Final Report

Predicting of the Stiffness of Cracked Reinforced Concrete Structures

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Summary

Cracking is inherent in design of reinforced concrete, and it influences the structure's durability and its appearance. If the cracks are too wide, the structure might not fulfill requirements with regard to durability and serviceability e.g. liquid tightness. Therefore a good design and detailing of a structure should be made to limit crack widths. But unexpected cracking might occur.

Many factors influence the cracking behavior of concrete structures: Cracks can not only be caused by imposed loads, but also by (partially) restrained imposed deformations. In the latter case there is an interaction between the forces generated and the stiffness of the structure, which is influenced by the cracking behavior: the more the stiffness is reduced by cracking, the lower the forces. It is difficult to make a design in which all influencing factors are taken into account. So, when structural modeling imposed deformations, engineers often reduce the uncracked stiffness when modeling the structure and designing the reinforcement. The question arises which reduction factor to use. In practice, Young's modulus is often reduced to 1/3 of its original value. Answering the question whether this is a suitable value is the main goal of the research.

The research focused on basic theories on cracking behavior. The tension stiffening law is used and it is researched from micro size to macro size, from cross-section to system, from effect to action. Finally, an appropriate stiffness reduction value is obtained. The procedure is:

- 1. By using the cross section stress balance, the accurate compression zone height will be obtained under both axial force and bending moment.
- 2. The elastic modulus is an important parameter related to the moment caused by restrained deformation. After the compression zone height is obtained, by using the Tension Stiffening Law, the Elastic Modulus in the crack is calculated.
- 3. After transferring the cross sectional stiffness into system stiffness, the accurate moment curvature curve and the design mean stiffness are obtained.

The design mean stiffness is not constant for different loading combinations. It is larger than one third of the uncracked stiffness when there is a tensile axial force and a high positive temperature gradient. On the other hand, the design mean stiffness might also be less than one third of uncracked stiffness. There is no difference for the loading sequence. That means that whether external loading or restrained deformation is applied first, the results will be same at the final state. After cracking, the non-linear response of the member investigated will influence the bending moment distribution. As a result, the bending moment in a cross-section is not only influenced by external loading and restrained deformation, but also by the stiffness distribution over the length of the member.

It is not suitable for engineers to always use one third of the uncracked stiffness to design the reinforcement since they might then underestimate the forces caused by the temperature gradient: It will be higher when there is an axial tensile force in combination with a high positive temperature gradient. A program to obtain the accurate value of the stiffness of a clamped beam is developed. This will help engineers to prepare a more accurate structural model.

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CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

Nowadays, concrete is one of the most important construction materials in the world. Concrete projects are distributed in many fields, such as buildings, tunnels, bridges and so on. Concrete is a kind of construction material with high compressive strength and a good durability, but a relatively low tensile strength. The tensile strength of concrete is much lower than its compressive strength. Cracks might occur in concrete at a low tensile stress.

Cracking is inherent in design of reinforced concrete. These cracks might influence the structure's durability and its appearance. If the cracks are too wide, the structure might not fulfill requirements with regard to durability and service ability e.g. liquid tightness. Therefore a good design and detailing of a structure should be made to limit crack widths.



Fig. 1-1 Cracks in a concrete structure

1.2 Problem description

In order to prevent the failure of a structure caused by cracking, a good understanding of cracking is required. Usually, cracks which have small width will not or hardly affect the structure. The crack width should therefore be controlled under a limit level.

Unexpected or excessive cracking might occur. An example is the new cast wall which is restrained at both sides at early age when there is no external loading on it. But still some cracks might occur as shown in Fig.1-2. The design of the wall is ok with regard to ULS design, but why are there some cracks? What is the reason for the formation of these cracks? Might these cracks influence the durability of the structure? How to model these cracks with the cracks together caused by the other actions?





Fig.1-2 Early age cracking on concrete structure

Generally, cracking can be caused by various kinds of reasons, such as external loading, restrained deformation, creep and so on. External loading and restrained deformation always are the main reasons of cracking. But codes often deal extensively with the first category which is the external loading, whereas the second category is hardly dealt with. Therefore, cracking caused by restrained deformation might be ignored by using codes to design. The cracking in the Fig.1-2 mostly is caused by the restrained deformation. Or cracks might be caused by a combination of external loading and restrained deformation. The key point is how to calculate the stiffness, for the stiffness is used to transfer a restrained deformation from an action to an effect on the structure. Before cracking, the stiffness will be constant as the stiffness of the uncracked cross-section, but if the concrete is cracked by external loading or restrained deformation, the stiffness of the structure also change. The stiffness will decrease as the cracking increases.

1.3 Goal of the research

The goal of this research is to find an expression of the structural crack width calculation for cracking caused by different action combinations in different structures, such as combinations of external loading and thermal deformation, or external moment and imposed curvature. There might be a difference in the order of the actions that occurred. So what is the difference between the imposed deformation first and the external loading first? Is there also any difference when an imposed deformation and an external load occur together?

Firstly cracking caused by an individual action should be investigated. After this, there is a problem about how to combine the individual actions. In order to solve the problem of action combination, the stiffness of the structure should be calculated exactly. Also the conversion from the structure action to the cross-sectional effect is another important point.



1.4 Research outline



Fig. 1-3 Outline of the research



CHAPTER 2

LITERATURES SURVEY



In the literatures survey chapter ,the main contents will be presented as two parts: "crack width calculation in different codes" and "theoretical model for the calculation the crack width".

The literature study on comparison with different codes includes four different codes to calculate the crack width.

- Dutch Code
- Eurocode
- American Code
- Chinese Code

The literature study on theoretical models is divided into four parts:

- Introduction of crack width control
- Causes of crack formation
- Crack combinations
- Height of compression zone
- Continuous theory to determine crack widths

2.1 Different codes in calculation crack

Nowadays, in an actual project, the crack width has always been calculated by following a Code. But there are differences between different codes, which depend on their different theories. Four different codes will be compared, namely the Dutch Code, EuroCode, American Code and Chinese Code.

In these codes, the theories for the calculation of crack width are not totally the same. Mostly, the equations are found by an empirical equation or a semi-theoretical and semi-empirical equation. Crack width control based on steel stress and bar diameter/spacing is derived from crack width equations, so their basis is the same in different codes. But the calculation methods or criteria in different code have a little difference. For example, in the American code the crack width will be controlled by controlling the reinforcement stress or bar spacing. In the Chinese code it will be controlled by calculating the crack width and comparing with the maximum width. In the Eurocode both methods are mentioned. For these two methods, the basic theory is the same. If the maximum crack width is substituted into the equation of the crack width calculation, the maximum steel stress will be obtained. So crack width control might be transferred into steel stress control which is much easier for an engineer to use.

2.1.1 Crack width calculation equations and comparison

In the Eurocode 1992-1-1 and the Chinese Code GB50009 is presented the method of directly calculating the crack width to control crack width. The equations to calculate the crack width are shown below.



In the Eurocode 1992-1-1 [4], the equation of calculating crack width is,

$$w_k = s_{r,\max} \left(\mathcal{E}_{sm} - \mathcal{E}_{cm} \right) \tag{1-1}$$

 $s_{r,\text{max}}$ is the maximum crack spacing $s_{r,\text{max}} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$ Eq.(7.11) in [4]

 \mathcal{E}_{sm} is the mean strain in the reinforcement under the relevant combination of loads,

including the effect of imposed deformations and taking into account the effects of tension stiffening. Only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered

 \mathcal{E}_{cm} is the mean strain in the concrete between cracks

Where

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \ge 0.6 \frac{\sigma_s}{E_s}$$
(1-2)

- σ_s is the stress in the tension reinforcement assuming a cracked section. For pretensioned members, σ_s may be replaced by $\Delta \sigma_p$ the stress variation in prestressing tendons from the state of zero of the concrete at the same level.
- α_e is the ratio E_s / E_{cm}

$$\rho_{p,eff}$$
 is $\frac{(A_s + \xi_1^2 A_p)}{A_{c,eff}}$

 k_1 is a factor dependent on the duration of the load

 $A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth, see Fig 2-1



Fig 2-1 Effective tension area of cross section

And in the Chinese Code GB50009 [7], the equation for calculating crack width is

$$w_{\max} = \alpha_{cr} \psi \frac{\sigma_{sk}}{E_s} (1.9c + 0.08 \frac{d_{eq}}{\rho_{ts}})$$
(1-3)

(1-4)

 $\alpha_{\rm cr}$ is the coefficient in [7] table 8.1.2-1

 ψ is a strain coefficient of steel between cracks. When $\psi < 0.2$, then $\psi = 0.2$. When

$$\psi > 1$$
, then $\psi = 1$. $\psi = 1.1 - 0.65 \frac{f_{tk}}{\rho_{te} \alpha_{sk}}$

 $\sigma_{
m sk}\,$ is calculated in [7] equation 8.1.3

 f_{tk} is the concrete tensile strength.

c is concrete cover, when c<20mm, then c=20mm; when c>65mm, then c=65mm.

 $ho_{
m te}$ is the ratio of reinforcement in the effective tension zone which is similar as

$$A_{c.eff}$$
 in Fig 2-1. When $\rho_{te} < 0.01$, then $\rho_{te} = 0.01$.

- d_{eq} is equivalent diameter of the reinforcement.
- V_i is bond coefficient in [7] table 8.1.2-2.

Also in the American code ACI 318-02 [5], there is an equation to calculate the crack width. This equation is based on the Gergely-Lutz equation [14]. This equation is derived by data fitting from many experiments. After Frosch's derivation [15], the equation used in ACI 318-02 is shown as below.

$$w_c = 2\beta \frac{\sigma_s}{E_s} \sqrt{(d_c)^2 + \left(\frac{s}{2}\right)^2}$$
(1-5)

 W_{e} is the crack width,

- β is the ratio of the distances to the neutral axis from the extreme tension fiber and from the centroid of the reinforcement,
- $\sigma_{\rm s}$ is calculated stress in the reinforcement at service loads,
- d_{c} is thickness of the concrete cover measured from the extreme tension fiber to the

center of the bar or wire located closest to it,

A is effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by

2



the number of bars or wires .

s is bar spacing.

In Eurocode 1992-1-1 [4] and the Chinese Code GB50009 [7], the allowable crack width will be determined by different exposure class and reinforcement condition. The allowable crack width is derived from Table 1 and Table 2 in Eurocode 1992-1-1 and Chinese Code GB50009.

The maximum crack width in Eurocode will be found in Table 2-1 [4].

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons					
	Quasi-permanent load combination	Frequent load combination					
X0, XC1	0,4 ¹	0,2					
XC2, XC3, XC4		0,2 ²					
XD1, XD2, XS1, XS2, XS3	0,3	Decompression					
 Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed. Note 2: For these exposure classes, in addition, decompression should be checked under the guasi-permanent combination of loads. 							

Table 2-1 Recommended values of w_{max} in Eurocode 1992-1-1 [4]

In Table 2-1, the exposure class is defined in Table 2-3 as below:



1 No risk of corrosion or attack X0 For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry Concrete inside buildings with very low air humidity 2 Corrosion induced by carbonation Concrete inside buildings with low air humidity XC1 Dry or permanently wet Concrete surfaces subject to long-term water contact
X0 For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry Concrete inside buildings with very low air humidity 2 Corrosion induced by carbonation Concrete inside buildings with low air humidity XC1 Dry or permanently wet Concrete inside buildings with low air humidity XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
X0 embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry Concrete inside buildings with very low air humidity 2 Corrosion induced by carbonation Concrete inside buildings with low air humidity XC1 Dry or permanently wet Concrete inside buildings with low air humidity XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry Corrosion induced by carbonation XC1 Dry or permanently wet Concrete inside buildings with low air humidity Concrete permanently submerged in water XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
attack For concrete with reinforcement or embedded metal: very dry Concrete inside buildings with very low air humid 2 Corrosion induced by carbonation XC1 Dry or permanently wet Concrete inside buildings with low air humidity Concrete permanently submerged in water XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
For concrete with reinforcement or embedded metal: very dry Concrete inside buildings with very low air humid 2 Corrosion induced by carbonation XC1 Dry or permanently wet Concrete inside buildings with low air humidity Concrete permanently submerged in water XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
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XC1 Dry or permanently wet Concrete inside buildings with low air humidity Concrete permanently submerged in water XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
XC1 Dry or permanently wet Concrete inside buildings with low air humidity Concrete permanently submerged in water XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
XC2 Wet, rarely dry Concrete permanently submerged in water Concrete surfaces subject to long-term water contact
XC2 Wet, rarely dry Concrete surfaces subject to long-term water contact
contact
Many foundations
XC3 Moderate humidity Concrete inside buildings with moderate or high
humidity
External concrete sheltered from rain
XC4 Cyclic wet and dry Concrete surfaces subject to water contact, not
within exposure class XC2
3 Corrosion induced by chlorides
XD1 Moderate humidity Concrete surfaces exposed to airborne chlorides
XD2 Wet, rarely dry Swimming pools
Concrete components exposed to industrial wate
containing chlorides
XD3 Cyclic wet and dry Parts of bridges exposed to spray containing
chlorides
Pavements
4 Corrosion induced by chlorides from sea water
XS1 Exposed to airborne salt but not in direct Structures near to or on the coast
contact with sea water
XS2 Permanently submerged Parts of marine structures
AS3 Tidal, spiasn and spray zones Parts of marine structures
5. Freeze/Thaw Attack
XF1 Moderate water saturation, without de-icing Vertical concrete surfaces exposed to rain and
agent reezing
XF2 Moderate water saturation, with de-icing agent Vertical concrete surfaces of road structures
Exposed to reezing and anomic denting agents
Ars High water saturation, without de-cing agents Honzontal concrete surfaces exposed to rain and
VE4 High water acturation with do joing agents or Dead and hides deales expand to do joing age
AP4 Fight water saturation with devicing agents of Road and bridge decks exposed to devicing agents of concrete surfaces exposed to device surfaces exposed to device surfaces exposed to device surfaces and Concrete surfaces exposed to device exposed to device surfaces exposed to device expos
Sea water Confidence Subject to direct spray
Solash zona of marina structures exposed to
freezing
6 Chemical attack
XA1 Slightly aggressive chemical environment Natural soils and ground water
according to EN 206-1, Table 2
XA2 Moderately aggressive chemical environment Natural soils and ground water
according to EN 206-1, Table 2
XA3 Highly aggressive chemical environment Natural soils and ground water

Table 2-3 Exposure class in Eurocode 1992-1-1

2 5.					
	Only reinforce	ed members in	Prestressed members in the		
Exposure	conc	crete	con	crete	
Class	Cracking	Crack width	Cracking	Crack width	
	control level	(mm)	control level	(mm)	
1	3	0.3	3	0.2	
2	3	0.2	2	Decompression	
3	3	0.2	1	Decompression	

And the maximum crack width in Chinese Code GB50009 will be found in Table 2-3.

Table 2-3 Recommended values of w_{max} in Chinese Code GB50010 [7]

In Table 2-3, the exposure class is defined as below:

Exposure Class 1: Normal environment indoor.

Exposure Class 2: Moist environment indoor or outdoor except in cold area and corrosion environment.

Exposure Class 3: The other exposure condition.

Compared with the above two tables, the Chinese code GB50009 seems more strictly than Eurocode 1992-1-1. And in Eurocode, it is divided with bonded tendons. But in Chinese Code GB50009, it is divided with whether contain pre-stressed reinforcement.

The crack width calculation equations in the codes and its influential factors are compared in the following Table 2-4. \blacktriangle illustrates that the factor is present the equation. \triangle illustrates that the factor is not in the equation but it is already considered in the equation.

1			
Influence factor	Direct method in	Chinese Code	Eq.(5) refer to
	Eurocode 1992-1-1	GB90005	ACI 318-02
Concrete cover			A
thickness			
Concrete tensile	A	Δ	
strength			
E modulus of steel			A
Steel stress	A		A
Reinforcement	Δ		Δ
diameter			
Bar spacing	Δ	Δ	A
Exposure	Δ	Δ	
environment			
Tension	A		
reinforcement ratio			
Effective tension		Δ	
area of concrete			

Table 2-4 Comparison the factors in crack width calculation equations of three codes

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From Table 2-4 and above equations, some conclusions will be obtained as below.

- 1> All the equations consider the concrete cover thickness, steel stress, steel diameter and bar spacing. Especially for the steel stress which directly influences the crack width.
- 2> Changing the steel diameter, bar spacing and reinforcement ratio has impact on the crack width. It will also respond to a change in the steel stress. So in American code ACI 318-02 and Dutch Code NEN6720, they use the method of controlling steel stress to control crack width is used.
- 3> This direct calculation method is more complicated compared with the other methods. When changing the reinforcement properties, it should be recalculated again.

2.1.2 Steel stress equations under crack width control and comparison

By using this method, it is only necessary to substitute the structural parameters and exposure parameters in to the equations to find out the maximum allowable steel stress. And comparing the steel stress with the maximum steel stress, it will let the engineers know whether it is sufficient. Or even it can use the steel stress and the maximum crack width to determine the maximum bar spacing or bar size.

The following tables will illustrate the different indirect crack width controls in different codes.

