.2*
3) Load effects (LE) are calculated *from unit-weight trucks, for all properties (P) of the sample space*
   Described in detail in *Section 9.1*
4) The results of 2) and 3) are coupled: load effects are assigned to the simulated trucks in each category, based on their known truck property and GVW

$$\mathrm{LE}_{\mathrm{sim,Pi}} = \frac{\mathrm{GVW}_{\mathrm{sim}}}{\mathrm{GVW}_{\mathrm{unit\ Pi}}} \mathrm{LE}_{\mathrm{Pi}}$$

   Described in detail in *Section 9.2*

5) The results of 4, simulated load effects for several years, the monthly, yearly or even 15-yearly maximum values can be gathered and analysed
   Described in detail in *Section 9.3*

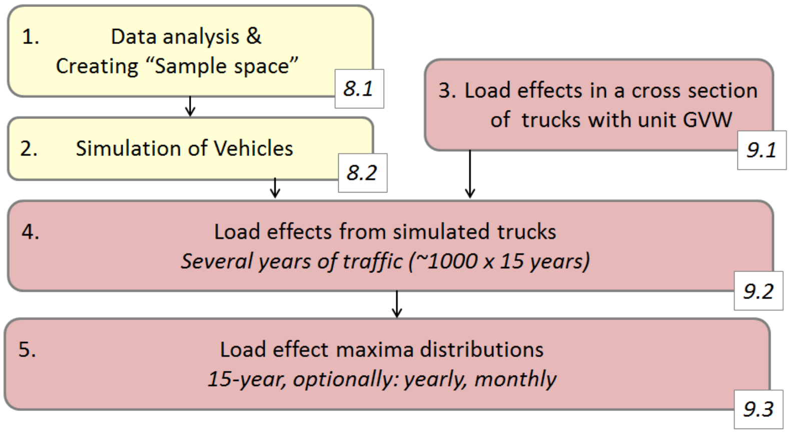


**Figure 18- Traffic loading analyis and simulation - Flow chart of chosen approach**

## 7.3 Fundamental assumptions

The proposed model for traffic loading and load effect analysis is based on some fundamental assumptions.

a. It is assumed, as in Steenbergen et al. (n.d.) that on a short bridge one single heavy vehicle will cause the extreme load effect and not multiple vehicles present on the bridge. The method is not directly applicable for multiple vehicles present on a bridge. Thought should be given to the cases where a more complex simulation model is necessary:

- Multiple vehicles in the same lane
- Multiple vehicles in parallel lanes

b. Dynamic effects of vehicles are not taken into account. Dynamic amplification shall be considered separately, as a stochastic variable (DAF) in the fully probabilistic calculation.

c. The proposed traffic loading simulation is only directly applicable if the relation between the load *(i.e. Q, GVW*) and the load effect (*M, V*) is linear.

d. The measured trucks properties (P) are assumed to represent the population of expected traffic. (i.e. it is assumed that the "worst case loading" can be found accurately by simulating only trucks with axle distributions which have been recorded already and extrapolated vehicle weights).

e. The accuracy of WIM measurements is considered as acceptable.
It is noted however, that observation of the measurement data suggests significant differences between measured loads, often over 10% differences between values for the same axle. For the current work, the average of these two values is used.

f. The initially proposed model was based on the assumption that after the vehicles are distributed to categories, there is no further correlation present between the truck property P and the gross vehicle weight GVW.

In the evaluation process comparing measurements with simulation results this assumption proved to be incorrect, therefore the model was adapted. This is described in detail in *Section 8.2.3.*

## 7.4 Available data

The background of the measurement program that was carried out in Rotterdam has been summarised in *Section 6.2.5*

As described there, measurement data of two months, 53 853 heavy vehicles was used as a basis for analysis in the current thesis.

### *7.4.1 Form of data*

The form of data available for use in the current thesis was a *.mat* file in MatLAB®. The columns relevant for the further analysis are:

- Number of axles
- Gross vehicle weight (2 measured values and an average given)
- Axle distances
- Axle load (2 measured values and an average given for each axle)

Furthermore, the use of vehicle category, vehicle sub-category and vehicle statistical category were investigated but does not contribute to the final result of the current thesis. It can nevertheless be a useful concept in further analysis.

### *7.4.2 Filtering measurements in Rotterdam*

Invalid data had been filtered out by the company performing the analysis. Further filtering, which resulted in the above mentioned 48 586 vehciels, was done based on the following criteria:

- Sum of axle loads and gross vehicle weight differ by over 150%
- Smallest axle distance is below 1.4 m.

Such measurements were eliminated from the dataset.

# 8 Analysis of WIM Measurements and Simulation of Traffic

## 8.1 Data analysis and creating sample space

### *8.1.1 Goal*

The goal of the data analysis is to create a *sample space* from which it will be possible to generate traffic using a Monte Carlo simulation model.

The sample space contains the following information:

- **Composition of traffic**, defined as the ratio of vehicle categories[28] within the total measured traffic.
- Statistical **distributions of gross vehicle weight** within each category, in the current work modelled by Gaussian mixture distributions.
- Within each category, the matrix of recorded **vehicle properties**, defined as a vector containing axle distance in meters and the ratio of GVW per each axle.
  Sampling from properties is done from empirical data, thus no continuous distribution functions will be used here.

In an adapted version of the model, which will be described in *Section 8.2.3*:

- The **threshold GVW values** splitting the truck population within a vehicle category to sub-classes.

  As will be described in *Section 8.2.3*, the adjusted model will take into account in an empirical way that within each category, there is a correlation between the vehicle property and the GVW. The vehicle properties are split into *blocks* based on the GVW with which they occurred. Therefore a fourth piece of information is necessary: the chosen threshold values.

### *8.1.2 Practicalities*

#### *Data preparation*

The WIM data is stored in a *.mat* file format, in an array of the size 48586 x 109. This data had originally been "cleaned" by TNO, meaning that un-realistic measurements were discarded.

To simplify working with categories, the data is first split to a *cell array* of *c x 1* cells, where *c* is the number of different categories. For example *c = 9* if axle categories 2 to 10 appear and c = 13 for statistical categories. Each cell of the array contains a matrix of $n_{ci}$ x 109, where $n_{ci}$ is the number of registered vehicles per category $c_i$. *Figure 19* gives an example of the resulting cell array in MatLab®.

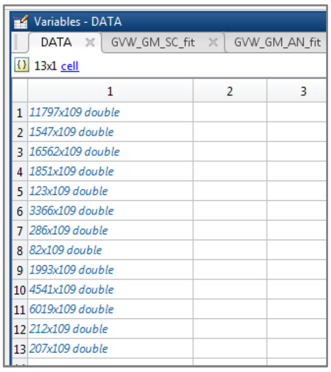


**Figure 19**
**Data split by statistical categories**

The MatLab® script '*DatasplitToCellArray*' performs this process. From this point on *"script"* refers to MatLab® script.

#### *Storing information for sample space*

As described in detail in *Section 8.2.1*, the sample space for the traffic simulation will consist of four parts. The information is stored in cell arrays, the first dimension of which corresponds to the total number of categories. Cell arrays of the sample space are described shortly below. The way of arriving to them is detailed in the following sections.

[28] Category: can be axle category, statistical category or other.

***CatRatio*** gives information of the ratio of each category of vehicles within the measured sample

***GVW_GM_AN_P*** contains the matrixes of “properties” where each row represents a measured vehicle: the first 11 columns correspond to axle weights as ratio of total weight, columns 12 – 21 correspond to axle distances. The column 22 will possibly contain vehicle distances.

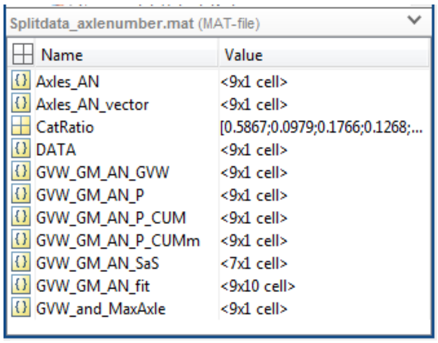


**Figure 20- Storing information for the sample space**

***GVW_GM_AN_P _CUM*** and ***GVW_GM_AN_P _CUM m*** are similar to *GVW_GM_AN_P,* but the axle distances are expressed as a cumulative value. The second matrix contains these distances in metres. These adjustments are useful for the code calculating load effects.

***GVW_GM_AN_fit*** contains fitted Gaussian mixture distributions to the GVW, from 1 to maximum 10 components, per category.

***GVW_GM_AN_SaS*** contains the selected Gaussian mixture fit for each category and will serve as the “sample space” for the simulation. It is not a must to have this array, as values can be selected also from *GVW_GM_AN_fit* directly.

Other elements of the array are used for comparing measurements and simulations, or for creating the fits. These are:

***Axles_AN*** and ***Axles_AN_vector*** are cell arrays contain the measured axle loads divided by vehicle category, the first stores the information in matrices, while the latter in vector format in order to simplify plotting.

***GVW_and_MaxAxle*** contains the measured GVWs coupled to the heaviest axle of the given vehicle, the latter expressed as a fractile. This information was necessary during the process of detecting the relation between the GVW and truck property within a vehicle category.

#### *Ratio of categories in total traffic*

The measured data has been split to categories in the previous step, now the ratio of categories in the total measured traffic is determined and saved as *CatRatio* to the same *.mat* file as where the “split” data is already stored.

The short script *‘CategoryRatios’* performs this process.

### *8.1.3 Probability distribution fits to gross vehicle weight per category*

Based on study of literature it seems reasonable to approximate the vehicle weight distributions by a Gaussian mixture distribution model. The various fitted distributions are stored in an array *GVW_GM_AN_fit* and saved. The script *‘GaussianFit_PerChosenCategory’* performs the fitting process. This step is a relatively simple way of creating several fits for the sub-groups of data. The appropriate fit for each category should be selected, GVW distributions of different vehicle categories are described with number of fits than the others.

#### *Selecting the appropriate fit*

1. As a first estimate this can be done by comparing histograms and plots of the mixture distributions, such as in *Figure 21*.

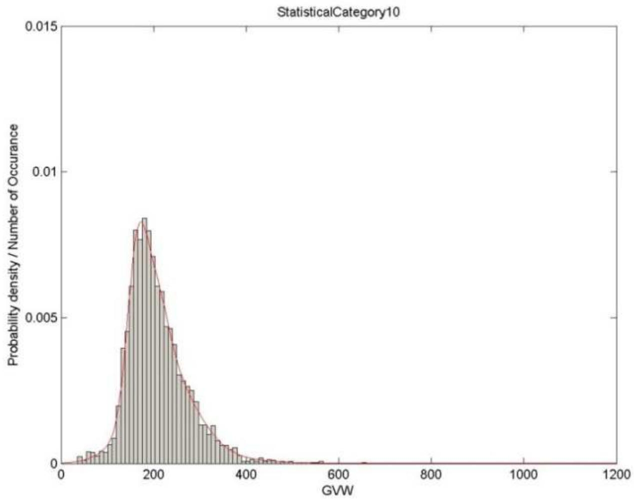


**Figure 21 - Vehicle Statistical category 10 - Mixture distribution fit and histogram of GVW [kN]**

2. As a second estimate, the cumulative exceedance frequency of the measurements can be plotted against the exceedance probabilities of the selected mixture distribution, such as in *Figure 22*. This way, the extremely high occurring values can be compared more accurately to the simulation.

The **tail data** is expected to have significant influence on the extreme values of the load effects – the highest gross vehicle weights will likely cause the highest load effects (with the other influencing factor being the truck property *P*). Therefore the fits should represent as closely as possible the extreme vehicle weights. In order to take this into consideration when choosing the number of Gaussian mixture distributions, the cumulative exceedance probabilities of the measured GVW data is plotted against (1-f($GVW_{fit}$)) for each category, where f($GVW_{fit}$) is the cumulative probability distribution of the Gaussian mixture distribution fitted to the data. An example is shown in *Figure 22.*

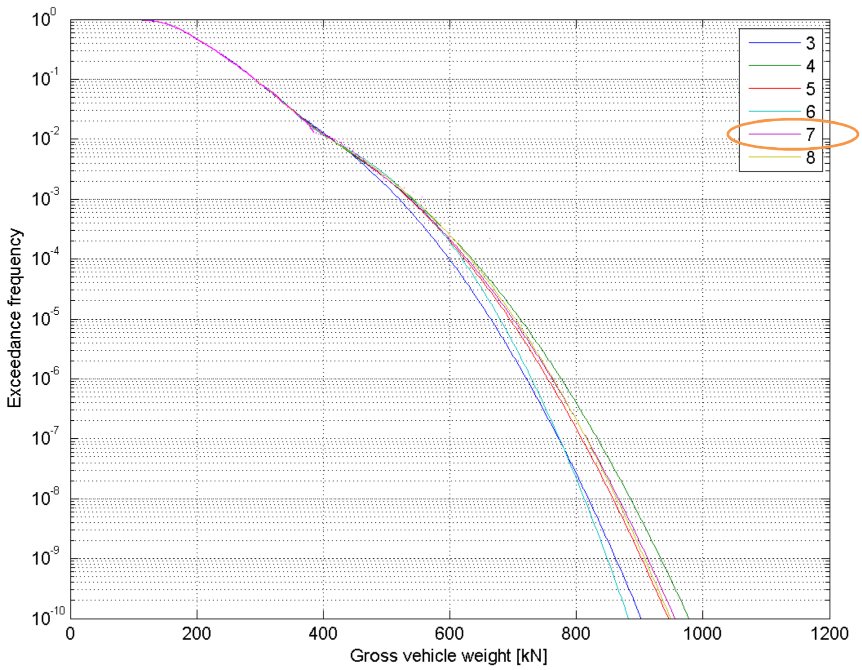


**Figure 22 – Vehicle Statistical category 10 – Mixture distribution fit and exceedence frequencies of GVW**

3. A more advanced option is to pay extra attention to the right tail of the distribution. This can be done with the help of truncated maximum likelihood estimation (O’Brien et al. 2010; Steenbergen et al. 2012)

The selected number of mixture distributions is summarized in *Table 1.* Figures with the distributions are added to *Appendix E* .

**Table 17 – Number of mixture distributions selected to describe GVW distributions per category**

| Axle Category | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Number of Gaussian distributions | 10 | 4 | 5 | 10 | 6 | 9 | 4 |

### *Is there a maximum GVW? - Truncation*

One may argue that a given GVW will never be exceeded, especially in a residential area. There are legal weight limits imposed which are likely to be exceeded, but it can be reasonably assumed that the exceedance will not be above a certain limit. In statistical terms this situation is represented by a truncated distribution.

Considering the two aspects of GVW-modelling mentioned above: the modelling of tail distribution and truncation of the fit the effect of these assumptions on the final load effect can be checked. Therefore four possibilities of modelling the gross vehicle weights can be considered, as visualised in ***Error! Reference source not found.***.

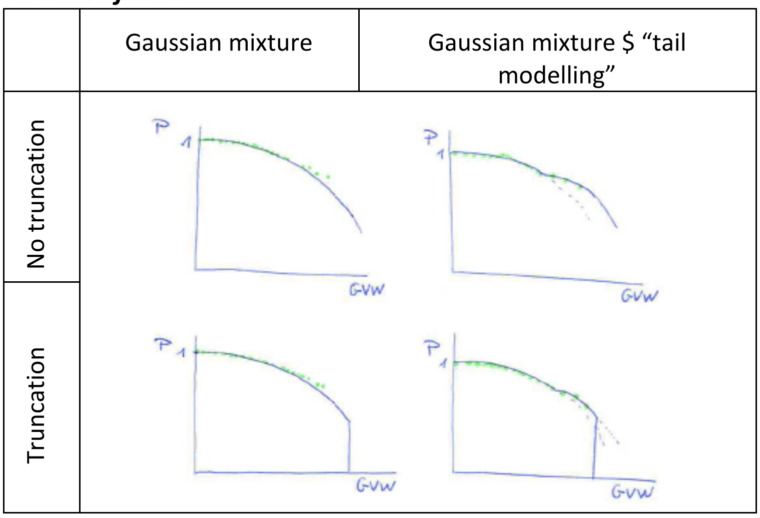


**Figure 23 - Fits to GVW -effect of tail fitting and truncation**

## *8.1.4 Vehicle property analysis and sample space*

### *Property - initial calculation*

For each recorded vehicle, axle distance in meters and the ratio of GVW per axle is taken and collected per category in a *Property Matrix*.
One such set of data is considered as a property $P_i$; all properties $P_i$ are recorded in subsequent rows of a matrix, per category.

The script *'AxlesInfo_PerChosenCategory'* performs this process, the matrices are saved in cells of the array *GVW_GM_AN_P.*

The truck properties are then transformed to cumulative axle distance. This way the algorithm for „running the trucks over the bridge” will be more simple. Moreover, the distances are converted to meters as in the measurements they are given in cm. The script *'AxlesInfo_Cumdist_meters'* performs this process, the results are saved in the array *GVW_GM_AN_P_CUMm*.

In the latter steps it will be shown that this model should be further adapted.

## 8.2 Traffic loading simulation

### 8.2.1 Goal

The goal of the traffic simulation is to obtain a number of trucks in the lifetime of a bridge, which are described by their properties relevant for calculating load effects. These properties are finally chosen to be the *Category*, the *Property* and the *Gross Vehicle Weight*.

### 8.2.2 Traffic simulation – Strategy

*Simulating traffic with full information*

The "crude" result would be the total traffic flow, expressed by:

- Axle loads [kN] *($A_{i,j}$)*
- Axle distances in [m] or [cm]

$$\overline{T_c} = \begin{bmatrix} A_{1,1} & A_{1,2} \cdots d_{1,1} & d_{1,j} \\ A_{2,1} & \ddots & \vdots \\ \vdots & & \\ A_{n,1} & \cdots & d_{n,j} \end{bmatrix}$$

**Figure 24 - Matrix of simulated traffic belonging to vehicle category *c* – Crude**

*$A_{i,j}$ is axle j of truck nr i; $d_{i,j}$ is axle distance between axle j and j+1 of truck i*

To reach this, the first step is to simulate the traffic (per category), where each truck is described by a row consisting of property P, that is:

- Axle loads, expressed in % of GVW ($a_{i,j}$)
- Axle distances in [m] or [cm]

The output of the traffic simulation could be a matrix containing this data, as visible in *Figure 25*.

$$\overline{P_c} = \begin{bmatrix} a_{1,1} & a_{1,2} \cdots d_{1,1} & d_{1,j} \\ a_{2,1} & \ddots & \vdots \\ \vdots & & \\ a_{n,1} & \cdots & d_{n,j} \end{bmatrix}$$

**Figure 25 - Matrix of simulated traffic belonging to vehicle category *c* - Normalised**

Each row of the simulated property matrix could then be coupled to a gross vehicle weight simulated from the Gaussian (or other) distribution model, taken from the sample space and depending on the category of the truck. If the simulated GVW per category is denoted as $\overline{\mathrm{GVW_c}}$ , using the descriptions above:

$$\mathrm{A_{i,j}} = \overline{\mathrm{GVW_c}}(\mathrm{j}) \cdot \mathrm{a_{i,J}}$$

However, this information is not all necessary and finally a more economic method is applied.

*Simulating traffic with reduced information*

Working with load effects of unit-weight trucks and assuming that a single heavy truck will be the decisive loading situation, it is not necessary to use all information that „describes" a property $P_i$. *An index* can be assigned *to each property,* from now on is denoted as ***Property index***, reducing the amount of data simulated and thus resulting in a computationally less expensive simulation.

The ***output of the traffic simulation*** will be the following matrix:

$$\overline{\mathrm{P_{cR}}} = \begin{bmatrix} \mathrm{P_{sim\_1}} & \mathrm{GVW_{sim\_1}} \\ : & : \\ \mathrm{P_{sim\_n}} & \mathrm{GVW_{sim\_n}} \end{bmatrix}$$

**Figure 26 - Matrix of simulated traffic belonging to vehicle category *c* - Reduced info., final**

*$P_{sim,1}$ is the property expressed with an index (integer)*

### *Number of trucks simulated*

The ideal case is to simulate several years of traffic, if the load effect from these could be calculated and several 15-year maxima (considering the typical requirement of NEN 8700 for existing bridges) could be registered, this data (i.e. population of 15-yar maximums) could be used directly to describe the load effect maxima distribution. If this is not possible, extreme value analysis will be necessary to arrive from block-maxima (such as monthly maxima) to 15-year maxima functions.

The starting information is that 25 000 trucks correspond to one month of traffic (from measurements). This can easily be changed for cases with different traffic flow.

**Table 18 - Number of trucks**

| years | 1/12 | 1 | 15 | 150 | 1500 |
|---|---|---|---|---|---|
| nr trucks | 2,5E+04 | 3,0E+05 | 4,5E+06 | 4,5E+07 | 4,5E+08 |

*5*10*$^{7}$ trucks, corresponding to 166 years of traffic can easily be simulated and saved in one *.m file*. The computation time is approximately two minutes on a normal laptop thus clearly not a bottleneck. Using an additional *for loop* the simulation can be done multiple times and saved in either separate columns of a matrix or in separate *.mat* files. In this way, 100 or more times 15 years of traffic can be simulated, thus it is expected that the necessary number of simulations for an accurate 15-year maxima distribution can be reached.

### *8.2.3 Simulation results versus measurements*

#### *Relation of vehicle property and gross vehicle weight*

After having run the traffic simulation based on the assumption 'f' of *Section 7.3*, namely that after having distributed the vehicles to categories the vehicle property and the gross vehicle weight is not further correlated, the measured and simulated axle weights were compared. An example of the exceedance-probability plot on a logarithmic scale is shown in *Figure 27.* A significant difference can be observed, thus it is necessary to adjust the model.

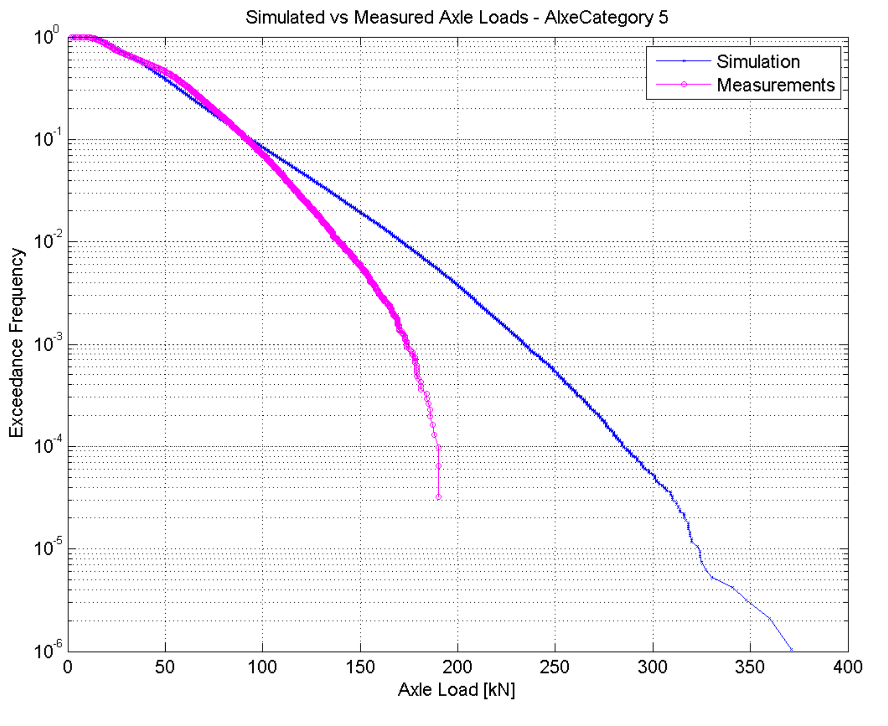


**Figure 27 - Comparison of measured and simulated naxle loads - Initial simulation model**

Before adjusting the model the reason for the deviation should be understood.  An axle load within a vehicle category is created in the simulation as the product of two numbers: the simulated GVW and the simulated axle weight ratio. The first value, the gross vehicle weight is based on sampling from a fitted dsitribution, thus the measurement and simulation results don't deviate significantly in case the GVW fit is correct. When creating the simulation strategy, as described in *Section 7.3,* the assumption was made that within one category there is no further correlation present between the GVW and the truck property. Now this assumption is challenged and the validity is further investigated.

Highest axle loads are caused by a combination of a high GVW and a high axle load ratio. For example, within axle category 2 there  is a measurement of a 5.5 tonns vehicle with axle weights 5.2 and 0.3 tonns respectively. This results in an axle property of [0.95 0.05 ... $d_1$], (where the first two elements refer to the ratio of GVW per axle and $d_1$ to the axle distance). If, instead of a 6-ton truck this property gets „coupled" during the simulation with a 30-ton truck, it would result in an extremely high axle load of 28.5 tonns (~285 kN). It is now disregarded that this measurement seems irrealistic and is very likely to contain an error.

The question arises whether there is a correlation between the heaviest axle load and the GVW. This is investigated with the following steps:

1. The heaviest axle of each observed vehicle, expressed as ratio of GVW is plotted against the measured GVW.
2. For a better estimation, a multi-modal bi-variate Gaussian mixture distribution is fitted to the two variables (maximum axle load per truck as % of GVW and GVW). As a rough estimate, a mixture of three bi-variate normal distributions is used.

   The scatter plots are extended with contour lines of the mixture distribution, visualising the correlations present. Surface plots are also created.

An example of the data visulaisation can be seen in *Figure 28, Figure 29 and Figure 30*. A negative correlation is visible, implying that on heavier vehicles the load is more evenly distributed between the axles, the heaviest axle carries a lower ratio of the GVW than for some of the lighter vehicles.

However it is not correct to speak of one single correlation coefficient because the data is not normally distributed.  If a multimodal bivariate normal distribution (Gaussian mixture for two variables) is fitted, the correlations within each bivariate distribution can be defined.


**Figure 28 - Scatter plot of measured GVW and heaviest axle, Vehicle category 5**

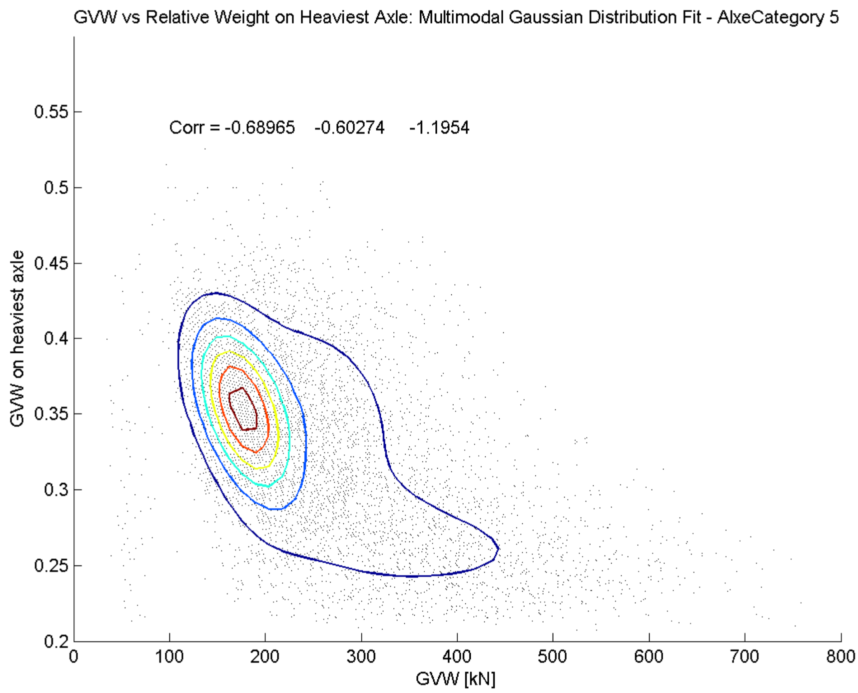


**Figure 29 - Scatter plot and contour lines of fitted mixture distribution GVW - heaviest axle, Vehicle Cat. 5**

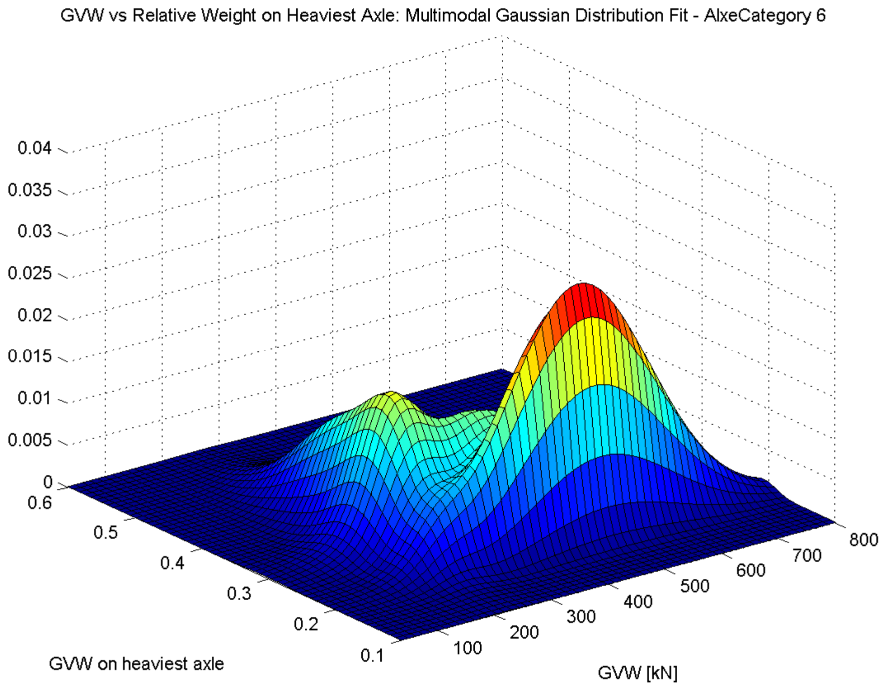


**Figure 30 - Surface plot of fitted mixture distribution GVW – heaviest axle, vehicle category 5**

It has been concluded that the assumption (f) made in *Section 7.3* does not sufficiently approximate reality, therefore the model has to be adjusted.

### *Possibilities to adjust the simulation model*

It has been concluded that the vehicle property cannot be considered independent from the gross vehicle weight of trucks within one category (whether categorised based on number of axles or by statistical categories).

This relation is considered in the model of Caprani (2005) as well. Caprani further divides axle categories to 50kN (5 Ton) intervals and within each interval uses different distributions for the axle weight ratios. In the current model this approach cannot be directly adapted because for the axle properties (%GVW and axle distance) discrete data points, an empirical sampling from measured properties is used instead of continuous statistical distribution of axle loads as in the model of Caprani.

Some possible approaches to **consider the observed relation between the GVW and axle load ratio**, resulting in the simulation of more realistic axle loads are listed below and a strategy adapted for the model is chosen.

1. Split the full data in sub-categories based on their heaviest axle or based on various existing classification models including more classes. Fit statistical distributions to the gross vehicle weights per sub-category and collect the truck properties per sub-category to form a sample space.

   The simulation is then carried out in the same way as in the proposed original model, but with more categories and the assumption (f) of *Section 7.3* is kept for the new classification.

   <u>Advantages</u>:

   - It is not excluded that a vehicle property appears with a high GVW while due to the more classes, the relation between the GVW and the type of truck is more realistic than for example when divided only by vehicle numbers
   - The simplicity of the load effect calculation can be kept

Disadvantages:

- When splitting to more sub-categories, the amount of GVW data-points per category decreases, thus the probability distribution fit may be less precise.

2. Split only the vehicle properties in sub-categories based on the GVW they occurred with, for example by 50 or 100 kN blocks. Use the GVW distribution fit to the full dataset within the category for the simulation.
An example is visualised in *Figure 32* and explained in the following sub-section.
    a. Once a certain GVW is simulated, couple it with a property from the interval where this GVW falls.
    b. In order to allow for a given "truck property" to appear with a higher GVW, the interval from which properties are sampled can be larger than the GVW interval in which the simulated value falls. For example in case of choosing intervals of 100kN, a simulated vehicle falls in the interval [200 300[ kN and it is coupled with a random property registered with trucks falling in the [100 400[ kN interval.

Advantages:

- The simplicity of the load effect calculation can be kept

Disadvantages:

- a. A truck property near the upper value of an interval cannot appear with a GVW value which is higher than measured. For example a property belonging to a 9.9 ton vehicle cannot be coupled with a GVW of 11 tons.
- b. The original ratio of vehicle properties is not kept in the sampling.
*This could somehow be adjusted → by some sort of constraint on the sampling to keep the ratios*

3. A maximum axle load can be "artificially" imposed, for example by re-sampling from the properties when an axle load above a certain threshold occurs.

Disadvantages:

- Calculating the axle load in each simulation step significantly increases the time necessary for the simulation.
- Re-sampling distorts the original ratio of properties unless it is corrected for in the model.

4. It may be possible to sample from a correlated multi-modal, multivariate distribution such as the fits to the heaviest axle – GVW variables.

Advantages:

- This may be the mathematically correct way to consider the relation between the truck property and the GVW

Disadvantages:

- Sampling from multivariate correlated data when one of the variables is discrete is a complex procedure.
- It cannot be said that the only or that the most relevant relation is between the heaviest axle and the GVW. Further correlations may be present with the other axles and / or axle distances as well.

### *Adjusted simulation model*

After considering the possibilities, the strategy 2.b, sub-division to blocks is taken further for the current model.

First a GVW block size for the sub-division of properties is chosen. This number should be kept reasonably small in order to allow for high GVW-s to appear with properties that contain high axle load ratios, keeping the method conservative. In the current model, intervals of 100 kN are chosen. For this an input matrix is created, containing the chosen thresholds within each category. Depending on the original measured data distribution, different categories may be distributed to a different number of sub-blocks. The matrix is visible in *Table 19* and is saved in the file *Thresholds.mat* as *Threshold*. The first row represents the thresholds but is not strictly necessary for the simulation; the first column gives the axle categories. For blocks that are not "used", elements with no value (*'NaN'*) are added.

**Table 19 - Threshold matrix for splitting properties by GVW [kN]**

| NaN | 100 | 200 | 300 | 400 | 500 | 600 |
|---|---|---|---|---|---|---|
| 2 | 100 | 200 | 300 | NaN | NaN | NaN |
| 3 | 100 | 200 | 300 | 400 | NaN | NaN |
| 4 | 100 | 200 | 300 | 400 | 500 | NaN |
| 5 | 100 | 200 | 300 | 400 | 500 | 600 |
| 6 | NaN | 200 | 300 | 400 | 500 | 600 |
| 7 | NaN | NaN | 300 | 400 | 500 | 600 |
| 8 | NaN | NaN | NaN | 400 | 500 | 600 |

The recorded vehicle properties are divided in sub-categories, based on the GVW they had occurred with. As mentioned in the *Section 'Possibilities to adjust the simulation model'*, this division can be done by allowing for properties only within the given GVW interval, or also for properties in the neighbouring categories. Both options are checked, as well as a third possibility which allows for properties from the respective GVW and interval and the interval to the "left".

The output is a cell array of c x m, where *c* is the number of original categories, now 7 for axle categories and *m* is the total number of intervals. Depending on the original measured data distribution, different categories may be distributed to a different number of sub-blocks. In practice this can be seen by the empty cell arrays.

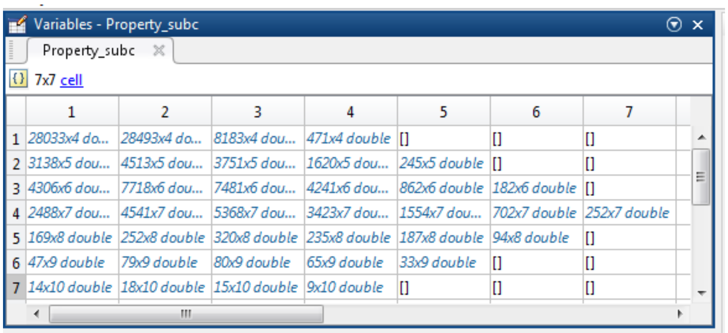


**Figure 31 - Vehicle properties divided to sub-categories, stored in a cell array**

This procedure is performed by the script '*Property_per_MultipleSubcategory_withaxles.m*' , the results are saved to *'Property_Sim.mat'*.

Within each category GVW-values are simulated from the statistical distributions defined in the previous step. Then, based on the value of the GVW a vehicle property is sampled from the adequate sub-category. This is done by two scripts: *'SimTraffic_adj_1_GVW'* and *'SimTraffic_adj_2_Property'*. The results are saved in the cell array *'SIMUL'* and stored in the files *'Sim_AN_multiSC_(i).mat'*.

An example is visualised in *Figure 32*. Imagine that within for example Axle Category 5 a vehicle weight of 345 kN is simulated. The second parameter describing this vehicle, the Vehicle Property, is then simulated from a sub-category of properties (and not from all recorded properties within the axle category, as in the initial model). The set of properties which we randomly choose from is those where the GVW of the original vehicle falls in either the interval [300;400[ or [200;400[ or [200;500[ kN.

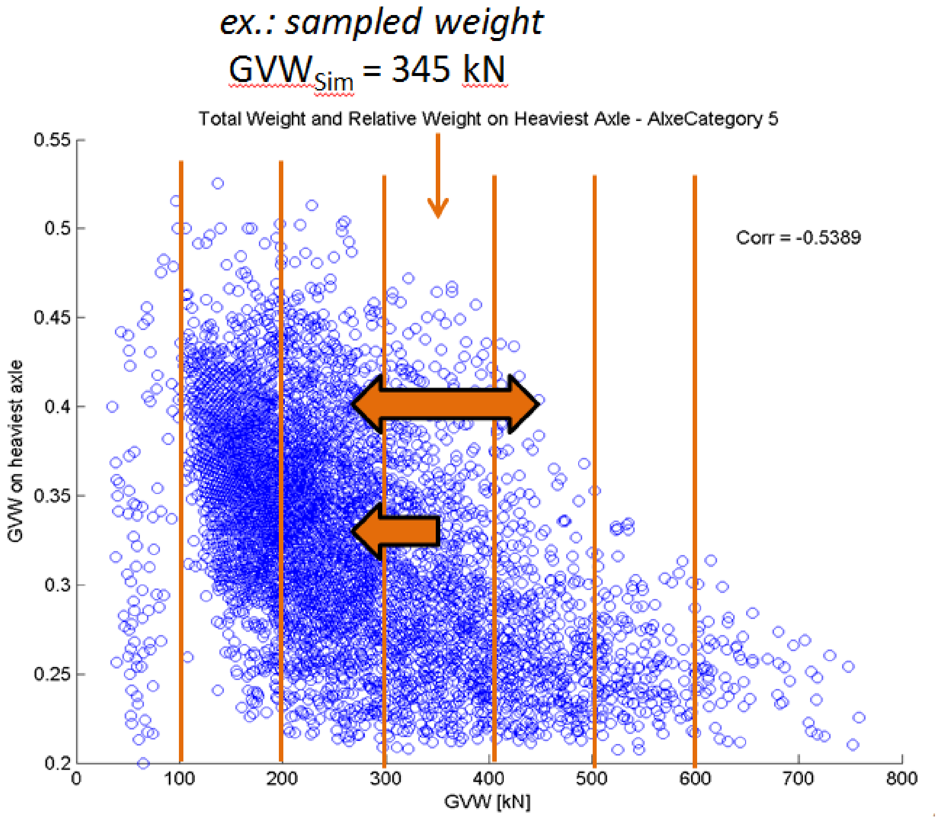


**Figure 32 - Example of sub-division of properties to blocks**

**Practicalities:**

In order to keep the simulation flexible, simulated GVW-s per category are saved in a cell array *'GVW_simul_sort'* within files *'Sim_ANcategories_(i).mat'*. Keeping the GVW and property simulation separated allows for re-use of the simulated data in case the categorisation of properties would change. Ordering simulated GVW-s within each cell (increasing or decreasing order) as well as converting the values to type *int32* reduces the memory need with a factor of over 60. In later calculations, the order of the rows can be shuffled (for example if block-maxima values are to be registered) and for multiplying with non-integer values the GVW matrix can be converted back to type *double*.

The second step of the traffic simulation is assigning a property index to each simulated vehicle, based on its' category and sub-category. Having previously ordered the GVW-s in increasing order makes this process significantly faster than by "choosing" the sample space of the properties one-by-one for each vehicle. The sorted GVW-s can be (temporarily) split to blocks and stored in cells of a cell array. For each block then a random sampling from the adequate property indices is run. Results are saved in the cell array *'SIMUL'* and stored in the files *'Sim_AN_multiSC_(i).mat'*.

The effect of adjusting the simulation model is checked by comparing the simulated and measured axle loads again, an example is visualised in *Figure 33.* It is visible that by even a limited number of sub-categories (7 for the axle category 5) the resulting axle loads are similar to the measured ones. Moreover, very high loads occur with a higher probability thus making the model conservative.

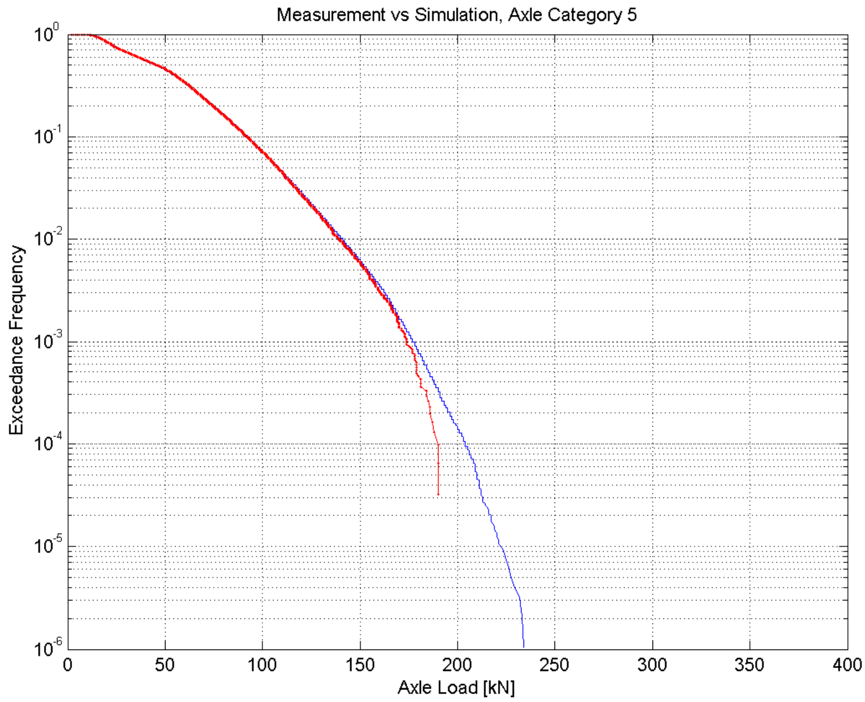


**Figure 33 - Comparison of measured and simulated axle loads - Adjusted simulation model (V1)**

# 9 Load Effects

## 9.1 Load effects from trucks with unit GVW

### *9.1.1 Goal*

The approach for load effect analysis is to determine the load effect in a **given cross section** from **each possible axle configuration**, for a **GVW of unity** assigned to each truck.

$$\mathrm{LE}_{\mathrm{sim,Pi}} = \frac{\mathrm{GVW}_{\mathrm{sim}}}{\mathrm{GVW}_{\mathrm{unit\ Pi}}} \mathrm{LE}_{\mathrm{Pi}}$$

Where

| | |
|---|---|
| $\mathrm{LE}_{\mathrm{sim,Pi}}$ | Load effect from simulated truck |
| $\mathrm{LE}_{\mathrm{Pi}}$ | Load effect from unit-weight truck |
| $\mathrm{GVW}_{\mathrm{sim}}$ | Simulated GVW |
| $\mathrm{GVW}_{\mathrm{unit\ Pi}}$ | Unit GVW |

The **maxima load effect is assigned to each property *Pi***. The property $P_i$ is described only by its index number. Later, if one wants to know what type of truck causes a certain load effect, the property can be looked up by this index number. This procedure can be carried out for various bridge lengths in various cross sections.

### *9.1.2 Load effect in cross section*

The load effect (LE) in a cross section can be defined with the help of an influence line.

For an arbitrary point on a structure the chosen load effect (shear, bending moment) can be defined as a function of the magnitude and location of the load. In the current examples the case of bending moment at mid-support of a simple supported beam is used, where the load effect can be described by an equation.

The function *'il_m_ss_Q'* calculates the bending moment in a simple supported beam caused by a point load. Input parameters are the length, the location of the cross section, the magnitude and the location of the point load.

### *9.1.3 Maximum load effect in cross section from one vehicle*

In the cross section of interest, for each "unit-weight truck", the magnitude of the maxima load effect as well as the location of the truck when this effect is reached are recorded. This is performed by the function *'max_le_cs_4'* which uses the load effect function described in the previous section. The input to this function is the beam length, location of cross section, the step size (in the current application this was defined as 20 cm, but can be changed) and the truck property consisting of axle loads and axle distances.

*Cross section versus full structure*

It is noted that with this method, the load effect maxima for **one specified cross section** is calculated. It is possible that for some axle configurations the maximum bending moment will not occur in the middle cross-section of the beam. In order to take this into account and to perform an analysis on the "system level", thus to determine the load effect maxima for the whole structure, an additional function or script can be used. This script should "loop" the load effect maxima function, for example *'max_le_cs_4',* over several cross sections. Then maxima in all cross sections or the absolute maxima caused by a given vehicle (axle load and distance combination) can be recorded.

### 9.1.4 Maximum load effect in cross section from several vehicles

The next step is to register the maxima load effects in a given cross section for each possible vehicle configuration. Looping the function of load effect maxima from one vehicle *'max_le_cs_4'* over several vehicle parameters maxima load effects (and coordinates of the given truck, if of interest) can be registered. Since the proposed methodology uses unit-weight trucks, in this way a load maximum effect can be coupled to each vehicle property. This maxima load effect for a unit-weight vehicle depends on the location of the cross section as well as the beam length.

The script *'max_le_moretrucks_morelengths'* calculates and registers the information (maximum load effect, location of vehicle when causing the maximum load effect) for beams of multiple lengths. This can be useful for a parameter study of various bridge lengths. The structure of the loop can easily be changed that instead of different beam lengths the load effect maxima in different cross sections of the same beam can be calculated.

The results are saved in files named according to the properties, *'LE_M_L(i)_CSmid_ST02_AN.mat'* in cell arrays *'B'*. The first dimension of *'B '*corresponds to the total number of categories. Each element of the array contains a matrix of LEmax and xLEmax and has the length equal to the number of trucks measured in the given category (i.e. number of "parameters" describing the class).

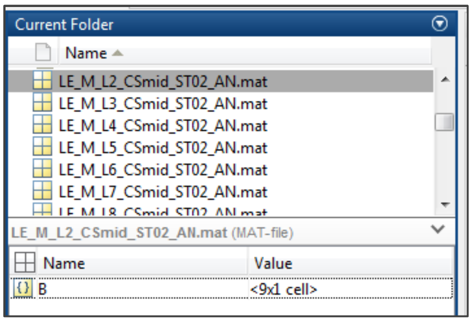


**Figure 34 - Storing maximum load effects of unit weight trucks**

## 9.2 Load effects from simulated trucks

### 9.2.1 Goal

The goal of this step is to determine the load effects from all simulated vehicles, for one chosen section of a given bridge (described by a specific influence line).

As the load effect from all possible (assumed) axle configurations is known for a unit GVW and collected in matrix $LE_{I,I}$, we can now assign to any simulated truck $T_s$ with property $P_i$ and gross vehicle weight $GVW_S$ a **load effect**:

$$\mathrm{LE_{s,i}} = \frac{\mathrm{GVW_s}}{\mathrm{GVW_{unit}}} \mathrm{LE_i}$$

This relation is true due to the linear relation between *GVW* and *LE*.

**<u>Limitations</u>:**

- The method is not directly applicable in case we allow for multiple vehicles to be present on a bridge. This is definitely a strong assumption[29], thus thought should be given to (and likely a more complex simulation carried out for):
    - Multiple vehicles in the same lane
    - Multiple vehicles in parallel lanes
- So far dynamic loading has not been taken into account

[29] The same assumption is used in (Steenbergen et al. n.d.)

- This method can only be used if the relation between the load (*Q, GVW – force*) and the load effect (*M, V, …*) is linear.

### 9.2.2 Determining load effects for over 100 years of traffic

The script *'max_le_simultrucks_multiple.m'* determines the load effect for trucks which were simulated. This step "couples" the simulated traffic, where a vehicle is described by its GVW and its property index, with the load effects caused by unit-weight trucks.

The principle behind the simulation is that it is sufficient to know only the GVW and property index of each truck and the load effect caused by unit weight trucks with all possible vehicle properties.

The output is stored in files '*LE_M_L(i)_CSmid_ST02_AN_Sim_ANcat_multiSC_(i)*', in a cell array *'LEsimul'*, where each cell corresponds to a category and contains a matrix of the load effects from the simulated traffic.

**Practicalities:**
The information *'P'* can be stored as well in a second column of the matrix in order to know which truck property caused the load effect, but it is not definitely necessary. As long as the load effects in *LE_SIMUL_M_L6_CSmid_ST02_AN* are stored in rows corresponding to the rows of trucks in the simulation file *SIMUL,* the truck properties can be „tracked back".

After having performed the multiplication, considering that the load effect values are in a "sufficient range" the output can be converted to and saved as type '*uint32',* saving memory space.

The simulated traffic could be ordered within each category by vehicle property index, this might further speed up the simulation process of "looking up" load effect values belonging to vehicle properties.

## 9.3 Load effect maxima

### 9.3.1 Goal

The overall goal is to **determine a 15-years maxima distribution function**. The first method consists of simulating several times 15-years maxima and fit an extreme value distribution to the results. The second method is to obtain several monthly- or yearly maxima values from simulations, fit an extreme value distribution to the results and then transform the distribution functions to a different return-period. In case of a Gumbel distribution this would consist of "shifting" the probability density function to the right.

### 9.3.2 Block maxima

#### Selecting block maxima

For both methods the first step is the collection of maximum values. The function *'maximasplit_indexed.mat'* collects block maxima values, for example monthly, yearly or 15-yearly maxima of each vehicle category and registers the row number from the simulation, so the truck causing the maximum load effect can be looked up. This function is used by script *'MaximaScript'* to collect data from several files and finally construct the matrix *'MaxLE'* in the file *'Max_LE_M_L6_CSmid_ST02_AN_Sim3_AxleNumber($i_{block}$)_'* . The structure of the output is shown in *Table 20.*

The assumption is made that within one time block the ratio of vehicle categories corresponds to the ratio of categories in the measured traffic. If a more sophisticated approach is taken, for example assuming that the ratio of categories is a discrete distribution, then this discrete distribution can be used for "sampling with replacement". Moreover, a parameter study could reveal what influence a changing category ratio has on the maxima distribution.

**Table 20- Structure of matrix *MaxLE* containing maxima data, example: 5-yearly maxima collected**

| | | | Counter 'j' | 1 | 2 | j | 30 |
|---|---|---|---|---|---|---|---|
| | | | Year | 1-5 | 6-10 | (j-1)*5+1 *to* j*5 | 145-160 |
| | Counter 'i' | Axle Category | Row / column of matrix | 1 | 2 | j | 30 |
| Load effect | 1 | 2 | 1 | | | | |
| | … | … | … | | | | |
| | i | i+1 | i | *Max. load effect in category 'i' (axle nr=i+1), year (j-1)*5+1 to j*5* | | | |
| | … | … | … | | | | |
| Index of truck | 1 | i+1 | catmax + 1 | | | | |
| | 2 | | catmax + 2 | | | | |
| | … | … | … | | | | |
| | i | i+1 | catmax+i | *Index of vehicle causing max. LE in category 'i'* | | | |
| | .. | | | | | | |
| Absolute Max | - | - | 2*catmax+1 | *Absolute maximum of year (j-1)*5+1 to j*5* | | | |
| | - | - | 2*catmax+2 | *Vehicle category of absolute maximum* | | | |

### *Data visualisation*

Data analysis should start with visualisation. Therefore the maxima values for various block maxima types are plotted in histograms. As an example, the yearly maxima of the moment at the mid-span of a 6 m long beam is plotted in *Figure 35,* based on division per axle category. The number in brackets refers to the number of simulated yearly maxima. Figures based on other block maxima are added to the Appendix.

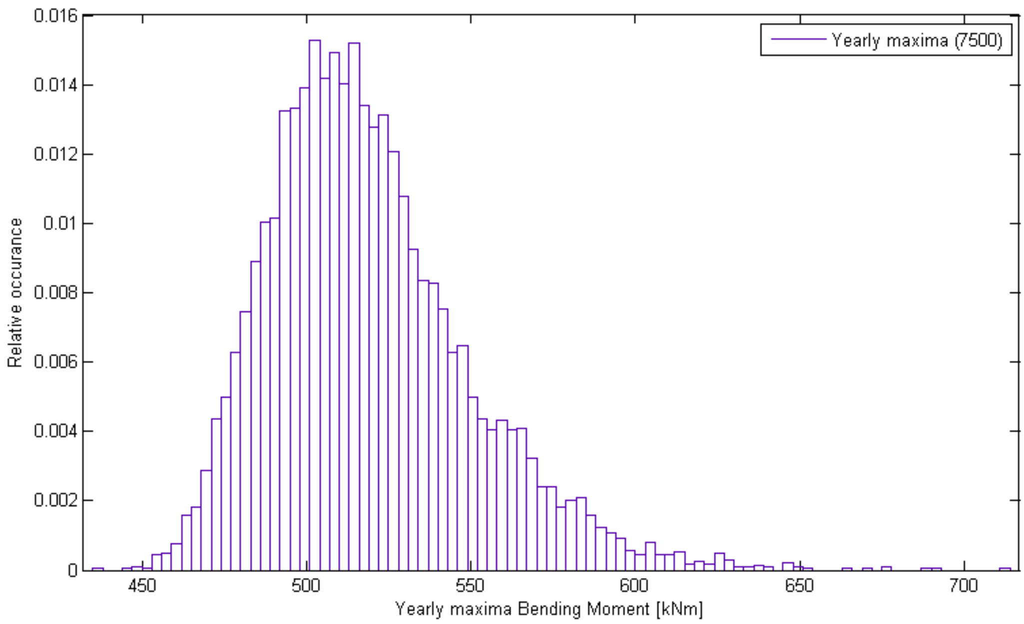


**Figure 35 – Distribution of yearly maxima of load effect at mid span of 6m span beam [kNm]**

### 9.3.3 15 year maxima distribution

*Fits to 15-year maxima data*

15-year periods were simulated 500 times, the results are visualised in the histogram in *Figure 36*. The number 500 refers to the number of simulated 15-year maxima.

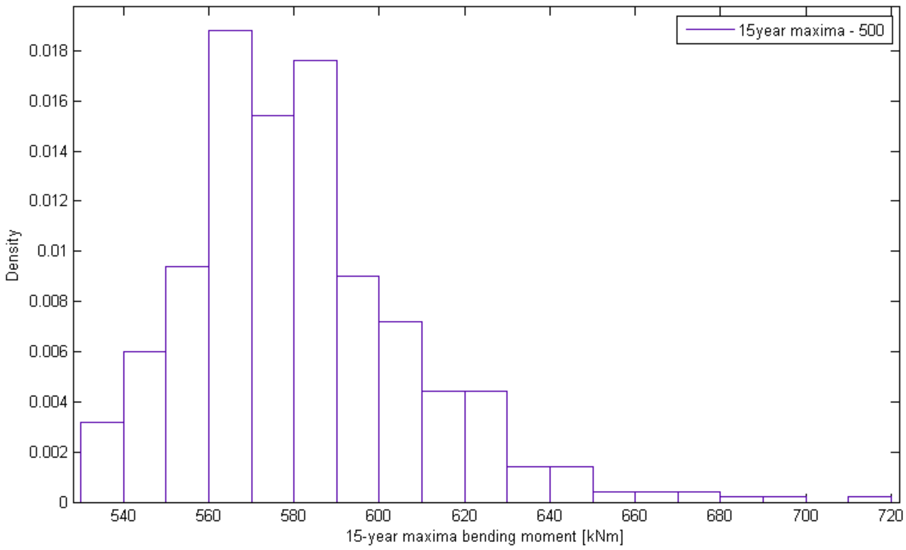


**Figure 36 - 15 year maxima bending moment histogram, 6m span simple supported beam [kNm]**

A probability distribution can be fitted to the collected data, for example with the help of the built-in algorithm of MatLab which uses maximum likelihood estimation.