Steel stress ²	Ma	aximum bar size (m	m]
[MPa]	w _k = 0,4 mm	w _k = 0,3 mm	w _k = 0,2 mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Notes: 1. The values in the table are based on the following assumptions:

c = 25mm; $f_{cteff} = 2,9$ MPa; $h_{cf} = 0,5$; (h-d) = 0,1h; $k_1 = 0,8$; $k_2 = 0,5$; $k_c = 0,4$; k = 1,1 $k_1 = 0,4$ and k' = 1,0

2. Under the relevant combinations of actions

Table 2-5 Maximum bar diameters for crack control in Eurocode 1992-1-1 [4]



Steel stress ²	Maximum bar spacing [mm]						
[MPa]	w _k =0,4 mm	w _k =0,3 mm	w _k =0,2 mm				
160	300	300	200				
200	300	250	150				
240	250	200	100				
280	200	150	50				
320	150	100	-				
360	100	50	-				

Table 2-6 maximum bar spacing for crack control in Eurocode 1992-1-1 [4]

		Clear Cover (in.)							
Steel Stress	3/4	1	1-1/4	1-1/2	1-3/4	2	2-1/2	3	
f _s = 36 ksi	12	12	11.88	11.25	10.68	10	8.75	7.5	
f _s = 40 ksi	10.8	10.8	10.38	9.75	9.18	8.5	7.25	6	

Table 2-7 maximum bar spacing for crack control in ACI 318-02 [5]

Environmental class		σ _s N/mm ²							
without prestressing steel	with prestressing steel	100	125	150	200	250	300	350	400
1	-	870	670	540	370	270	200	155	120
2	1	620	470	370	245	170	120	85	55
3, 4 en 5	2	370	270	200	120	70	40	10	~
-	3, 4 en 5	120	70	35	-	-	-	-	-
Voor omhulde strengen volgens NEN 3868:1991 is de kolom "zonder voorspanstaal" van toepassing.									

Table 2-8 Maximum bar diameters for crack control in NEN6720 [6]

2.1.3 Conclusion

From above it can be seen that there are two different methods to compute crack width. One method is to calculate the crack width directly and compare with the maximum width. On the other hand, detailing requirements with regard to bar diameter or bar spacing linked to steel stress are linked with the crack width equation, acquired by presenting this equation in a different form. The latter method is more convenient for engineers, which do not need to calculate the crack width.



2.2 General literatures

2.2.1 Introduction of crack width control

Generally, a concrete crack is generated when the stress in the concrete is larger than the cracking stress. So in a reinforced concrete cross-section the concrete carries the compressive stress and the reinforcement has to carry the tensile stress. In the beginning of the crack stage, if too little reinforcement is used, the crack can be too wide, even if the cracking force is only exceeded to a small extend. So we also have to define a maximum crack width value to check whether the crack in the structure is sufficient.

2.2.2 Causes of cracks

Though there are many reasons for cracking, the main reasons are external loading and restrained deformation.

2.2.2.1 External Loading

From [1, 2], the axial force – strain diagram of a reinforced concrete tension member is obtained see Fig.2-1. From this diagram it can be seen that there are four stages of cracking behavior.



Fig. 2-1 The axial force - strain relation diagram in a reinforced concrete

Stage I is the uncracked stage. In this stage concrete does not crack, and the axial

2



force is smaller than the cracking force of the concrete N_{cr} . So in this stage, the equivalent stiffness is equal to the concrete stiffness.

$$\mathbf{E}_{b}\mathbf{A}_{m} = (E\mathbf{A})_{CS} \tag{2-1}$$

 E_{h} is the concrete elastic modulus

 A_m is the equivalent area of the section which is transfer the steel area into

concrete area by times $\frac{E_s}{E_c}$.

Stage II is the crack development stage. This stage only occurs under the condition of $\epsilon_{fdc} > \epsilon > \epsilon_{cr}$ and $N = N_{cr}$. When the imposed strain is larger than the cracking strain, the crack will occur, and in the whole stage II the axial force will be equal to cracking force. So in this stage, the equivalent stiffness will be computed by the following equation,

$$E_b A_m = \frac{N_{cr}}{\varepsilon_m}$$
(2-2)

 \mathcal{E}_m is the mean steel strain

Stage III is the crack widening stage and the crack pattern is fully developed. In this stage, the number of cracks will be constant while their width will increase. The tensile force will be fully carried by the steel, and the bond force will transfer part of the force from the steel to the concrete. In this stage, the equivalent stiffness will be calculated by the following equation,

$$E_b A_m = \frac{(\varepsilon_m + \Delta \varepsilon)}{\varepsilon_m} E A^{II}$$
(2-3)

 EA^{II} is the steel stiffness only.

 \mathcal{E}_m is the mean steel strain.

 $\Delta \mathcal{E}_{ts}$ is the tension stiffening.

Stage IV is the final stage. In this stage, the force reaches the yield strength of the steel. The deformation will increase when the force remains unchanged. The equivalent stiffness will be calculated by the following equation,

$$\mathbf{E}_{b}\mathbf{A}_{m} = \frac{N_{sy}}{\varepsilon_{m}} \tag{2-4}$$



2.2.2.2 Imposed strain

Mostly the response caused by an imposed strain is similar to the axial force without two major differences.

Firstly, the force caused by an imposed deformation dose not exceed the stiffness of the tensile member in stage I times the imposed strain,

$$N_{\Delta\varepsilon} \le (EA)_{cs} \Delta\varepsilon \tag{2-5}$$

This stage it only present for small imposed deformations, because the imposed deformation must be limited to

$$\Delta \varepsilon \le \varepsilon_{cr} \tag{2-6}$$

 \mathcal{E}_{cr} is cracking strain of concrete

So mostly $(EA)_{cs}\Delta\varepsilon$ will be larger than $N_{\Delta\varepsilon}$.

On the other hand, the length of the stage II largely depend on the reinforcement ratio. Because the external loading is constant in this stage, so the lower reinforcement ratio will cause the longer in this stage.

2.2.3 Loading combination

In actual projects, there usually is not only one action that will act on the structure. External loading and deformation will often take place together.

From the previous part the substantive force or deformation calculation method and theory are obtained. But if two or more different types of force and deformation together are combined, what will happen? From [1], several examples will be demonstrated here.

2.2.3.1 Axial force and Imposed strain in tensile member

1> The external load will occur before the restrained deformation.

$N > N_{cr}$:

If the external load is larger than the cracking force, then the crack pattern will be fully developed which directly in the 3rd stage in Fig.4. After that, the imposed deformation will be added. The existing crack will become larger due to the imposed deformation. From [1], to calculation of crack width, there is not a purely theoretically exact method. In this method, the incremental crack width is

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the mean crack spacing times the imposed strain at the level of reinforcement, which is also equivalent to the steel stress of $E_s \varepsilon_v$.

$$N < N_{cr}$$

On the other hand, if the external load will not exceed the cracking force, the member will be in the 1st stage in Fig.2-1. The crack is caused by the following imposed strain. This is mostly a not fully developed crack pattern, since a fully developed crack pattern will be found for a really large imposed strain.

2> The external load will occur after the imposed deformation.

$$\Delta \varepsilon > \varepsilon_{fdc}$$

If the imposed deformation is larger than the fully developed crack pattern strain ϵ_{fdc} , an increased of the external load will result in an increase of the stress

in the steel. So the increase of the steel stress is N_F / A_s . So the total

stress
$$\sigma_s = \sigma_{s,cr}(at N_{cr}) + \frac{N_F}{A_s}$$

 $\Delta \varepsilon < \varepsilon_{fdc}$

The crack pattern now is not fully developed due to the imposed deformation. So the following external load will cause the fully developed crack pattern mostly. In [1], the calculation method can be that the resulting steel stress in a crack is: if the concrete is not cracked under imposed deformation $(\Delta \varepsilon E_c + N_F)/A_s$; if the concrete is cracked under imposed deformation $(N_{cr} + N_F)/A_s$

2.2.3.2 Bending moment and imposed strain

1>Bending moment before imposed strain

If the bending moment is larger than the crack bending moment, there will be a fully developed crack pattern. This is similar as in section the 2.2.3.1, the widening of existing cracks will be caused by the imposed strain.

If the bending moment is smaller than the cracking bending moment, there will be a not fully developed crack pattern. So in [1], the fully developed crack pattern will mostly develop, and the resulting steel stress is the steel stress due to

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the bending moment plus the stress due to the imposed deformation times the incremental stiffness from stage III $M_q / (zA_s) + E_s \varepsilon_v$.

2>Bending moment after imposed strain

If the imposed strain is larger than the cracking strain, the bending moment causes an increase of the steel stress equal to $M_q / (zA_s)$. The stress due to the strain will be calculated by using Fig.2-1.

On the other hand, if the imposed strain is smaller than the cracking strain,the crack pattern will stay in the not fully developed pattern. From [1], the resulting steel stress in a crack is $\sigma_{s,cr} + M_q / (zA_s)$

From the above we can obtain that a critical part in a loading combination is the definition of the stiffness. In general, the stiffness will be estimated in practice. But this value is often not exactly correct. Considering this, we will work out a more exact result in the latter part of the thesis.



CHAPTER 3

CALCULATION OF COMPRESSION ZONE

HEIGHT



In this section, the calculation of compression zone height in different condition and with different forces will be illustrated.



Fig 3-1 Compression zone height in a concrete element

The compression zone is the compressive area of the cross section after cracking. It will bear the compressive stress due to moment or a normal force. Also in the cross section equilibrium, the compressive force taken by the compression zone is equal to the tensile force taken by tension reinforcement for pure bending. If the compression zone height is equal to zero, that means the cross section is all under tensile stress. On the other hand, if the compression zone height reaches its maximum value which is equal to h, the section is all under a compressive stress.

The compression zone height is a very important parameter in the concrete cross section calculation. The stiffness of the cross section after cracking largely depends on the compression height. So obtain the exact value of the compression zone height is necessary.

In order to obtain the compression zone height in a crack, we need to calculate in a cracking cross section as Fig 3-2. The Normal force balance $\sum N = 0$ and moment balance $\sum M = 0$ will be used in the calculation for solving the compression zone height. And the cross section parameters and materials parameters also are needed for solving the compression zone height. The calculation process will be found in the following sections and Appendix.1.

The compression zone height calculation will be divided into several conditions as below:

- 1> Only moment & only tension reinforcement (bottom reinforcement)
- 2> Moment and Normal force & only tension reinforcement (bottom reinforcement)
- 3> Only moment & both compression and tension reinforcement
- 4> Moment and Normal force & both compression and tension reinforcement

At last, the general equation for solving the compression zone height will be obtained.





Fig 3-2 Cross section in a crack

3.1 Compression zone height under only tension reinforcement and only bending moment

In this condition, there is only bending moment and tension reinforcement. So the cross section parameters are shown in Fig 3-3 as below.



Fig 3-3 Cross section M & only tension reinforcement

By using the normal force $\sum N = 0$, the compression zone height will be calculated. The stress and strain distribution is shown in Fig 3-4. The process of obtaining the

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compression zone height is illustrated later.



Fig 3-4 Stress and strain of the cross section

By using the normal force equilibrium $\sum N = 0$, which is derived from the stress distribution in Fig 3-4 the following relationship will be obtained:

$$\sum N = 0 \rightarrow N_c - N_s = 0$$

Where,

The concrete area force $N_c = \frac{bx}{2} E_c \varepsilon_c$

The tension reinforcement force $N_s = A_s E_s \varepsilon_s$

So the relationship can be rewritten as

$$\frac{bx}{2}E_c\varepsilon_c - A_sE_s\varepsilon_s = 0 \tag{3-1}$$

In this equation,

- b is the cross section width
- x is the compression zone height
- A_s is the tension reinforcement area
- E_c is the elastic modulus of concrete
- $E_{\scriptscriptstyle S}\,$ is the elastic modulus of tension reinforcement
- \mathcal{E}_c is the concrete strain
- \mathcal{E}_s is the tension reinforcement strain



The relationship between concrete strain \mathcal{E}_c and reinforcement strain \mathcal{E}_s can be

derived by the strain relation in the Fig 3-4 as below

$$\mathcal{E}_s = \frac{d-x}{x} \mathcal{E}_c \tag{3-2}$$

If we substitute Eq. (3-2) into Eq. (3-1), the following relationship will be obtained:

$$\varepsilon_c \left(\frac{bx}{2}E_c - A_s E_s \frac{d-x}{x}\right) = 0 \longrightarrow \frac{bx}{2}E_c - A_s E_s \frac{d-x}{x} = 0$$
(3-3)

Finally, the compression zone height x will be obtained by solving the Eq. (3-3) and the result is as below.

$$x = \frac{-A_s E_s + \sqrt{A_s E_s (A_s E_s + 2E_c bd)}}{E_c b}$$
(3-4)

If we use the ratio of E-modulus $\alpha_e = \frac{E_s}{E_c}$ and reinforcement ratio $\rho = \frac{A_s}{bd}$ in the

equations, the Eq. (3-4) will be rewritten as below.

$$x = d(-\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho})$$
(3-5)

3.2 Compression zone height under both compression and tension reinforcement with only bending moment

In this condition, there is only bending moment with both compression and tension reinforcement. So the cross section basic parameters are shown in Fig 3-5 as below.



Fig 3-5 Cross section M & both compression and tension reinforcement



By using the normal force $\sum N = 0$, the compression zone height will be calculated. The stress and strain distribution is shown in Fig 3-6. The process of obtaining the compression zone height is illustrated later.



Fig 3-6 Stress and strain of the cross section

By using the normal force equilibrium $\sum N = 0$, which is derived from the stress distribution in Fig 3-8 the following relationship will be obtained:

$$\sum N = 0 \longrightarrow N_c + N_{scomp} - N_s = 0$$

Where,

The concrete area force $N_c = \frac{bx}{2} E_c \varepsilon_c$

The tension reinforcement force $N_s = A_s E_s \varepsilon_s$

The compression reinforcement force $N_{scomp} = A_{scomp} \mathcal{E}_{scomp} \mathcal{E}_{scomp}$

So the relationship can be rewritten as

$$\frac{bx}{2}E_c\varepsilon_c + A_{scomp}E_{scomp}\varepsilon_{scomp} - A_sE_s\varepsilon_s = 0$$
(3-6)

In this equation,

- b is the cross section width
- x is the compression zone height

 A_{s} is the tension reinforcement area

 A_{scomp} is the compression reinforcement area

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- E_c is the elastic modulus of concrete
- $E_{\scriptscriptstyle S}\,$ is the elastic modulus of tension reinforcement
- E_{scomp} is the elastic modulus of compression reinforcement
- \mathcal{E}_c is the concrete strain
- \mathcal{E}_s is the tension reinforcement strain
- \mathcal{E}_{scomp} is the compression reinforcement strain

The relationship between concrete strain \mathcal{E}_c , tension reinforcement strain \mathcal{E}_s and compression reinforcement strain \mathcal{E}_{scomp} can be derived by the strain relation in the Fig 3-6 as below

$$\varepsilon_s = \frac{d-x}{x} \varepsilon_c \tag{3-7}$$

$$\varepsilon_{scomp} = \frac{x - c_u}{x} \varepsilon_c \tag{3-8}$$

If we substitute Eq. (3-7) and Eq. (3-8) into Eq. (3-6), the following relationship will be obtained:

$$\varepsilon_{c}\left(\frac{bx}{2}E_{c} + A_{scomp}E_{scomp}\frac{x - c_{u}}{x} - A_{s}E_{s}\frac{d - x}{x}\right) = 0$$

$$\rightarrow \frac{bx}{2}E_{c} + A_{scomp}E_{scomp}\frac{x - c_{u}}{x} - A_{s}E_{s}\frac{d - x}{x} = 0$$
(3-9)

Finally, the compression zone height x will be obtained by solving the Eq. (3-9) and the result is as below.

$$x = \frac{-A_s E_s - A_{scomp} E_{scomp} + \sqrt{A_s^2 E_s^2 + 2A_s A_{scomp} E_s E_{scomp} + 2E_c b dA_s E_s + A_{scomp}^2 E_{scomp}^2 + 2E_c b c_u A_{scomp} E_{scomp}}{E_c b}$$

If we use the ratio of E-modulus $\alpha_e = \frac{E_s}{E_c}$, $\alpha_{escomp} = \frac{E_{scomp}}{E_c}$ and reinforcement

ratio
$$\rho = \frac{A_s}{bd}$$
, $\rho_{scomp} = \frac{A_{scomp}}{bd}$ in the equations, the Eq. (3-10) will be rewritten as

below.

$$x = d(-\alpha_e \rho - \alpha_{escomp} \rho_{scomp} + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho \alpha_{escomp} \rho_{scomp} + (\alpha_{escomp} \rho_{scomp})^2 + 2\alpha_{escomp} \rho_{scomp} + 2\alpha_e \rho)}$$

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(3-10)

(3-11)

3.3 Compression zone height under only tension reinforcement with both bending moment and normal force

In this condition, there is both normal force and bending moment with only tension reinforcement. So the cross section basic parameters are shown in Fig 3-7 as below.



Fig 3-7 Cross section M and N & only tension reinforcement

By using the normal force $\sum N = 0$ and $\sum M = 0$, the compression zone height will be calculated. The stress and strain distribution is shown in Fig 3-8. The process of obtaining the compression zone height is illustrated below.