As an initial try, a Generalised Extreme value Distribution, a Gumbel distribution (particular case of GEV with k = 0), and mixture of both 10 and 20 normal distributions are fitted and plotted. The probability density functions of the fitted distributions are shown in *Figure 37.*

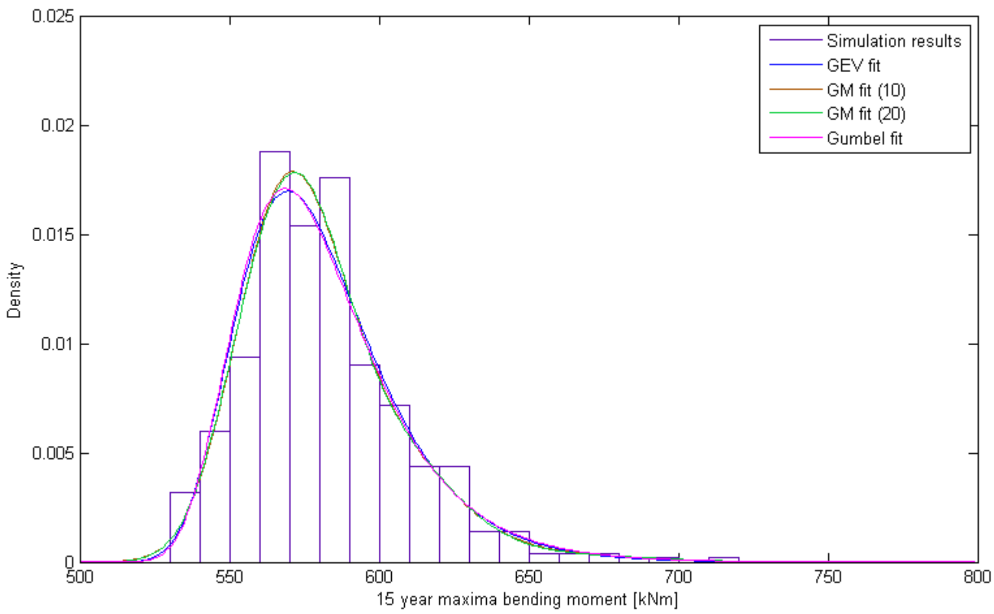


**Figure 37 - 15 year bending moment histogram and probability distribution fits [kNm]**

The distribution most accurately modelling the **tail data** should be chosen, as the failures will occur due to the most extreme loads. *Figure 38* and *Figure 39* show the relevance of the tail-modelling considering various distribution types.

The fitted generalised extreme value distribution is Type III "short-tail", thus less conservative than the Gumbel distribution in the tail part. [30]

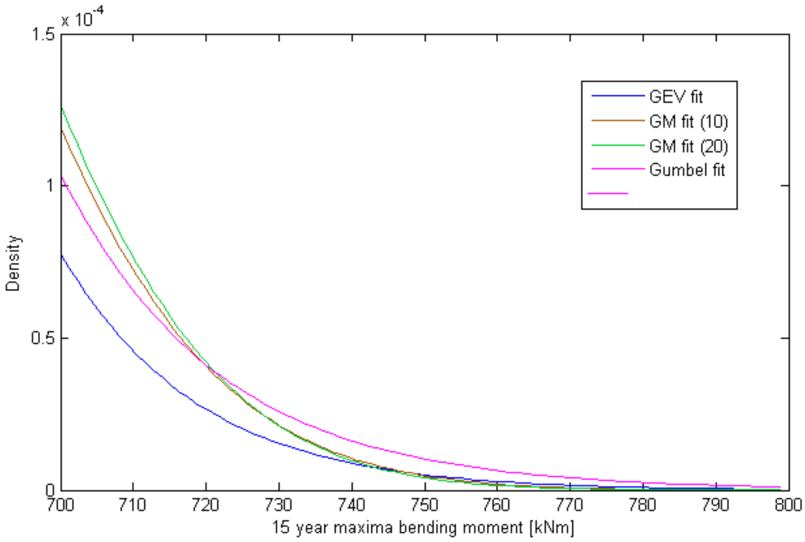


**Figure 38 - Tail of probability distribution fits to 15 year bending moment maxima**

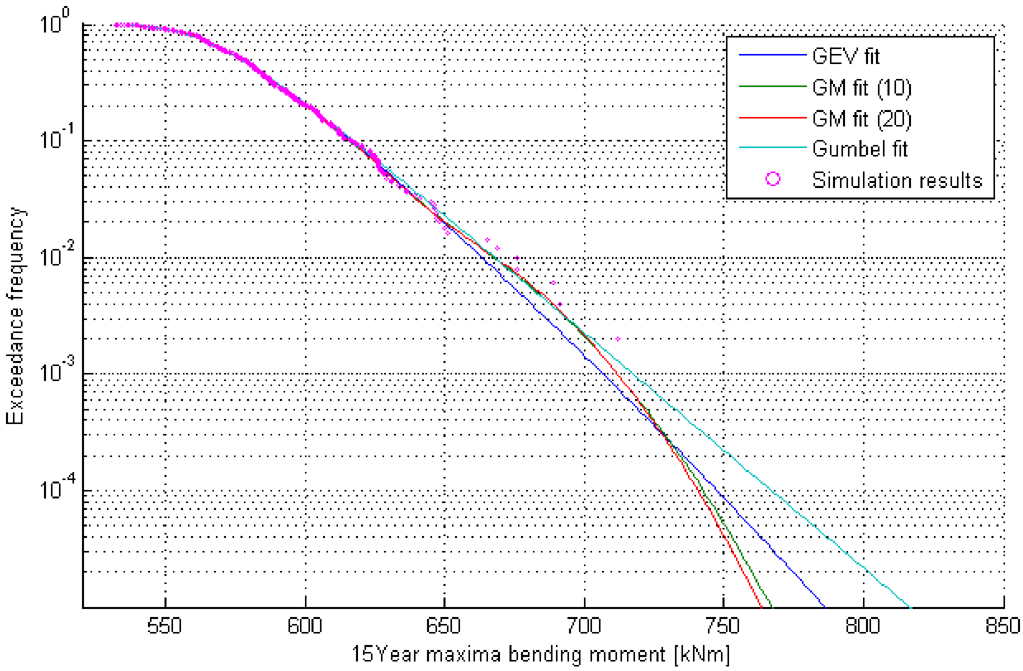


**Figure 39 - Exceedance-probability plot of probability distribution fits to 15 year bending moment maxima**

The selected distribution **can be used directly as input to a full probabilistic analysis** of the **given cross section**.

[30] It is noted that the fit made by the MatLab tool gives a shape parameter k < 0 for Type II and k > 0 for Type III distributions. In the consulted literature the analytical format of the GEV distribution is defined oppositely thus giving k > 0 for Type II k < 0 for Type III distributions.

### *Load effect values for some non-exceedance probabilities*

Another possible application may be determining a design load effect based on the generalised importance factor α (-0.7) and the required reliability β (2.5), where the exceedance probability is:

$$\Phi(\beta\alpha) = 4\%$$

However, for this further model uncertainty and dynamic effects should be taken into account. Values belonging to 96% non-exceedance probability therefore cannot be directly compared to load effect results from loading of Eurocode / NEN 8700 & 8701.

The values belonging to some non-exceedance probabilities are collected in *Table 21*. Although the values should not be directly compared, as stated above, the design load effect values from the Eurocode model are given in *Table 22* as reference.

**Table 21- 15-year bending moment maxima for various non-exceedance probabilities**

| Non-exceedance probability | 96,0% | 99,0% | 99,9% |
|---|---|---|---|
| GEV (k=-0.02) | 635,66 | 663,25 | 706,64 |
| Gumbel | 637,56 | 667,74 | 717,41 |
| GM model - 10 normals | 630,4 | 667,5 | 712,5 |
| GM model - 20 normals | 634,7 | 668,0 | 712,0 |

**Table 22 - Maximum bending moment on 6m long, 3 m wide beam from Eurocode LM1[31]**

| With and without reduction factors of NEN 8700 / EN 1 | | Without reduction | Reduction 0.8 |
|---|---|---|---|
| From axles (Q load) | kNm | 802 | 642 |
| M from q | kNm/m | 45 | 36 |
| | kNm | 134 | 107 |
| M total | kNm/m | 312 | 249 |
| | kNm | 936 | 748 |

### *Statistical uncertainty*

As described in *sections 5.3.2* and *5.3.3*, the parameters of the fitted distributions are stochastic variables themselves, because they contain uncertainty. One way to quantify this is by assigning a *standard error* (i.e. standard deviation of parameter) to the parameters of the distribution. This is also the measure used by MatLab *dfittool*, together with the covariance[32].

For example, the standard errors related to the fitted Gumbel distribution are given in *Table 23.*

**Table 23 - Statistical uncertainty in load effect maxima**

| | Value (mean) | Standard error (st.dev.) |
|---|---|---|
| μ | 568,7 | 1,015 |
| σ | 27,63 | 0,7433 |

It is visible that the standard errors are not very high compared to the mean, therefore the confidence bound of for example 95% is expected to be relatively narrow.

[31] It should be noted that these load effects are maxima of all cross sections.
[32] The diagonal of the covariance matrix contains the variances, i.e. the squared standard deviations while the the other elements refer to the relationship between the parameters

The standard deviations correspond to degrees of confidence, as described in *Section 5.3.3*. As an example, the9 5% confidence intervals in relation to the mean and standard deviation of the fitted Gumbel distribution are plotted in *Figure 40.* It can be observed that the uncertainty in the standard deviation of the Gumbel distribution is more influential on the overall result than the uncertainty of the mean.

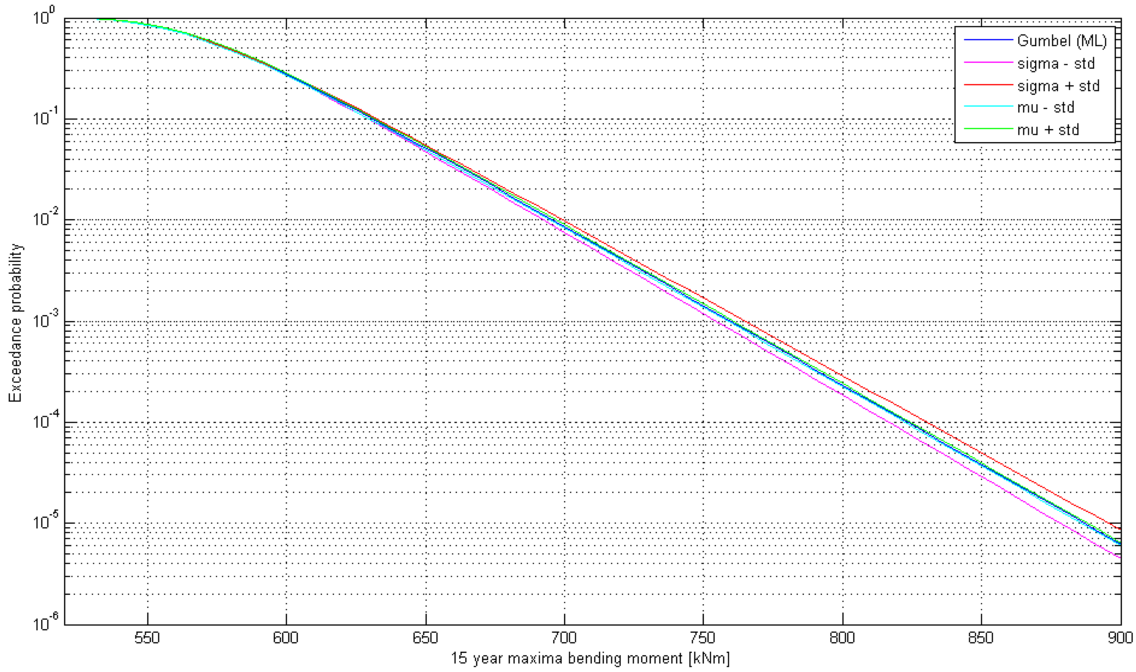


**Figure 40 - 15 year load effect maxima 95% confidence intervals**

In the range of the design values (see previous sub-section*)*, i.e. at probabilities of non-exceedance $P_{NE}$~0.96 (probability of exceedance $P_E$~0.04) the deviation from the determined distribution is very small. Therefore the influence of the parameter uncertainty on the overall reliability result is expected to be reasonably low.

Here it is mentioned that for a correct procedure, a similar analysis of statistical uncertainty should be carried out within any step of the load analysis and simulation process and should finally be considered in the fully probabilistic analysis by an additional stochastic parameter.

### *Chosen distribution – input to probabilistic analysis*

For further evaluation of the example of the 6m span beam the Gumbel distribution is chosen. The reason for this are:

- The Gumbel distribution models more accurately the tail data than the fitted generalised extreme value distribution
- It is the most conservative from the models – as a first try it seems reasonable to stay on the conservative side rather than use the mixture of normal distributions.

The parameters of the fitted **Gumbel distribution** are: $\sigma$ = 21,53 $\mu$ = 568,7 [kNm].
Using the formulas for the relation between the mean and the standard deviation, these are respectively: Mean = 581; Std = 27,63. This gives a CoV = 0,0475. The values, converted to Nm (as this will be used in the probabilistic calculation) are summarized in *Table 24.*

**Table 24 - 15-years load effect maxima distribution from traffic load, 6m span beam**

| | | Distribution | Dim. | m (Mean) | σ (St. Dev.) | CoV | P1 (Parameter) | P2 (Parameter) |
|---|---|---|---|---|---|---|---|---|
| Load from simulation | M | Gumbel | Nm | 5,811E+05 | 27613 | 0,05 | 5,69E+05 | 21530 |

This data can be used directly as input to a fully probabilistic calculation.

## 9.4 Summary and evaluation of traffic load analysis and simulation

A traffic load model has been developed with the final aim to arrive to a probability distribution of life-time maxima load effect, which can be used directly in the probabilistic analysis of a short-span bridge as input on the loading side.

The specific context of the thesis work is short-span city bridges. For the load modelling, the fact that the bridges are short (below 20 meters) is most relevant from this. Firstly, this makes it necessary to consider more adequately axle loads- and distances than for a long-span bridge. On the other hand, this fact also leads to simplifications: it can be reasonably assumed that the governing load effects will be caused by a single heavy vehicle crossing the bridge. Therefore one of the main assumptions of the model could be that only these individual cases were considered. This main assumption is also supported by the fact that city bridges are considered. The likelihood of two heavy vehicles driving behind each other is significantly lower than on a highway, decreasing the probability that the maximum load effect will be caused by multiple vehicles on a bridge. Nevertheless, in certain areas such “multiple presence events” may happen, thus further research could investigate these.

The data analysis and simulation model is based on five steps, as described in *Section 7.2.2.* As the process of simulating traffic (i.e. vehicles described by GVW, axle load- and distance) is independent of the structure, once traffic is simulated for a given time period it can be used for various lengths of structures and various load effect functions, depending on the cross section and load effect of interest. The constraint is that the relationship between the GVW (or more precisely the total load present on the bridge) and the load effect should be linear.

The developed model was based on the attempt to avoid extrapolation of load effects: in this case it would not be possible to conclude what vehicle configuration causes the most extreme loads. Moreover, the method of extrapolation is expected to have a significant impact on the final result. Therefore a simplified simulation model was chosen with which it was possible to simulate 7500 years of traffic and the resulting load effects on a personal computer in a reasonable time. The “cost” of the simplification was that only vehicle configurations (axle distances and relative axle loads) which have been measured could be considered. The assumption is made that the measurement data used sufficiently represents expected vehicle types. This assumption could be validated in further research by making a similar analysis using new measurements, or by some kind of boot-strap process using only the available measurement data.

A relation was observed between the proportionally heaviest axle load on a vehicle and the GVW. Therefore the model was adjusted in order to “match” the axle load distribution resulting from the simulation model to the measurement data. This process resulted in a sufficient agreement between the simulation results and measurements. With the process applied in the current simulation method however, the proportion of vehicle properties within an axle category is somewhat distorted. In a further developed model this distortion could be accounted for.

The model is two-dimensional, i.e. lateral distribution is not considered. In a further developed model this could also be considered in a stochastic way.

As a result, load effect maxima distribution functions (moment and shear) can be determined in an arbitrary cross section of a slender structure with a span of up to 20 meters. This distribution function can be used directly in the fully probabilistic reliability analysis of the modelled structure. Unless significant statistical- or model uncertainties must be considered, the example values calculated for a structure of 6m span indicate that the load effects will lie below the values that would result from applying Eurocode loads.

# Section III

# Application and Evaluation

# 10 Probabilistic Analysis of Simple Supported Beam Using Traffic Loading Input

## 10.1 Task description

### *10.1.1 General information*

The aim of the example is to show whether a probabilistic analysis with Monte Carlo simulation, with loading input gained from traffic loading analysis gives a more economic result than calculation according to the Eurocode. This can be done in two main steps: first parameters of an optimal beam should be determined, sec

Secondly the beam can be analysed using Monte Carlo simulation (or any other fully probabilistic method).

*Parameters of optimal beam*

For such a comparison first the parameters of an optimal beam are determined, where optimal is defined as the design resistance being equivalent to the design load.

$$\mathrm{M_{Ed}} = \mathrm{M_{Rd}}$$

The design bending moment broken down to variable load and self-weight is:

$$\mathrm{M_{Ed}} = \mathrm{M_{Q\ Ed}} + \mathrm{M_{G\ Ed}} = \mathrm{M_{Q\ char}} \cdot \gamma_\mathrm{Q} + \mathrm{M_{G\ char}} \cdot \gamma_\mathrm{G}$$

As the context of the current research is the evaluation of existing structures, the appropriate reliability requirements given in the norms (Eurocode, NEN 8700 and 8701) are taken into account for this optimisation. Reliability requirements can be given as a maximum acceptable failure probability ($P_{f.max}$) or as a required reliability ($\beta_{min}$). The relation of these two measures is:

$$\beta = -\phi(P_f)$$

The above mentioned requirements are given in the form of adequate partial safety factors which are based on the relation between level II and level III methods, as described in *Section3.4.1*. Optionally, the optimisation calculation can be based on traffic loading multiplied with reduction factors [33]($\psi_t$, $\alpha_{q,}$ $\alpha_{trend}$), which are allowed for by Eurocode 1 and NEN 8700 / 8701 and are expected to be used in practice when re-evaluating a structure in The Netherlands. From this point these shall be denoted as *'factors'.*

*Reliability analysis of beam with traffic loading input*

After having determined the cross-section parameters necessary to safely carry the loading prescribed by the norms (i.e. fulfil equation $M_{Ed} = M_{Rd}$), a fully probabilistic calculation can be set up. The basis of such a calculation, as described in detail in *Section 2.2,* is the ***reliability equation***:

$$Z = R - S$$

Failure occurs when the load *S* exceeds the resistance *R,* thus when *Z* takes on a negative value. Probabilistic analysis investigates the ***probability of failure*** , which is the probability that the reliability function takes on a negative value.

[33] 'Factors': $\psi_t$, $\alpha_{q,}$ $\alpha_{trend}$ Current codes in Netherlands (NEN 8700) allow for reduction of loading for a shorter reference period, traffic trend (shorter time thus lower traffic increase than anticipated for design loads of code) and traffic intensity (less vehicles therefore smaller exceedance probabilities).

Both the resistance *R* and the load (or load effect) *S* are described by several variables, including model uncertainties too. The basic variables are of a stochastic nature, described by probability distribution functions. Their values can be gained from the Probabilistic Model Code (JCSS 2001), here also their relation with nominal values used in a semi-probabilistic calculation (i.e. a calculation using partial safety factors, according to codes) is described. Besides, for steel structures, the work of Cajot et al. (2005) provides additional guidance for the choice of appropriate parameters. As the basic variables are stochastic, the values of R, S and Z will also be.

In the probabilistic calculation the result of the traffic loading analysis process, the load effect maxima distribution function is used as load variable corresponding to the variable load. Dynamic amplification should also be considered as an additional stochastic variable, as a multiplying factor of the load effect. This is described in *Section 6.5*. The self-weight should correspond to what was used in the semi-probabilistic calculation (based on $\chi$), information of the variability can be taken from the Probabilistic Model Code (JCSS 2001). Finally, model uncertainties on both the resistance and load side should be included. The model uncertainties used in the TNO report for Rijkswaterstaat (Steenbergen et al. 2012) were summarized in *Section 6.4.3.* Some of these are considered in the current analysis as well. A summary is given in *Table 25*.

**Table 25 - Model uncertainties in TNO report and in the current work**

| | Mean | CoV | Source | In the current work |
|---|---|---|---|---|
| DAF vehicle | 1 | 0 | Within report | Considered |
| DAF bridge | 1,1 | 0,05 | Within report | Considered |
| Statistical uncertainty | 1 | 0,05 | Assumption | *Not quantified* |
| Spatial spread<br>(Typical location in Nl. less loaded than the location of measurements) | 0,86 | 0,07 | TNO 98-CON-R1813 | Not necessary, because location-specific loading is taken |
| Trend factor 15 years | 1,1 | 0,1 | TNO 98-CON-R1813 | Not considered |
| Load effect | 1 | 0,1 | JCSS PMC | Considered |

The result of a Monte Carlo simulation will be a failure frequency, which is approximately equal to the failure probability ($P_{f\_MC}$) . The later can also be expressed as a reliability index ($\beta_{MC}$). Optionally, results of the reliability function *Z* can be visualised in a histogram or the simulated resistance – load values (R-S pairs) in a scatter plot where the line R = S represents the so called *failure boundary*. An example is given in *Figure 41.*

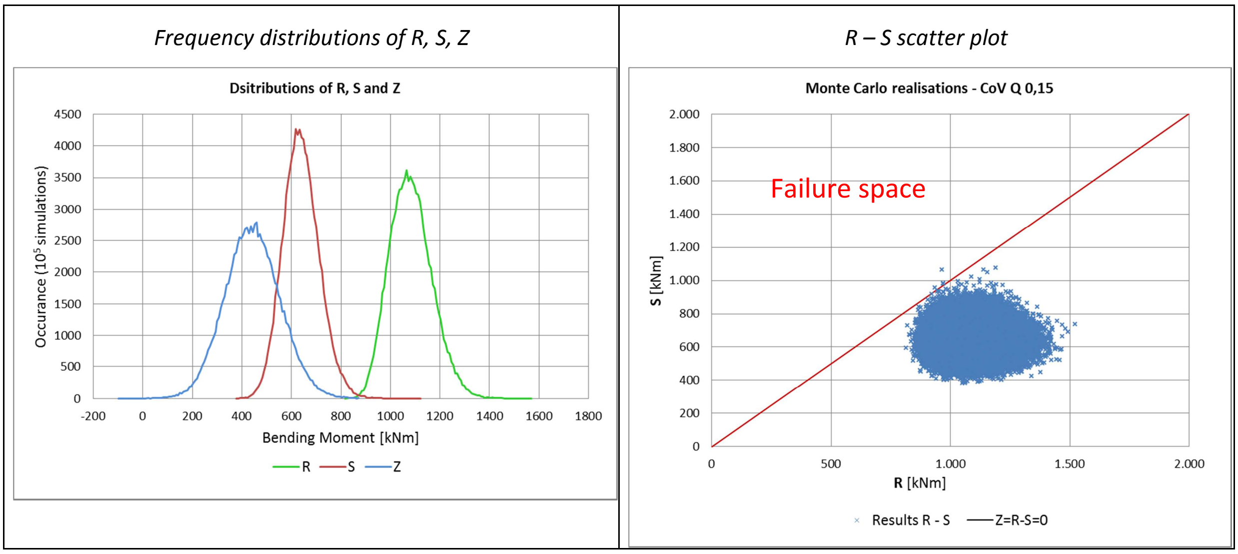


**Figure 41- Example of visualising results of Monte Carlo simulation**

### Comparison of results

To compare the results of the deterministic and fully probabilistic calculation, the failure probability or reliability index resulting from the probabilistic calculation should be compared to the required reliability index. If the result of the probabilistic calculation is a lower failure probability / higher reliability index than the requirement, it is shown that the fully probabilistic calculation including traffic loading data input is favourable.

Expressed in a formula using the reliability index, the main question is:

$$\beta_{MC} >^{?} \beta_{min}$$

Expressed with failure probability:

$$P_{f\ MC} <^{?} P_{f\ max}$$

## 10.1.2 Reliability requirement

The reliability requirement mentioned in the previous section is necessary for both the "optimal design" and for comparison of the semi-probabilistic and probabilistic calculation. The requirement is based on the remaining life of the structure as well as the consequence class[34].

The values for this example are chosen based on typical values used for a structure in Rotterdam:

| | |
|---|---|
| Consequence class | 2 |
| Remaining life | 15 years |
| Reliability level | 'a*fkeur' (NEN 8700)* |
| Required reliability | $\beta = 2.5$ |

Therefore partial factors of loads are:

| | |
|---|---|
| $\gamma_S$= 1,1 | Partial safety factor for traffic load, NEN 8700 |
| $\gamma_G$= 1,1 | Partial safety factor for self-weight, NEN 8700 |
| *factors* = 0.8 and 1 | Reduction factors, according to EN1991 and NEN 8700. ($\alpha_t$, $\alpha$ ,$\psi$)<br>*The value 0.8 is selected based on typical values used in Rotterdam* |

## 10.1.3 Design load of the beam according to European and Dutch norms

The design load for the input $M_{Ed}$ consists of traffic loading and self-weight. The current example does not consider wind, seismic, accidental etc. loading.

### Traffic loading

Load Model (LM) 1 and 2 are to be used, which are described in *Section 6.3.2.* An algorithm in Visual Basic was developed [35] which calculates the maxima bending moment in a simple supported beam for vehicles of various axle loads and -spacing. The results are summarized in *Appendix F.*

For the current example of the 6m span simply supported beam, the relevant design values are:

| | |
|---|---|
| Without reduction factor | $M_{Q\ Ed}$ = 936 kNm |
| With a reduction factor of 0,8 | $M_{Q\ Ed}$ = 748 kNm |

### Self-weight

Various ratios of live-load to self-weight are taken into account, expressed with the factor $\chi$:

[34] As well as the type of loading: when wind loading is dominant, in some cases the reliability requirement is lower. (Steenbergen & Vrouwenvelder 2010)

[35] Based on the work of ing. Bas Govindasamy (Ingenieursbureau Gemeente Rotterdam)

$$\chi = \frac{M_{Q\,char}}{M_{Q\,char} + M_{G\,char}} = \frac{M_{Q\,Ed}/\gamma_Q \cdot \text{factors}}{M_{Q\,Ed}/(\gamma_Q \cdot \text{factors}) + M_{G\,Ed}/\gamma_G}$$

As $M_{Q\,Ed}$ is known, self-weight can be added as a parameter for various $\chi$ ratios. Technically this is taken into account in a slightly different way for the steel and the concrete beam, which is described in the relevant sections.