By using the normal force equilibrium $\sum N = 0$ and $\sum M = 0$, which are derived from the stress distribution in Fig 3-8 the following relationship will be obtained:

$$\sum N = 0 \rightarrow N_s - N_c = N$$
$$\sum M = 0 \rightarrow N_s (d - \frac{h}{2}) + N_c (\frac{h}{2} - \frac{x}{3}) = M$$

Where,

The concrete area force $N_c = \frac{bx}{2} E_c \varepsilon_c$

The tension reinforcement force $N_s = A_s E_s \varepsilon_s$

h is the total height of the cross section

d is the distance between the reinforcement and the top of the cross section

So the relationship can be rewritten as

$$A_s E_s \varepsilon_s - \frac{bx}{2} E_c \varepsilon_c = N \tag{3-12}$$

$$\frac{bx}{2}E_c\varepsilon_c(\frac{h}{2}-\frac{x}{3})+A_sE_s\varepsilon_s(d-\frac{h}{2})=M$$
(3-13)

In this equation,

b is the cross section width

x is the compression zone height

 A_s is the tension reinforcement area

- E_c is the elastic modulus of concrete
- E_s is the elastic modulus of tension reinforcement
- \mathcal{E}_c is the concrete strain
- \mathcal{E}_s is the tension reinforcement strain

The relationship between concrete strain \mathcal{E}_c , tension reinforcement strain \mathcal{E}_s and compression reinforcement strain \mathcal{E}_{scomp} can be derived by the strain relation in the Fig 3-8 as below

$$\mathcal{E}_s = \frac{d-x}{x} \mathcal{E}_c \tag{3-14}$$

If we substitute Eq. (3-14) and Eq. (3-15) into Eq. (3-16), the following relationship will be obtained:

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$$\mathcal{E}_{c}\left(A_{s}E_{s}\frac{d-x}{x}-\frac{bx}{2}E_{c}\right)=N$$
(3-15)

$$\varepsilon_{c}\left(\frac{bx}{2}E_{c}\left(\frac{h}{2}-\frac{x}{3}\right)+A_{s}E_{s}\frac{d-x}{x}\left(d-\frac{h}{2}\right)\right)=M$$
(3-16)

From equations (3-15) and (3-16), the relationship between M/N ration and compression zone height x is as below:

$$\frac{bx}{2}E_{c}(\frac{h}{2}-\frac{x}{3})+A_{s}E_{s}\frac{d-x}{x}(d-\frac{h}{2})}{A_{s}E_{s}\frac{d-x}{x}-\frac{bx}{2}E_{c}}=\frac{M}{N}$$
(3-17)

So we can obtain the value of compression zone height x by given a certain M / N value.

If we use the ratio of E-modulus $\alpha_e = \frac{E_s}{E_c}$, the moment and normal force ratio $e = \frac{M}{N}$

and reinforcement ratio $\rho = \frac{A_s}{bd}$, in the equations, the Eq. (3-17) will be rewritten as below.

$$\frac{\frac{x}{2d}(\frac{h}{2} - \frac{x}{2}) + \alpha_e \rho \frac{d - x}{x}(d - \frac{h}{2})}{\alpha_e \rho \frac{d - x}{x} - \frac{x}{2d}} = e$$
(3-18)

3.4 Compression zone height under both compression and tension reinforcement with both normal force and bending moment

In this condition, there is both normal force and bending moment with both compression and tension reinforcement. So the cross section basic parameters are shown in Fig 3-9 as below.





Fig 3-9 Cross section N and M & both compression and tension reinforcement

By using the normal force $\sum N = 0$ and $\sum M = 0$, the compression zone height will be calculated. The stress and strain distribution is shown in Fig 3-10. The process of obtaining the compression zone height is illustrated below.



Fig 3-10 Stress and strain of the cross section

By using the normal force equilibrium $\sum N = 0$ and $\sum M = 0$, which are derived from the stress distribution in Fig 3-10 the following relationship will be obtained:

$$\sum N = 0 \rightarrow N_s - N_c - N_{scomp} = N$$
$$\sum M = 0 \rightarrow N_s (d - \frac{h}{2}) + N_{scomp} (\frac{h}{2} - c_u) + N_c (\frac{h}{2} - \frac{x}{3}) = M$$

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Where,

The concrete area force $N_c = \frac{bx}{2} E_c \varepsilon_c$

The tension reinforcement force $N_s = A_s E_s \varepsilon_s$

The compression reinforcement force $N_{scomp} = A_{scomp} \mathcal{E}_{scomp} \mathcal{E}_{scomp}$

- h is the total height of the cross section
- d is the distance between the reinforcement and the top of the cross section

So the relationship can be rewritten as

$$A_{s}E_{s}\varepsilon_{s} - \frac{bx}{2}E_{c}\varepsilon_{c} - A_{scomp}E_{scomp}\varepsilon_{scomp} = N$$
(3-19)

$$\frac{bx}{2}E_c\varepsilon_c(\frac{h}{2}-\frac{x}{3}) + A_{scomp}E_{scomp}\varepsilon_{scomp}(\frac{h}{2}-c_u) + A_sE_s\varepsilon_s(d-\frac{h}{2}) = M$$
(3-20)

In this equation,

- b is the cross section width
- x is the compression zone height
- A_s is the tension reinforcement area

 A_{scomp} is the compression reinforcement area

- E_c is the elastic modulus of concrete
- E_s is the elastic modulus of tension reinforcement
- E_{scomp} is the elastic modulus of compression reinforcement
- \mathcal{E}_c is the concrete strain
- \mathcal{E}_s is the tension reinforcement strain
- \mathcal{E}_{scomp} is the compression reinforcement strain

The relationship between concrete strain \mathcal{E}_c , tension reinforcement strain \mathcal{E}_s and compression reinforcement strain \mathcal{E}_{scomp} can be derived by the strain relation in the Fig 3-10 as below

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c \tag{3-22}$$


$$\varepsilon_{scomp} = \frac{x - c_u}{x} \varepsilon_c \tag{3-21}$$

If we substitute Eq. (3-7) and Eq. (3-8) into Eq. (3-6), the following relationship will be obtained:

$$\mathcal{E}_{c}\left(A_{s}E_{s}\frac{d-x}{x}-\frac{bx}{2}E_{c}-A_{scomp}E_{scomp}\frac{x-c_{u}}{x}\right)=N$$
(3-22)

$$\mathcal{E}_{c}\left(\frac{bx}{2}E_{c}\left(\frac{h}{2}-\frac{x}{3}\right)+A_{scomp}E_{scomp}\frac{x-c_{u}}{x}\left(\frac{h}{2}-c_{u}\right)+A_{s}E_{s}\frac{d-x}{x}\left(d-\frac{h}{2}\right)\right)=M$$
 (2-23)

Finally, the compression zone height x will be obtained by solving the Eq. (3-9) and the result is as below.

$$\frac{A_{s}E_{s}(\frac{d-x}{x})(d-\frac{h}{2}) + \frac{bx}{2}E_{c}(\frac{h}{2}-\frac{x}{3}) + A_{scomp}E_{scomp}(\frac{x-c_{u}}{x})(\frac{h}{2}-c_{u})}{A_{scomp}E_{scomp}(\frac{x-c_{u}}{x}) - A_{s}E_{s}(\frac{d-x}{x}) + \frac{bx}{2}E_{c}} = \frac{M}{N} \quad (3-24)$$

If we use the ratio of E-modulus $\alpha_e = \frac{E_s}{E_c}$, $\alpha_{escomp} = \frac{E_{scomp}}{E_c}$ and reinforcement

ratio $\rho = \frac{A_s}{bd}$, $\rho_{scomp} = \frac{A_{scomp}}{bd}$ in the equations, the Eq. (3-10) will be rewritten as

below.

$$\frac{\alpha_e \rho(\frac{d-x}{x})(\frac{h}{2}-\frac{x}{2}) + \frac{x}{2d}(\frac{h}{2}-\frac{x}{3}) + \alpha_{escomp}\rho_{scomp}(\frac{x-c_u}{x})(\frac{h}{2}-c_u)}{\alpha_{escomp}\rho_{scomp}(\frac{x-c_u}{x}) - \alpha_e\rho(\frac{d-x}{x}) + \frac{x}{2d}} = e \quad (3-25)$$

3.5 The compression zone height equation of Noakowski

From [17], Noakowski obtained an equation of calculation compression zone height. In his equation he moved the centroidal axis to the position where the area moment equal to 0 after cracked which is shown in Fig 3-11. His basic theory is using the area inertia equal to 0 and the stress at compression zone height position equal to 0 to obtain the relationship between compression zone height and force.



Fig 3-11 Stress and strain in situation I and II by Noakowski

In Fig 3-11, situation I is the uncracked stage, the centroidal axis is at position $\frac{h}{2}$. After

moved, the centroidal axis from o to o'. The position of o' is the stress equal to 0 only under moved bending moment after cracked. After calculation [17], the compression zone height equation is shown as Eq. (3-26).

$$\frac{k_x^4 + 4Ak_x^3 - 12Bk_x^2 + 12Ck_x + 12AC - 12B^2}{-6k_x^3 - 18Ak_x^2 - 12(A^2 - B)k_x + 12AB} = \eta_2$$
(3-26)

In this equation,

$$A = \alpha_e (\eta_s + \eta_{scomp})$$
$$B = \alpha_e (\eta_s + k_u \eta_{scomp})$$
$$C = \alpha_e (\eta_s + k_u^2 \eta_{scomp})$$

 α_e is the modulus ratio between reinforcement and concrete $\alpha_e = \frac{E_s}{E_c}$



 η_s is the area ratio between tension reinforcement and concrete $\eta_s = \frac{A_s}{A_{c.eff}}$

 η_{scomp} is the area ratio between compression reinforcement and concrete $\eta_{scomp} = \frac{A_{scomp}}{A_{c,eff}}$

Where $A_{c,eff} = bd$

 k_u is the related height from the reinforcement to bottom $k_u = \frac{c_u}{d}$

 η_2 is the moment and normal force ratio after moved the centroidal axis,

$$\eta_2 = \eta_1 + k_{x0} - \frac{(2B + k_x^2)}{2(A + k_x)}$$

 $\eta_{\mathrm{l}}~$ is the moment and normal force ratio before moved the centroidal axis,

$$\eta_1 = \frac{e}{d} = \frac{M}{dN}$$

 k_{x0} is the ratio of centroidal position and total height, in rectangular cross section, it is always 0.5.

3.6 Example by using two method of calculation compression zone height

There is a cross section shown in Fig 3-12. The external normal force and moment is applied on it. Both top and bottom have the reinforcement. The two methods in section 3.4 and 3.5 will be applied to calculate in this example.



Fig 3-12 Cross section of example

The basic parameters of the cross section and the external force are shown as below:

$$E_{c} = 16000 N / mm^{2}$$

$$E_{scomp} = 2 \cdot 10^{5} N / mm^{2}$$

$$n = \frac{E_{s}}{E_{c}} = 12.5$$

$$h = 605 mm$$

$$d = 550 mm$$

$$c_{u} = 55 mm$$

$$A_{c,eff} = bd = 550 \times 400 = 220000 mm^{2}$$

$$A_{s} = 1000 mm^{2}$$

$$A_{scomp} = 1000 mm^{2}$$

$$\eta_{s} = \frac{1000}{220000} = 0.00455$$

$$e = \frac{M}{N} = 1434.4 mm$$

$$\eta_{1} = \frac{M}{dN} = -2.608$$

Substituting the above values into the Eq. (3-25) and Eq. (3-26), the compression zone height will be obtained. The process of calculation is shown in Appendix 1.

After calculation, the results of Noakowski equation and Eq. (3-25) are similar which are



165mm and 164.99mm respectively. The difference between Eq. (3-26) and Eq. (3-25) is that, for Eq. (3-25) the basic theory is the cross section strain and stress balance. But for Eq. (3-26), Noakowski use the area moment equal to 0 and stress balance after moved centroidal axis to obtain the compression zone height equation. The result obtained by both methods is similar, so that either of the methods can be used in compression zone height calculation.

The η_1 -k_x curves by using equation (3-25) and (3-26) are plotted as Fig 3-13 and Fig 3-14. In these figures, η_1 is the moment and normal force ratio, and k_x is related height of compression zone height.



Fig 3-13 η_1 -k_x relationship curve by using Eq. (3-25)



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Fig 3-14 η_1 -k_x relationship curve by using Eq. (3-26)

CHAPTER 4

CALCULATION OF THE STIFFNESS



Stiffness is a very important parameter in crack width calculation. Always the stiffness can be easily found if only one type of action is applied on the element. But if there are two or more different actions on the structure, such as external loading and restrained deformation, the calculation of stiffness is much more complicated.

For obtaining the mean stiffness of the element $(EI)_m$ and $(EA)_m$, the Tension Stiffening

Law [2, 3, 16, 17] is applied. The moment-curvature relationship in Fig 4-1 is used to obtain a realistic determination of the bending moment or normal force. Also after having obtained the stiffness of the element, every parameters of the element in both the uncracked and the cracked stage will be known. So the mean stiffness of the element is the key parameter in the future calculation.



Fig 4-1 Tension Stiffening Law moment-curvature relationship

By using Tension Stiffening Law in Fig 4-1, the stiffness calculation process is divided into several steps in Fig 4-2 as below, and it will be described detailed in the following sections.





Fig 4-2 Process of mean stiffness calculation

4.1 Cracking Force

In the cracking force calculation, there are three conditions. For every condition, there is a different expression for the cracking force.

1> Only bending:

Cracking moment $M_{cr} = W \cdot f_{ct}$

W is the area moment of the cross section in the uncracked stage

- f_{ct} is the concrete cracking tensile strength
- 2> Only Normal force

Cracking normal force $N_{cr} = A \cdot f_{ct}$

A is the area of the cross section in the uncracked stage

3> Both bending and normal force

Cracking moment with a constant normal force $M_{cr} = W \cdot \left(f_{ct} - \frac{N_0}{A} \right)$

 $N_o > 0$ for tension.



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4.2 Bending stiffness in a crack

From Fig 4-1, the stiffness in a crack $(EI)_{s,cr}$ is equal to the mean stiffness of the element in stage II.

$$(EI)_{s\,cr} = E_c I^{\ II} \tag{4-1}$$

And the inertia modulus in a crack is equal to

$$I'' = \frac{bx^3}{12} + bx \cdot (\frac{x}{2})^2 + \alpha_e A_s (d-x)^2$$
(4-2)

- b is the cross section width
- x is the compression zone height
- $A_{\rm s}$ is the tension reinforcement area

$$\alpha_e$$
 is the ratio of E-modulus where $\alpha_e = \frac{E_s}{E_c}$

So that the stiffness in a crack should be

$$(EI)_{s,cr} = E_c I^{II} = E_c \cdot (\frac{bx^3}{12} + bx \cdot (\frac{x}{2})^2 + \alpha_e A_s (d-x)^2)$$
(4-3)

The stiffness in a crack can also be calculated by another method as below

$$E_{c}I^{II} = E_{s}A_{s}(d - \frac{1}{3}x)(d - x)$$
(4-4)

By using the same theory, the axial stiffness also will be obtained as below.

$$(EA)_{s,cr} = E_c A^{II} = E_c bx$$

4.3 Difference of the centroidal axis Δx after moved

The centroidal axis moves from the original axis which always is at position $\frac{h}{2}$ in the uncracked stage to the area moment equal to 0 after cracking which is shown in Fig 3-11. By using the area moment equal to 0 to determine the position of the centroidal axis after cracking. The difference Δx is

$$\sum W_o^{II} = 0 \Longrightarrow \Delta x = \frac{2B + k_d^2}{2(A + k_d)}$$
(4-5)

Where

 $A = \alpha_e(\eta_s + \eta_{scomp})$

$$B = \alpha_e (\eta_s + k_u \eta_{scomp})$$

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 α_e is the modulus ratio between reinforcement and concrete $\alpha_e = \frac{E_s}{E_e}$

 η_s is the area ratio between tension reinforcement and concrete $\eta_s = \frac{A_s}{A_{c,eff}}$

 η_{scomp} is the area ratio between the compression reinforcement and the

concrete
$$\eta_{scomp} = \frac{A_{scomp}}{A_{c,eff}}$$

Where $A_{c,eff} = bd$

 k_d is the related height of compression zone $k_d = \frac{x}{d}$

4.4 Tension stiffening value

From Fig 4-1, the distance $\Delta \kappa (\Delta \varepsilon)$ between the mean moment-curvature line and the cracked cross section moment-curvature line is called the tension stiffening effect. The value of $\Delta \kappa (\Delta \varepsilon)$ describes the magnitude of the tension stiffening effect. Between the cracks the bond stresses are active and the concrete takes over part of the tension from the steel. Thus the reinforcement is being stiffened by the concrete. [16, 17]. Fig 4-3 shows the strains for calculating the crack spacing and the average strains. [18]



Fig 4-3 Strains for calculating the crack spacing and the average strains

By using the mean bond law from the Model Code [18], Fig 4-4 and Fig 4-5 show the



basic theory to obtain the tension stiffening value. In Fig 4-4, κ_1 means the maximum

curvature in a cracked section at the beginning of the crack formation stage. By using the mean bond law [18], the mean curvature in a cracked section should be

 $\kappa_{m1} = 0.44\kappa_1$ by using the theory in CEB-FIP Model Code [18]. So that the

 $\Delta \kappa = (1 - 0.44)\kappa_1 = 0.56\kappa_1$. And the transfer length is $\frac{a_1}{2}$. In Fig 4-5, the element is in fully developed crack pattern. So the transfer length will be changed to

 $\frac{a}{2} = 0.75 \frac{a_1}{2}$ [2]. So the tension stiffening factor $\Delta \kappa$ also will be changed.



Fig.4-4 Curvature at crack section in beginning of crack formation stage by mean bond law



Fig 4-5 Curvature at crack in fully developed crack pattern by mean bond law

The value of the tension stiffening factor at a fully developed crack pattern is equal to

$$\Delta \kappa = \kappa_{s.cr} - \kappa_{fdc} = 0.75 \Delta \kappa_1 = 0.42 \kappa_{s,cr}$$
(4-6)

$$\Delta \varepsilon = \varepsilon_{s,cr} - \varepsilon_{fdc} = 0.75 \Delta \varepsilon_1 = 0.42 \varepsilon_{s,cr} \tag{4-7}$$

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Where

$$\kappa_{s,cr}$$
 is the curvature at a cracked section $\kappa_{s,cr} = \frac{M + N\Delta x}{(EI)_{s,cr}}$

$$\mathcal{E}_{s,cr}$$
 is the strain at a cracked section $\mathcal{E}_{s,cr} = \frac{N}{(EA)_{s,cr}}$

4.5 Calculation of the mean stiffness

In the calculation of the mean stiffness of a element, if there is only one action on the element it is easily obtained by the tension stiffening law [1, 2, 16, 17]. But if there is two or more different actions on the element, the method should be followed as presented below. The black line is the moment-curvature effect line in total element. The red line is the moment-curvature effect in the cracked cross section. The blue line is the moment-curvature effect in the cracked cross section without ΔM which is caused by moving the centroidal axis.



Fig 4-5 Moment-curvature relationship by applying both moment and normal force

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The procedure to obtain the mean moment-curvature relationship of whole element which is shown as black line in Fig 4-5 is described as below:

- 1> Defining the M_{cr} value and position in Y axis.
- 2> Calculating the stiffness at a crack $(EI)_{s.cr}$.
- 3> Using the stiffness at a crack, the slope of the blue line in Fig 4-5 can be defined.
- 4> Adding the extra moment due to normal force ΔM
- 5> The position of red line in Fig 4-5 will be defined by using ΔM .
- 6> Calculating the tension stiffening value $\Delta \kappa$.
- 7> By using tension stiffening law, the position of κ_{fdc} will be found.
- 8> At last, the mean moment-curvature relationship of whole element shown as black line will be obtained.

From Fig 4-5, the there are 3 different stages in the cracking development: uncracked stage, crack formation stage and fully developed crack stage. So each stage will have different expression of mean stiffness.

Uncracked stage:
$$(EI)_m = E_c I_c$$
 (4-7)

Crack formation stage: $(EI)_m = \frac{M}{\kappa_m}$ $\kappa_m = \kappa_s - \Delta \kappa$ $\kappa_s = \frac{M}{(EI)_{s,cr}}$ (4-8)

Fully developed crack stage:
$$(EI)_m = \frac{M_2}{\kappa_m} = \frac{M_1 - N\Delta x}{\kappa_s - \Delta \kappa}$$
 (4-9)

 M_1 is the bending moment after moving the centroidal axis

$$M_1 = (\kappa_m + \Delta \kappa)(EI)_{s,cr} = \kappa_s(EI)_{s,cr}$$

 M_2 is the original moment without moving the centroidal axis

 ΔM is the extra moment caused by moving centroidal axis $\Delta M = N\Delta x$ Δx is the moving distance of centroidal axis

 $\Delta \kappa$ is the tension stiffening effect $\Delta \kappa = 0.42 \kappa_{s.cr}$

 $\kappa_{s,cr}$ is the curvature at the cracked cross section under cracking moment

 κ_s is the curvature at the cracked cross section at the load considered

 $(EI)_{s,cr}$ is the stiffness in a crack Eq. (4-5).



CHAPTER 5

BEAM UNDER DEAD LOAD AND

TEMPERATURE GRADIENT

After discussed about the cross section problem, in this chapter, the calculation based on a simple structure will be illustrated. In order to illustrate it more clearly, a clamped beam will be discussed in this chapter to show the characteristics under different loading combination. As solving this problem, the theory of the simple element can be used in some complicated real projects.

The simple element is shown in Fig 5-1. It is a clamped beam with tension reinforcement under both dead load and temperature gradient effects. The dead load can be transferred as a uniform distribute load q, and the temperature gradient is ΔT . So there will be a bending moment due to self-weight, and also a bending moment due to thermal difference. A big problem is that how to combine these two actions together, and is there any difference among different combinations? The following calculation will illustrate it.



Fig 5-1 Clamped beam under both dead load and temperature gradient

For a combination of dead load and temperature gradient, there are many various situations. Such as dead load is given first, and then the temperature gradient is loaded, or exchange the order. For different situations, there will be different procedures and results respectively. So using an appropriate calculation procedure is essential to obtain correct results. All the different situations are listed as below:

- 1. $M \rightarrow \Delta T$
 - 1.1 M \rightarrow Uncracked
 - 1.1.1 $\Delta T \rightarrow$ Uncracked
 - 1.1.2 $\Delta T \rightarrow$ Not fully developed cracked pattern
 - 1.1.3 $\Delta T \rightarrow$ Fully developed cracked pattern
 - 1.2 M \rightarrow Cracking at ends only
 - 1.2.1 $\Delta T \rightarrow$ Ends Cracking increased
 - 1.2.1.1 No cracking at middle span
 - 1.2.1.2 Cracking at middle span
 - 1.2.2 $\Delta T \rightarrow$ Ends cracking decreased
 - 1.2.2.1 No cracking at middle span
 - 1.2.2.2 Cracking at middle span
 - 1.2.2.3 Cracking at middle span but ends cracking disappeared
 - 1.3 M \rightarrow Cracking at both ends and middle span



- 1.3.1 $\Delta T \rightarrow$ Ends cracking increased
 - 1.3.1.1 Middle span cracking is not disappeared
 - 1.3.1.2 Middle span cracking is disappeared
- 1.3.2 $\Delta T \rightarrow$ Ends cracking decreased
 - 1.3.2.1 Ends cracking is not disappeared
 - 1.3.2.2 Ends cracking is disappeared
- 2. $\Delta T \rightarrow M$
 - 2.1 $\Delta T \rightarrow$ Uncracked
 - 2.1.1 M \rightarrow Uncracked
 - 2.1.2 M \rightarrow Cracking at ends
 - 2.1.3 M \rightarrow Cracking at both ends and middle span
 - 2.1.4 M \rightarrow Cracking at middle span
 - 2.2 $\Delta T \rightarrow$ Not fully developed cracked pattern
 - 2.2.1 M \rightarrow Cracking at ends
 - 2.2.2 M \rightarrow Cracking at both middle span and ends
 - 2.2.3 M \rightarrow Cracking at middle span
 - 2.3 $\Delta T \rightarrow$ Fully developed cracked pattern
 - 2.3.1 M \rightarrow Cracking at ends
 - 2.3.2 M \rightarrow Cracking at both middle span and ends
 - 2.3.3 M \rightarrow Cracking at middle span
- 3. ΔT & M
 - 3.1 Uncracked
 - 3.2 Cracked
 - 3.2.1 Cracking at ends
 - 3.2.2 Cracking at both middle span and ends
 - 3.2.3 Cracking at middle span

In order to illustrate it, one typical condition will be discussed in the following section. And one example will be calculated to prove this theory.

5.1 M $\rightarrow \Delta T \& M \rightarrow$ cracking at both ends $\& \Delta T \rightarrow$ enlarge the cracking at ends and no cracking at middle span

In this situation, the moment due to uniformly distribute load q will be added first. The beam will crack at both ends for this moment. Then the temperature gradient is loaded, the cracking at ends will increase. It is shown as Fig 5-2.





Fig 5-2 Loading procedure

So the effect caused by dead load only need to be analyzed first, then added the temperature gradient.

5.1.1 Dead load effect.

Dead load of a beam is uniformly distributed. So it can be recognized as a uniform distributed load q. The moment due to dead load should be a parabola line. In order to make the calculation easier, the linear line is instead of parabola line. The moment curve due to dead load is shown as Fig 5-3. The moment due to dead load at end is equal to $\frac{1}{12}ql^2$ and at mid span is equal to $\frac{1}{24}ql^2$.



Fig 5-3 Moment curve due to dead load

In the first step, the cracking length due to moment will be estimated. The length is the area where moment is larger than cracking moment on an uncracked beam which is also shown in Fig 5-3. This estimated length is called L_2 , and this value will be used in the future calculation. Usually the estimated cracking length can be used as the area where the moment is larger than cracking moment on an uncracked beam. The cracking moment can be calculated as

$$M_{cr} = \frac{1}{6}bh^2\sigma_{cr}$$
(5-1)

Since the cracking length is estimated, the moment curve due to dead load will be calculated as below.

In order to obtain the exact moment due to dead load, the following method will be used. Firstly, this uniformly distributed load can be divided into two parts: one is dead load on a simple support beam, and the other one is a simple support beam with a constant moment which is shown as Fig 5-4. By using this method, the clamped beam will be translated into two simple support beam problems with the rotation at ends equal to 0. It is much easier to calculate the moment curve.





Fig 5-4 Transition from clamped beam to simple support beam

In the Fig 5-4, EI_1 is the stiffness of uncracked part, EI_2 is the stiffness of cracked part which is not fixed and $M_{q,end}$ is the moment at both ends.

Considering the first part which is the dead load on the simple support beam, by using symmetry, the stiffness curve, the moment curve and the curvature curve can be easily calculated as shown in the Fig 5-5:



Fig 5-5 Stiffness, moment and curvature of first part

From the second graph in the Fig 5-5, the stiffness in the cracked section can be calculated which is specified in chapter 3 and 4.

By applying the equation (3-4), (3-10), (3-17), and (3-14), the compression zone height of the cracked section can be drawn. In this case the compression zone height will be calculated as below,

$$x = \frac{-A_{s}E_{s} + \sqrt{A_{s}E_{s}(A_{s}E_{s} + 2E_{c}bd)}}{E_{c}b}$$
(5-2)

After obtained the compression zone height, the stiffness on the cracked section will be calculated by using equation (4-3) as below

$$(EI)_{s,cr} = E_c I^{II} = E_c \cdot (\frac{bx^3}{12} + bx \cdot (\frac{x}{2})^2 + \alpha_e A_s (d-x)^2)$$
(5-3)

By using the knowledge in chapter 4, the stiffness at position Lx should be calculated as below:



$$(EI)_{x} = \frac{M_{end,DL} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{M_{end,DL} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{(EI)_{scr}} + \Delta\kappa}$$
(5-4)

Where

$$\Delta \kappa = 0.42 \kappa_{scr}$$
$$\kappa_{scr} = \frac{M_{cr}}{(EI)_{scr}}$$

The final curvature at point 1, 2, 3 can be calculated as below:

$$\kappa_1 = \frac{M_{L_2}}{EI_2} \tag{5-5}$$

$$\kappa_2 = \frac{M_{L_2}}{EI_1} \tag{5-6}$$

$$\kappa_3 = \frac{M_{mid}}{EI_1} \tag{5-7}$$

The rotation at cracked section with a certain position L_x is

$$\kappa_{x,cracked} = \frac{\frac{qLL_x}{2} - \frac{qL_x^2}{2}}{(EI)_x}$$
(5-8)

And the rotation at uncracked section with a certain position L_x is

$$\kappa_{x,uncracked} = \frac{\frac{qLL_x}{2} - \frac{qL_x^2}{2}}{(EI)_c}$$
(5-9)

The rotation at end due to dead load on the simple support beam is the uncracked part plus cracked part. So the uncracked part rotation is φ_{L1} , and the uncracked part rotation is φ_{L2} , so the total rotation at end should be

$$\varphi_{q,end} = \varphi_{L1} + \varphi_{L2} \tag{5-10}$$



Where

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$$\varphi_{L1} = \int_{L2}^{\frac{L}{2}} \frac{qLL_x}{(EI)_c} - qL_x^2 dL_x$$
(5-11)

$$\varphi_{L2} = \int_{0}^{L2} \frac{\frac{qLL_{x}}{2} - qL_{x}^{2}}{\frac{M_{end,DL} + \frac{qLL_{x}}{2} - qL_{x}^{2}}{\frac{M_{end,DL} + \frac{qLL_{x}}{2} - qL_{x}^{2}}{(EI)_{scr}} + \Delta\kappa}$$
(5-12)

Considering the second part which is the constant moment on the simple support beam which is shown in Fig 5-6, by using symmetry, for the moment on the simple support beam, the stiffness curve, the moment curve and the curvature curve are shown in Fig 5-7:



Fig 5-6 The second part which is only moment applied on the beam





Fig 5-7 Stiffness, moment and curvature of second part

From the fourth graph in Fig 5-6, the curvature at cracked and uncracked parts can be calculated as below:

$$\kappa_{M1} = \frac{M_{q,end}}{EI_1} \tag{5-13}$$

$$\kappa_{M2} = \frac{M_{q,end}}{EI_x} \tag{5-14}$$

Similar as before, the rotation at the end due to moment at this case is:

$$\varphi_{M_{q,end},end} = \varphi_{M1} + \varphi_{M2} \tag{5-15}$$

Where

$$\varphi_{M1} = \kappa_{M1} L_1$$
$$\varphi_{M2} = \int_0^{L_2} \kappa_{M2} dL_x$$

By substituting Equation (5-4), the expression of the total rotation due to moment can be written as

$$\varphi_{end,M} = \left(\frac{L_{1}}{(EI)_{c}} + \int_{0}^{L_{2}} \frac{1}{\frac{M_{end,DL} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{M_{end,DL} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{(EI)_{scr}}} dL_{x}\right)M_{q,end}$$
(5-16)

Because it is a clamped beam, there is no rotation at the end. So the rotation due to dead load plus the rotation due to constant moment should be equal to 0:

$$\varphi_{q,end} + \varphi_{M_{q,end},end} = 0 \tag{5-17}$$

Substituting (5-10) and (5-16) into (5-17), the moment at end due to dead load on a clamped beam should be

$$M_{q,end} = \frac{-\varphi_{end,DL}}{\frac{L_1}{(EI)_c} + \int_0^{L_2} \frac{1}{\frac{M_{end,DL} + \frac{qLL_x}{2} - \frac{qL_x^2}{2}}{\frac{M_{end,DL} + \frac{qLL_x}{2} - \frac{qL_x^2}{2} + \Delta\kappa}}$$
(5-18)



So the moment curve due to dead load should be shown as Fig 5-8:



Fig 5-8 Moment curve due to dead load

For the cracking length L_2 is estimated, so find the moment value at L_2 from the curve in the Fig 5-8.

$$M_{L_2} = \left(\frac{L_2}{L} - 1\right) \cdot \frac{1}{2} qL^2 + M_{q,end}$$
(5-19)

And then compare M_{L_2} with M_{cr} , if they are equal, then the estimation is correct. Otherwise, re-estimate L_2 until $M_{L_2} = M_{cr}$.

5.1.1 Temperature gradient effect.

Caused by the dead load effect, the beam has already changed to three parts, two cracked parts and one uncracked part. The cracking length due to dead load is known

as L_2 after several iterations.

The moment due to temperature gradient should be constant as the Fig 5-9. But the value should be calculated by using the rotation at end which is equal to 0. So by using the similar method in 5.1.2, the clamped beam with temperature gradient can be divided into two parts as in the Fig 5-10.

Fig 5-9 Moment due to a temperature gradient on an uncracked beam



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Fig 5-10 Transition from clamped beam to simple support beam

In order to obtain the actual moment, the cracking length L_3 also should be estimated first.

In section graph in Fig 5-10, there is a constant moment on the simple support beam. Similar with Dead load section, by using symmetry, for the moment on the simple support beam, the stiffness curve, the moment curve and the curvature curve are shown in Fig 5-11:



Fig 5-11 Stiffness, moment and curvature of beam

From the fourth graph in Fig 5-11, the curvature at each part can be calculated as below:

$$\kappa_{1} = \frac{M_{\Delta T,end}}{EI_{2}}$$

$$\kappa_{2} = \frac{M_{\Delta T,end}}{EI_{1}}$$
(5-20)
(5-21)

Similar as before, the rotation at end due to moment at this case is:

$$\varphi_{M_{\Delta T,end},end} = \varphi_1 + \varphi_2 = M_{\Delta T,end} \left(\frac{L_1}{E_c I_c} + \int_0^{L_3} \frac{1}{(EI)_x} dL_x \right)$$
(5-22)

Where

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$$(EI)_{x} = \frac{M_{end,DL} + M_{end,\Delta T} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{M_{end,DL} + M_{end,\Delta T} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{(EI)_{ser}} + \Delta\kappa}$$
(5-23)

In third graph in Fig 5-10, there is a rotation at end on the simple support beam. The value of this rotation can be directly calculated by the temperature gradient and temperature gradient coefficient as below:

$$\varphi_{rotation} = \kappa_{\Delta T} (L_1 + L_3) \tag{5-24}$$

From equation (5-24) and (5-22), the total rotation should be equal to 0 for it is a clamped beam.

$$\varphi_{M_{\Lambda T end},end} + \varphi_{rotation} = 0 \tag{5-25}$$

Substitute (5-22) and (5-24) into (5-25), the moment due to temperature gradient will be obtained:

$$M_{\Delta T} = \frac{\kappa_{\Delta T} (L_{1} + L_{3})}{\frac{L_{1}}{E_{c} I_{c}} + \int_{0}^{L_{3}} \frac{1}{\frac{M_{end,DL} + M_{end,\Delta T} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{M_{end,DL} + M_{end,\Delta T} + \frac{qLL_{x}}{2} - \frac{qL_{x}^{2}}{2}}{(EI)_{scr}} dL_{x})}$$
(5-26)

At last, plus the moment due to temperature gradient and dead load, the final moment curve will be obtained. Then the moment at position L_3 should be calculated and compare with cracking moment M_{cr} . If they are same, that means the estimated cracking length L_3 is correct. If not same, the cracking length L_3 should be estimated again until they are same.



5.2 Example:

There is an example which will be used by Tension Stiffening Law to calculate the exact moment distribution shown in Fig 5-12. This is a clamped beam with dead load and a constant temperature gradient. All the parameters of the beam and load are shown as below.