### *10.1.4 Steps of probabilistic analysis with Monte Carlo simulation in Excel®*

According to Steenbergen et al. (2012) the steps of a probabilistic analysis are:

1. Determining required reliability
2. Quantification of uncertainty
3. Determining values for all uncertainties
4. Verification

In the current case, using Monte Carlo simulation (with Excel), the steps are further detailed as:

1. Determine the required reliability
2. a. Determine all uncertainties
   b. Write down the reliability equation, which consists of a load *(S )*and a resistance *(R)* model

   The steps 2 a. and b. somewhat "go together", because in order to determine the uncertainties the model of the mechanical behaviour of the structure should be known. For example, the engineer should be aware that the yield strength $f_y$ of a reinforcement bar has an influence on the resistance of a reinforced concrete beam and should then consider that the value of $f_y$ is uncertain.
3. Assign values to the uncertainties:
   a. Type of probability distribution for each parameter that is present in the reliability equation

      When the probability distribution type is determined, it is known how many values are needed to describe this distribution. In the most simple case, when a value can be considered deterministic, this is a mean or nominal value. For example a normal distribution is described by two parameters, while a generalized extreme value by 3. Information about distribution functions and the relations between their parameters are given in *Appendix B.*

   b. Parameters that fully describe the stochastic variables

   c. Practicality for carrying out Monte Carlo simulation: collect all information about the stochastic variables (their parameters and types of distribution) a spreadsheet.
4. Carry out the Monte Carlo simulation
   a. As a preparatory step, in a spreadsheet create a column for each stochastic variable.

   b. Generate several[36] random values of each stochastic variable, denoting the number of simulations with *n*.
      A random number on the interval [0;1], *RAND()* in Excel®, is an exceedance probability. There are now 3 options to arrive to the value of the stochastic variable which belongs to the given exceedance probability:

[36] Refer to *Appendix A* for more information about the necessary number of Monte Carlo simulations

i. Use a built-in function of Excel®, for example *NORM.INV* with the arguments *probability; mean; standard deviation*

ii. Use an analytical formula which gives the relation between the distribution parameters and the *inverse cumulative distribution function.* This formula is exact for some cases, such as the exponential distribution while for others it is an approximate formula, such as the normal distribution. These formulas usually include one or two random numbers and the parameters of the distribution.

For example the exponential distribution the value *X* of the stochastic variable belonging to a certain probability *p* is:

$$X = F^{-1}(p) = \frac{-\ln(1-p)}{\lambda}$$

For a normal distribution an option to determine the value *X* of the stochastic variable is:

$$X = \mu_X + \sigma_X \sqrt{(-2\ln X_{u,1})} cos(2\pi X_{u,2})$$

Where $X_{u,1}$ *and* $X_{u,2}$ are realisations of a uniformly distributed random variable on the interval [0;1].

In practice, the appropriate formulae should be used in the spreadsheet, substituting *RAND()* to the value of probability *p* or $X_{u,I}$ respectively.

Analytical expressions for some other distributions are given in *Appendix B.*

iii. In case an analytical distribution function is not known or neither an Excel nor an analytical formula exists for random number generation, alternative methods must be sought for. Some examples of such a situation are briefly mentioned:
- The distribution may be discrete, distributed uniformly in a simple case (think about rolling a dice) or in another form.
- The distribution may be truncated.
- The distribution may be empirical measurement data, possibly extrapolated or not

c. Knowing *n* values of each stochastic variable, the reliability equation can be constructed *n* times as well[37].

5. Evaluate the results – determine failure frequency and reliability index
In step 4.c. several values (n) for the reliability equation were gained. Let the number of failure events, i.e. when Z<0 be $n_{fail}$. The failure frequency, which is approximately equal to the failure probability can now be expressed as:

$$P_f \cong f_{fail} = \frac{n_{fail}}{n}$$

From $P_f$ the reliability index β can directly be calculated.

6. Verification
Determine whether the required reliability index is reached / allowed failure probability not exceeded

*Illustrations of the process are to be added to the Appendix.*

[37] In fact by creating different combinations of the simulated values, the reliability equation could be evaluated more than *n* times.

### 10.1.5 Comment on practical implication

In the examples, the bending moment resulting from traffic loading is to be used. This bending moment has been calculated on a 2-dimensional beam model, concentrating the axle loads as a point load. The structures in the following two basic examples are also modelled as 2-dimensional (slender) structures.

In case of a real structure, the respective load might be carried by multiple structural elements, for example in a beam grid steel bridge. At this point however, for the theoretical example this is not considered. Attention is given to not arrive to a completely unrealistic size for the beam.

For the case of the concrete beam, a 3m wide structure shall be taken into account and modelled as a one-way spanning slab (thus in practice as a beam). The live load therefore is to be distributed respectively.

## 10.2 Steel beam example

### 10.2.1 Semi-probabilistic calculation: dimensioning optimal beam

#### Basic data

The current example determines the necessary cross section properties of a steel beam that shall carry the traffic loading and various ratios of self-weight. The basic data of the beam is:

Span $\quad l = 6{,}0\ m$

Steel S235 - Characteristic yield strength $\quad f_{yk} = 235\ \frac{N}{mm^2}$

#### Loading

The characteristic value of the moment from self-weight, based on the definition of $\chi$ is:

$$M_{Gk} = \left(\frac{1}{\chi} - 1\right) M_{Qk}$$

Or similarly for the design value:

$$M_{G\ Ed} = \gamma_G \left(\frac{1}{\chi} - 1\right) \frac{\mathrm{M_{Q\ Ed}}}{\gamma_Q \cdot \mathrm{factors}}$$

Considering a simple supported beam where the self-weight is represented by a uniformly distributed load (UDL), the characteristic value of the self-weight is:

$$g_k = \frac{8\ M_{Gk}}{l^2}$$

The value $M_{Qk}$ is known from LM 1 loading, thus values of $M_{gk}$, $M_{Ed}$ and $g_{nom}$ can be determined for various ratios of traffic load to total load. For a beam of 6 m span in the example, these values are summarized in *Table 26*.

**Table 26- Design bending moment on 6 m span steel beam for various traffic- to total load ratios $\chi$ [kNm]**

| Variable load effect (traffic) | factor | $MQ_{char}$ | 850,9 | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1,0 | $MQ_{Ed}$ | 936,0 | | | | | |
| | 0,8 | | 748,8 | | | | | |
| Ratio of variable load | | $\chi$ | 0,40 | 0,50 | 0,60 | 0,70 | 0,80 | 0,86 |
| Moment from self-weight | | $Mg_{tot\ char}$ | 1276,4 | 850,9 | 567,3 | 364,7 | 212,7 | 135,0 |
| Self-weight [kN/m] | | $g_{tot\ char}$ | 283,6 | 189,1 | 126,1 | 81,0 | 47,3 | 30,0 |
| Total load | 1,0 | $M_{Ed} = M_{Rd}$ | 2340,0 | 1872,0 | 1560,0 | 1337,1 | 1170,0 | 1084,5 |
| | 0,8 | | 2152,8 | 1684,8 | 1372,8 | 1149,9 | 982,8 | 897,3 |

### Resistance

The design yield strength of steel is:

$$f_{yd} = f_{yk}/\gamma_s$$

For a steel beam in bending: $\gamma_R = 1{,}0$

The resistance of a steel beam in bending is:

$$M_{Ed} = w_{nom} f_{yd}$$

Where $w_{nom}$ is the cross section modulus of a steel section. In the current example it doesn't matter whether it is the plastic or elastic resistance.

### Cross section properties

Knowing the design bending moment, the cross section properties of the 'optimal beam' are determined, based on requirements described in *Sections 10.1.2 and 10.1.3*.

The cross section size, expressed as section modulus is:

$$w_{nom} = \frac{M_{Ed}}{f_{yd}}$$

The resulting nominal cross-section modulus $w_{nom}$ for various ratios of self-weight and live load are given in *Table 27*.

**Table 27 - Nominal section modulus of 6 m span beam for various traffic traffic- to total load ratios $\chi$**

| $\chi$ | | | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
|---|---|---|---|---|---|---|---|---|---|
| $w_{nom}$ [1E-3 m3] | factors | 1,0 | 9,96 | 7,97 | 6,64 | 5,69 | 4,98 | 4,43 | 3,98 |
| | | 0,8 | 9,16 | 7,17 | 5,84 | 4,89 | 4,18 | 3,63 | 3,19 |

### Summary

The parameters of the optimal steel beam have been determined, based on the requirements of the Eurocodes and NEN 8700 / 8701 considering that the beam is part of an existing structure belonging to consequence class 2 and with a remaining lifetime of 15 years.

The determined parameters, namely $w_{nom}$ nominal cross section modulus shall now be taken as basis for a fully probabilistic calculation.

## *10.2.2 Reliability equation*

The reliability equation is the basis of any probabilistic calculation, as described in *Section 2.2.*
For bending moment at the mid-span of a steel beam it consists of the following components:

Resistance: $R_M = \theta_R \cdot w \cdot f_y$

Load: $S_M = \theta_S (g\frac{l^2}{8} + DAF \cdot M_{WIM})$

Where:

- $M_{WIM}$ is the load effect maxima for the given reference period, gained from the analysis of the weigh-in motion measurements (result of *Part II,* refer to *Section 9.3.3*).
- DAF is the dynamic amplification factor (refer to *Section 6.5*)

Thus the reliability equation is:

$$Z_{steel\ M} = \theta_R \cdot w\, f_y - \theta_S (g\frac{l^2}{8} + M_{WIM})$$

### *10.2.3 Reliability with Monte Carlo simulation* [38]

#### *Input to Monte Carlo analysis: Resistance side*

*Table 28* gives the input parameters used for the evaluation of failure probability for the steel beam. In **bold** values given by the Probabilistic Model Code of the Joint Committee on Structural Safety (*JCSS PMC)*. The yield strength ($f_y$) distribution of steel is a lognormal distribution truncated at the 2,3% characteristic value. The nominal value of the section modulus (w) is calculated in the previous step, here and example belonging to a reduction factor of 0,8 and a live-load to total load ratio of $\chi$=70% is given. For the simple failure mode of a steel beam in bending there is no resistance uncertainty ($\theta$R), thus it can either be neglected or taken as 1.

**Table 28 - Input to probabilistic analysis of steel beam in bending - Resistance**

| | | Distribution | Dim. | Nominal value | m (Mean) | σ (St. Dev.) | CoV | P1 (Param.) | P2 (Param.) | trunc. point |
|---|---|---|---|---|---|---|---|---|---|---|
| Yield str. | $f_y$ [39] | Lognormal | *N/m²* | 2,350E+08 | 2,708E+08 | 1,896E+07 | **0,07** | 19,415 | 0,0699 | 0,023 |
| Section modulus | **w** | Normal | *m³* | *4,893E-03* | *4,893E-03* | *1,957E-04* | **0,04** | - | - | |
| Resistance uncertainty | $\theta_R$ | - | - | 1 | - | - | - | - | - | |

#### *Input to Monte Carlo analysis: Load side*

Load variables are summarised in *Table 29*. The input for live loading ($M_Q$) is the distribution gained from the load effect analysis (result of *Part II,* refer to *Section 9.3.3*). The self-weight (g) changes with $\chi$, just as the section modulus among the resistance parameters. Here the example belonging to $\chi$=70% is taken. Load effect uncertainty ($\theta$S) is taken into account, the value of which is based on experience at TNO. The dynamic amplification factor is based on considerations detailed in *Section 6.5.*

**Table 29 - Input to probabilistic analysis of steel beam bending – Loading**

| | | Distr. | Dim. | Nominal value | m (Mean) | σ (St. Dev.) | CoV | P1 (Parameter) | P2 (Parameter) |
|---|---|---|---|---|---|---|---|---|---|
| Load from simulation | M | Gumbel | Nm | - | 5,811E+05 | 27613 | 0,05 | 5,69E+05 | 21530 |
| Dynamic amplification | DAF | Normal | - | - | 1,1 | 0,1 | 0,11 | - | - |
| Span of beam | L | det | m | 6,00 | 6,00 | - | - | - | - |
| Self weight | g | Normal | N/m | 8,104E+04 | 8,10E+04 | 1,62E+03 | 0,02 | - | - |
| Load effect unceratainty | θSQ, θSg | Normal | - | - | 1 | 0,05 | 0,05 | - | - |

### *10.2.4 Results and conclusion of reliability calculation*

The result of the simulation for various live-load – self-weight ratio sis tabulated in *Table 30.* The results give a higher reliability than the required $\beta$ = 2,3.

[38] All input variables are assumed to be non-correlated in the current analysis.

[39] For details on the yield strength distribution of steel and the truncated lognormal distribution refer to *Appendix C.*

**Table 30 - Results of Monte Carlo Simulation with traffic laoding input data – steel beam**

6m span, bending moment at mid cross section.

| Factor | χ | 0,40 | 0,50 | 0,60 | 0,70 | 0,80 | 0,90 | 1,00 |
|---|---|---|---|---|---|---|---|---|
| 1 | β | 3,97 | 4,08 | 4,09 | 4,15 | 4,00 | 3,98 | 3,92 |
| 0,8 | | 2,94 | 2,87 | 2,79 | 2,67 | 2,53 | 2,41 | 2,28 |

It is visible that reliability indices for the beam evaluated with a fully probabilistic analysis (Monte Carlo simulation) are above the required β =2.5, the reliability index corresponding to the requirement according to which the beam was designed, for traffic load ratios below approximately 85%. This means that the probabilistic analysis and inclusion of traffic loading data provides a more economic result than "checking" the beam according to Eurocode / NEN loading with semi-probabilistic methods. However if solely traffic loading is taken into account with the current parameters, the analysis does not prove to be beneficial.

This means that for the current beam (6m) with the given input parameters the application of traffic loading data is not more favourable than using the Eurocode loading with a possibly allowed reduction factor of 0.8. It is noted however that the standard deviation of the dynamic amplification factor was taken as 0.1, which may be slightly conservative as in some TNO reports 0.05 is chosen.

The "spread" of the load and resistance functions for two different ratios of live-load to self-weight, with and without load reduction factors are shown in *Figure 42* (a histogram of simulated values with connected data points) and *Figure 43* (scatter plot of simulated load- resistance pairs with failure boundary).

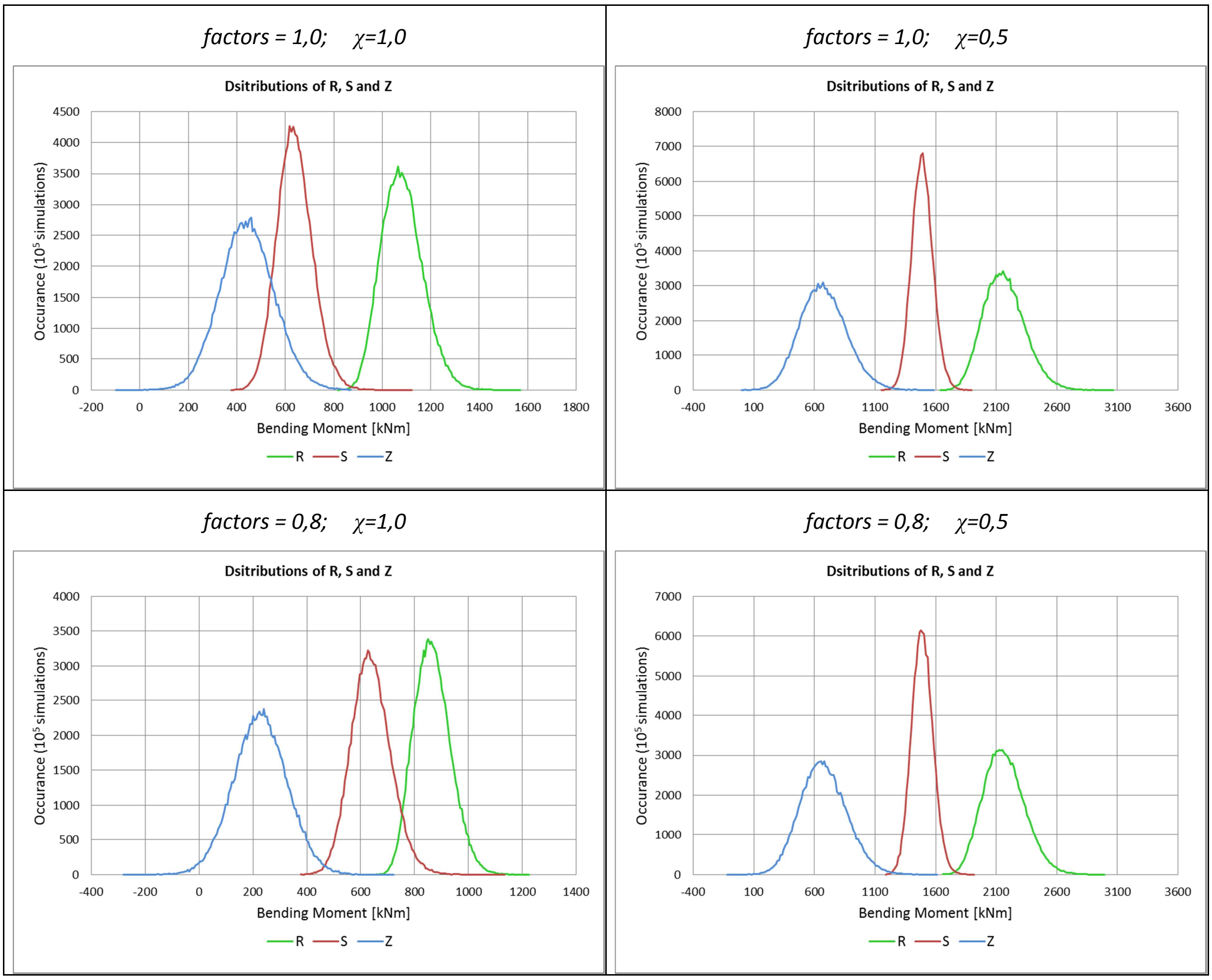


**Figure 42 - Distributions of Resistance, Load and Z - 6m steel beam, various design criteria**

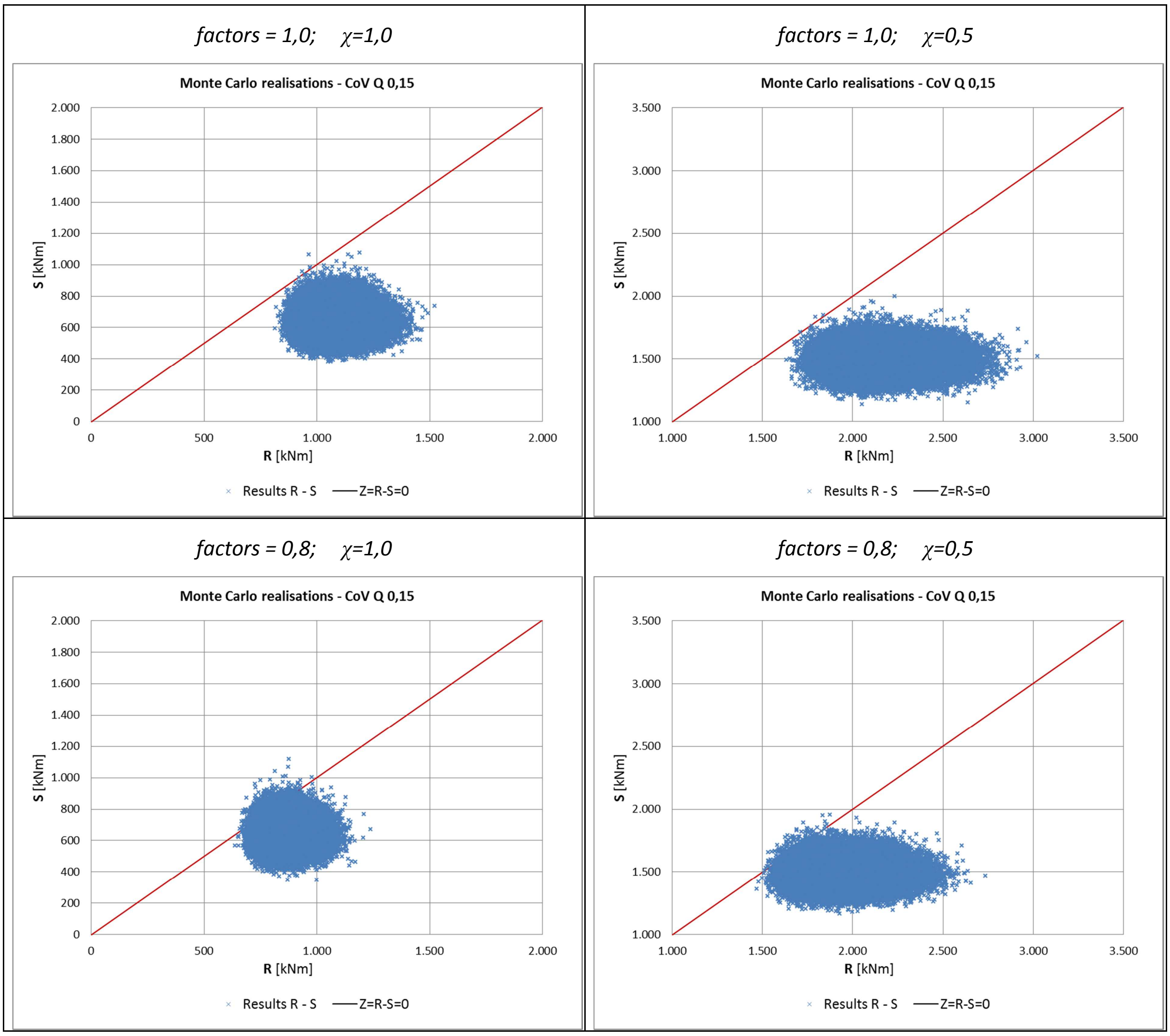


**Figure 43 - Resistance - Load scatter plots, result of MC analysis - 6m steel beam, various design crietria**

In the figures it is visible that the spread of the resistance is in the range of the spread of the load. For a steel beam this can be explained by the high certainty of both the material- and the strength model (for simple bending). Intuitively it would be expected that the uncertainty in the load will be more significant. With the current load model this is not the case. The CoV of the Gumbel distribution for example is only 0.05. Higher standard deviation in model uncertainty would create to a larger spread, as well as the inclusion of a stochastic parameter for the statistical uncertainty. In this case the reliability indices would decrease.

## 10.3 Concrete beam – Example

### *10.3.1 Semi-probabilistic calculation: dimensioning optimal beam*

#### *Basic data*

The current example determines the necessary cross section properties of a reinforced concrete beam that shall carry the traffic loading and various ratios of self-weight (including its own weight). The basic data of the beam is given in *Table 31*.

**Table 31- Initial parameters for concrete beam design**

| **Beam data for design** | | | |
|---|---|---|---|
| Yield strength | fyk | $N/m^3$ | 5,00E+08 |
| | fyd | $N/m^3$ | 4,35E+08 |
| Compression | fck | $N/m^3$ | 3,00E+07 |
| strength | $\alpha_{cc}$ | - | 0,85 |
| *C30/35* | fcd | $N/m^3$ | 1,70E+07 |
| Reinforcement | $\Phi$ | m | 0,02 |
| Area rebar | As | $m^2$ | 3,14E-04 |
| Concrete cover | c | m | 0,02 |
| **Beam data for load effects** | | | |
| Span | L | m | 6,00 |
| Height | h | m | 1,20 |
| Width | w | m | 1,00 |
| Unit weight | $\gamma$c.weight | $N/m^3$ | 25000 |

#### *Loading*

Traffic loading is to be taken into account as described in *Section 10.1.3.*

The self-weight of the structure can be expressed in two parts: the self-weight of the concrete beam ($g_c$ and the caused bending moment $M_{Gc}$) and the self-weight of other parts, for example the surface layers, rails, etc. ($g_2$ and $M_{G2}$).
The total characteristic load effect from the self-weight is:

$$M_{G\,char} = \mathrm{M_{Gc\,char}} + \mathrm{M_{G2\,char}}$$

Thus, according to definition, $\chi$ can be written as:

$$\chi = \frac{\mathrm{M_{Q\,char}}}{\mathrm{M_{Q\,char}} + \mathrm{M_{Gc\,char}} + \mathrm{M_{G2\,char}}}$$

Knowing the self-weight if the slab, the resulting load effect is:

$$\mathrm{M_{Gc\,char}} = \frac{g_{c\,char}\, l^2}{8}$$

The characteristic value of the load effect from variable loading is:

$$M_{Q\,char} = \frac{\mathrm{M_{Q\,Ed}}}{\gamma_{\mathrm{Q}} \cdot \mathrm{factors}}$$

Thus, for various $\chi$ factors the self-weight of "other parts" expressed in moment and in UDL is:

$$\mathrm{M_{G2\,char}} = \left(\frac{1}{\chi} - 1\right) \mathrm{M_{Q\,char}} - \mathrm{M_{Gc\,char}} = \left(\frac{1}{\chi} - 1\right) \mathrm{M_{Q\,char}} - \frac{g_{c\,char}\, l^2}{8}$$

$$\mathrm{g_{2\,char}} = \frac{8\left(\frac{1}{\chi} - 1\right) \mathrm{M_{Q\,char}}}{l^2} - g_{c\,char}$$

The uniformly distributed loads $g_{2\ \mathrm{char}}$ and $g_{c\ char}$ are expressed in units [kN/m]. Knowing the height and width of the beam the uniformly distributed load of the concrete self-weight is:

$$g_{\mathrm{c\ char}} = \rho_c\ \mathrm{hw}$$

In the current example the moment caused by the variable loading is carried by a one meter width beam.

The value $M_{Q\ Ed}$ is known from LM 1 loading, and $g_{c\ char}$ from the equation above. Values of the "other" self-weight ($g_2$) and total bending moment ($M_{Ed}$) can be determined for various ratios of traffic load to total load. For a beam of 6 m span in the example, these values are summarized in *Table 32.*

**Table 32 - Design bending moment on 6 m span concrete beam** (h=1,2m, w=1,0m) **for various traffic- to total load ratios χ**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Variable load effect (traffic) | $MQ_{char}$ | factor | 805,9 | | | | | |
| | $MQ_{Ed}$ | 1,0 | 936,0 | | | | | |
| | | 0,8 | 748,8 | | | | | |
| Ratio of variable load | χ | | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 |
| Total self-weight | $Mg_{tot\ char}$ | | 1276,4 | 850,9 | 567,3 | 364,7 | 212,7 | 135,0 |
| Beam self-weight | $Mg_{1\ char}$ | | 112,5 | | | | | |
| Other self-weight | $Mg_{2\ char}$ | | 1163,9 | 738,4 | 454,8 | 252,2 | 100,2 | 22,5 |
| Total load | $M_{Ed} = M_{Rd}$ | 1,0 | 2340,0 | 1872,0 | 1560,0 | 1337,1 | 1170,0 | 1084,5 |
| | | 0,8 | 2152,8 | 1684,8 | 1372,8 | 1149,9 | 982,8 | 897,3 |
| Total self-weight | $g_{tot\ char}$ | | 283,6 | 189,1 | 126,1 | 81,0 | 47,3 | 30,0 |
| Beam self-weight | $g_{1\ char}$ | | 30,0 | | | | | |
| Other self-weight | $g_{2\ char}$ | | 253,6 | 159,1 | 96,1 | 51,0 | 17,3 | 0,0 |

## *Resistance*

The design compressive strength of concrete according to Eurocode 2 is:

$$f_{cd} = \propto_{cc} f_{ck}/\gamma_c$$

Where

$\propto_{cc}$= 0.85 — Long-term effects on compressive strength and unfavourable effects of the way the load is applied. 0.85 for bending and axial load, may be 1.0 for other phenomena.

The resistance of a concrete beam in bending can be derived. The sketch of the dimensions and internal forces is given in *Figure 44*.