Fig 5-12 Clamped beam with dead load and temperature gradient

The basic parameters:

$$E_{c} = 30000 N / mm^{2}$$

$$E_{s} = 2 \cdot 10^{5} N / mm^{2}$$

$$n = \frac{E_{s}}{E_{c}} = 12.5$$

$$\sigma_{cr} = 3N / mm^{2}$$

$$L = 10000mm \quad h = 605mm \quad d = 550mm \quad c_{u} = 55mm$$

$$A_{c,eff} = bd = 550 \times 400 = 220000mm^{2}$$

$$A_{s} = 4000mm^{2} \qquad \eta_{s} = \frac{4000}{220000} = 0.0182$$

$$q = 40N / mm \quad \Delta T = 25^{\circ}C \qquad \alpha = 10^{-5} / {^{\circ}C}$$

By using the calculation method in 5.1, the final moment distribution will be obtained. See workings in Appendix 2. The final moment distribution is shown in the Fig 5-13 as below. In this example, the cracking moment is equal to $2 \times 10^8 Nmm$. The moment at end and middle span are $4.387 \times 10^8 Nmm$, $0.6135 \times 10^8 Nmm$ respectively. The cracking length is 1400mm at both end, and there is no cracking at midspan.





Fig 5-13 The moment distribution on the beam [Nmm]

In order to compare the moment distribution with other condition, the moment distribution under following condition will be also calculated and compared.

A. Calculation with the uncracked stiffness in the whole span.



Fig 5-14 Moment distribution with uncracked stiffness B. Calculation with the 1/3 uncracked stiffness in whole span. [Nmm]

4.066*10^8 Mor 169mm 4.066*10^8 4.066*10^8 0.9333*10^8

Fig 5-16 Moment distribution with 1/3 uncracked stiffness whole span [Nmm]



CHAPTER 6

CALCULATION INCLUDING NORMAL

FORCE



Actually, there are many actions on a structure in real projects. Due to many complicated factors, moment and normal stress are the direct effects on the structure. All the loading actions are transformed into effects on the structure. If the complicated system can be easily analysis. In Chapter 5, the clamped beam under both temperature gradient and external loading is discussed. These actions cause moments only, whereas a normal stress will be caused by a normal force. After having applied the normal force, the reinforcement ratio and compression zone height will be changed. Also the moment distribution and stiffness will be changed as the reinforcement ratio and compression zone height changed. So if there is a normal force on the clamped beam, what will be changed? Will one third of the uncracked stiffness still be suited for designing the reinforcement correctly?



Fig 6-1 Clamped beam under temperature gradient, uniformly distributed load and normal force

In this chapter, the clamped beam will be also used, and the normal force is considered too, which is shown in Fig 6-1. With the similar method of chapter 5, the final state of the moment curve and mean stiffness will be obtained. The specific procedure is shown as Fig 6-2 as shown below.





There is another matter to which attention should be paid attention that the source of the



normal force. Usually the normal force is a mechanic load, but there is another important part which might be ignored easily. That is the temperature difference between construction temperature and operating temperature. Due to the difference between two states, some shrinkage or expansion will occur. It also will produce a normal stress. Thus in this chapter, the normal force's source will be considered as both mechanic load and temperature difference.

6.1 Analysis procedure

The procedure is similar to the one used in chapter 5, though a little more complicated since including the normal force implies that there is one more procedure in the calculation of the compression zone height.

In order to make the calculation procedure much easier, both dead load effect and temperature gradient effect will be considered together in this chapter which is different from that in chapter 5. In this way, the moment of each individual effect is not read directly, but is the result that can be calculated in one formula. The procedure is described below.

6.1.1 Determine the compression zone height

If the reinforcement is unknown, the reinforcement ratio should be determined by cross-section equilibrium.But the compression also is unknown, so the reinforcement ratio and compression zone height need to be estimated first. In the estimated process, one empirical formula is given as below which is close to the correct value.

$$A_{s} = \frac{M_{\max}}{0.9 \cdot \sigma_{s} \cdot d} + \frac{N_{T} \cdot 0.39 \cdot h}{0.9 \cdot d \cdot \sigma_{s}}$$
(6-1)

In this equation, the value of $M_{\rm max}$ contain both moment due to external load and temperature gradient. The moment due to temperature gradient can be calculated by using one third of the uncracked stiffness first which is an estimated value but close to the accurate value.

After estimated the compression zone height and reinforcement, the following two equations should be checked. If they are not equal, the compression zone height and reinforcement should be estimated iterative again until equation (6-2) and (6-3) are tenable.

$$\frac{\frac{x}{2d}(\frac{h}{2} - \frac{x}{2}) + \alpha_e \xi(\frac{d - x}{x})(d - \frac{h}{2})}{\alpha_e \xi(\frac{d - x}{x}) - \frac{x}{2d}} = \frac{M_{\text{max}}}{N_T}$$
(6-2)



$$A_{s} = \frac{M_{\max} + N_{T}(\frac{h}{2} - \frac{x}{3})}{\sigma_{s}(d - \frac{x}{3})}$$
(6-3)

When equation (6-2) and (6-3) are tenable, the required reinforcement A_s and compression zone height x are obtained. The two values will be used in the future calculation.

With the value of compression zone height and reinforcement ratio, the stiffness at cracked cross-section will be obtained by equation (6-4) as below.

$$EI_{scr} = \left(E_c\left(\frac{bx^3}{12} + bx(\frac{x}{2})^2 + \frac{E_s}{E_c}A_s(d-x)^2\right)\right)$$
(6-4)

6.1.2 End rotation calculation

The clamped beam will be divided into 3 parts which is shown in Fig 6-3: simple support beam with moment at ends; simple support beam with a rotation at end; simple support beam with uniform distributed load on the beam. The length of cracked area will be estimated. The length of end cracked area is estimated as L_2 , also the length of middle cracked area is estimated as L_3 . These two parameters should be given the estimated values which will be used in the future calculation. At the end of the calculation, these will be checked and iterative.



Fig 6-3 Transition from clamped beam to simple support beam

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After estimated the length of cracked zone, the mean stiffness can be obtained by using equation (6-4) by tension stiffening law. According to section 4.5, the stiffness

at position L_x will be obtained as below.

$$EI_{x} = \frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{EI_{scr}} + \Delta\kappa}$$
(6-5)

Before calculation, the moment at end M_{end} should be estimated too. Using this estimated moment in the calculation of rotation at end.

For the uniformly distributed load on the beam, with the similar method in chapter 5, by using symmetry system the stiffness, moment and curvature distribution will be shown in Fig 6-4 as below.



Fig 6-4 Stiffness, moment and curvature distribution

From Fig 6-4, the curvature at section 1, 2 and 3 can be obtained with the stiffness and moment.

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$$\kappa_{x} = \frac{\frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{EI_{x}}$$
(6-6)

Then the rotation at end due to the uniformly distributed load will be obtained by equation (6-6), which is shown as below:

-

Total rotation of section 2:

$$\phi_{L2,DL} = \int_{0}^{L_{2,estimated}} \frac{\frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}} dL_{x}$$
(6-7)
$$\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}} + \Delta\kappa$$

Total rotation of section 1:

$$\phi_{L1,DL} = \int_{L_{2,estimated}}^{L_{1}+L_{2,estimated}} \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2} dL_{x}$$
(6-8)

Total rotation of section 3:

$$\phi_{L1,DL} = \int_{L_1+L_{2,estimated}}^{\frac{L_{total}}{2}} \frac{\frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}} dL_x$$
(6-9)
$$\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2} + \Delta\kappa}$$

The total rotation at end of the whole beam:

$$\phi_{end,DL} = \phi_{L1,DL} + \phi_{L2,DL} + \phi_{L3,DL}$$
(6-10)

For the second part which is a simply supported beam with a moment at its ends, a similar method can be used to obtain the rotation at the end. The stiffness, moment and curvature distribution are illustrated in Fig 6-5 below.





Fig 6-4 Stiffness, moment and curvature distribution

The total rotation at end in this situation is

$$\phi_{end,M} = -M_{end} \left(\int_{0}^{L_{2,estimated}} \frac{1}{\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{EI_{scr}}} dL_x + \frac{L_{1}}{\frac{L_{1}}{E_c I_c}} + \int_{L_{1}+L_{2,estimated}}^{\frac{L_{total}}{2}} \frac{1}{\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{EI_{scr}}} dL_x \right)$$

$$(6-11)$$

$$\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{\frac{-M_{end} + \frac{qL_{total}L_x}{2} - \frac{qL_x^2}{2}}{EI_{scr}}} dL_x)$$

For the third part which is only a rotation at the end of the simply supported beam, the rotation can be easily calculated by the temperature gradient.

$$\phi_{end,\theta} = \kappa_{\Delta T} \, \frac{L_{total}}{2} \tag{6-12}$$



6.1.3 End moment and moment due to temperature gradient

In order to obtain the moment at end, the boundary condition of the beam is needed. Because it is clamped beam, there is no rotation at end. The sum of rotation at end of all three parts should be equal to zero which is shown in equation (6-13). If it is simple support beam or one end simple support and the other end clamped, the boundary condition will be changed.

$$\phi_{end.DL} + \phi_{end,M} + \phi_{end,\theta} = 0 \tag{6-13}$$

By using the boundary condition, the relationship of moment and other parameters will be obtained. Substitute equation (6-10), (6-11) and (6-12) into (6-13), the equation of moment at end will be obtained as below.

$$M_{end} = \frac{-\kappa_{\Lambda T} \frac{L_{total}}{2} - \int_{0}^{L_{2,columed}} \frac{\frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-\frac{M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}}{-\frac{M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-\frac{M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{-\frac{M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}}{-\frac{M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}}{-\frac{M_$$

This end moment should be compared with estimated end moment. These two values should be equal, so that the estimation is correct. If not, the end moment should be estimated again, until the two values are equal. In order to check the estimated value, the moment at position L_2 and L_3 should be calculated as equation (6-15) and (6-16) as below.

$$M_{L2} = -M_{end} - \frac{qL_{total}L_{2,estimated}}{2} + \frac{qL_{2,estimated}}{2}$$
(6-15)

$$M_{L3} = -M_{end} - \frac{qL_{total}(L_{2,estimated} + L_1)}{2} + \frac{q(L_{2,estimated} + L_1)^2}{2}$$
(6-16)

If in the estimated stage, one of L_2 and L_3 is equal to 0, which means the cracks is occurred at ends or middle span. So the moment related to the edge of cracking zone should be equal to cracking moment. The moment of L_2 and L_3 which is not equal to 0 should be equal to M_{cr} . If not, the cracking length should be


estimated again, until $M_{L2(L3)} = M_{cr}$.

If the cracking occurred at both ends and middle span, the estimation of L_2 and L_3 are both not equal to 0. Now the moment at L_2 and L_3 should be both equal to cracking moment M_{cr} . If not, estimating both values again, until the moment at L_2 and L_3 are equal to M_{cr} .

After three parameters' iteration, the end moment and cracking zone position will be determined. The moment curve will be obtained by using these. By using the similar method, the moment due to temperature gradient will be obtained as below. In this calculation, only second and third parts of Fig 6-3 are used. Because the temperature gradient on clamped beam can be divided into a rotation at end on simple support beam and a moment at end on simple support beam.

$$M_{\Delta T} = \frac{-\kappa_{\Delta T} \frac{L_{total}}{2}}{\int_{0}^{L_{2,estimated}}} \frac{1}{\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{EI_{scr}}} dL_{x} + \frac{L_{1}}{E_{c}I_{c}} + \int_{L_{1}+L_{2,estimated}}^{\frac{L_{total}}{2}} \frac{1}{\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{\frac{-M_{end} + \frac{qL_{total}L_{x}}{2} - \frac{qL_{x}^{2}}{2}}{EI_{scr}}} dL_{x}$$

$$(6-17)$$

6.2 Design stiffness and mean stiffness

In the structural modeling process, there are many influential factors, such as: shrinkage, expansion or some external loading. For the influence of restrained imposed deformation, there is an interaction between the forces generated and the stiffness of the structure, which is influenced by the cracking behavior: the more the stiffness is reduced by cracking, the lower the forces. It is difficult to make a design in which all influencing factors are taken into account. So, when structural modeling imposed deformations, engineers often reduce the uncracked stiffness when modeling the structure and designing the reinforcement. The question arises which reduction factor to use. In practice, Young's modulus is often reduced to 1/3 of its original value.

Actually, this value is unlikely to be appropriate for all conditions. Due to the design procedure, the beam is considered as an uncracked beam which only reduced the stiffness to 1/3. But in practice, the beam's geometry is not linear. Due to the nonlinear of geometry, the moment curve due to uniformly distributed load will be changed. If there is some



cracking at ends, the stiffness at end will be reduced and the stiffness of middle span has no changes. The system looks like a beam with two plastic hinges at end which is shown in Fig 6-5.



Fig 6-5 Stiffness distribution on cracked beam

So the moment distribution due to a uniformly distributed load will alter. A constant moment change related to the geometry of the cracked zone should be added to the original moment curve. For example, if there are some cracks at the both ends of clamped beam, the moment curve will move down relative to the original moment curve in Fig 6-6.

As the cracking length increase, the changed moment value M_x will increase. But after the cracking length reach 1/4 of the total length, the changed moment value will start to decrease. When the whole beam is cracked, the changed value will be again reduced to 0.



Fig 6-6 Moment translational movement with end cracked

Oppositely, if the cracking is occurred at middle span, the stiffness of the middle span zone will reduce. So the stiffness at ends will be higher than middle span. For this



situation, the clamped beam will be looked like two cantilever beams. Then the moment at end will increase, and the moment and middle span will decrease due to the lower stiffness. The moment curve trend has no changes, so there will be an upward translational movement of the moment curve compared with the original uniformly distributed loading moment. Similar with cracking at ends, after the cracking length is more than half total length, the translational movement of the moment will stop increase but start to decrease. When the total beam is cracked, the moment curve will be changed to the original geometry which is the moment distribution on uncracked beam.



Fig 6-7 Moment translational movement with middle span cracked

From Fig 6-6 and Fig 6-7, it is seen clearly that in the final state the moment is not only combined with the original uniformly distributed loading moment and the temperature gradient moment, but also with a moment decrease or increase due to the nonlinear properties of the beam.

$$M_{end} \neq M_{\Delta T} + \frac{1}{12} q L_{total}^2$$
$$M_{end} = M_{\Delta T} + \frac{1}{12} q L_{total}^2 + M_x \qquad (6-18)$$

So two definitions of stiffness are given as below:

Mean stiffness: Average stiffness including all uncracked and cracked zone. It is the actual



mean stiffness of the whole beam. This value directly influences the moment due to restrained imposed deformation.

$$(EI)_{actual} = \frac{M_{\Delta T}}{\kappa_{\Delta T}}$$
(6-19)

Design stiffness: The stiffness difference between the original uniformly distributed loading moment and the moment at final stage. Both the moment due to temperature gradient and the moment changes due to the nonlinear geometry of beam are considered. This value is not actual stiffness. It can be used in the design procedure where engineers need moment estimation due to temperature gradient but with uncracked beam estimation. This value can be directly compared with 1/3 uncracked stiffness which is used in actual projects. In the future of the report, this value will be considered as mean stiffness.

$$(EI)_{design(mean)} = \frac{M_{\Delta T} + M_x}{\kappa_{\Delta T}}$$
(6-19)

After obtained the moment due to temperature gradient in equation (6-17) and (6-19), the actual stiffness of the beam can be easily calculated with the curvature of temperature gradient. For the mean stiffness, the value $M_{\Delta T} + M_x$ can be obtained by the equation (6-20). Then the mean stiffness also can be easily calculated.

$$M_{\Delta T} + M_{x} = M_{end} - \frac{1}{12} q L_{total}^{2}$$
(6-20)

The value in equation (6-19) can be instead of 1/3 uncracked stiffness in the structural modeling and the reinforcement design procedure. This value is more accurate than 1/3 uncracked stiffness. By using the 1/3 uncracked stiffness, there might always be some unexpected cracking or too much reinforcement might be used. If the stiffness in equation (6-19) is used, one can avoid these problems.

6.3 Comparison of mean stiffness with different situations

In order to find the difference between mean stiffness and 1/3 uncracked stiffness, and the influence factors of mean stiffness, different situation will be discussed and compared in this chapter. The trend of mean stiffness with different value of different parameters is very important for engineers to understand this topic. It can help engineer to improve their design.

6.3.1 Comparison with different temperature gradient

Different temperature gradient may cause different magnitude and direction moment. The moment due to temperature gradient depend on the actual stiffness of beam and the magnitude of temperature difference. In this section, comparisons



of different temperature gradient under 3 different situations are discussed: no normal force, tensile normal force and compressive normal force. All the calculation sheets and moment curves are shown in Appendix 4.

When there is no normal force on the beam, and the temperature gradient and related mean stiffness are shown as Table 6-1 as below.

T (° C)	EIm(N=0)(*10 ¹ 3Nmm ²)
-80	8.289
-70	8.116
-60	7.924
-50	7.862
-40	8.041
-30	8.938
20	7.343
30	9.229
40	10.017
50	10.84
60	11.11
70	11.31
80	11.27
90	11.02

Table 6-1 Temperature gradient and related mean stiffness with no normal force

So the relationship between temperature gradient and mean stiffness can be illustrated by the curve in Fig 6-8.







T (° C)	EIm(N=400) (*10 ¹³ Nmm ²)
-80	9. 902
-70	9.916
-60	9.919
-50	9.912
-40	10.2
-30	10.79
-25	11.45
15	9.458
20	10.84
30	12.23
40	12.8
50	13.04
60	13.03
70	12.88
75	12.62
80	12.46

When the normal force on the beam is positive, which is a tensile normal force with the value equal to 400kN, and the temperature gradient and related mean stiffness are shown in Table 6-2 below.

Table 6-2 Temperature gradient and related mean stiffness with 400kN tensile normal force

And the relationship between temperature gradient and mean stiffness can be illustrated by the curve in Fig 6-9.



Fig 6-9 Relationship between temperature gradient and mean stiffness with 400kN tensile normal force

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When the normal force on the beam is negative, which is a compressive normal
force with the value equal to 400kN, and the temperature gradient and related
mean stiffness are shown in Table 6-3 below.