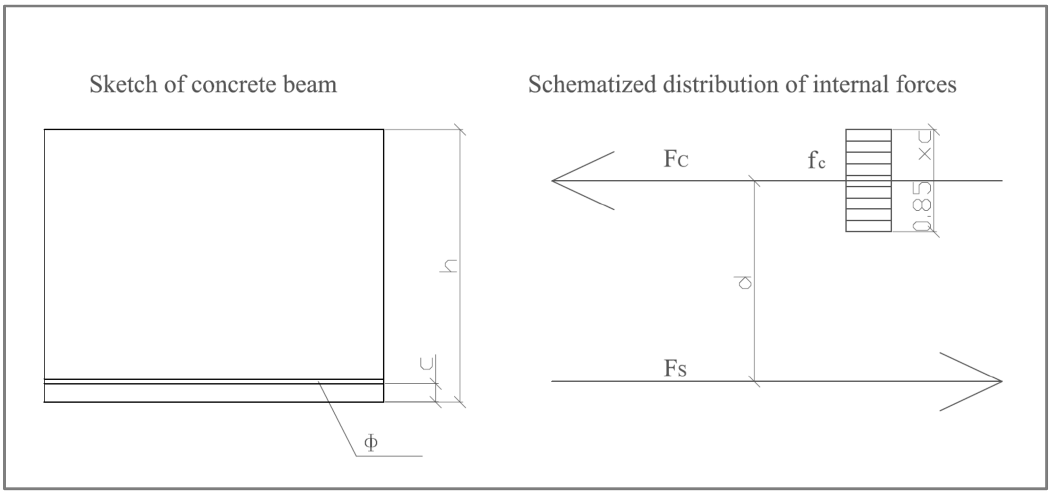


**Figure 44- Sketch of concrete beam cross section in bending**

Horizontal equilibrium: $$F_{sd} = F_{cd}$$

Where the forces in the steel and concrete respectively:

$$F_{sd} = nA_s f_{yd} = \frac{nA_s f_{yk}}{\gamma_s}$$

$$F_{cd} = x_c \cdot 0{,}85 f_{cd}$$

Where $x_c$ is the distance of the neutral axis from the top of the beam, which can be expressed as:

$$x_c = \frac{F_{cd}}{0{,}85 f_c} = \frac{n\,A_s\,f_{yd}}{0{,}85 f_{cd}}$$

Moment resistance: $M_{Rd} = d \cdot F_{sd}$

The internal level arm is: $d = h - c - 0{,}5 \cdot \phi - 0{,}5 \cdot x_c$

Therefore the moment resistance can be expressed as:

$$M_{Rd} = \left(h - c - 0{,}5\phi - 0{,}5\frac{n\,A_s\,f_{yd}}{0{,}85 f_{cd}}\right)\frac{n\,A_s\,f_{yd}}{0{,}85 f_{cd}}$$

### *Cross section properties*

For a concrete structure there are multiple design parameters, as also visible in the equation of the moment resistance (height of the structure, concrete and steel strength, reinforcement bar diameter, number of reinforcement bars). In practice the reinforcement area is to be determined given the necessary moment resistance, the other parameters are decided in advance. If the necessary reinforcement area (number of bars) is too large, the other parameters, first typically the bar diameter, or if the values are unacceptable for some reason then the beam height may be changed.

In the current example the necessary number of reinforcement bars is chosen as a design parameter, after a bar diameter had been selected.

The “design” requirement:

$$M_{Rd} = M_{Ed}$$

The design bending moment resistance has been expressed with the beam properties in the previous section . The number of bars per meter *‘n’* in a beam can be determined from the equation:

$$A \cdot n^2 + B \cdot n + C = 0$$

Where:

$$A = \frac{0{,}5\,A_s\,f_{yd}}{0{,}85 f_{cd}}$$

$$B = -(h - c - \frac{\phi}{2})$$

$$C = \frac{M_{Rd}}{A_s f_{yd}} = \frac{M_{Ed}}{A_s f_{yd}}$$

The necessary number of reinforcement bars per meter width is defined. The number is not rounded up to an integer, because the aim of the example is to use a limit state ($M_{Ed} = M_{Rd}$) as starting point. For various live-load to total-load ratios, which result in various design mending moments (summarized in *Table 32.*) the resulting values of *n* are summarized in *Table 33*.

**Table 33 - Necessary number of reinforcement bars** (n)

| | | | $\chi$ | 0,40 | 0,50 | 0,60 | 0,70 | 0,80 | 0,86 |
|---|---|---|---|---|---|---|---|---|---|
| factors | 1 | n | | 15,63 | 12,33 | 10,18 | 8,67 | 7,55 | 6,98 |
| | 0,8 | | | 14,30 | 11,03 | 8,91 | 7,42 | 6,31 | 5,75 |

### Summary

The parameters of the optimal concrete beam have been determined, based on the requirements of the Eurocodes and NEN 8700 / 8701 considering that the beam is part of an existing structure belonging to consequence class 2 and with a remaining lifetime of 15 years.

The determined parameters shall now be taken as basis for a fully probabilistic calculation.

### *10.3.2 Reliability equation*

The reliability equation is the basis of any probabilistic calculation, as described in *Section 2.2.*

Based on the relations derived in *Section 10.3.1*, the bending moment resistance and the load can be expressed using variables which were already introduced. The difference compared to the semi-probabilistic calculation is that the ***variables*** are now ***stochastic***, the value of each is described by a statistical distribution. The other difference is the inclusion of ***model uncertainties*** (in semi-probabilistic calculation the partial safety factors include the effects of this) in the reliability equation, which are denoted by $\theta$ and are described in detail in *Section 2.2.3.* Furthermore, in determining the load effect a ***dynamic amplification factor*** (DAF) is taken into account. More of this (DAF) can be read in *Section 6.5*.

Bending moment resistance at mid-span of a concrete beam:

Resistance:

$$R_{c\,M} = \theta_R n \frac{\pi\phi^2}{4} f_y (h - c - \frac{\phi}{2} - 0{,}5n \frac{\pi\phi^2}{4} \frac{f_y}{0{,}85 f_c})$$

Load:

$$S_M = \theta_{SQ} M_{WIM} \cdot DAF + \theta_{Sg} \left( g_{tot} \frac{L^2}{8} \right)$$

Where:

- $M_{WIM}$ is the load effect maxima for the given reference period, gained from the analysis of the weigh-in motion measurements (result of *Part II,* refer to *Section 9.3.3*).
- DAF is the dynamic amplification factor (refer to *Section 6.5*)

Thus the reliability equation is:

$$Z_{cM} = \theta_R n \frac{\pi\phi^2}{4} f_y \left( h - c - \frac{\phi}{2} - 0{,}5n \frac{\pi\phi^2}{4} \frac{f_y}{0{,}85 f_c} \right) - \theta_{SQ} M_{WIM} \cdot DAF + \theta_{Sg} \left( g_{tot} \frac{L^2}{8} \right)$$

### *10.3.3 Reliability with Monte Carlo simulation*

All input variables are assumed to be non-correlated in the current analysis.

#### Input to Monte Carlo analysis: Resistance side

*Table 34* gives the resistance input parameters used for the evaluation of failure probability of the concrete beam. The design is based on $\chi$=0.7, which changes the value of the design parameter n (number of reinforcement bars / m). *Table 35* gives further details about the origin of the parameters as well as their calculation method and "meaning".

**Table 34 - Input to probabilstic analysis of concrete beam in bending – Resistance**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Basic compressive strength | C30/35 | Distribution | Dim. | m' | n' | s' | v' | log mean | log std |
| Option 1: logstudent | $f_{c0}$ | logstudent | N/mm$^2$ | 3,85 | 3 | 0,09 | 10 | 3,85 | 0,1232 |
| | | | | Nominal value | m (Mean) | σ (St.Dev.) | CoV | P1 (Param.) | P2 (Param.) |
| Option 2: lognormal | $f_{c0}(2)$ | Lognorm | N/mm$^2$ | - | 47,351 | 5,858 | 0,12370 | 3,85 | 0,1232 |
| Long-term effects | $\alpha$ | Determ | - | 0,85 | 0,85 | | - | | |
| Variation in situ - test | $\lambda$ | Determ | - | 0,96 | 0,96 | | - | | |
| | Y1,j | Lognorm. | - | - | 1 | 0,06 | 0,06 | -0,002 | 0,0599 |
| Number of bars / m | n | Determ. | - | 8,671 | 8,671 | | - | | |
| Reinforcement | $\Phi$ | Determ. | m | 0,02 | 0,02 | | - | | |
| Yield str. | $f_y$ | Lognorm. | N/m$^2$ | 5,00E+08 | 5,60E+08 | 3,00E+07 | 0,0536 | 20,142 | 0,0535 |
| Area rebar | $A_s$ | Normal | m$^2$ | 3,14E-04 | 3,14E-04 | 6,28E-06 | 0,02 | - | |
| Concrete cover | c | Normal | m | 0,02 | 0,02 | 0,005 | 0,25 | - | |
| Beam depth | h | Determ. | m | 1,2 | 1,2 | | - | | |
| Beam depth "deviation" | Y | Normal | m | - | 0,003 | 0,01 | | - | |
| Resistance unceratainty | $\theta_R$ | Lognorm. | - | - | 1,2 | 0,18 | 0,15 | -0,005 | 0,0998 |

**Table 35 - Concrete beam resistance properties - explanation**

| Variable | | Distribution | Source | Description |
|---|---|---|---|---|
| Yield strength bar | fy | Lognormal | PMC III. - 3.2<br>Static Properties of Reinforcing Steel | *Distribution is truncated at 2.3% value*<br>*Detailed description in Appendix* |
| Compression str. | fc0,<br>fc,<br>$\alpha$, $\lambda$ | *Logstudent* | PMC III. - 3.1 - Concrete Properties<br>ISO 2394 - Annex D | *Refer to Appendix* |
| Beam depth | $h_{nom}$ | Deterministic | PMC III. - 3.10 - Dimensions<br>3.10.1. External dimensions of concrete coomponents | *"Dimensional deviations of a dimension X are described by statistical characteristics of its deviations Y from the nominal value $X_{nom}$" (JCSS)*<br>$Y = X - X_{nom}$<br>2 formulas for mean and stand.dev. As function of $X_{nom}$ (p.33) |
| *Variable part of beam depth* | $Y_h$ | Normal | | |
| Reinforcement | $\Phi$ | Determ. | - | *Deterministic only for the calculation of effective depth* |
| Area rebar | As | Normal | PMC III. - 3.2<br>Static Properties of Reinforcing Steel | |
| Concrete cover | c | Deterministic | PMC III. - 3.10 - Dimensions<br>3.10.2. Concrete Cover | *See beam depth for main concept*<br>*Variability of concrete cover effects the effective depth. One option to take this into account is by adding a variable Y to the calculated effective depthto get its stochastic form* |
| Effective depth | a= c+Φ/2 | Deterministic | | |
| | $Y_a$ | Normal | | |
| Resistance unceratainty | θR | Lognormal | PMC III. - 3.9 - Model Uncertainties<br>3.9.3 Recommendations for practice | *"Including effects of normal and shear forces"* |
| Density conc. | $\gamma$c | Normal | PMC II. - 2.1 Self Weight<br>2.1.4 Weight density | - |

### *Input to Monte Carlo analysis: Load side*

Load variables are summarised in *Table 36*. The input for live loading ($M_Q$) is the distribution gained from the load effect analysis (result of *Part II,* refer to *Section 9.3.3*). The self-weight (g) changes with $\chi$, just as the section modulus among the resistance parameters. Here the example belonging to $\chi$=0.7 is taken. Load effect uncertainty ($\theta$S) is taken into account, the value of which is based on experience at TNO. The dynamic amplification factor is based on considerations of *Section 6.5*.

**Table 36 - Input to probabilistic analysis of concrete beam - Loading**

| LOAD VARIABLES | | Distr. | | Nominal | μ (Mean) | σ (St. Dev.) | CoV | Location (b) | 1/α |
|---|---|---|---|---|---|---|---|---|---|
| Moment from traffic | $M_{traffic}$ | Gumbel | *N* | - | 5,81E+05 | 2,76E+04 | 0,0475 | 5,69E+05 | 21522,8 |
| Dynamic amplification | DAF | Normal | - | - | 1,1 | 0,05 | 0,0455 | - | |
| Span of beam | L | Det. | *m* | 6,00 | 6 | - | | | |
| Width of beam | w | Det. | *m* | 1,00 | 1 | - | | | |
| Density conc. | γc | Normal | *N/m³* | 25000 | 25000 | 1000 | 0,04 | - | |
| Self weight concrete | g | Normal | *N/m* | 3,00E+04 | 3,00E+04 | 1,20E+03 | 0,04 | - | |
| Other self-wieght | g2 | Normal | *N/m* | 5,10E+04 | 5,10E+04 | 2,04E+03 | 0,04 | - | |
| Load effect unceratainty | θSQ<br>θSg | Normal | - | - | 1 | 0,07 | 0,15 | - | |

## *10.3.4 Results and conclusion of reliability calculation*

The result of the simulation for various live-load – self-weight ratio sis tabulated in *Table 37.*

**Table 37 - Results of Monte Carlo Simulation with traffic laoding input data – concrete beam**
6m span, bending moment at mid cross section.

| Factor | χ | 0,40 | 0,50 | 0,60 | 0,70 | 0,80 | 0,86 |
|---|---|---|---|---|---|---|---|
| 1 | β | 3,63 | 3,86 | 3,95 | 4,13 | 4,19 | 4,22 |
| 0,8 | | 3,15 | 3,24 | 3,22 | 3,22 | 3,22 | 3,20 |

The reliability indices for the beam evaluated with a fully probabilistic analysis (Monte Carlo simulation) are significantly higher than the required β =2.5, the reliability index corresponding to the requirement according to which the beam was designed. This means that the probabilistic analysis and inclusion of traffic loading data provides a more economic result than “checking” the beam according to Eurocode / NEN loading with semi-probabilistic methods.

The “spread” of the load and resistance functions for two different ratios of live-load to self-weight, with and without load reduction factors are shown in *Figure 45* (a histogram of simulated values with connected data points) and *Figure 46* (scatter plot of simulated load- resistance pairs with failure boundary).

In the figures, it is visible that the “spread” of the resistance is relatively large compared to the spread of the load.

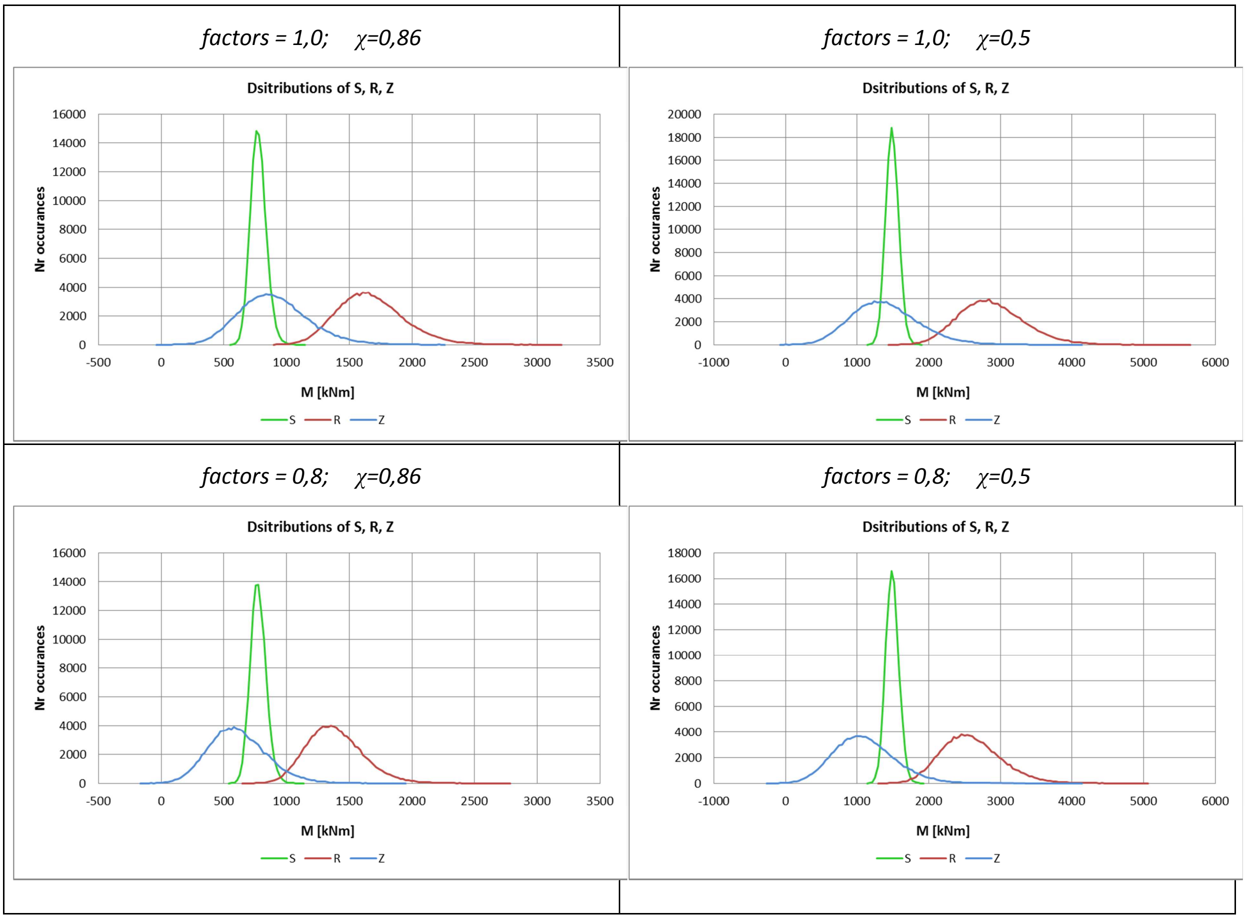


**Figure 45- Distributions of Resistance, Load and Z - 6m concrete beam, various design criteria**

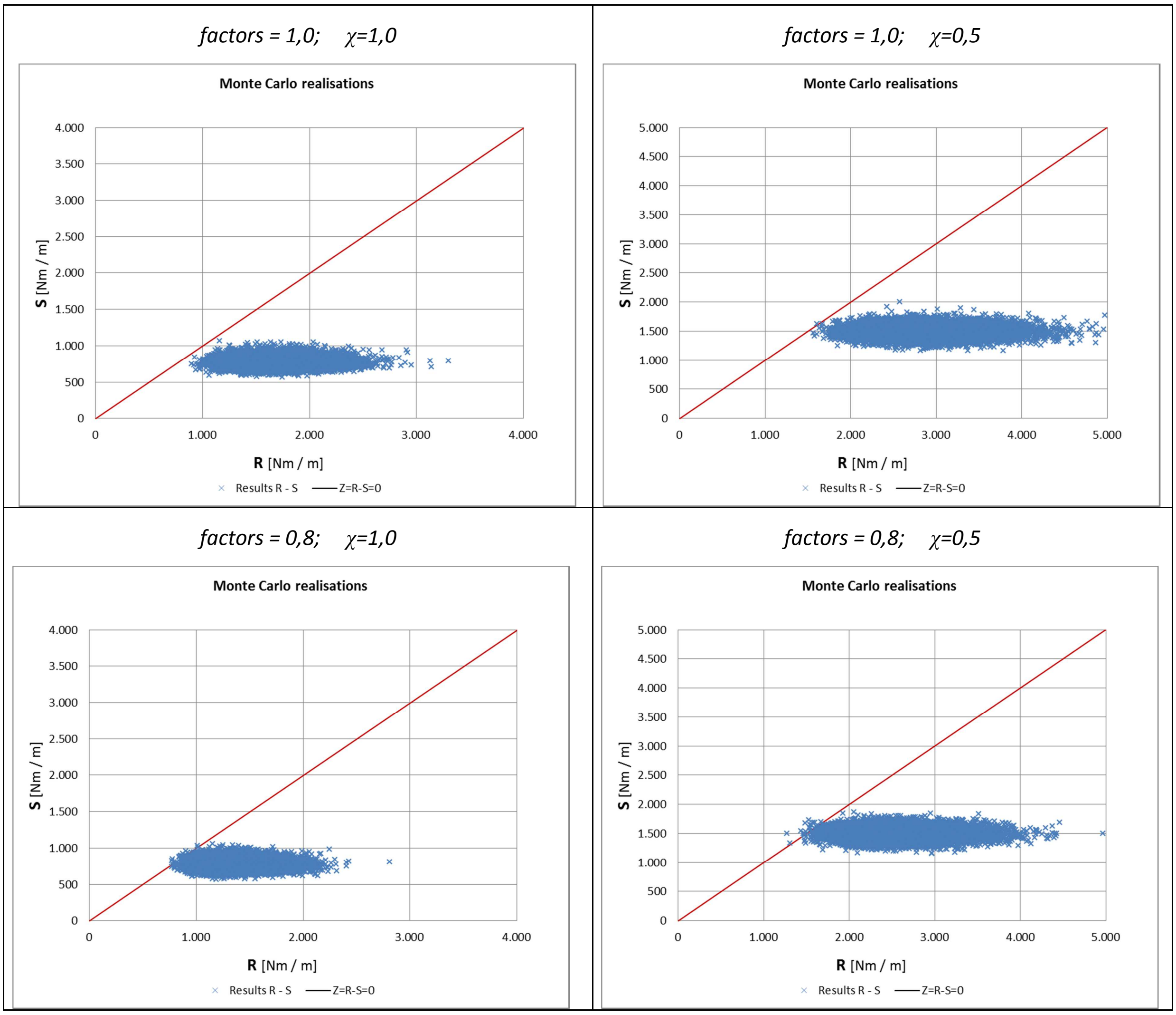


**Figure 46 - Resistance - Load scatter plots, result of MC analysis, 6m concrete beam, various design criteria**

# 11 Conclusions and Recommendations

## 11.1 Conclusion

In this thesis the applicability of Monte Carlo simulation for existing city bridges with the inclusion of weight-in-motion measurements was investigated.

The currently applicable norms allow for the use of probabilistic methods. Therefore the applicability of Excel ( a tool commonly used in the daily practice of an engineering office) for elementary structural reliability problems was determined and found possible. Input data can be easily changed and output can be visualised, moreover the accuracy is sufficient. For the target reliabilities in the context of existing urban bridges (typically $\beta$=2.5 for consequence class 2 and 15 years remaining life) the required maximum failure probability is in a range which can be simulated without difficulty.

It was recognised that input for the resistance side *R* of the reliability equation, Z = R-S , data is readily available. However, a probabilistic traffic load model for short-span bridges had to be developed in order to arrive to stochastic input for the load side *S*.

WIM data analysis and the application for traffic loading has been studied extensively and a method was proposed and worked out to arrive in a relatively simple way to life-time load effect maxima functions. The aim was to gain a distribution function which could be then used in a fully probabilistic reliability analysis (Monte Carlo Simulation). The model was compared to measurement results and after some adjustment corresponded well to the available data. Currently the result of the load effect maxima function is available for one specific structure: a simply supported beam with a span of 6 meters. The simulation can be reproduced easily for various spans and load effects due to the adaptability of the code and the possibility to combine its different “blocks”.

The reliability analysis for both a steel and a concrete beam with the above mentioned 6 m span was performed. As a first step the beam was designed as optimal for a reliability index of 2.3 and was then evaluated with a Monte Carlo simulation. For both structures the result was a higher reliability than that required by the norms: for the steel beam was in the range 2.3 – 2.9, while the concrete beam in the range 3.2 – 3.8. It can be concluded on one hand, that partial safety factors for concrete structures (for bending moment resistance) are relatively conservative in comparison to those of steel structures. The application of WIM data and a fully probabilistic analysis, under certain assumptions (*Section 7.3*), was therefore shown to provide less conservative results than a semi-probabilistic analysis with the load models of Eurocode.

### *Overview of research questions*

After having formulated a problem statement, aims of the thesis work were set, which were broken down to specific research questions. In the following, these are listed and answered briefly, with reference to relevant sections of the thesis work.

***I)*** *Gain overview of methods in structural reliability analysis;*

a. *What methods of structural reliability analysis are available?*
b. *How are these methods applicable with respect to building codes and regulations?*
c. *What are the advantages and disadvantages of each method and when are they applicable?*

These questions were answered in Part I – Background, in Sections XXYYZZ

***II)*** *Determine relevant structure types and failure modes;*

a. *What are “typical” structures among bridges in Rotterdam?*
b. *Are there common failure modes and if yes, what are these?*
c. *Is there potential for applying Monte Carlo analysis to investigate these failure modes?*

After having considered on the one hand the complexity of the bridge park of Rotterdam, on the other hand the relation between the type of failure investigated and the load modelling, it was decided to model a simple limit state of global failure due to bending moment. For global analysis in cases when the structure can be "reduced" to a slender structure, the developed codes can be applied easily for bending moment and shear failure. The model could be further developed to consider transverse distribution over the bridge. Fatigue effects have not been investigated in the thesis due to the completely different nature of the load model which would be necessary.

***III)*** *Model the relevant (or otherwise chosen) structural failures;*

*a. How can the resistance of these failures be modelled?*

Bending-moment failure was investigated, using well-known basic relations of structural engineering. Input for the resistance parameters could be found in codes, some values (concrete compressive strength and steel yield strength) are elaborated in more detail in appendices.

*b. What is the result of the analysis without input traffic loading data?*

No probabilistic traffic load model was directly available to use as input to the traffic loading analysis. What could be done was to assume a Gumbel-distributed load (or load effect) and find the CoV which provides a reliability index equivalent to those in the code.

***IV)*** *Analyse and interpret traffic loading*

*a. Convert weigh-in-motion data to traffic loading;*

*i. What does WIM data represent and how is it related to standard load models?*

*ii. What is the best strategy for analysis of the data, with respect to data interpretation and extrapolation?*

*b. Convert traffic loading to load effects;*

*i. How can the loading be converted to load effects?*

*ii. What are the possibilities to use these load effects in simulation?*

This goal has been fulfilled in Chapters 7 – 9. The sub-question *a.i.* is covered in *Section 6,* which also includes analysis of the strategies adapted in literature. The chosen strategy is presented in *Section 7*. The sub-goals *a.* and *b.* are reached in steps described according to *Sections 8* and *9* respectively.

***V)*** *Determine structural reliability of relevant (or otherwise chosen) failure modes and a chosen specific case;*

a. Incorporate load effects in limit state equations and carry out analysis;

This is done in *Section 10*, for a steel and concrete beam.

***VI)*** *Evaluation of applied methods with respect to precision, usability and usefulness;*

*a. Are the methods applicable in practice?*

Yes, the methods of Monte Carlo analysis (using Excel) is applicable in practice, as demonstrated with the examples of Section 10 and as summarized also above.

*b. What are the limitations?*

Monte Carlo analysis does not provide insight to the influence coefficients of the stochastic parameters. This means that the influence of input parameters on the *variance* of the output cannot be directly determined. Either the parameters of the distribution functions should be changed manually to arrive to some insight about their influence or a separate FORM analysis has to be carried out (in Excel or using reliability software) to gain this information, if necessary.

The developed traffic load model is only applicable for structures where the load can simply be reduced to a slender structure (for example to a beam). To consider a (random) lateral distribution, the model should be further developed.

c. *What are costs and benefits in comparison to semi-probabilistic calculations?*

It has been shown that, especially in case of a concrete structure, the results of a Monte Carlo analysis can be expected to be less conservative than an evaluation with semi-probabilistic methods. The benefit in this case may be that an existing structure is proven sufficiently safe and traffic limitation, refurbishment or other measures can be avoided.

A further advantage is that location-specific load data can be used, in comparison to calculation according to the norms.

The disadvantage of the current model that it is not directly applicable for a more advanced structural model. Naturally, the time required to

d. *What are costs and benefits in comparison to other, level II or III probabilistic assessment methods?*

As stated already, in comparison to a FORM analysis a Monte Carlo simulation does not give direct information about sensitivities. On the other hand, as the distribution functions are not approximated, the result of a simulation may be more reliable. Furthermore, visualisation can be used to give insight to the "spread" of the total load and resistance functions.

As the input parameters of even the presented simple reliability equations are in the form of several different probability distribution functions, an analytical solution is not preferable over the proposed numerical method – especially in the context of "daily engineering practice".

## 11.2 Recommendations for further research

Recommendations for further research are divided in two categories. Firstly, recommendations are given related to verifying assumptions on which the current research is based. Secondly, an outlook is given on possibilities for “expanding” the usability of both WIM data analysis and Monte Carlo simulation in structural reliability analysis within an engineering office.

### *11.2.1 Investigating validity of assumptions*

It is recommended to check the validity of splitting vehicle properties within an axle category to sub-categories based on GVW for the sampling. Although with this method simulated axle loads tend to correspond to the measurements (and are conservative at the high values), it is not fully proven that the simulation method is a “safe estimate”. This could be done by plotting simulated load effects against load effects from measurements. Besides, instead of sub-categorising within a bigger category based on the GVW, the readily available sub-categories from the WIM measurements could be used.

Vehicles within categories in which a low number of measurements had been carried out could be included in further analysis. The influence of including the information of vehicle properties of such truck (for example 10-axle vehicles) in the 8-axle category could be investigated. Another solution for including such vehicles would be to assume a similar distribution for the GVW as for 8-axle trucks and possible increase the simulated GVW with a factor.

It could be further investigated whether truck configurations are adequate representation of a population in a category. One way of doing this would be to use the data of highways or measurements from other municipalities to describe a truck population, couple it with GVW distributions in Rotterdam and investigate the difference in resulting load-effect maxima distributions.

The influence of adequate tail modelling of GVW-s can be further investigated, using truncated maximum-likelihood procedure. The results from this more precise modelling could then be compared to those resulting from fits with Gaussian mixture distributions.

In several places in literature the fundamental assumption is mentioned that one very heavy truck on a bridge is dominant . The validity of this assumption however is could be checked.

### *11.2.2 Outlook*

It is of practical relevance to investigate the influence of weight limitation of vehicles. This can be done by modelling GVW with a truncated distribution function.

The current model allows to investigate what the dominant trucks are which are expected to cause the maximum load effects. This can be further analysed and visualised.

Looking at multiple cross sections of the beam instead of one specific cross-section, it can be interesting to know how the distribution function of load effect maxima changes.

Using the developed procedure, further calculations can be carried out on various beam lengths. Shear effects can also be investigated directly.

A suggestion for the further development of the model, it is recommended to investigate the possibility to consider lateral load distribution on a structure and upgrade the traffic load- or load effect model. In this way the probabilistic calculation could be used for structures which cannot be simplified to a beam or where the semi-probabilistic analysis was done on a more complex FEM model and further “gain” with a probabilistic analysis cannot be expected if carried out on a simple beam.

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# Appendices

# A. Needed number of Monte Carlo simulations

In order to determine the applicability of a Monte Carlo simulation with the proposed general software (Excel, MathCad), the question was investigated whether the necessary number of simulations can be reached? On one hand, it is relevant to know how many simulations are necessary, on the other hand, how many simulations can Excel / MathCad handle?

## Theoretical background [40]

Relative error in simulation:

$$\varepsilon = \frac{\frac{n_f}{n} - P_f}{P_f}$$

The expected value of the relative error is 0. The standard deviation is:

$$\sigma_\varepsilon = \sqrt{\frac{1 - P_f}{n \times P_f}}$$

With enough simulations, based on the Central Limit Theorem, the error is normally distributed. Then the probability that the error is smaller than E is:

$$P(\varepsilon < E) = \phi\left(\frac{E}{\sigma_\varepsilon}\right)$$

With a reliability of $\Phi(k)$ the relative error is smaller than $E = k\sigma_\varepsilon$.

Thus the needed number of simulations, <u>for an expected k and E</u> is:

$$n > \frac{k^2}{E^2}\left(\frac{1}{P_f} - 1\right)$$

## Requirement

JCSS (Vo.1, p.15):

A. Overestimation of the reliability due to use of an approximate calculation method shall be within limits generally accepted for the specific type of structure

B. The overestimation of the **reliability index** should not exceed 5 % with respect to the target level.

Practice:

C. $\beta$ rounded up to one integer. This is a more strict requirement than the 5% in JCSS PMC, if target probabilities in the range of existing structures are used.

Diamantidis et al. (2012)

D. "A general rule for the specification of the number of simulations is relatively simple. (…) n should be about two orders greater than the number of simulations "expected to give" one failure (inverse of failure probability).

Reason: coefficient of variation $w_{pf}$ of the failure probability can be estimated by

$$w_{pf} = (1 - P_f)^{0,5} \cdot (n \cdot P_f)^{-0,5}$$

The graphs below show the outcome comparing requirement B and C.

[40] Based on CUR-publicatie 190 (1997)

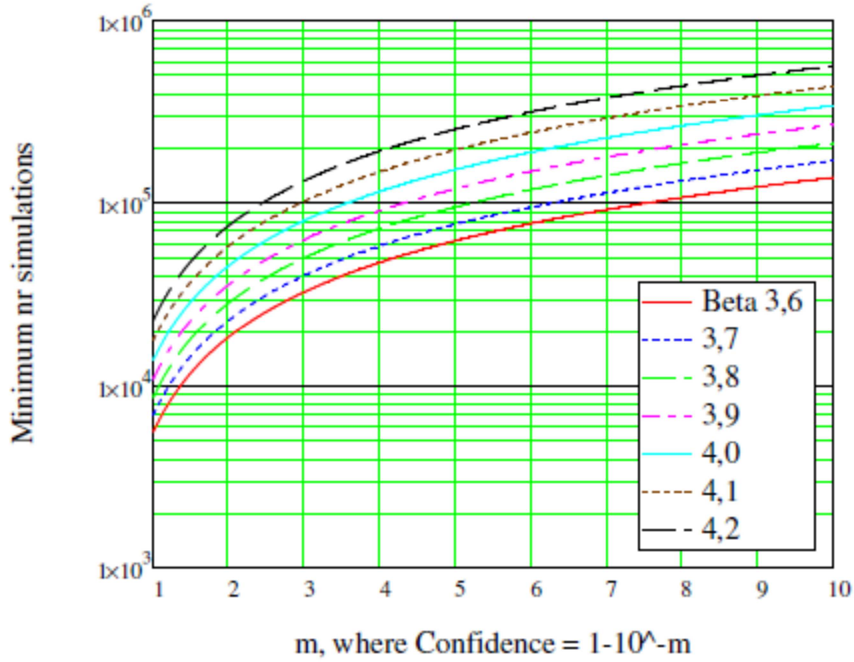


**Figure 47 - Number of needed simulations, JCSS (5% difference in β) recommendation**

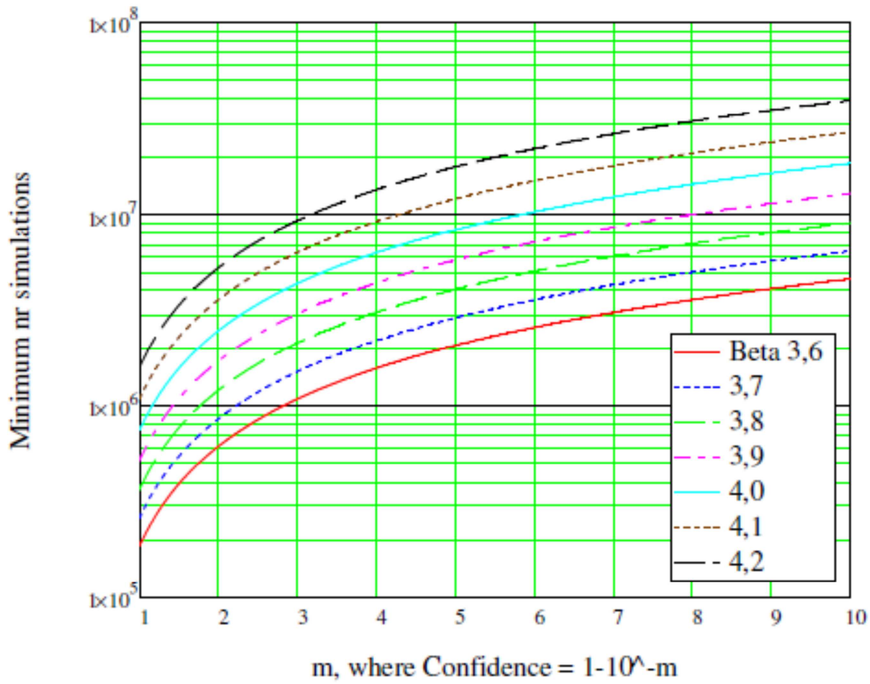


**Figure 48 - Number of needed simulations, "practical" punctuality (β rounded to 1 integer)**

The answer for the 1st question seems to be: number of simulations needed appears to be in the range of $10^5$ : $10^7$. The final number depends mainly on which error requirement is actually used, and with what confidence should this error not be exceeded?

*For the final thesis version: when in the range $\beta$=2,5 – 3,3, how many simulations are needed?*

In practice, Excel could handle (in a tabulated format – thus not with Macros) up to 500 000 rows of simulation. These can be run multiple times or multiple columns can be created. The capacity also depends on the amount of input variables of course.

## B. Distributions

*To be added in final version – besides description of the distribution, the analytical formula for the inverse cumulative probability will be given.*

# C. Steel yield strength

## *Models of JCSS, ProQua and analytical*

There are two seemingly different models available in recommendations (in Europe) for the probabilistic modelling of steel yield strength. One is given in JCSS PMC (JCSS 2001), the other in a more recently developed recommendation *'Probabilistic Quantification of Steel Structures… '* (Cajot et al. 2005).Moreover, the steel yield strength can be described by the simple analytical expression of the lognormal distribution. In the current section these models are introduced, as well as the relation between the characteristic value (which is taken into account in semi-probabilistic calculations) and the probability distribution function of the yield strength. The results of the models are compared.

The probability distribution is modelled in each case with a lognormal distribution, which is visualised below together with the nominal value.

*Add figure of lognormal distribution with the nominal value*

### *Statistical model of JCSS PMC*

The relation between the nominal (value used in the semi-probabilistic calculation) and mean value of the steel strength is described in *JCSS PMC* as follows:

$$\mu_{fy} = f_{ysp} \cdot \alpha \cdot e^{-u \cdot v} - C$$

Factors are explained below, example values are given in bold. These are the values used in the calculation of *Section 10.2.*

$f_{ysp}$ Yield strength: code-specified nominal value – thus in case of S235 **235N/mm$^2$**

$\alpha$ Spatial position factor; 1,05 for hot-rolled webs, **1,0** otherwise

$u$ Factor related to the fractile of distribution, used to describe the distance between the code-specified nominal value and the mean value; Range of -1,5 to **-**2,0 (production by EN standards). For example: *-2 corresponds to the fractile 0,023; -1,5 corresponds to ~0,07*

$v$ Coefficient of variation; **0,07**

$C$ Constant reducing yield strength due to excluding "weak" samples; recommended: 20 MPa
The reduction introduced by the constant *C* represents the fact that the mean value of distribution which is truncated (from the left) also shifts to the left. In case such a distribution is used, the "input" to the probabilistic calculation should also be a truncated lognormal distribution.

### *Statistical model of 'ProQua'*

According to this document the following model is suggested, based on extensive research on properties of structural steel:

$$\mu_{fy} = f_{ysp} + k_s \sigma$$

$$\sigma = 0{,}08 \cdot \mu_{fy}$$

➔ $\sigma = \frac{0{,}08 f_{ysp}}{1-0{,}08 k_s}$

$k_s$ Range of 2 to 2,5, depending on execution control; (2,0 for non-regular, 2,5 for good quality control)

### *Analytical Model*

The yield strength is modelled with a truncated lognormal distribution, where the **nominal value is the truncation point of 0,023**.

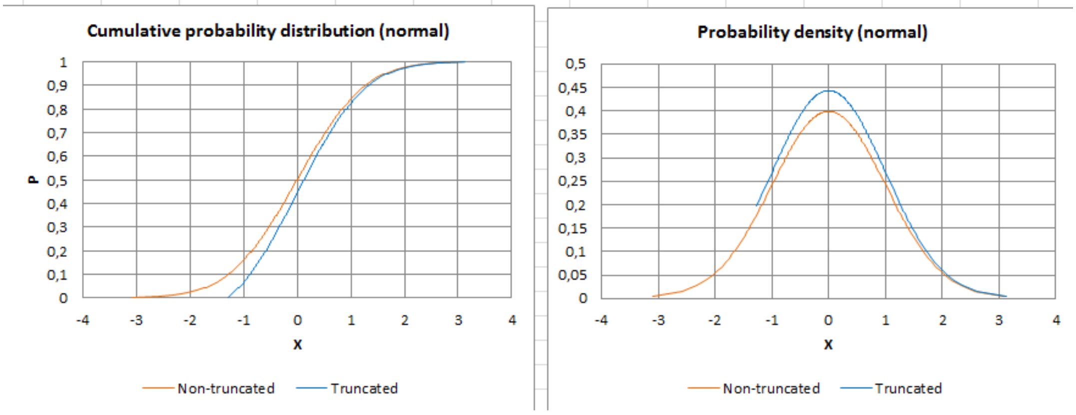


**Figure 49 - Example of truncated distribution**

The following input parameters of the lognormal distribution are known:

- Nominal value at p = 0,023 $f_{yp\%}$ = 235 N//*mm*$^2$
- Coefficient of variation CoV = 0,07

This information is enough to define a lognormal distribution, which is described by 2 parameters P1 and P2. The following relations hold:

$$P_2 = \sqrt{\ln(CoV^2 + 1)}$$

$$P_1 = \ln\left(f_{yp\%}\right) - P_2 \phi^{-1}(p)$$

The mean and standard deviation of the distribution can then be computed as follows:

$$\mu = exp\left(P_1 + \frac{P_2^2}{2}\right)$$

$$\sigma = \sqrt{exp(2P_1 + P_2^2)(exp(P_2^2) - 1)}$$

As an example, the following table summarizes values describing lognormal distributions with various *p [%]* values, same $f_{yp}$ and same CoV.

**Table 38 - Truncated lognormal distributions**

| Nominal value | m (Mean) | σ (St. Dev.) | P1 (Parameter) | P2 (Parameter) | Point of trunc. PDF |
|---|---|---|---|---|---|
| **235,0** | **277,18** | 19,40 | 5,622 | 0,069914 | 0,01 |
| | **275,54** | 19,29 | 5,616 | | 0,0125 |
| | **274,17** | 19,19 | 5,611 | | 0,015 |
| | **272,99** | 19,11 | 5,607 | | 0,0175 |
| | **271,95** | 19,04 | 5,603 | | 0,2 |
| | **270,84** | **18,96** | 5,599 | | 0,023 |
| | **270,17** | 18,91 | 5,597 | | 0,025 |
| | **269,40** | 18,86 | 5,594 | | 0,0275 |
| | **268,68** | 18,81 | 5,591 | | 0,03 |
| | **267,39** | 18,72 | 5,586 | | 0,035 |
| | **266,25** | 18,64 | 5,582 | | 0,04 |
| | **265,22** | 18,57 | 5,578 | | 0,045 |
| | **264,29** | 18,50 | 5,575 | | 0,05 |

### *Comparison*

The three models give slightly different results, as shown in *Table 39*.

**Table 39 - Comparison of yield strength models**

| | **JCSS PMC** | | | | | **ProQua** | | |
|---|---|---|---|---|---|---|---|---|
| | **hot rolled web** | | **other** | | | | *Control–* ? | Control+ ? |
| Alfa | 1,05 | 1,05 | 1 | 1 | 1 | ks | 2 | 2,5 |
| u | -2 | -2 | -2 | -2 | -1,5 | σ | 22,381 | 23,500 |
| V (CoV) | 0,07 | 0,07 | 0,07 | 0,07 | 0,07 | → v | 0,095 | 0,1 |
| C | **20** | 0 | **20** | 0 | 0 | | | |
| fy nom | 235 | 235 | 235 | 235 | 235 | | 235 | 235 |
| **Mu fy** | 263,83 | 283,83 | 250,31 | 270,31 | 261,02 | | 279,76 | 293,75 |
| P(fynom) | *5,26 %* | *0,38 %* | *19,28 %* | *2,46 %* | *7,12 %* | | *1,61 %* | *0,29 %* |

# *D.* Concrete properties, compressive strength [41]

## General information

The *Probabilsitic Model Code (PMC) Part 3: Resistance models* (JCSS 2001) gives detailed information about the calculation of concrete properties. Another useful document is the relevant *ISO-2394* standard (Technical Commtitee ISO/TC 98 1998) which describes in more detail the background of the suggested distributions and gives guidance on updating with information from measurements. In the examples worked out to the PMC (Vrouwenvelder et al. 2002) two concrete structures are calculated as well, a beam and a multi-story building.

According to the PMC, similarly as in the Eurocode, all basic properties of concrete are related to the **basic concrete compression strength,** $f_{c0}$, which is the compressive strength of a standard test specimen (cylinder). From this, the **in-situ compressive strength** $f_c$ can be determined, taking into account the concrete age at loading time, the duration of loading and the spatial variability. The further properties can then be calculated from this value. In case of a probabilistic model further variability should also be considered.

## Basic concrete compressive strength

The first step for any calculation requiring resistance or elastic properties is to determine the **basic concrete compression strength.**

The distribution of this variable is lognormal, provided that its parameters are determined from an *ideal infinite sample*. In reality, a lognormal distribution can be taken also when a "sufficiently high number of samples" is available. Let's call this approach of determining the distribution of $f_{c0,ij}$ ***Method 1***.

A lognormal distribution is related to a normally distributed variable $X_{ij}$ by:

$$f_{c0,ij} = \exp(X_{ij})$$

In a genera case however, the amount of samples is never infinite and not always "sufficiently high" (which will be defined later). Then the base distribution of $X$ cannot be taken as normal, but should be approximated using a *Student's t distribution*. The relation between this base–distribution and the distribution of $f_{c0,ij}$ , described in the equation above, still holds. Let's call this option ***Method 2*** *and* the resulting type of distribution of $f_{c0,ij}$ a ***"log-student"*** distribution. With this a lognormal distribution is understood, where the base distribution instead of a normal distribution is a Student's-t distribution. The "log-student" distribution can be expressed as:

$$F_{c0,ij} = \exp\left(F_{t_{v''}}\left(\ln\left(\frac{X}{m''}\right)\frac{1}{s''\sqrt{\left(1+\frac{1}{n''}\right)}}\right)\right)$$

While a value at a given fractile is determined as:

$$f_{c0,ij} = \exp\left(m'' + t_v s''\sqrt{\left(1+\frac{1}{n''}\right)}\right)$$

Where $t_v$ is the Student's t-variate with $v$ degrees of freedom.

[41] Section based on the Probabilistic Model Code (JCSS 2001) and on ISO 2394-1998 (Technical Commtitee ISO/TC 98 1998)

**Explanation of $t_v$**

The Student's t variate, $t_v(p)$ is the value of the inverse cumulative Student's t distribution with *v* degrees of freedom (i.e. corresponding to *v* experiment results) corresponding to probability *p*.

In excel: T.INV(*p;v*)
ex.: T.INV(0,05;10) = -1,8125 T.INV(0,95;10) = 1,8125
While a normal distribution would be:
NORM.S.INV(0,05) = -1,665

From table: F, v → read value of $t_v$
ex.: F = 0,95; v = 10 → $t_v$ = 1,812

Specific about a Student's-t distribution is that it "corresponds" to a normal distribution with a degree of uncertainty associated both to its mean and standard deviation. These are represented by the coefficients *n'* and *v'* (prior information) or *n''* and *v''* (posterior information). The coefficient *n* can be understood as a "hypothetical number of observations" from which the mean value *m* was determined. Similarly, *v* is called "degrees of freedom" and corresponds to the number of (real or hypothetical) tests based on which the standard deviation *s* was determined (number of tests – 1). In the case of analysis of test results usually *v=n-1*.

A Student's-t distribution can also represent a distribution which is not based on a specific number of tests. In this case *n'* and *v'* can be chosen independently of each other. For the case of basic concrete compression strength the JCSS PMC (JCSS 2001) gives recommendations for these values. The mean value of the "base distribution" has higher uncertainty than the standard deviation, which is a typical situation according to ISO 2394 (Technical Commtitee ISO/TC 98 1998). Therefore the "number of hypothetical tests" corresponding to the mean, *n'*=3 (or 4 for some pre-cast elements) is lower than the value corresponding to the standard deviation *v'*=10.

If no prior information is available (i.e. test results), the values in *Table 42* can be used for the student distribution.

**Table 40 - Prior parameters for concrete basic strength distribution ($f_{c0}$) [MPa]** (JCSS 2001)

| Concrete type | Concrete grade | Parameters | | | |
|---|---|---|---|---|---|
| | | m' | n' | s' | v' |
| Ready mixed | C15 | 3.40 | 3.0 | 0.14 | 10 |
| | C25 | 3.65 | 3.0 | 0.12 | 10 |
| | C35 | 3.85 | 3.0 | 0.09 | 10 |
| | C45 | 3.98 | 3.0 | 0.07 | 10 |
| | C55 | - | - | - | - |
| Pre-cast elements | C15 | - | - | - | - |
| | C25 | 3.80 | 3.0 | 0.09 | 10 |
| | C35 | 3.95 | 3.0 | 0.08 | 10 |
| | C45 | 4.08 | 4.0 | 0.07 | 10 |
| | C55 | 4.15 | 4.0 | 0.05 | 10 |

***Method 1***

As mentioned previously, a normal distribution can be taken as "base-distribution" for $f_{c0,ij}$, i.e. $f_{c0,ij}$ is lognormal distributed if "infinite amount of measurements" are available. It is also allowed to use a lognormal distribution if "sufficient amount of information" is available, for which PMC gives the following value: n''v'' > 10. In this case the (logarithmic) mean value can be taken as m'' and the (logarithmic) standard deviation as

$$\sigma = s'' \sqrt{\left(\frac{n''}{n''-1} \cdot \frac{v''}{v''-2}\right)}$$

Using either of the two methods, the value of the basic compressive strength can therefore be determined, corresponding to a given probability *p.*

If multiple cross sections are evaluated within one member, correlation has to be taken into account between the values of the basic concrete compression strength in points *j* and *k*.

**Comparison method 1 and 2**

Both methods for determining the concrete compressive strength have been used when evaluating a simply supported concrete beam in bending. It was observed that the reliability of the beam when calculated by the two methods was very slightly different.

### *In-situ concrete compressive strength*

When a specific cross section is to be checked, the appropriate value to use in the resistance model is the **in-situ compressive strength** of concrete. The value at one particular point *'i'* in a particular structure *'j'* contains variability due to the variability of the basic concrete compression strength and also due to "additional causes", for example different curing at different points in the structure. When calculating the value of this in-situ strength in a particular point therefore two stochastic variables are present in the formula: $f_{c0,ij}$,, the basic concrete compression strength, which has already been determined and $Y_{1,j}$, a stochastic variable accounting for the "additional variations". For values of the latter, the suggestions given by PMC is:

$Y_{1,j}$ Lognormal Mean: 1,0 CoV: 0,06

Furthermore, the concrete age at loading time and the duration of loading are also accounted for through the variable $\alpha(t,\tau)$, the value of this is deterministic according to the PMC.

The compressive strength at point *i* of structure *j* can be determined based on the following formula:

$$f_{c,ij} = \alpha(t,\tau)\left(f_{c0,ij}\right)^{\lambda} Y_{1,j}$$

In a general case, when no measurements of the concrete strength are carried out and the concrete type is known, random values for concrete strength can be simulated in the steps summarized in *Table 41*. The Excel formulas are also given.

**Table 41 - Steps of generating random values of concrete compressive strength**

| | |
|---|---|
| **1** | Knowing the concrete class, select appropriate distribution parameters m', n', s', v' from JCSS PMC / Part III / Table 3.1.2 |
| **2** | **Simulation 1:** Determine tv(p,v) where v = v'and p=RAND(). Use excel function "*t.inv*".<br>Formula: T.INV(RAND(),v)<br>Comment: the formula uses "left-tail" distribution, to probability p therefore use (1-p) in the formula. |
| **3** | Calculate value of basic concrete compression strength fc0ij from 1. and 2. |
| **4a** | Determine coefficients $\alpha(t,\tau)$ and $\lambda$ |
| **4b** | Take values for Y1j from JCSS PMC / Part III / Table 3.1.1<br>Calculate parameters of lognormal distribution from the known mean and CoV (m, s)<br>**Simulation 2:** determine Yij. Use excel function "*lognorm.inv*".<br>Formula: LOGNORM.INV(RAND();m;s) |
| **5** | Calculate the value of concrete compressive strength $f_{c,ij}$ from 3., 4,a and 4.b |

### *Other parameters*

Once the value of the in-situ compressive strength is known, other relevant properties of the concrete can be calculated.

The values contain a further stochastic component $Y_{i,j}$ which reflect variation due to factors that are not accounted for within the compressive strength. For the modulus of elasticity and compression strain further information of the creep and loading situation is needed, this can be accounted for in a deterministic way. The values which can be calculated are summarized in *Table 42*.

**Table 42 - Concrete properties which can be derived from the compressive strength $f_c$**

| Property | | Other values needed for calculation |
|---|---|---|
| Tensile strength | $f_{ct}$ | $Y_{2,j}$ – Variability |
| Modulus of elasticity | $E_c$ | - $Y_{3,j}$ and $Y_{4,j}$ – Variability<br>- $\beta_d$ - ratio of permanent to total load<br>- $\varphi(t,\tau)$ - creep coefficient (to be determined by "modern code") |
| Ultimate compression strain | $\varepsilon_u$ | |

**Formulas**:

Tensile strength

$$f_{ct,ij} = 0.3 f_{c,ij}{}^{2/3} Y_{2,j}$$

Modulus of elasticity

$$E_{c,ij} = 10.5 f_{c,ij}{}^{1/3} Y_{3,j} \frac{1}{(1+\beta_d \varphi(t,\tau))}$$

Ultimate compression strain

$$\varepsilon_{u,ij} = 6 \cdot 10^{-3} f_{c,ij}{}^{-1/6} Y_{4,j} (1+\beta_d \varphi(t,\tau))$$

### *Values based on the JCSS / ISO model in comparison to values of Eurocode*

Based on the method described above, the values for some concrete types are calculated and summarized in *Table 43*. In this calculation the "spatial" variability $Y_{ij}$ is not considered.

**Table 43 - Characteristic adn eman values of concrete strength JCSS PMC - Eurocode**

| | Concrete types | | | |
|---|---|---|---|---|
| | C15 | C25 | C35 | JCSS ex.4. C35 |
| **m'** | 3,40 | 3,65 | 3,85 | 3,85 |
| **n'** | 3 | 3 | 3 | 3 |
| **s'** | 0,14 | 0,12 | 0,09 | 0,12 |
| **v'** | 10 | 10 | 10 | 6 |
| Fractile for char. value | 0,05 | 0,05 | 0,05 | 0,05 |
| | **Calculated by Method 1.** (lognormal) | | | |
| mean (logarithmic) | 3,4 | 3,65 | 3,85 | 3,85 |
| Stand.dev. (logarithmic) | 0,1917 | 0,1643 | 0,1232 | 0,18 |
| Mean | 30,52 | 39,00 | 47,35 | 47,76 |
| Stand.dev. | 5,90 | 6,45 | 5,86 | 8,66 |
| CoV | 0,193 | 0,165 | 0,124 | 0,181 |
| Characteristic value 1. (0,05) | 21,86 | 29,36 | 38,37 | 34,95 |
| | **Calculated by Method 2.** (logstudent) | | | |
| Charactieristic value 2. (0,05) | 22,35 | 29,93 | 38,93 | 35,90 |
| *Difference of charcetristic values* | 2,26% | 1,93% | 1,45% | 1,45% |
| | **Eurocode values** | | | |
| Mean | 23 | 33 | 43 | 43 |
| **Char. value** | 15 | 25 | 35 | 35 |

*Examples from literature*

Values in some examples in literature, which are based on the Probabilistic Model Code are summarized in *Table 44*.
It is strange that similar values correspond to different concrete types within the JCSS publications. The reason for this difference has not been found to date.

**Table 44 - Concrete strength distribution parameters in literature**

| | Concrete type | Distribution | Char. [MPa] | Mean [MPa] | Stand. dev. [MPa] | CoV | Uses long-term reduction α? | |
|---|---|---|---|---|---|---|---|---|
| JCSS Example 1 | C 20/25 | Lognormal | - | 30 | 5 | 0,17 | no | - |
| JCSS Example 4 | C 35 | Logstudent | - | 30 | 5,40 | 0,18 | yes | 0,85 |
| Holicky et. al. 2008 | | Lognormal | 30 | 37,5 | *5* | *0,13* | yes | 0,85 |

# E. Gross Vehicle Weight Distribution Fits

The chosen  number of Gaussian mixtures to represent the statistical distribution of vehicle weights per category is summarized in the following table.