T (° C)	EIm(N=-400) (*10 ¹ 3Nmm ²)
-80	6.945
-70	6.709
-60	6. 469
-50	6. 423
-40	6.828
-30	8.104
-25	9. 325
30	6. 562
40	8.047
50	8.887
60	9.448
70	9.812
80	10.02
90	10.08
100	9.919

Table 6-3 Temperature gradient and related mean stiffness with 400kN compressive normal force

And the relationship between temperature gradient and mean stiffness can be illustrated by the curve in Fig 6-10.



Fig 6-10 Relationship between temperature gradient and mean stiffness with 400kN compressive normal force



From Fig 6-8 to Fig 6-10, the moment curve due to temperature gradient is similar as double parabola curve. When the temperature gradient is negative, which means upper temperature is larger than lower temperature, the mean stiffness will increase as the value of temperature gradient decrease. Oppositely, when the temperature gradient is positive, which means upper temperature is less than lower temperature, the mean stiffness will increase as the value of temperature as the value of temperature gradient is positive, which means upper temperature is less than lower temperature, the mean stiffness will increase as the value of temperature gradient is positive.

Fig 6-8 to 6-10 are combined in one graph which is shown in Fig 6-11, it is clearly seen that the value of mean stiffness compared with 1/3 uncracked stiffness. Under the condition with positive temperature gradient and tensile normal force, the mean stiffness is larger than 1/3 uncracked stiffness, which means the actual moment due to temperature gradient will be larger than the estimated moment due to temperature gradient will be larger than the estimated by estimated temperature gradient moment under 1/3 uncracked stiffness is not enough to carry the actual moment, the cracks might be happened then. It is dangerous when there is both positive temperature gradient and tensile normal force existed. Obviously in this condition the 1/3 uncracked stiffness is not suitable to be used to design the reinforcement. More accurate design stiffness should be calculated by using equation (6-19).



Fig 6-11 Relationship between temperature gradient and mean stiffness

On the other hand, when there is no normal force or compressive normal force with negative temperature gradient, from Fig 6-11, the mean stiffness is much less than 1/3 uncracked stiffness. Under this circumstance, the structure designed by 1/3 uncracked stiffness is over safe. So less reinforcement can be used compared with the reinforcement designed by 1/3 uncracked stiffness. From Table 6-4 as below, whether using 1/3 uncracked stiffness to estimate moment due to temperature gradient is safe will be illustrated.



	High negative temperature gradient	Low negative temperature gradient	Low positive temperatur e gradient	High positive temperature gradient
Tensile normal force	Safe	Safe	Safe or dangerous	Dangerous
No normal force	Over safe	Safe	Over safe	Safe
Compressive normal force	Over safe	Safe	Over safe	Over safe

Table 6-4 Different combination of temperature gradient and normal force

6.3.2 Comparison with different normal force

Different normal force will influence the reinforcement ratio of the structure and the compression zone height of cross-section. These factors will direct or indirect influence the mean stiffness of the structure. Different normal force under different temperature gradient will be compared in this section. Due to the curve of the relationship between normal force and mean stiffness looks like linear line, the regression of the curve will be obtained. All the calculation sheets and moment curves are shown in CD-ROM.

When the temperature gradient is negative with the value of -40 degree, which means the upper temperature is higher than lower temperature of the structure, the normal force and the related mean stiffness are shown as Table 6-5 as below.

N (kN)	EIm(T=-40) (*10 ¹³ Nmm ²)
-800	5.953
-600	6. 328
-400	6.828
-200	7.353
0	8.041
200	8.95
400	10.02
600	10.73
800	11.65

Table 6-5 Normal force and related mean stiffness with -40° C temperature gradient

From Table 6-5, the relationship between normal force and mean stiffness can be illustrated as curve as Fig 6-12.



Fig 6-12 Relationship between normal force and mean stiffness with a -40° C temperature gradient

By using software Origin, the curve and equation can be regressed as Fig 6-13 and equation (6-21).





$$(EI)_m = 8.42811 + 0.00366N \tag{6-21}$$

When the temperature gradient is negative with the value of -60 degree, which means the upper temperature is higher than lower temperature of the structure, the normal force and the related mean stiffness are shown as Table 6-6 as below.

N (kN)	EIm(T=-60) (*10 ¹ 3Nmm ²)
-800	5. 219
-600	5.844
-400	6.469
-200	7.135
0	7.924
200	8.802
400	9.919
600	11. 12
800	12.64

Table 6-6 Normal force and related mean stiffness with -60° C temperature gradient

From Table 6-6, the relationship between normal force and mean stiffness can be illustrated as curve as Fig 6-14.



Fig 6-14 Relationship between normal force and mean stiffness with a -60° C temperature gradient

By using software Origin, the curve and equation can be regressed as Fig 6-15 and equation (6-22).



Fig 6-15 Regression of the mean stiffness-normal force curve

 $(EI)_m = 8.34133 + 0.00451N \tag{6-22}$

When the temperature gradient is negative with the value of 40 degree, which means the upper temperature is less than lower temperature of the structure, the normal force and the related mean stiffness are shown as Table 6-7 as below.

N (kN)	EIm(T=+40) (*10 ¹ 3Nmm ²)
-800	5. 797
-600	6.922
-400	8.047
-200	9.047
0	10.017
200	11.42
400	12.8
600	14.3
800	15.92

Table 6-7 Normal force and related mean stiffness with 40° C temperature gradient

From Table 6-7, the relationship between normal force and mean stiffness can be illustrated as curve as Fig 6-16.



Fig 6-16 Relationship between normal force and mean stiffness with a 40° C temperature gradient

By using software Origin, the curve and equation can be regressed as Fig 6-17 and equation (6-23).



Fig 6-17 Regression of the mean stiffness-normal force curve

$$(EI)_m = 10.47444 + 0.00621N \tag{6-23}$$

From Fig 6-12, Fig 6-14 and Fig 6-16, the trend is clearly seen that the mean stiffness will increase as the normal force increase. From equation (6-21) to (6-23), the slope and constant value (when normal force is 0) will increase as the positive temperature gradient increase and negative temperature gradient decrease which is

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announced similar in previous section. So as the same result with previous section, it is dangerous for engineers to use 1/3 uncracked stiffness to estimate the moment due to temperature gradient when positive temperature gradient and tensile normal force are happened together.



Fig 6-18 Relationship between normal force and mean stiffness

6.3.3 Comparison with different q load

If the uniformly distributed load is different, the cracking pattern and length will be different too. In this case, the uniformly distribute load is the only mechanic load which directly influence the moment value. So the cracking position of the clamped beam will be influence by the uniformly distributed load. If the q load is not big enough, the cracks will always at ends or middle span which is also depend on the temperature gradient. But if the q load is big enough and the temperature gradient is suitable, there will be cracking at both ends and middle span. What is happened in this situation? Is there any difference with only one part cracking?

So in this section, in order to make all cracking phenomenon happened, different q load will be compared under different temperature gradient. In this case there is no normal force added, so mostly the obtained mean stiffness is less than 1/3 uncracked stiffness. The main point of this section is to find out the difference of mean stiffness with different cracking pattern. All the calculation sheets and moment curves are shown in CD-ROM.

When the temperature gradient is negative with the value of -40 degree, which means the upper temperature is higher than lower temperature of the structure, the uniformly distributed load and the related mean stiffness are shown as Table 6-8 as below.



q (N/mm)	EIm(T=-40) (*10 ¹³ Nmm ²)	Cracking length(mm)
100	11.13	218*2 mid
150	9. 306	325*2 mid
211.5	8.041	420*2 mid
250	7.406	468*2 mid 7*2 end
300	7. 563	570*2 mid 88*2 end
350	7.969	635*2 mid 147*2 end
400	8.5	680*2 mid 191*2 end

Table 6-8 Uniformly distributed load with related mean stiffness and cracking length with -40° C temperature gradient

From Table 6-8, the relationship between uniformly distributed load and mean stiffness can be illustrated as curve as Fig 6-19



Fig 6-19 Relationship between uniformly distributed load and mean stiffness

When the temperature gradient is negative with the value of -60 degree, which means the upper temperature is higher than lower temperature of the structure, the uniformly distributed load and the related mean stiffness are shown as Table 6-9 as below.

q (N/mm)	EIm(T=-60) (*10 ¹³ Nmm ²)	Cracking length (mm)
100	11.25	455*2 mid
150	10.44	500*2 mid
211.5	10.017	685*2 mid
250	10.19	716*2 mid
300	10. 38	747*2 mid 7*2 end
350	10.56	791*2 mid 67*2 end
400	10.87	820*2 mid 113*2 end

Table 6-9 Uniformly distributed load with related mean stiffness and cracking length with -60° C temperature gradient





From Table 6-9, the relationship between uniformly distributed load and mean stiffness can be illustrated as curve as Fig 6-20.



Fig 6-20 Relationship between uniformly distributed load and mean stiffness

When the temperature gradient is negative with the value of 40 degree, which means the upper temperature is higher than lower temperature of the structure, the uniformly distributed load and the related mean stiffness are shown as Table 6-10 as below.

q (N/mm)	EIm(T=40) (*10 ¹ 3Nmm ²)	Cracking length (mm)
100	8. 167	298*2 end
150	7.983	363*2 end
211.5	7.924	424*2 end
250	7.958	453*2 end
300	8.083	484*2 end
350	8.542	511*2 end
400	9.167	532*2 end

Table 6-10 Uniformly distributed load with related mean stiffness and cracking length with 40° C temperature gradient

From Table 6-10, the relationship between uniformly distributed load and mean stiffness can be illustrated as curve as Fig 6-21.



Fig 6-21 Relationship between uniformly distributed load and mean stiffness

Fig 6-22 is the combination of Fig 6-19 to 6-21. In Fig 6-22, when temperature gradient is positive in this case, there is only cracking at end, so the mean stiffness will increase as the q load increase. If the temperature gradient is negative in this case, there will be cracks at both ends and middle span when the value of temperature gradient is large enough. So as shown in Fig 6-22, the curve with negative temperature gradient has a transition point when the ends start to crack. When the cracking is only occurred at middle span, the mean stiffness decrease as the q load increase, but after the cracking is occurred at middle span and ends, the mean stiffness start to increase as the q load increase.



Fig 6-22 Relationship between q load and mean stiffness

CHAPTER 7

BIO-DIESEL PROJECT



The Bio-diesel project is an oil industry project. A bracing structure is designed to support the oil tanks on it. Around the slab there is a wall to keep the oil from tanks inside in case of an emergency. As shown in the Fig 7-1 which is a corner part of Bio-diesel project, the oil tank is direct on the slab. In this tank, there is some high temperature liquid oil. For the oil tank is made of steel, so the high temperature will directly have impact at the top of the slab.



Fig 7-1 A corner of Bio-diesel project

7.1 Project analysis

Due to the structure and condition of the project, there are some load actions on it: dead load of slab, dead load of tank, storage loading of oil in the tank, temperature gradient due to higher temperature at upper of slab and a temperature difference between construction period and using period. Due to these load actions, the corresponding load effects are: uniformly distributed load, normal force and temperature gradient.

The normal force on the slab is caused by the temperature difference between construction period and operating period. So a definition will be given of the reference temperature. A normal force will be caused by temperature difference at a specific time or within a specific environment; a standard should be made to distinguish the normal force value caused by temperature difference. So the temperature in the construction period will be considered as reference temperature. During the operating period, the gap between

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reference temperature and operating temperature will cause a normal force which is shown in the Fig 7-2 as below.



Fig 7-2 Reference temperature and normal force caused by it

So in this case the axial loading effect is not only caused by a mechanic normal force, but also by a temperature difference.

Even with a proper design of the reinforcement in the Bio-diesel project, there might still be some unexpected cracking in the slab. Engineers ofter assume that this phenomenon has no problem for the whole design procedure. The estimated moment due to temperature gradient in design process is based on1/3 of the Young modulus. From Chapter 6 it is clearly seen that the reinforcement is underestimated when tensile normal force and positive temperature gradient has been applied. When structural modeling imposed deformations, engineers often reduce the uncracked stiffness to 1/3 of uncracked stiffness when modeling the structure and designing the reinforcement. This is the question why the reinforcement might not be enough to carry the load in fact.

More accurate design is needed in this project to prevent unexpected cracking occurring. The method mentioned in chapter 6 can be used to make a more exact estimation of reduced Young's modulus. Then a superior reinforcement design can be made by the new Young's modulus.

7.2 Redesign of the project

In the original design of Bio-diesel project, some estimations are used to analyse: no cracking of the slab, Young's modulus is reduced to 1/3 of its original value due to many influenced factors. Under this estimation, a reinforcement ratio can be calculated. Actually, this reinforcement is not proper to every condition. A better design of reinforcement is needed to prevent unexpected cracking or reduce the reinforcement amount. As to this case, the model is a slab. In order to simplify the calculation, the most



unfavorable part of the slab will be recognized as a clamped beam to analysis.

From the stress distribution as a result of a Scia Engineering, the most unfavorable part is the inner part of the edge of the tank which is shown in Fig 7-3. All the parameters at that part will be used in the clamped beam calculation. The calculation procedure is shown in Appendix 5.



Fig 7-3 the stress distribution by Scia Engineering

So the calculation result is the mean stiffness in design should be used as $1.02 \times 10^{14} \,\mathrm{N} \cdot \mathrm{mm}^2$, and where the 1/3 of uncracked stiffness is $1.076 \times 10^{14} \,\mathrm{N} \cdot \mathrm{mm}^2$.



1/3 uncracked stiffnessMean design stiffnessFig 7-4 Comparison of stress distribution with different design stiffness

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When compared with the stress distributions due to the two design stiffnesses in the Fig 7-4, the difference between both is not too wide. The reason is that the temperature gradient is negative in this case and the tensile force is not very strong. So the actual design mean stiffness should not be larger than 1/3 of the uncracked stiffness.

If the normal force is changed, what's happened with the same reinforcement? So a normal force equal to 600kN, 400kN, 200kN, -200kN, -400kN and -600kN was also calculated, and the result is shown in Fig 7-5. The calculation procedures are shown in Appendix 6.



Fig 7-5 comparison with different normal force

From the Fig 7-5, the higher the tensile normal force added, the higher the mean design stiffness that should be used. If the normal force is higher than 600kN in this case, 1/3 of uncracked stiffness will be a considerable underestimated of actual stiffness.

On the other hand, if the temperature gradient is changed from negative to positive, what will happen then? The calculation working is included in the CD-files. After calculation, the mean design stiffness will change from $1.02 \times 10^{14} \,\mathrm{N \cdot mm^2}$ to $1.28 \times 10^{14} \,\mathrm{N \cdot mm^2}$ as the temperature gradient is changed from -40 degree to +40 degree.

So the stress distribution will be changed. The stress distribution under two mean design





stiffnesses is shown in the Fig 7-6 as below.



It is clearly seen that the stress is much higher by using mean design stiffness than using 1/3 of uncracked stiffness. So in order to make a better design of Bio-diesel project, the mean design stiffness is suggested to be instead of 1/3 of uncracked stiffness in reinforcement design.



CHAPTER 8

PROGRAM FOR OBTAINING MEAN

STIFFNESS



In order to make the calculation simply, some easier method is needed to obtain. Due to the mean stiffness calculation procedure is too complicated, it is impossible for engineering to calculate the mean stiffness every time in each project. According to the experience, some solutions can be used:

- Calculation table: Making a calculation sheet which includes all the parameter and different situations. Engineers can directly search and find the corresponding value.
- Calculation program: A calculation program of computer can be developed. Engineers input all the parameters to the program, and the results will be obtained automatic.
- Safe reduction factor: Because 1/3 Young's modulus is dangerous to be used in the design procedure, so a safer reduction factor can be given to instead of 1/3.

From the above three solutions, each of them has positive and negative. For calculation table, the parameters of the case are a little too much which is around 8 parameters. So the table should be very complicated, even there are some appendix tables. Engineers cannot find the result in a short time. For the calculation program, it should be designed by some computer language software. So the computer language should be also good at. For the safe reduction factor, this is the simplest method. But in some other conditions, it might be too much to be used. The reinforcement ratio will be over designed; the budget also will be higher than actual.

Finally, the calculation program is chosen to be focused on. The software Matlab and C++ language will be used to design this program. Firstly, the Mathcad equations should be translated to Matlab input as Fig 8-1.



Fig 8-1 Translation from Mathcad file to Matlab file



After input all the data to the Matlab program, combined with C++ language, the program of mean stiffness calculation will be obtained. As shown in Fig 8-2, in this program all the parameters will be input in the left window. The results will be shown in the right window such as: cracked zone, moment at end and mean stiffness. All the input files and program will be included in the CD-ROM.



Fig 8-2 Program for calculation of mean stiffness



Fig 8-3 Example of calculation

This program is only under the condition of clamped beam which mean both the ends are clamped. Under other conditions such as one end clamped and one end simple support, or both ends are simple support are not developed due to time limited and my computer language is not very good. So in the future, the rest conditions will be developed by me or





someone else. These programs can help engineers to make a better design.

Fig 8-4 Condition of the program developed

CHAPTER 9

CONCLUSION AND RECOMMENDATIONS



9.1 Conclusion:

Compression zone height

- If there is no axial loading on the cross-section, the compression zone height will be constant when the cross-section is cracked. This value is due to the cross-section properties but not due to the external loading or imposed deformation.
- The compression zone height is influenced by the ratio of moment and normal force. The relationship looks like a double parabola as in Fig 3-14.