**Table 45 - Number of normal distributions chosen to describe the GVW per axle category**

| Axle Category | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Number of Gaussian distributions | 10 | 4 | 5 | 10 | 6 | 9 | 4 |

*Figure 50*-57 show the exceedance probability diagrams of various Gaussian mixture distributions, fitted to the measured gross vehicle weight data per category.

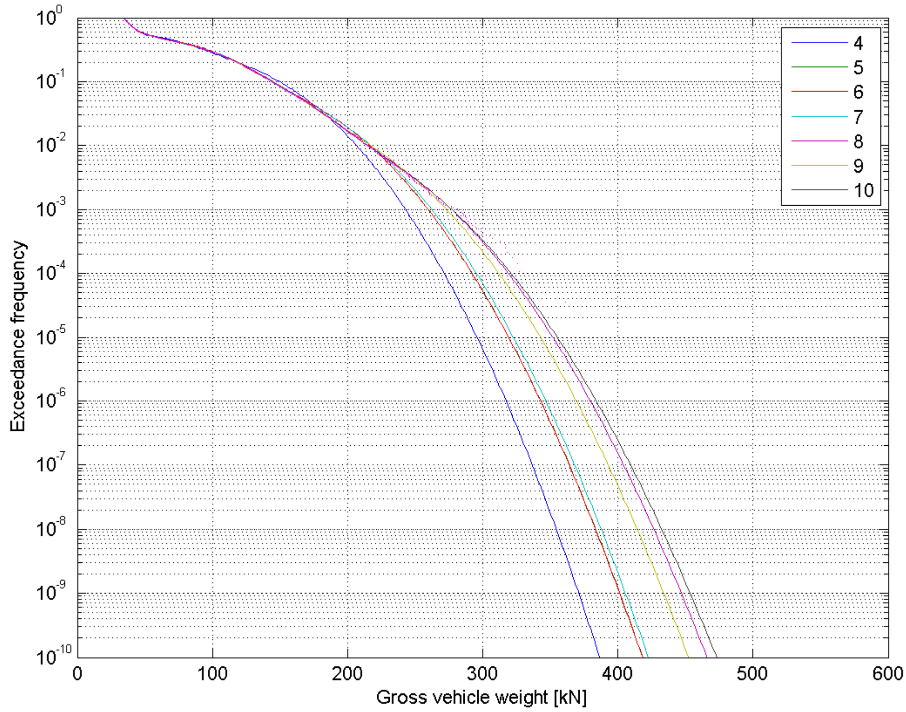


**Figure 50- Gaussian mixture distribution fits to GVW od vehicles, Axle Category 2**

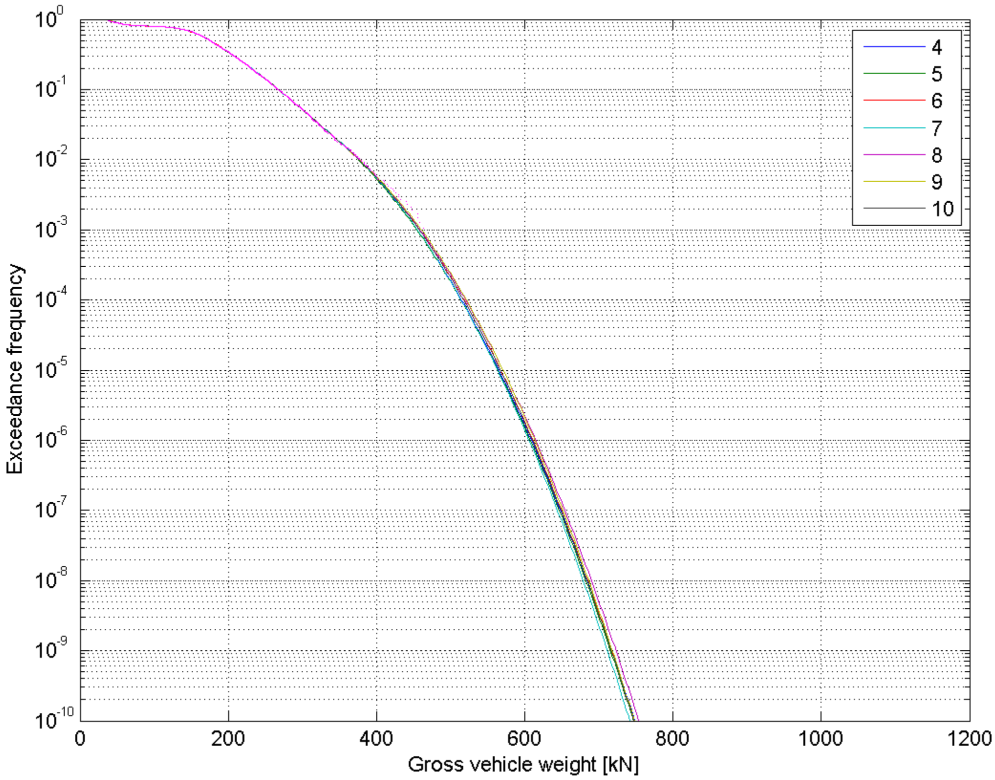


**Figure 51 - Exceedance probabilities and fits - Axle category 3**

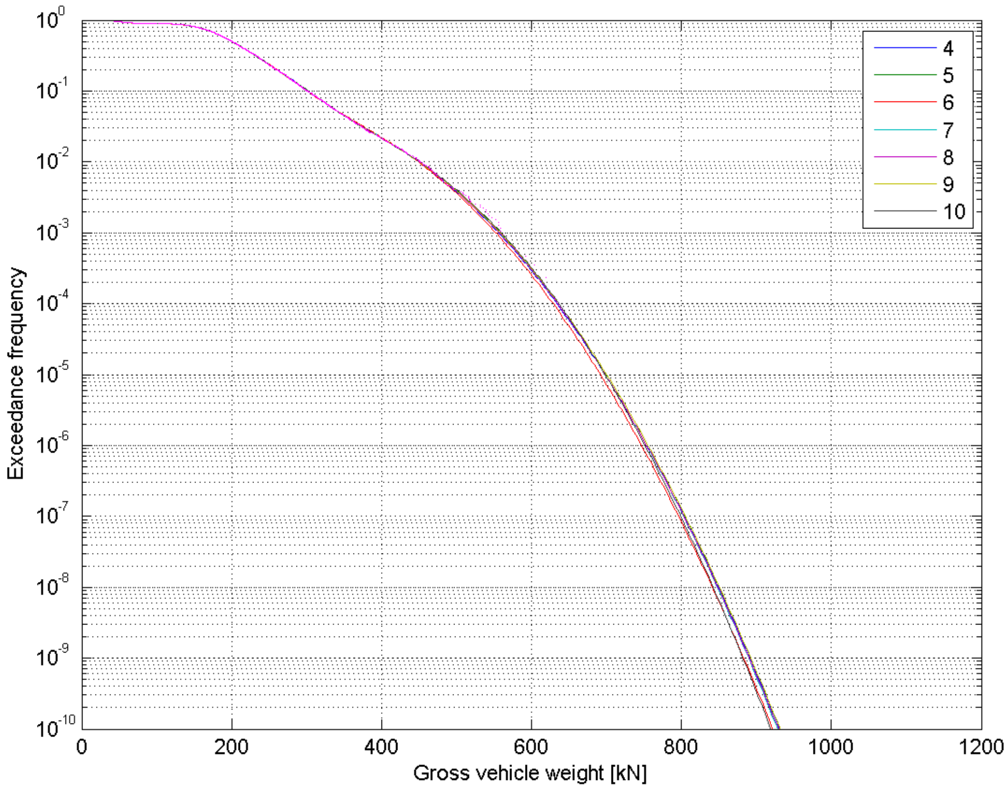


**Figure 52 - Exceedance probabilities and fits - Axle category 4**

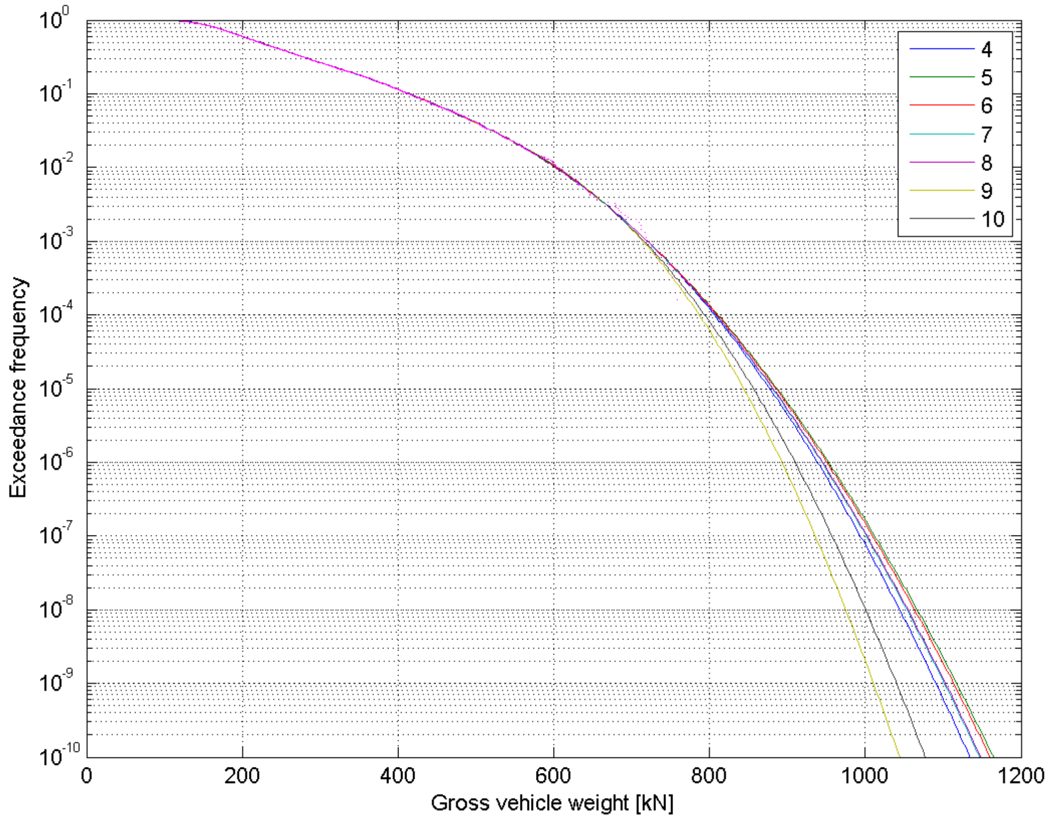


**Figure 53 - Exceedance probabilities and fits - Axle category 5**

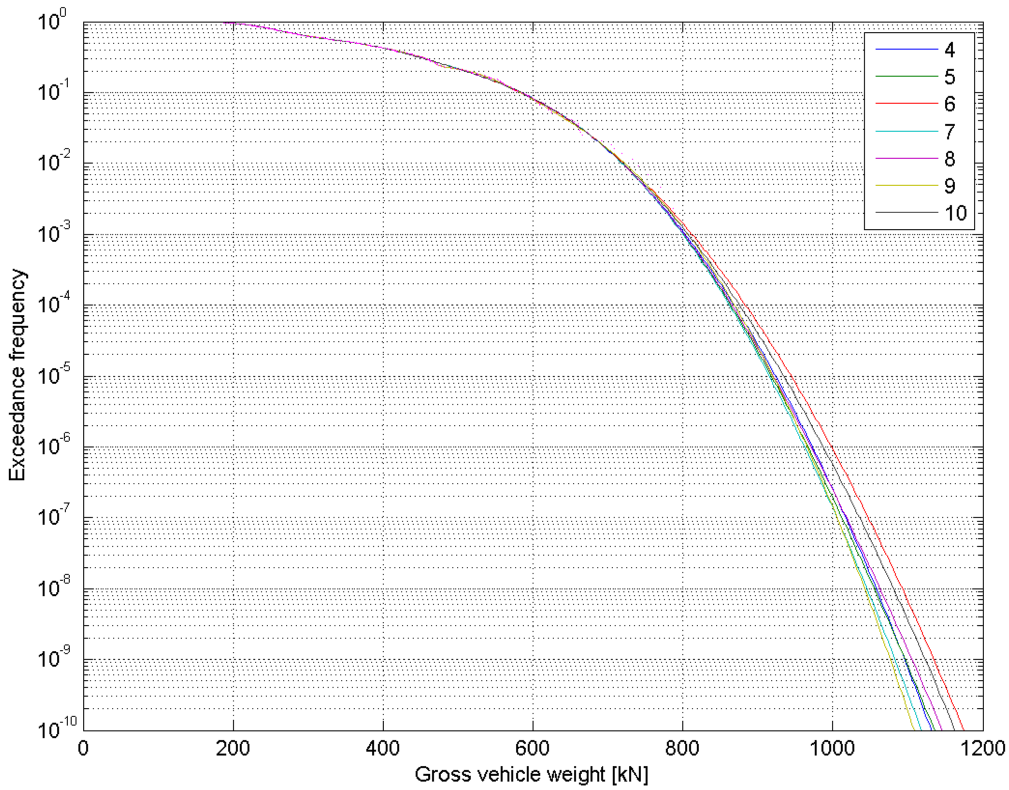


**Figure 54 - Exceedance probabilities and fits - Axle category 6**

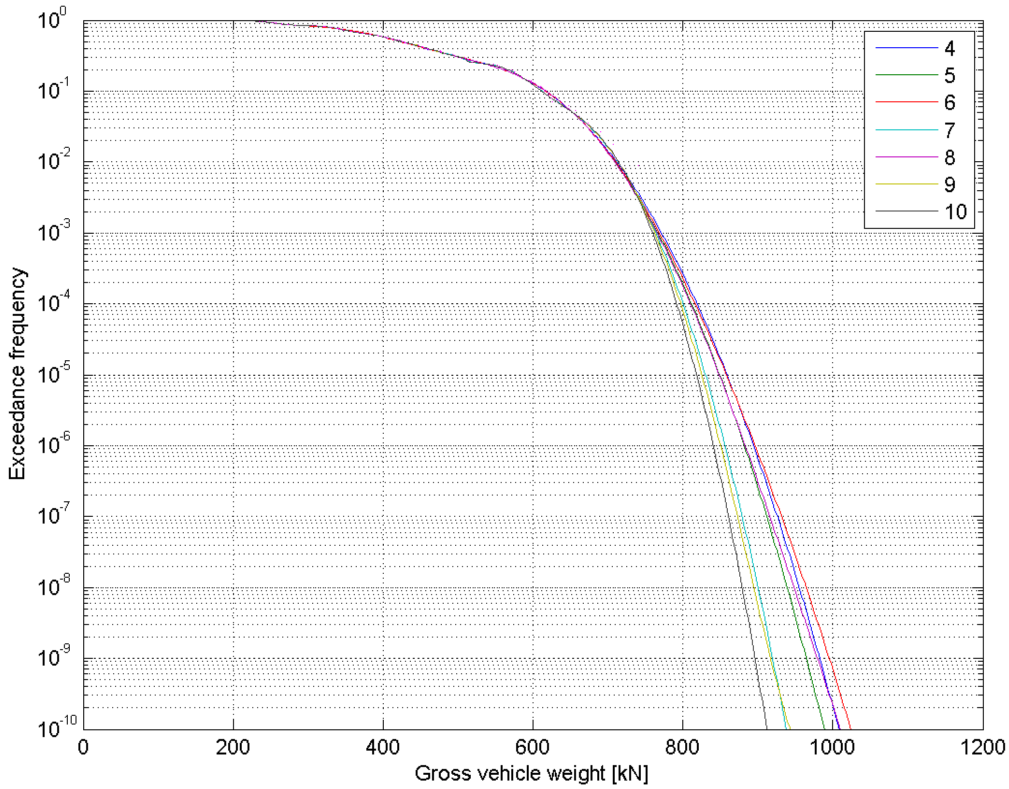


**Figure 55 - Exceedance probabilities and fits - Axle category 7**

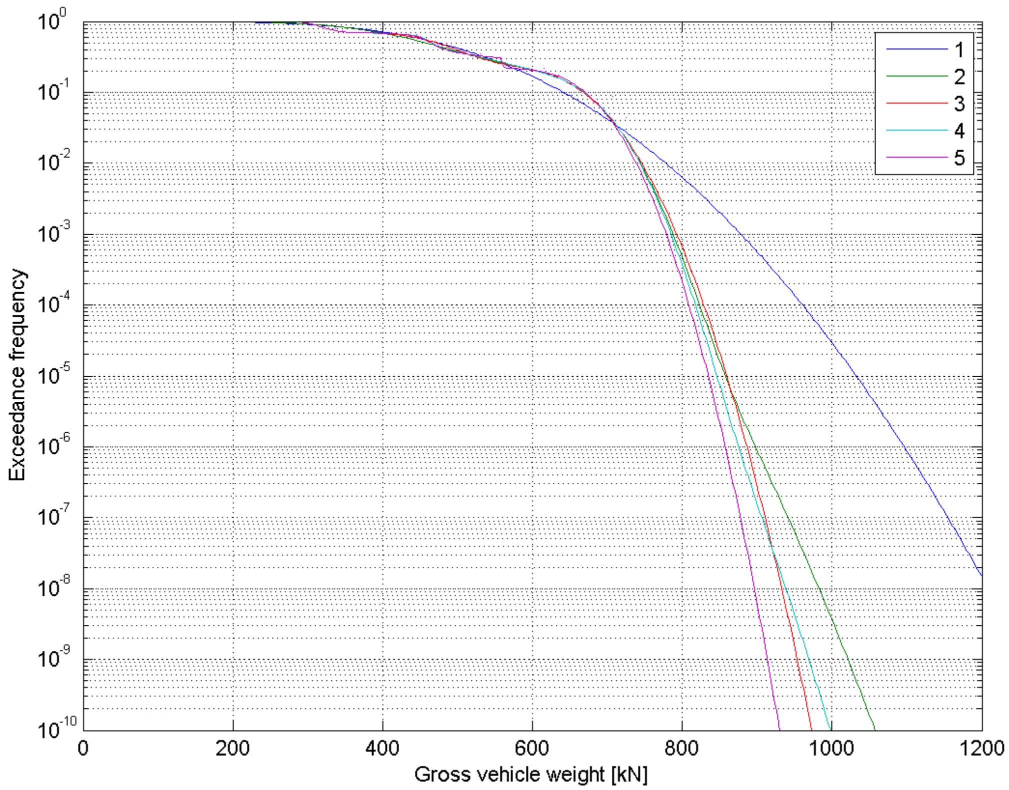


**Figure 56 - Exceedance probabilities and fits - Axle category 8**

# F. Maximum Load Effects – Eurocode

## *Eurocode load model 1*

The loading described by Eurocode for general and local verifications consists of a uniformly distributed load and a tandem system.

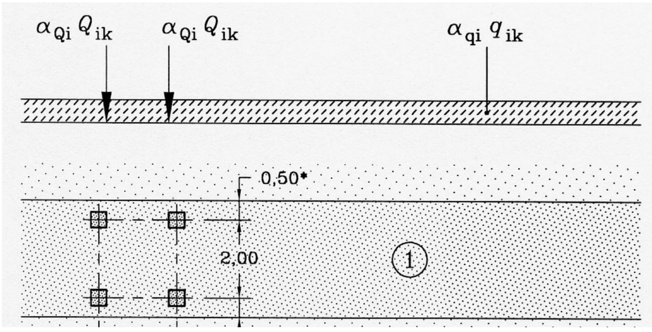


**Figure 57 - Eurocode load model 1**

For lane number 1 (which represents the heaviest loaded traffic lane):

- Q = 300 kN — Axle load (2 wheels)
- q = 9 kN/m$^2$ — Uniformly distributed load

Reliabilty requirements:

- Consequence class — 2
- Remaining life — 15 years
- Reliability level — *'afkeur' (NEN 8700)*
- Required reliability — $\beta$ = 2,5

Therefore partial factors:

- $\gamma_S$= 1,1 — Partial safety factor for traffic load, NEN 8700
- *fac* = 0,8 and 1 — Reduction factors, according to EN1991 and NEN 8700. ($\alpha_t$, $\alpha$ ,$\psi$)
  *The value 0,8 is selected based on typical values used in Rotterdam*

## *Load Effects*

Maximum bending moments on a simple supported beam of various lengths from the Eurocode Load Model 1 are tabulated.

**Table 46 - Maximum bending moment from EC loading, CC2, 15y remaining life**

| Maximum moment LM1 on simple supported structure [kNm]; 1 lane, width: 3m | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | l [m] | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| From axles | kNm | 161,7 | 316,8 | 476,9 | 801,9 | 1129,4 | 1457,9 | 1787,0 | 2116,2 | 2445,7 | 2775,3 | 3105,0 |
| $M_d$ from Q | kNm | 162 | 317 | 477 | 802 | 1129 | 1458 | 1787 | 2116 | 2446 | 2775 | 3105 |
| $M_d$ from q | kNm/m | 5 | 11 | 20 | 45 | 79 | 124 | 178 | 243 | 317 | 401 | 495 |
| | kNm | 15 | 33 | 59 | 134 | 238 | 371 | 535 | 728 | 950 | 1203 | 1485 |
| $M_d$ total | kNm/m | 59 | 117 | 179 | 312 | 456 | 610 | 774 | 948 | 1132 | 1326 | 1530 |
| | kNm | 177 | 350 | 536 | 936 | 1367 | 1829 | 2322 | 2844 | 3396 | 3978 | 4590 |
| "Weight" | kN | **719** | **749** | **779** | **838** | **898** | **957** | **1016** | **1076** | **1135** | **1195** | **1254** |
| Maximum moment for 1 truck on simple supported structure [kNm] **LM2** | | | | | | | | | | | | |
| $M_d$ tot/m | kNm/m | 73 | 110 | 147 | 220 | 293 | 367 | 440 | 513 | 587 | 660 | 733 |
| $M_d$ tot | kNm | 220 | 330 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 | 1980 | 2200 |

**Table 47 - Maximum bending moment from EC loading, CC2, 15y remaining life, total reduction factor 0.8**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Other factors:** | Gamma | 1,1 | Reduction | 0,8 | Total | 0,88 | | | | | | |
| Maximum moment LM1 on simple supported structure [kNm]; 1 lane, width: 3m | | | | | | | | | | | | |
| | l [m] | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| From axles | kNm | 129,4 | 253,4 | 381,5 | 641,5 | 903,5 | 1166,4 | 1429,6 | 1693,0 | 1956,6 | 2220,2 | 2484,0 |
| $M_d$ from Q | kNm | 129 | 253 | 381 | 642 | 904 | 1166 | 1430 | 1693 | 1957 | 2220 | 2484 |
| $M_d$ fromq | kNm/m | 4 | 9 | 16 | 36 | 63 | 99 | 143 | 194 | 253 | 321 | 396 |
| | kNm | 12 | 27 | 48 | 107 | 190 | 297 | 428 | 582 | 760 | 962 | 1188 |
| $M_d$ total | kNm/m | 47 | 93 | 143 | 249 | 365 | 488 | 619 | 758 | 906 | 1061 | 1224 |
| | kNm | 141 | 280 | 429 | 748 | 1094 | 1463 | 1857 | 2275 | 2717 | 3183 | 3672 |
| "Weight" | kN | **576** | **599** | **623** | **671** | **718** | **766** | **813** | **861** | **908** | **956** | **1003** |
| Maximum moment for 1 truck on simple supported structure [kNm] **LM2** | | | | | | | | | | | | |
| $M_d$ tot/m | kNm/m | 59 | 88 | 117 | 176 | 235 | 293 | 352 | 411 | 469 | 528 | 587 |
| $M_d$ tot | kNm | 176 | 264 | 352 | 528 | 704 | 880 | 1056 | 1232 | 1408 | 1584 | 1760 |

These values are used as benchmark within comparison calculations.

# G. Interim Calculations

During the  development of the traffic loading analysis process, interim calculations were performed with the goal to arrive to (preliminary) conclusions about the interpretation of WIM measurements. These are not directly related to the “end result” but were relevant in arriving to the final loading model and determining how complex it actually must so that relevant conclusions can be drawn.

The aim of the interim calculation was to determine, if possible, whether analysis of the WIM data will lead to load effects that are expected to be lower or higher than the load effects from Eurocode loading.  To achieve this, an initial comparison was carried out three levels:

1) Total “weight” present on the bridge
2) Load effects from the design vehicle if $GVW_d$ is uniformly distributed on various base lengths
3) Load effects from the design vehicle if $GVW_d$ is distributed on various axle numbers and distances

Comparison was done with Eurocode Load Model 1 and 2, the loading is described in *Appendix F.*

The following beam structure is considered:

Simply supported beam, maximum bending moment on the structure
Spans: 2 – 20 m
Breadths: 3 m *As this is the lane width given in Load Model 1 of EN1*

Input for loading:

Consequence class 2
Remaining life 15 years
Reliability level *‘afkeur’ (NEN 8700)* → Required **β = 2,5**

In the initial check only one vehicle of the maximum gross vehicle weight was considered.

## *Loads based on WIM measurements*

WIM measurements have been analysed by TNO . (Huibregste et al. 2014) and both **design axle load** and **design vehicle weight** have been determined. The gross vehicle weight has been described by a mixture of 10 normal distributions and the design weight was determined based on the given exceedance probability (which is a function of the required reliability index and the design life).

$GVW_d$ = 1010 kN Design gross vehicle weight for 15 years

*Design* axle weights were determined in a similar way.

$Q_d$ = 219 kN Design maximum axle load for 15 years

## *Comparison of total weight*

In *Table 1* total force present on the bridge ( 2*Q + w*l*q) is given.

**Table 48 - Total force present on structure [kN] according to EC LM1; 1 lane, 3m width**

| Reduction factor | Span | l [m] | 2 | 3 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1,0 | Weight | kN | 719 | 749 | 779 | 838 | 898 | 957 | *1016* | *1076* | *1135* | *1195* | *1254* |
| 0,8 | | kN | 576 | 599 | 623 | 671 | 718 | 766 | 813 | 861 | 908 | 956 | 1003 |

The first comparison shows that the design GVW is usually  higher, than the total weight of LM1.

However, this comparison does not give too much information, because it is expected that the heaviest truck will be distributed on multiple axles, thus for example will not be fully present on a very short bridge.

*Moments from GVW - assumed uniformly distributed*

In order to calculate the load effect, a model for the spatial distribution of the GVW has to be assumed. This can be in the form of concentrated or distributed loads, or a combination of these. As a first assumption, the maximum GVW is converted to a uniformly distributed load. For this, various **base lengths** are chosen, between 4 – 17,5 m.

In the analytical model used for analysing WIM data for Rijkswaterstaat, at TNO base lengths of 12,5m and in an earlier report 17,5 m were used (Steenbergen et al. 2012). The current graduation project examines shorter bridges, moreover in this comparison only one truck on a bridge is modelled. Therefore it is reasonable to check shorter base lengths, resulting in higher load effects.

The load effects from the various assumed base lengths as and from Eurocode LM1 are visualised in the figure below.