Stiffness distribution on a cracked beam

- Stiffness depends on the moment of inertia, and inertia depends on the compression zone height. So the stiffness of the cracked area is influenced by the compression zone height.
- Mean stiffness which is used to calculate the moment due to temperature is the average value of the whole beam. The stiffness distribution on the cracked beam is non-linear.
- The normal force can also be caused by the temperature difference between the construction period and using period. A negative temperature difference can result in shrinkage; a positive temperature difference can result in expansion.

Stiffness used in design procedure

- When structural modeling imposed deformations, engineers often reduce the uncracked stiffness when modeling the structure and designing the reinforcement. A new definition is given of a mean design stiffness which is to be used in design reinforcement.
- This reduction factor already considers all the influential action, such as shrinkage, expansion and creep. Usually a factor as 1/3 is used, but this value is not suitable for all the conditions. Some unexpected cracking may occur.
- The actual mean stiffness is larger than one third of the uncracked stiffness when there is a tensile axial force and a high positive?? temperature gradient. On the other hand, the design mean stiffness might also be less than one third of the uncracked stiffness.

Loading sequence

• There is no difference for the loading sequence. That means that whether external loading or restrained deformation is applied first, the results will be same at the final state.

Moment distribution influenced factors

• After cracking, the non-linear response of the member investigated will influence the bending moment distribution. As a result, the bending moment in a cross-section is not only influenced by external loading and restrained deformation, but also by the stiffness distribution over the length of the member.

9.2 Recommendations

R



Different conditions imply different reinforcement

- In the condition with positive temperature gradient and tensile normal force, more reinforcement should be used compared with the reinforcement designed with the reduction factor of Young's modulus of 1/3, and especially when the temperature gradient and normal force are high. In order to prevent some unexpected cracking, a more accurate mean stiffness should be calculated
- In the condition with a negative temperature gradient and a compressive normal force, less reinforcement can be used compared with the reinforcement designed with the reduction factor of Young's modulus to 1/3.
- In the condition with negative temperature gradient and a tensile normal force, or positive temperature gradient and compressive normal force, the mean stiffness is almost similar to 1/3 of the uncracked stiffness.

Program developed

• A program to calculate the mean stiffness on the clamped beam is developed, which can help the engineers in the design procedure.

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Appendix 1

For Noakowski's equation (23),

$$\begin{array}{ll} c_{\mathbf{u}} \coloneqq 55 & h \coloneqq 605 & b \coloneqq 400 & d \coloneqq 550 & k_{\mathbf{u}} \coloneqq \frac{c_{\mathbf{u}}}{d} & k_{\mathbf{u}} \equiv 0.1 \\ E_{s} \coloneqq 200 \cdot 10^{3} & E_{c} \coloneqq 16 \cdot 10^{3} & n \coloneqq \frac{E_{s}}{E_{c}} & n \equiv 12.5 \\ A_{s} \coloneqq 1000 & A_{scomp} \coloneqq 1000 & A_{ceff} \coloneqq b \cdot d & A_{ceff} \equiv 2.2 \times 10^{5} \\ \eta_{s} \coloneqq \frac{A_{s}}{A_{ceff}} & \eta_{s} \equiv 4.545 \times 10^{-3} & \eta_{scomp} \coloneqq \frac{A_{scomp}}{A_{ceff}} & \eta_{scomp} \equiv 4.545 \times 10^{-3} \\ A_{ceff} & n \in (\eta_{s} + \eta_{scomp}) & A \equiv 0.114 \\ B \coloneqq n \cdot (\eta_{s} + k_{\mathbf{u}} \cdot \eta_{scomp}) & B = 0.063 \\ C_{w} \coloneqq n \cdot (\eta_{s} + k_{\mathbf{u}}^{2} \cdot \eta_{scomp}) & C \equiv 0.057 \\ \eta_{x} \coloneqq 0.5 \\ Given \end{array}$$

$$\frac{k_{x}^{\ 4} + 4 \cdot A \cdot k_{x}^{\ 3} - 12 \cdot B \cdot k_{x}^{\ 2} + 12 \cdot C \cdot k_{x} + 12 \cdot A \cdot C - 12 \cdot B^{2}}{-6 \cdot k_{x}^{\ 3} - 18 \cdot A \cdot k_{x}^{\ 2} - 12 \cdot \left(A^{2} - B\right) \cdot k_{x} + 12 \cdot A \cdot B} = \eta$$

$$k_x := 0.3$$
 $x := k_x \cdot d$ $x = 165$

The compression zone height is 165mm



CHECK

$$\eta_2 := \frac{k_x^4 + 4 \cdot A \cdot k_x^3 - 12 \cdot B \cdot k_x^2 + 12 \cdot C \cdot k_x + 12 \cdot A \cdot C - 12 \cdot B^2}{-6 \cdot k_x^3 - 18 \cdot A \cdot k_x^2 - 12 \cdot (A^2 - B) \cdot k_x + 12 \cdot A \cdot B}$$

+

$$\eta_2 = -2.317$$
 $y_n := \frac{n}{2} - x$ $y_n = 137.5$

$$e_2 := \eta_2 \cdot d$$
 $e_2 = -1.275 \times 10^3$
 $k_{x1} := \frac{x}{d}$ $k_{x1} = 0.3$

$$k_{xx} := \frac{2 \cdot B + k_{x1}^{2}}{2 \cdot (A + k_{x1})} \qquad k_{x} = 0.26$$
$$\eta_{1} := \eta_{2} - \left(\frac{h}{2 \cdot d} - k_{x}\right) \qquad \eta_{1} = -2.608$$

For equation (22),

$$\begin{array}{ll} A_{s} \coloneqq 1000 & A_{scomp} \coloneqq 1000 & A_{ceff} \coloneqq b \cdot d & A_{ceff} = 2.2 \times 10^{5} \\ \\ \eta_{s} \coloneqq \frac{A_{s}}{A_{ceff}} & \eta_{s} = 4.545 \times 10^{-3} & \eta_{scomp} \coloneqq \frac{A_{scomp}}{A_{ceff}} & \eta_{scomp} = 4.545 \times 10^{-3} \\ \\ e_{x} \coloneqq 1434.4 \end{array}$$

Given

$$\frac{\mathbf{A}_{\mathbf{s}} \cdot \mathbf{E}_{\mathbf{s}} \cdot \left(\frac{\mathbf{d} - \mathbf{x}}{\mathbf{x}}\right) \cdot \left(\mathbf{d} - \frac{\mathbf{h}}{2}\right) + \frac{\mathbf{b} \cdot \mathbf{x}}{2} \cdot \mathbf{E}_{\mathbf{c}} \cdot \left(\frac{\mathbf{h}}{2} - \frac{\mathbf{x}}{3}\right) + \mathbf{A}_{\text{scomp}} \cdot \mathbf{E}_{\text{scomp}} \cdot \left(\frac{\mathbf{x} - \mathbf{c}_{\mathbf{u}}}{\mathbf{x}}\right) \cdot \left(\frac{\mathbf{h}}{2} - \mathbf{c}_{\mathbf{u}}\right)}{\left[\mathbf{A}_{\text{scomp}} \cdot \mathbf{E}_{\text{scomp}} \cdot \left(\frac{\mathbf{x} - \mathbf{c}_{\mathbf{u}}}{\mathbf{x}}\right) - \mathbf{A}_{\mathbf{s}} \cdot \mathbf{E}_{\mathbf{s}} \cdot \left(\frac{\mathbf{d} - \mathbf{x}}{\mathbf{x}}\right)\right] + \frac{\mathbf{b} \cdot \mathbf{x}}{2} \cdot \mathbf{E}_{\mathbf{c}}} - \mathbf{e}_{\mathbf{x}} = 0$$

 $Find(x) \rightarrow (164.99556716383340929 - 3.231174267785264355e-25i$

So the compression zone height is 165mm



And check

$$\begin{aligned} x &:= 165.0 \\ y_n &:= \frac{h}{2} - x \\ y_n &= 137.5 \end{aligned}$$
Given
$$\begin{aligned} \frac{A_s \cdot E_s \cdot \left(\frac{d-x}{x}\right) \cdot \left(d-\frac{h}{2}\right) + \frac{b \cdot x}{2} \cdot E_c \cdot \left(\frac{h}{2} - \frac{x}{3}\right) + A_{scomp} \cdot E_{scomp} \cdot \left(\frac{x-c_u}{x}\right) \cdot \left(\frac{h}{2} - c_u\right) \\ \hline \left[A_{scomp} \cdot E_{scomp} \cdot \left(\frac{x-c_u}{x}\right) - A_s \cdot E_s \cdot \left(\frac{d-x}{x}\right)\right] + \frac{b \cdot x}{2} \cdot E_c \end{aligned}$$
Find(e_x) $\rightarrow 1434.1438356164383562$
e₂ $= e_x + y_n$
e₂ $= 1.572 \times 10^3$
 $\eta_2 := \frac{e_2}{d}$
 $\eta_2 = 2.857$

Appendix 2

This is the calculation procedure of mean stiffness on a clamped beam.







Firstly, the dead load on the clamped beam can be divided into two parts:


$$M_{x} = M_{end.DL} + \frac{q \cdot L_{total} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}$$



Moment at simple support beam should be

$$M_{Lx} = \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}$$

$$EI_x = \frac{M_x}{\frac{M_x}{EI_{scr}} - \Delta \kappa}$$

$$EI_x = \frac{M_{end.DL} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{M_{end.DL} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{EI_{scr}} + \Delta \kappa}$$
where the as

Such as

$$M_{L2} := \frac{q \cdot L_{total} \cdot L_2}{2} - \frac{q \cdot L_2^2}{2} = 9.166 \times 10^7 \cdot N \cdot mm$$

$$EI_{L2} \coloneqq \frac{M_{end.DL} + M_{L2}}{\frac{M_{end.DL} + M_{L2}}{EI_{scr}} + \Delta\kappa} = 3.92 \times 10^{14} \cdot \text{N} \cdot \text{mm}^2$$

$$EI_{end} := \frac{M_{end.DL}}{\frac{M_{end.DL}}{EI_{scr}} + \Delta\kappa} = 3.419 \times 10^{14} \cdot N \cdot mm^2$$
$$E_c \cdot I_c = 1 \times 10^{15} \cdot N \cdot mm^2$$

Considering the first part which is dead load on the simple support beam









Curvature in uncracked section is

$$\kappa_{\text{x.uncracked}} = \frac{\frac{q \cdot L_{\text{total}} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}}{E_{\text{c}} \cdot I_{\text{c}}}$$

And curvature in cracked section is

$$\kappa_{\text{x.cracked}} = \frac{\frac{q \cdot L_{\text{total}} \cdot L_{\text{x}}}{2} - \frac{q \cdot L_{\text{x}}^2}{2}}{EI_{\text{x}}}$$

So the rotation at the end is



$$\phi_{end.DL} := \phi_{L1} + \phi_{L2} = 1.705 \times 10^{-3}$$



Then considering the second part which is the moment at end on a simple support beam



By using symmetry:



The curvature at fouth graph will be obtained as below:

$$\kappa_{M1} \coloneqq \frac{M_{qend}}{E_c \cdot I_c} \qquad \qquad \kappa_{M2} \coloneqq \frac{M_{qend}}{EI_x}$$



So the rotation at end is

 $\phi_{M1} := \kappa_{M1} \cdot L_1$

$$\phi_{M2} := \int_{0}^{L_2} \kappa_{M2} \, dL_x$$

$$\phi_{end.M} := \phi_{M1} + \phi_{M2}$$

$$\phi_{\text{end.M}} = \left(\frac{L_1}{E_c \cdot I_c} + \int_{0}^{L_2} \frac{1}{\frac{M_{\text{end.DL}} + \frac{q \cdot L_{\text{total}} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{M_{\text{end.DL}} + \frac{q \cdot L_{\text{total}} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{M_{\text{end.DL}} + \frac{q \cdot L_{\text{total}} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{1}{EI_{\text{scr}}} + \Delta \kappa}} \right) \cdot M_{\text{qend}}$$

For it is clamped beam, so

 $\phi_{end.M} + \phi_{end.DL} = 0$

So the moment at end due to dead load is

$$M_{qend} := \frac{-\Phi_{end.DL}}{\frac{L_1}{E_c \cdot I_c} + \int_{0}^{L_2} \frac{1}{\frac{M_{end.DL} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{M_{end.DL} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{M_{end.DL} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{H_{scr}}{EI_{scr}} + \Delta \kappa}} dL_x$$









2. Temperature Gradient Effect

Firstly, $\kappa_{fdc} - \kappa_{cr}$ is needed to be calculated.

So

$$\kappa_{\text{scr}} \coloneqq \frac{M_{\text{cr}}}{\text{EI}_{\text{scr}}} = 7.82 \times 10^{-4} \frac{1}{\text{m}} \qquad \qquad \Delta \kappa_{\text{scr}} \coloneqq 0.42 \cdot \kappa_{\text{scr}} = 3.284 \times 10^{-4} \frac{1}{\text{m}}$$

$$\kappa_{\text{fdc}} \coloneqq \kappa_{\text{scr}} - \Delta \kappa = 4.536 \times 10^{-4} \frac{1}{\text{m}} \qquad \qquad \frac{M_{\text{cr}}}{A_{\text{s}} \cdot \left(d - \frac{x}{3}\right)} = 114.091 \cdot \frac{N}{\text{mm}^2}$$

$$\kappa_{\text{cr}} \coloneqq \frac{M_{\text{cr}}}{E_{\text{c}} \cdot I_{\text{c}}} = 2 \times 10^{-4} \frac{1}{\text{m}}$$

$$\kappa_{\text{fdc}} - \kappa_{\text{cr}} = 2.536 \times 10^{-4} \frac{1}{\text{m}}$$

And

$$\kappa_{\Delta T} = 2.2 \times 10^{-4} \cdot \frac{1}{m} \le \kappa_{fdc} - \kappa_{cr} = 2.536 \times 10^{-4} \cdot \frac{1}{m}$$

(1) The cracking length due to temperature gradient also should be estimated

The estimated final cracking length should be

L_{3.estimated} := 1200mm

So the estimated final cracking length is 918mm

Iteration of L3:

L₃ := 1400mm



(2) Calculation of the mean stiffness

Then for this beam is a clamped beam, so the rotation at end should be equal to 0.



So the rotation at end from the secon graph should be:

$$\phi_{\mathbf{M}.\Delta \mathbf{T}} = \left(\frac{1}{\mathbf{E}_{\mathbf{c}} \cdot \mathbf{I}_{\mathbf{c}}} \cdot \mathbf{L}_{1} + \int_{0}^{\mathbf{L}_{3}} \frac{1}{\mathbf{E}\mathbf{I}_{\mathbf{x}}} \, d\mathbf{L}_{\mathbf{x}}\right) \cdot \mathbf{M}_{\Delta \mathbf{T} \mathbf{end}}$$

where

$$EI_{x} = \frac{M_{qend} + M_{\Delta Tend} + \frac{q \cdot L_{total} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}}{\frac{M_{qend} + M_{\Delta Tend} + \frac{q \cdot L_{total} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}}{EI_{scr}} + \Delta \kappa}$$





So the moment due to temperature gradient is 5.778*10*7Nmm

After iteration the moment is changed to 6.27*10^7Nmm

The moment at L3 position is

$$M_{L3} := M_{\Delta Tend} + M_{qend} + \frac{q \cdot L_{total} \cdot L_3}{2} - \frac{q \cdot L_3^2}{2} = -1.979 \times 10^8 \cdot N \cdot mm$$
$$M_{cr} = 2 \times 10^8 \cdot N \cdot mm$$
$$L_3 = 1.4 \times 10^3 \cdot mm$$

So the final cracking length :

$$L_3 = 1.4 \times 10^3 \cdot mm$$

The final moment at end:

$$M_{end} := M_{\Delta Tend} + M_{qend} = -4.387 \times 10^8 \cdot N \cdot mm$$





Appendix 3

Betondoorsnede								Betonst	erkteklass	ic	C28/35	
Breedte b =	1000	mm						$f'_{ek} =$	35 N/	/mm ²		
Hoogte h =	500	mm						$f'_{\rm b} =$	21 N/	/mm ²		
Trekzijde	Aantasti	ng door chemicaliën f _b = 1,40 N/mm ²										
Milicuklasse:	XA2	Matig agressieve chemische omgeving						$f_{bm} =$	2,74 N/	/mm ²		
Betondekking c =	50	mm	toeslag:	0	mm			Betonst	aalsoort		FeB500	
Verdeelwapening =	0	mm	c _{min} =	30	mm			$f_s =$	435 N/	/mm ²		
Drukzijde	Corrosic	ingeleid door										
Milieuklasse:	XC2	Nat, zelden d	roog									
Betondekking c =	50	mm	toeslag:	0	mm		•	••	•••	•••	••	
Verdeelwapening =	0	mm	c _{min} =	25	mm							
							•	••	• • •	••	• •	
Belastingen		explosie/stoo	ot/botsing:	nee								
M _{mp} =	164,0	kNm N _{mp} =	900	kN (trek)								
$M_d =$	210,0	$kNm N_d =$	1200	kN (trek)								
M _v =	0,0	$kNm N_v =$	0	kN								
Toegepaste wapening												
	trekzijd	c						0	lrukzijde			
	¢	h.o.h.		A _s			¢		h.o.h.		A _s	
staven laag 1	25	- 100	=	4909	mm ²		25	-	100	=	4909	mm²
staven laag 2	0	- 0	=	0	mm²		0	-	0	=	0	mm ²
			A _{stot} =	4909	mm²					$A_{stot} =$	4909	mm ²
afstan	d tussen l	laag 1 en laag 2	x =	0	mm					x =	0	mm
		nuttige h	noogte d =	438	mm					d =	63	mm
Controle wapening art. 8.	1.1	2			2							
A _{stot} =	4909	mm	A _{s;ben} =	2668	mm 2							
x_ =	61,80	mm	A _{smin} =	2668	mm ⁻	-	accoord					
			A _{s;max} =	16146	mm²							
$M_u > M_d ==>$	612,8	kNm >	210	kNm		-	accoord	1				
Controle rotatiecapaciteit	art. 8.1.3	3										
k _{x;max} =	0,535	5		224.01			n.v.t.					
d _{sgem} =	438	mm .	- X _{u;max} -	254,01	mm							
x _{spiralend;rep} =	43,70	mm	$\mathbf{x}_{u;d} =$	-2,18	mm							
.			_		_				1.	geen vo	orspanstaal	aanwezig
Controle scheurwijdte art. 8.7			w =	0,15	mm = ma	IX. too	laatbare s	cheurwije	dte	к ₁ =	1875	
Opgelegde vervo	ormingen:	nee	$\Delta \sigma =$	0,0	N/mm ²			k _{e\$} =	1,67	k ₂ =	375	
φ _{iem;nantw} =	25,0	mm	σ, =	180,1	N/mm ²			k _{e,s} =	1,29	k3 =	30000	
s _{aatw} =	100,0	mm	σ _{sr} =	0,0	N/mm ²			$m_1^{0.5} =$	1,00	k _e =	1,00	
σ _b =	3,89	N/mm ²	$f_{bm} =$	2,74	N/mm ²		\rightarrow	toetsing	volgens ar	t. 8.7.2		
toetsing volgens art 8.7.2		Volledig ont	troon									
$\phi_{im,toel} =$	10,4	x 1,67 =	17,4	mm								
s _{toel} =	78	x 1,29 =	101	mm								
$\sigma_s + \Delta \sigma_s =$	180	N/mm ² <	435	N/mm ²		-	toctsing	volgens	art 8.7.2 a	ccoord!	I	



Appendix 4

These figures are the moment and moment due to temperature gradient under different temperature gradient situations.