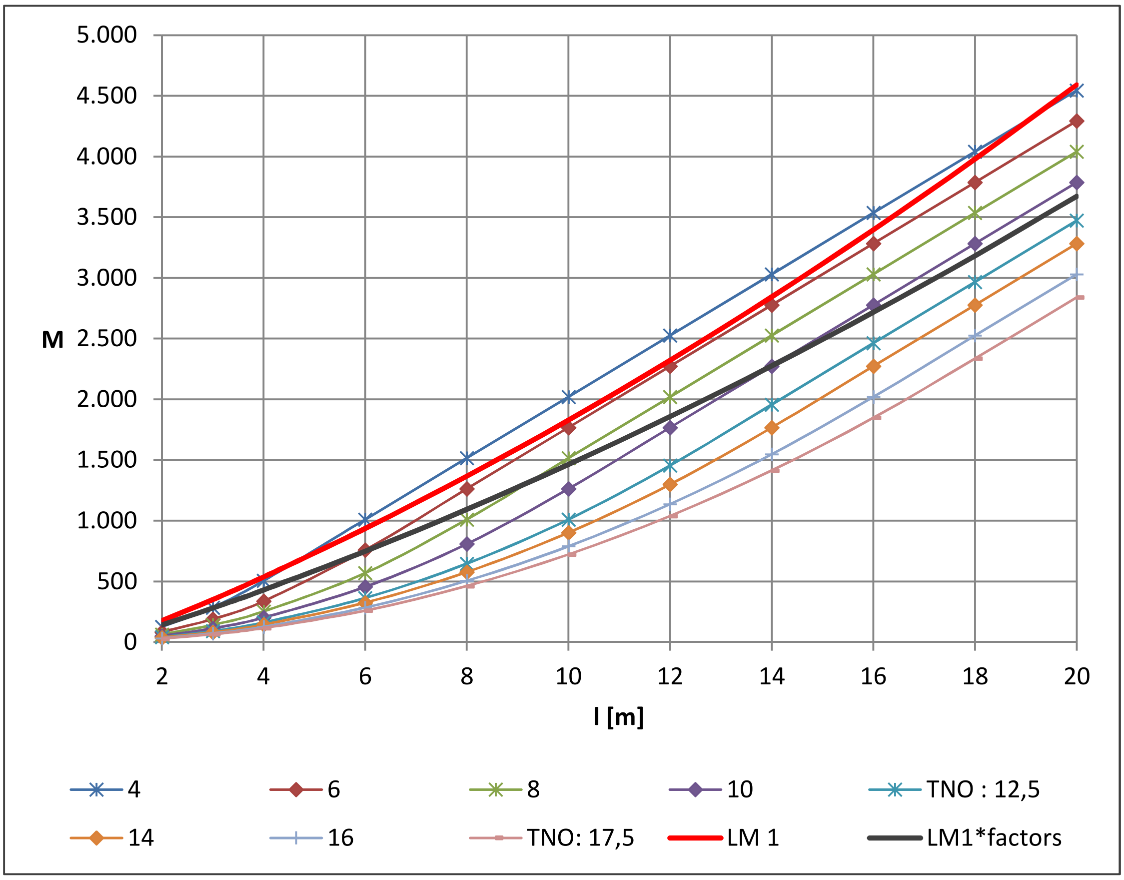


**Figure 58 - Moment from design GWV 1010 kN, various base length [kNm]** - *1 traffic lane, width: 3 m*

**Interpretation:**

- LM1 without load reduction factors creates larger load effects than the GVW in almost all cases of assumed "base lengths".
- If the GVW is distributed on a base length of 4m (which is very short, a minimum axle distance in trucks is approximately 1,2 – 1,3m), the calculated load effect is larger for most cases, than the load effect created by LM1.
- If load reduction factors may be taken into account, then the load effect from LM1 is not clearly on the conservative side.

- **Whether the load effect from the measured data is more or less than the load effect from LM1, is strongly dependent on the assumption made for the spatial distribution of the GWV on the bridge**. It was expected "intuitively" that for a short bridge, the axle configuration will be of high importance. The current simple parameter study shows that without investigating this aspect a clear conclusion about reducing the load cannot be drawn.

Therefore, the next step is to gain better insight to the distribution of axles and axle loads in relation to the gross vehicle weight.

### *Moments from GVW – distribution as point loads*

The design axle load is 216 kN, thus it can be assumed that a truck with 1010 kN of design weight will have minimum 5 axles. This is on the conservative side, but it might give a second impression about the loading.
The maximum moments caused by 2, 3, 4 and 5 axles of the maximum axle weight are calculated, taking into account 3 different axle distances. The results are compared with the load effects from Eurocodes. As an example, the result of the calculation with 5 axles can be seen in *Figure 59.*

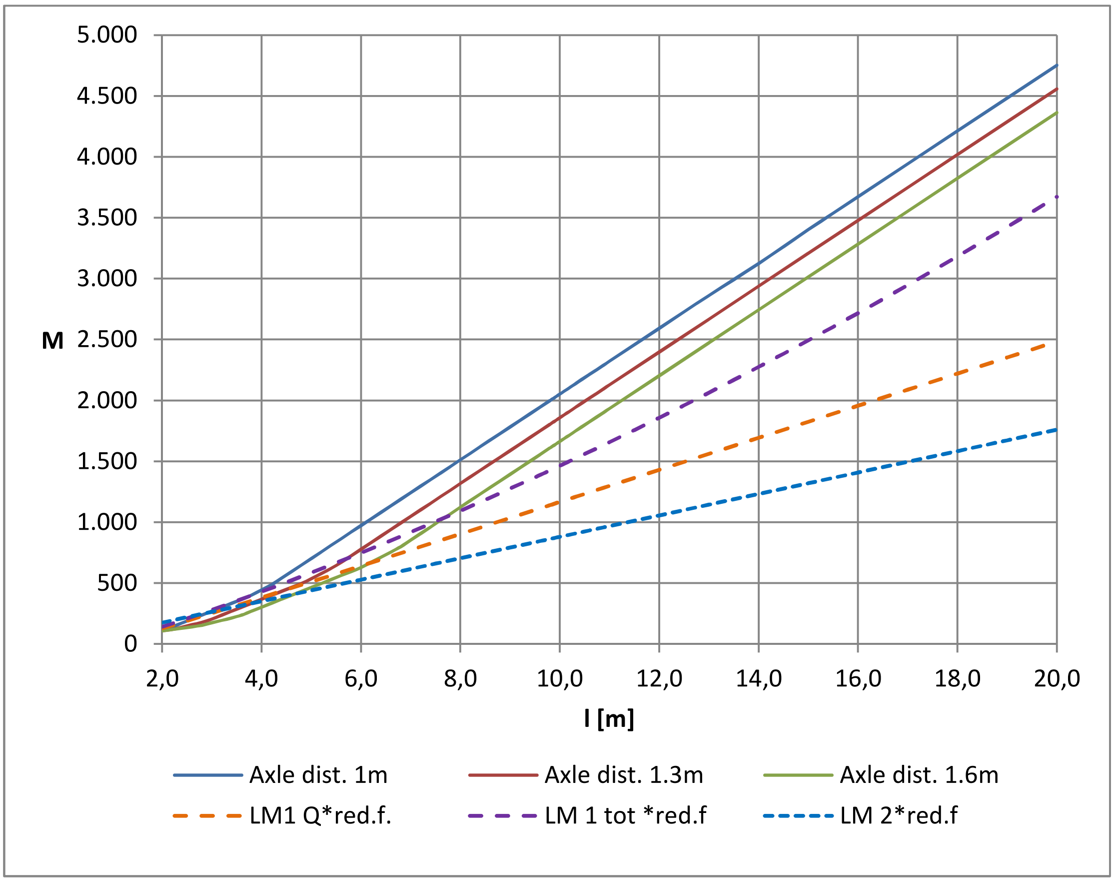


**Figure 59 - Moment from 5x216 kN axles, various axle distance [kNm]**

**Interpretation**

- For very short bridges, dependent on the axle distances (up to 3 – 4m), the design axle loads placed "very close" to each other give a more favourable result than the EC load models. (However, in this case the statical model of a beam is probably also not correct.)
- For bridges in the medium range (from 3-5 to 5-7m) whether the load effect is more or less conservative than that calculated from the Eurocode, depends strongly on the axle distances.

- For the “longer” bridges (from 5-7 to 20m) the results of a very heavy vehicle, in the range of the design gross vehicle weight, the weight of which is distributed over 5 closely spaced axles is always much more unfavourable than that of the EC load models.

  The assumption of such a vehicle is definitely conservative, thus **a more realistic axle weight distribution and axle distances should be considered**. This will follow in the data analysis.

### *Semi-probabilistic estimation*

After having started with the WIM data analysis process, at the point of having information about the load effects from unit weight trucks, an attempt was made to gain insight to the possible design load effect. Knowing the maxima load effect of a unit weight truck in a given vehicle class, we can check what load effect a “very” high load would give for this “worst case” truck type. Therefore we couple the worst axle configuration and the resulting $LE_{unit\_max\_class\_i}$ with a **design load per category $GVW_{d\_class\_i}$** . The later will be defined similarly to the design GVW for the full population, but taking into account the ratios of truck classes. Therefore the $P_d$ design exceedance probability for each truck will be different than $P_d$ for the full population.

**Table 49 - Design GVW per vehicle category [kN]**

| Axle number | | | | |
|---|---|---|---|---|
| | Catratio | Pd / catratio | nr of mix | GVWd (manual)[42] |
| 2 | 0,586671 | 1,52E-08 | 10 | 429 |
| 3 | 0,097929 | 9,09E-08 | 4 | 650 |
| 4 | 0,176594 | 5,04E-08 | 5 | 820 |
| 5 | 0,126847 | 7,02E-08 | 10 | 965 |
| 6 | 0,009036 | 9,85E-07 | 6 | 1000 |
| 7 | 0,002305 | 3,86E-06 | 9 | 836 |
| 8 | 0,000473 | 1,88E-05 | 4 | 839 |
| *9* | *0,000103* | *8,65E-05* | - | - |
| *10* | *4,12E-05* | *2,16E-04* | - | - |

This check can give an indication about *“whether we will win or not”*, but it does not represent reality for several reasons. (!)
Assumptions made:

- The highest load in a given design category is not likely to correspond to the worst case axle configuration → this makes our assumption conservative
- The traffic load categories are assumed to be constant (actually this assumption will be held for most of the thesis work) → this might be non-conservative
- At this point, the trucks with 9 and 10 axles are not considered, because there is not enough data to create fits to the GVW distributions. This could be partially overcome by merging the GVW-data for categories of eight or more axles, or taking the GVW fit of eight-axles and multiplying them with a factor of for example 9/8 and 10/8.

The results are summarized in the following table

[42] “Manual” refers to the fact that the values corresponding to the exceedance-probabilities are found manually by inserting “guess values” to the cumulative probability distribution of the mixture model.

**Table 50 - Moment caused by dominant truck with design GVW per category**

| Axle nr | Length 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 203 | 298 | 406 | 497 | 615 | 708 | 830 | 923 | 1044 | 1137 | 1259 | 1352 |
| 3 | 235 | 341 | 475 | 581 | 715 | 863 | 1031 | 1188 | 1356 | 1513 | 1681 | 1838 |
| 4 | 265 | 412 | 547 | 693 | 829 | 1011 | 1182 | 1383 | 1561 | 1769 | 1952 | 2166 |
| 5 | 254 | 361 | 507 | 649 | 871 | 1081 | 1323 | 1537 | 1780 | 1994 | 2252 | 2476 |
| 6 | 248 | 355 | 499 | 605 | 750 | 914 | 1077 | 1255 | 1467 | 1679 | 1931 | 2175 |
| 7 | 161 | 242 | 330 | 410 | 539 | 659 | 793 | 913 | 1071 | 1233 | 1397 | 1562 |
| 8 | 113 | 159 | 241 | 332 | 434 | 538 | 646 | 789 | 945 | 1094 | 1251 | 1399 |

The results compared to the maximum load effect from EC load models are plotted below. It is noted that the EC LM load effects are maxima for the full beam, thus not always the middle cross section.

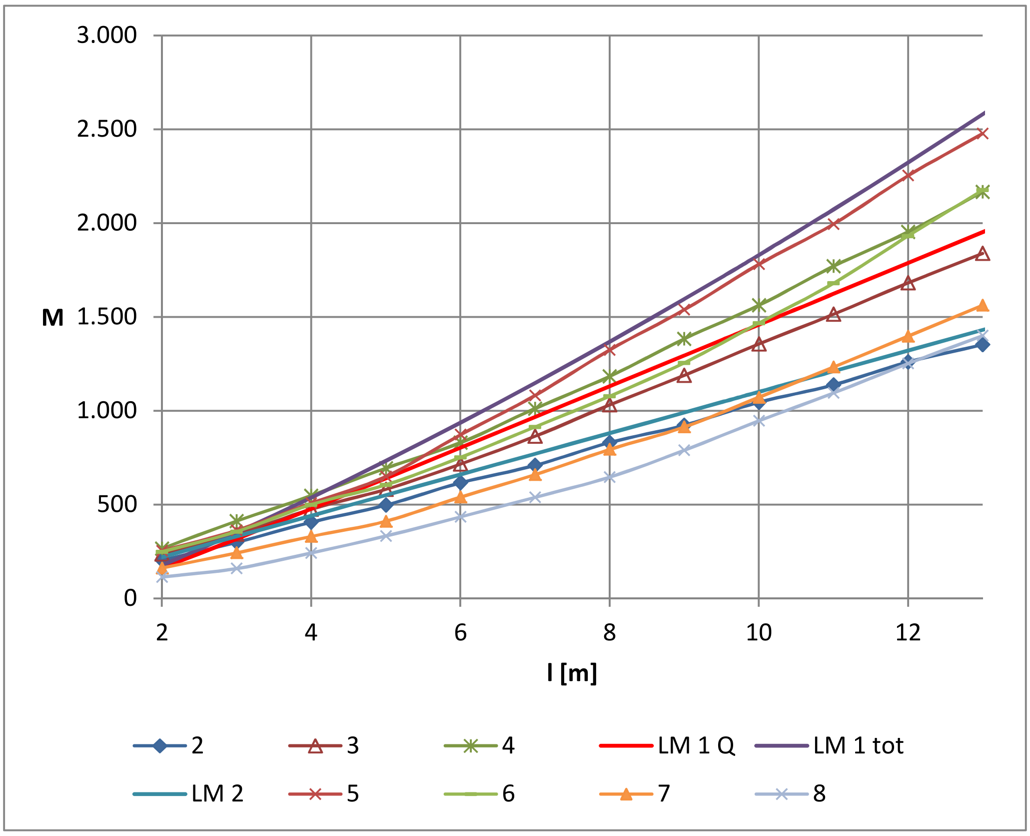


**Figure 60 - Moment by dominant truck with design GVW per cat. and max. LM1 effect**

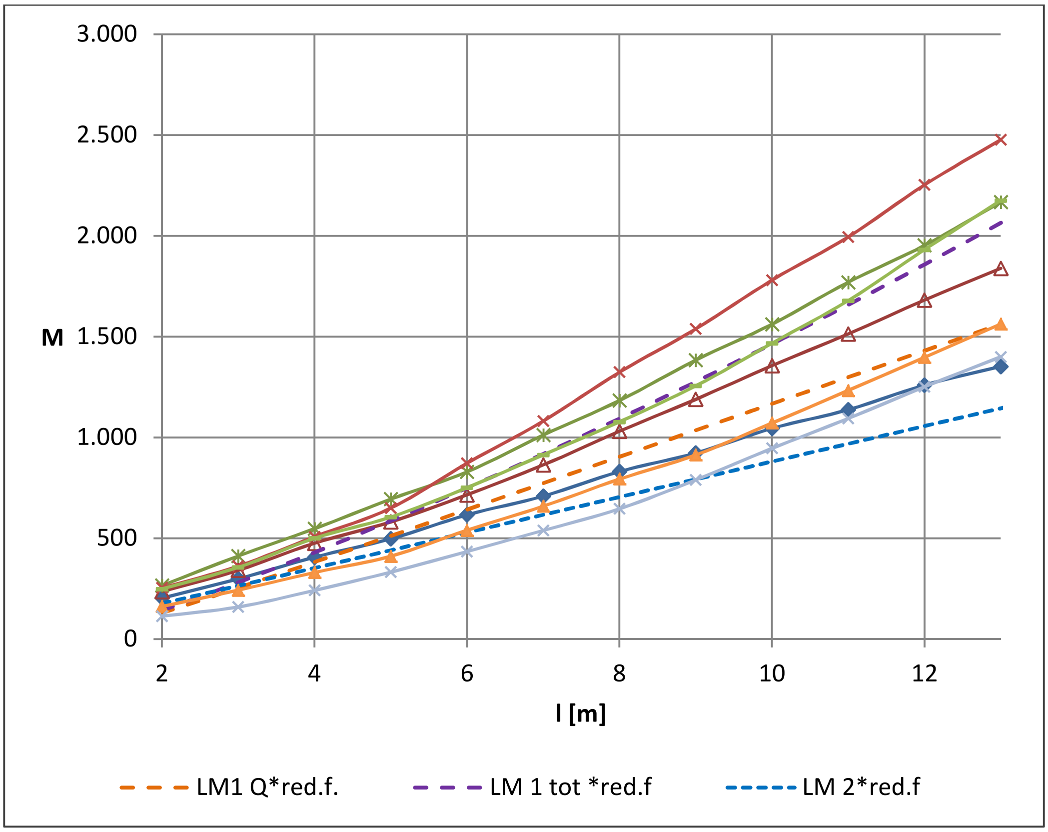


**Figure 61 - Moment by dominant truck with design GVW, per cat. and max. LM1 effect**

# H. Codes

This appendix collects the relevant codes which were developed for traffic loading analysis, simulation, load effect analysis and simulation. *Table 51* gives an overview of scripts, functions and their output. The detailed explanation of the processes can be found in Chapters 8 and 9.

**Table 51 - Scripts and functions for traffic laoding analysis**

| Script / Function | | Output | | | |
|---|---|---|---|---|---|
| **Name** | **Action** | **.mat file** | **variables** | **properties** | **info** |
| **0.** | **DATA PREPARATION** | | | | |
| DatasplitToCellArray | Splits meausrement data to cells of a cell array, by given category.<br>Categorisation by nr of axles, by statistical category etc. possible | Splitdata_axlenumber.mat<br>(Splitdata_statcat.mat)<br>… | DATA {} - cell array | <nrcatx1 cell><br>where nrcat = number of categories<br>in cell (double) | |
| **1.** | **CREATING SAMPLE SPACE** | | | | |
| CategoryRatios | Calculates % of categories in total data and stores it in a vector | Splitdata_axlenumber.mat<br>Splitdata_statcat.mat<br>… | CatRatio - array | <1 x nrcat><br>(double) | |
| AxlesInfo_PerChosen-Category | Creates a matrix of truck properties withing each category, consisting of %GVW per axle and axle distances.<br>Stores the result in a cell array similarly to DATA{} | Splitdata_axlenumber.mat<br>Splitdata_statcat.mat<br>… | GVW_GM_AN_P {} - cell array<br>(GVW_GM_SC_P) | <nrcatx1 cell><br>in cell (double) | |
| Axlesinfo_Cumdist_-meters | Axle distances in the truck property matrix are cumulated per vehicle and transformed from cm to meters.<br>Results are saved in a cell array similarly to the previous step | Splitdata_axlenumber.mat<br>Splitdata_statcat.mat<br>… | GVW_GM_AN_P_CUMm {} - cell array<br>(GVW_GM_SC_P_CUMm) | <nrcatx1 cell><br>in cell (double) | |
| GaussianFit_Per-ChosenCategory | Creates Gaussian mixture distribution models of a given range of mixture component numbers, per category | Splitdata_axlenumber.mat<br>Splitdata_statcat.mat<br>… | GVW_GM_AN_fit {} - cell | <nrcat x nrGauss cell><br>where nrGauss = number of fitted Gaussian mixtures.<br>in cell (double) | Each cell contains a probability distribution object.<br>Cells contain GMdistributions of 1,2, … nrGauss normal distributions respectively |

| Script / Function | | Output | | | |
|---|---|---|---|---|---|
| **Name** | **Action** | **.mat file** | **variables** | **properties** | **info** |
| - | Select manually the adequate number of normal distributions describing the data, per category, and collect the probability distribution objects in a separate cell array | Splitdata_axlenumber.mat<br>Splitdata_statcat.mat<br>… | GVW_GM_AN_SaS {} - cell | <nrcatx1 cell><br>in cell (double) | |
| **2.** | **TRAFFIC SIMULATION** | | | | |
| SimulateTraffic3_-<br>multiple | Simulate trucks per category, in the ratio 'catratio'. Each simulated truck is described by a GVW and a 'Property index'. | Sim3_AxleNumber(i)<br>where i is a counter: several .mat files contain simulated trucks as the data is too much to save in one file | SIMUL {} - cell | <nrcatx1 cell><br>in cell (double) | Cells of *SIMUL* contain matrices of several rows (4.5-E7) and two columns. Each row represents a truck and contains the infocmration: GVW, Property index |
| **3.** | **BEAM ALGORITHMS - LOAD EFFECTS OF UNIT-WEIGHT TRUCKS** | | | | |
| *il_m_ss_Q* | function - load effect<br>input arguments are: beam length, cross section location, Q load, location of Q load | - | [LE] | 1 value (double) | Load effect<br>*Now: Moment (m) on simple supported beam (ss) from point load (Q)* |
| *max_le_cs4* | function - calculates maximum load effect in a cross section from one truck, which is described by parameter P<br>input arguments are: beam length, cross section location, step size, property<br>calls *il_m_ss_Q* | - | [LEmax, xLEmax] | 2 element row vector (double) | - maximum load effect<br>- location of truck at maximum load effect |
| max_le_moretrucks_morelengths | Calculates maximum load effect in a cross section from a multitude of unit-weight trucks with different properties, for bridges of various lengths | LE_M_L6_CSmid_ST02_AN<br>separate .mat files per beam length<br><br>could be separate .mat files per CS, LE (ex. Moment, Shear) | B {} | <nrcatx1 cell> | Cells of *B* contain matrices with number of rows corersponding to the truck properties.<br>Each row contains the vector [LEmax, xLEmax] |

| Script / Function | | Output | | | |
|---|---|---|---|---|---|
| **Name** | **Action** | **.mat file** | **variables** | **properties** | **info** |
| **4.** | **LOAD EFFECTS OF SIMULATED TRUCKS** | | | | |
| max_le_simultrucks_multiple | Calculates maximum load effect in a cross section from the simulated trucks<br>Uses result of steps 2. and 3.<br>Saves result in a multitude of cell arrays | LE_M_L6_CSmid_ST02_AN_Sim3_-AxleNumber(i)<br>several .mat files | LEsimul {} - cell | <nrcatx1 cell><br>in cell (double) | Cells of *LEsimul* contain load effects.<br>Each row corresponds to the same cell and row of the array SIMUL, thus information about the truck type causing the maximum load effect can be "tracked back" if necessary |
| max_le_simultrucks_multiple_-integerformat | Same as previous but before saving .mat file converts data to uint16 format | *Same* | *Same* | *Same* | *Approximately 1/6 size of previous version* |
| Double_to_Integer | Changes the format of matrices in an array contained by several .mat files<br>--> can be used to convert large data of LE_M... file to smaller | LE_M_L6_CSmid_ST02_AN_Sim3_-AxleNumber_int16_(i)<br>several .mat files | Lesimul_int16 {} - cell | <nrcatx1 cell><br>in cell (uint16) | |
| **5.** | **MAXIMA ANALYSIS** | | | | |
| *maximasplit_indexed* | function - finds maxima of results of simulation<br>Input values are: data to analyse, size of a block, total vehicles per block, ratio of categories | - | [MaxLE] | matrix (double), described in *Table XX in Thesis* | Contains block maxima per category, the 'index' of the truck casuing it (row number in simulation), the total maxima per block and the category of the vehicle causing it |
| MaximaScript | Calls the *MaximaSplit_Indexed* function based on given input, and saves the resulting matrix | Max_LE_M_L6_CSmid_ST02_AN_-Sim3_AxleNumber... | MaxLE | <a x bsim> (double)<br>- a: nrcat*2 +2<br>- bsim: number of times a block is simulated<br>ex. 100x15years maxima -> simnr = 100 | *See above* |

# I. Some Details of Literature Study

## Determining optimal cut-off load for fit to "tail data"

This section summarizes the two methods recommended in (Steenbergen et al. 2012) for finding the optimal cut-off load when fitting an extreme value distribution to the "tail" of a dataset. The methods were used in the report for finding design values of axle loads, as described in *Section 6.4.3*.

### Determining the distribution parameters for a given cut-off load

Parameters of one or more selected extreme value distributions are determined using maximum likelihood estimation (MLE) *(Section5.3.4 )*.

The MLE procedure must be adapted due to using the cut-off.
If the maximum likelihood formula is written as: [43]

$$L_{\underline{y},i}\left(\underline{y};\underline{a}_i\right) = \prod_{j=1}^{m} f_{y,i}\left(y_j;\underline{a}_i\right) \qquad i = 1,2$$

Where $i$ is an index denoting the distribution type (in this case Gumbel and Weibull)
$y_j$ are the measured axle loads, *m* number of measurements
$a_i$ is the vector of parameters belonging to distribution type *i*

By re-ordering measurement data according to the cut-off level and denoting it as:

$y_1,\ldots,y_k$ measured loads $\leq y_0$
$y_{k+1},\ldots,y_m$ measured loads $> y_0$

As mentioned before, the total data set cannot be adequately described by a single distribution. Yet it is necessary to consider the data below the cut-off load. For example, it is relevant whether 75% or 99% of the measurements fall below a chosen cut-off load. It will have a different implication if 1000 measured axles are above 280 kN from 10 000 or from 100 000 measurements in total. However as we are not interested how these lower values are distributed, only the probability that these are fall below the cut-off load has to be considered.

The likelihood function is adapted and for each distribution type can be written as:

$$L_{\underline{y},i}\left(\underline{y};\underline{a}_i\right) = \left\{F_{y,i}\left(y_0,\underline{a}_i\right)\right\}^k \prod_{j=k+1}^{m} f_{y,i}\left(y_j;\underline{a}_i\right) \qquad i = 1,2$$

For practical reasons the *log-likelihood calculations* are applied.

### Optimal cut-off load – 'Scientific method'

For determining the final optimal distribution function type, the *Bayesian approach* is used, with an assumed uniform a-priori distribution (i.e. 0.5 – 0.5 probability for both assumed distribution types, Gamma and Weibull, given the data *y*). Brief summary of the theoretical background is given in *Section 5.3.4.*

[43] As in *Section 5.3.4* but different notation

### *Optimal cut-off load – 'Practical method'*

As described previously, distribution parameters are determined for various cut-off loads. Based on relations described in *Section 3.4* , the design value of the axle load (a value of the distribution function belonging to a certain non-exceedance probability) is determined for each case. Data pairs of cut-off load and design axle load can now be plotted in a graph, as visualised in *Figure 62.*

Now how do we decide on which cut-off load to choose? Here the so called *boot-strapping* is applied:

1. For each of the found distributions (different parameters per cut-off load), simulate multiple data samples (*m*, ex. 20). The simulated data sample should be of the same size as the original amount of measured data (*n*, ex. 200 000). The result is *m* datasets of *n* data points per each cut-off load.
2. Per cut-off load, determine the design value (i.e. design axle load) from each of the m datasets, using the method of maximum likelihood. Thus *m* design values for each cut-off load are obtained. The obtained design values, due to the randomness in the simulation process will have a certain spread for each cut-off load. Assuming a normal distribution of the design load, determine the mean and standard deviation of each set of design loads.
3. Make plots of cut-off load – design load as in *Figure 62*.
   The horizontal line represents the mean of the calculated design loads, the vertical lines represent a "band width" of 2 standard deviations of the design loads.

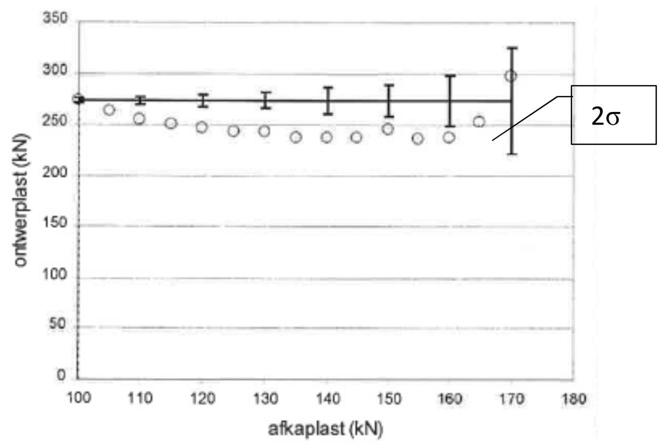


**Figure 62 - Cut-off load vs. design load plot, as result of boot-strap process**

4. The chosen distribution type and cut-off load (thus the resulting design load) are considered acceptable if the values of the design load calculated from cut-off loads higher than the optimal cut-off fall in the range of 2 σ from the design load. Therefore the example of *Figure 62* is not acceptable.
   The lowest of cut-off load which satisfies the 2 sigma "bandwidth criteria" criteria described above , is chosen.