Fig A-1 Temperature gradient is -40





Fig A-2 Temperature gradient is -80



Fig A-3 Temperature gradient is -70





Fig A-4 Temperature gradient is -60

174.4

216.81





Fig A-5 Temperature gradient is -50



Fig A-6 Temperature gradient is -30









Fig A-8 Temperature gradient is -20





Fig A-9 Temperature gradient is -10











Fig A-11 Temperature gradient is +30





Fig A-12 Temperature gradient is +40





Fig A-13 Temperature gradient is +50



Fig A-14 Temperature gradient is +60





Fig A-15 Temperature gradient is +70





Fig A-16 Temperature gradient is +80







Fig A-18 Temperature gradient is +100



Appendix 5

This is the calculation procedure of Bio-diesel project.

General calculation of a clamped beam with cracking



$$M_{max} := 170.98 \text{kN} \cdot \text{m}$$

$$N_T := 400 kN$$

Calculation the design reinforcement and compression zone height:

$$\sigma_{s} := 200 \frac{N}{mm^{2}}$$

$$A_{s} := \frac{M_{max}}{0.9 \cdot \sigma_{s} \cdot d} + \frac{N_{T} \cdot 0.41 \cdot h}{0.9 d \cdot \sigma_{s}} = 3.123 \times 10^{3} \cdot mm^{2} \qquad \xi := \frac{A_{s}}{A_{cd}} = 6.94 \times 10^{-3}$$

Compression zone height:

x := 73mm

$$\frac{\frac{x}{2d} \cdot \left(\frac{h}{2} - \frac{x}{2}\right) + \alpha_{e} \cdot \xi \cdot \frac{(d-x)}{x} \cdot \left(d - \frac{h}{2}\right)}{\alpha_{e} \cdot \xi \cdot \left(\frac{d-x}{x}\right) - \frac{x}{2 \cdot d}} = 0.423 \,\mathrm{m}$$
$$\frac{M_{max}}{N_{T}} = 0.427 \,\mathrm{m}$$

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$$A_{s1} \coloneqq \frac{M_{max} + N_{T} \cdot \left(\frac{h}{2} - \frac{x}{3}\right)}{\sigma_{s} \cdot \left(d - \frac{x}{3}\right)} = 3.069 \times 10^{3} \cdot mm^{2}$$

For the slab and tank gravity

For the storage load

$$q_{\text{storage}} := 116 \frac{\text{kN}}{\text{m}^2} \cdot \frac{\text{L}_{\text{total}}}{2 \cdot \text{L}_{\text{total}}} = 1.74 \times 10^5 \cdot \frac{\text{N}}{\text{m}} \qquad \frac{1}{8} \cdot q_{\text{storage}} \cdot \text{L}_{\text{total}}^2 = 1.957 \times 10^5 \cdot \text{N} \cdot \text{m}$$

For the floor load

$$q_{slab} := \frac{\rho \cdot \frac{L_{total} \cdot L_{total}}{2} \cdot h}{L_{total}} = 1.875 \times 10^4 \cdot \frac{N}{m}$$

So the uniform distributely load should be

$$q := \frac{\rho \cdot \frac{L_{total} \cdot L_{total}}{2} \cdot h}{L_{total}} + q_{storage} + q_{slab} = 211.5 \cdot \frac{N}{mm} - \frac{1}{8} \cdot q \cdot L_{total}^{2} = 2.379 \times 10^{8} \cdot N \cdot mm$$

$$\Delta T := -40C$$
 $\alpha := \frac{10^{-5}}{C}$ $\kappa_{\Delta T} := \frac{\Delta T \cdot \alpha}{h} = -8 \times 10^{-4} \frac{1}{m}$

1. Estimating the cracking length.





$$\begin{split} \mathbf{M}_{cr} &\coloneqq \frac{1}{6} \cdot \mathbf{b} \cdot \mathbf{h}^2 \cdot \boldsymbol{\sigma}_{cr} = 1.25 \times 10^8 \cdot \mathbf{N} \cdot \mathbf{mm} \\ \mathbf{L}_1 &\coloneqq \frac{1}{2} \cdot \mathbf{L}_{total} - \mathbf{L}_{2.estimated} - \mathbf{L}_{3.estimated} = 923 \cdot \mathbf{mm} \end{split}$$

L1 is uncracking length, L2 and L3 are cracked length.

EI1 is uncracking stiffness, EI2 and EI3 are cracked stiffness

Cracked stiffness:

$$EI_{scr} := \left[E_{c} \cdot \left[\frac{b \cdot x^{3}}{12} + b \cdot x \cdot \left(\frac{x}{2}\right)^{2} + \frac{E_{s}}{E_{c}} \cdot A_{s} \cdot (d - x)^{2}\right]\right] = 9.28 \times 10^{13} \cdot N \cdot mm^{2}$$

Cracking aera stiffness:

$$\kappa_{\text{scr}} \coloneqq \frac{M_{\text{cr}}}{\text{EI}_{\text{scr}}} = 1.347 \times 10^{-3} \frac{1}{\text{m}}$$
$$\Delta \kappa \coloneqq 0.42 \kappa_{\text{scr}} = 5.657 \times 10^{-4} \frac{1}{\text{m}}$$



$$EI_{x} = \frac{-M_{end} + \frac{q \cdot L_{total} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}}{\frac{-M_{end} + \frac{q \cdot L_{total} \cdot L_{x}}{2} - \frac{q \cdot L_{x}^{2}}{2}}{EI_{scr}} + \Delta\kappa}$$

where Mend is estimated

$$M_{end.estimated} := -77.1 \cdot kN \cdot m = -7.71 \times 10^7 \cdot N \cdot mm$$

$$M_{mid} := \frac{1}{8} \cdot q \cdot L_{total}^2 + M_{end.estimated} = 1.608 \times 10^8 \cdot N \cdot mm$$

-

Then calculate the stiffness at L2 and L3 parts

Considering the second part which is dead load on the simple support beam







Then by using symmetry



So the rotation at end is $\frac{1}{3} \cdot E_c \cdot I_c = 1.076 \times 10^{14} \cdot N \cdot mm^2$

$$\begin{split} \varphi_{L2:DL} &:= \begin{pmatrix} L_{2:estimated} & \frac{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{-M_{end.estimated} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{-M_{end.estimated} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{El_{scr}} dL_x = 0 \\ \varphi_{L1:DL} &:= \int_{L_{2:estimated}}^{L_{1}+L_{2:estimated}} \frac{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{E_c \cdot I_c} dL_x = 3.327 \times 10^{-4} \\ \varphi_{L3:DL} &:= \int_{L_{2:estimated}}^{L_{total}} \frac{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}} dL_x = 3.327 \times 10^{-4} \\ \varphi_{L3:DL} &:= \int_{L_{1}+L_{2:estimated}}^{L_{total}} \frac{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{-M_{end.estimated} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}} dL_x = 1.65 \times 10^{-3} \\ \frac{-M_{end.estimated} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{-M_{end.estimated} + \frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x}{2} - \frac{q \cdot L_x^2}{2}}{\frac{q \cdot L_{total} \cdot L_x} - \frac{q \cdot L_x}{2}}{\frac{q \cdot L_{total} \cdot L_x} - \frac{q \cdot L_x}{2}} - \frac{q \cdot L_x} - \frac{q \cdot L_x}{2}}{\frac{q \cdot L_{total} \cdot L_x} - \frac{q \cdot L_x}{2}}}{\frac{q \cdot L_x} - \frac{q \cdot$$

 $\phi_{end.DL} := \phi_{L1.DL} + \phi_{L2.DL} + \phi_{L3.DL} = 1.982 \times 10^{-3}$

Then considering the fisrt part which is the moment at end on a simple support beam





By using symmetry:



So the rotation at end is

$$\begin{split} \varphi_{M1} &= \kappa_{M1} \cdot L_1 \\ \varphi_{M2} &= \int_0^{L_{2}, estimated} \kappa_{M2} \, dL_x \qquad \varphi_{M3} = \int_{L_1 + L_{2}, estimated}^{L_{total}} \kappa_{M3} \, dL_x \end{split}$$

$$\phi_{end.M} = \phi_{L1.M} + \phi_{L2.M} + \phi_{L3.M}$$





For the rotation part

$$\phi_{\text{end},\theta} := \kappa_{\Delta T} \cdot \frac{L_{\text{total}}}{2} = -1.2 \times 10^{-3}$$

So the total rotation should be equal to 0

$$\phi_{end,DL} + \phi_{end,M} + \phi_{end,\theta} = 0$$

Then M_{end} will be obtained.





 $M_{end.estimated} = -7.71 \times 10^7 \cdot N \cdot mm$

The moment at section L2 and L3

$$M_{L2} = -M_{end} + \frac{q \cdot L_{total} \cdot L_2}{2} - \frac{q \cdot L_2^2}{2}$$

$$M_{L3} := -M_{end} - \frac{q \cdot L_{total} \cdot (L_{2.estimated} + L_{1})}{2} + \frac{q \cdot (L_{2.estimated} + L_{1})^{2}}{2} = -1.257 \times 10^{8} \cdot N \cdot mm$$

 $M_{cr} = 1.25 \times 10^8 \cdot N \cdot mm$

Iterative part

Here $\,{\rm M}_{L3}\,\,$ and $\,{\rm M}_{L2}\,\,$ should be equal to Mcr

$$M_{cr} = \frac{1}{6} \cdot \mathbf{b} \cdot \mathbf{h}^2 \cdot \sigma_{cr}$$

And M_{end} should be equal to $M_{end.estimated}$


The results

 $M_{end} = -7.706 \times 10^{7} \cdot N \cdot mm$

 $M_{mid} = 1.608 \times 10^8 \cdot N \cdot mm$

$$\begin{split} \mathbf{M}_{\Delta \mathrm{T}} &\coloneqq \frac{\Phi_{\mathrm{end},\theta}}{\prod_{2,\mathrm{estimated}}^{\mathrm{L}_{2},\mathrm{estimated}}} = -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{N} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{mm}} \\ &= -1.064 \times 10^{8} \cdot \mathrm{M} \cdot \mathrm{M}$$

$$\mathbf{M}_{\Delta} \coloneqq \$1.57 \mathbf{k} \mathbf{N} \cdot \mathbf{m} = \$.157 \times 10^7 \cdot \mathbf{N} \cdot \mathbf{m} \mathbf{m}$$

$$\begin{aligned} \mathrm{EI}_{\mathrm{m}} &\coloneqq \frac{\mathrm{M}_{\Delta}}{-\kappa_{\Delta \mathrm{T}}} = 1.02 \times 10^{14} \cdot \mathrm{N} \cdot \mathrm{mm}^2 \\ \mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{\mathrm{c}} &= 3.229 \times 10^{14} \cdot \mathrm{N} \cdot \mathrm{mm}^2 \qquad \qquad \frac{1}{3} \cdot \mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{\mathrm{c}} = 1.076 \times 10^{14} \cdot \mathrm{N} \cdot \mathrm{mm}^2 \end{aligned}$$

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Appendix 6

Concrete property:

 $E_{c} := 31000 \frac{N}{mm^{2}} \qquad b := 1000 \text{mm} \qquad h := 800 \text{mm}$ $c_{c} := 50 \text{mm} + 30 \text{mm} = 80 \text{mm} \qquad d := h - c = 720 \text{mm} \qquad A_{ceff} := b \cdot d = 7.2 \times 10^{5} \text{mm}^{2}$ $I_{c} := \frac{1}{12} \cdot b \cdot d^{3} = 3.11 \times 10^{10} \text{mm}^{4} \qquad F_{c} \cdot I_{c} = 9.642 \times 10^{14} \text{N} \text{mm}^{2}$

$$W_{c} := \frac{1}{6} \cdot b \cdot h^{2} = 1.067 \times 10^{8} \cdot mm^{3} \qquad f_{ct} := 2.8 \frac{N}{mm^{2}} + \frac{1}{6} \cdot b \cdot h^{2} = 1.067 \times 10^{8} \cdot mm^{3} \qquad f_{ct} := 2.8 \frac{N}{mm^{2}}$$

Reinforement property:

$$E_s := 200000 \frac{N}{mm^2}$$
 $A_{sc} := 4909mm^2$ $A_{st} := 2094mm^2$

A. Only Bending:

The compression zone height:

Due to Chapter 3.2 equation (3-10)

$$x := \frac{-A_{st} \cdot E_{s} - A_{sc} \cdot E_{s} + \sqrt{A_{st}^{2} \cdot E_{s}^{2} + 2 \cdot A_{sc} \cdot A_{st} \cdot E_{s}^{2} + 2 E_{c} \cdot b \cdot d \cdot A_{st} \cdot E_{s} + A_{sc}^{2} \cdot E_{s}^{2} + 2 \cdot E_{c} \cdot b \cdot c \cdot A_{sc} \cdot E_{s}}{E_{c} \cdot b} = 117.8 \cdot mm$$

Bending stiffness in a crack (Chapter 4.2 equation (4-3)):

$$\alpha_{e} := \frac{E_{s}}{E_{c}} = 6.452$$

$$EI_{scr} := E_{c} \cdot \left[\frac{b \cdot x^{3}}{12} + b \cdot x \cdot \left(\frac{x}{2} \right)^{2} + \alpha_{e} \cdot A_{st} \cdot (d - x)^{2} \right] = 1.688 \times 10^{14} \cdot \text{N} \cdot \text{mm}^{2}$$



Cracking Force (chapter 4.1):

$$M_{cr} := W \cdot f_{ct} = 2.987 \times 10^8 \cdot N \cdot mm$$

Difference of the centroidal axis (chapter 4.3 equation (4-5)):

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$$\begin{split} \eta_{st} &\coloneqq \frac{A_{st}}{A_{ceff}} = 2.908 \times 10^{-3} \qquad \eta_{sc} \coloneqq \frac{A_{sc}}{A_{ceff}} = 6.818 \times 10^{-3} \\ k_u &\coloneqq \frac{c}{d} = 0.111 \qquad k_d \coloneqq \frac{x}{d} = 0.164 \\ A_{ceff} &\triangleq \alpha_e \cdot (\eta_{st} + \eta_{sc}) = 0.063 \\ B &\coloneqq \alpha_e \cdot (\eta_{st} + k_u \cdot \eta_{sc}) = 0.024 \\ C_{cv} &\coloneqq \alpha_e \cdot (\eta_{st} + k_u^2 \cdot \eta_{sc}) = 0.019 \\ \Delta x &\coloneqq \frac{2B + k_d^2}{2(A + k_d)} = 0.164 \end{split}$$

Tension stiffening value (chapter 4.4 equation 4.6):

$$M := \mathbf{M}$$

$$\kappa_{scr} := \frac{\mathbf{M}}{\mathrm{EI}_{scr}} = \mathbf{M}$$

$$\Delta \kappa := 0.42 \kappa_{scr} = \mathbf{M}$$

$$\kappa_{fdc} := 0.58 \kappa_{scr} = \mathbf{M}$$

Mean stiffness (chapter 4.5):





$$EI_{mean} := \frac{M}{\left(\frac{M - M_{cr}}{EI_{scr}} + \kappa_{fdc}\right)} = \cdot N \cdot mm^2$$



B. Tension and Bending

Compresssion zone height (chapter 3.4 equation (3-24)):

$$x1 := 157mm \qquad k_{x} := \frac{x1}{d} = 0.218 \qquad k_{x0} := 0.5$$

$$\left[\frac{k_{x}^{-4} + 4 \cdot A \cdot k_{x}^{-3} - 12 \cdot B \cdot k_{x}^{-2} + 12C \cdot k_{x} + 12A \cdot C - 12B^{2}}{-6k_{x}^{-3} - 18A \cdot k_{x}^{-2} - 12(A^{2} - B) \cdot k_{x} + 12A \cdot B} - k_{x0} + \frac{(2 \cdot B + k_{x}^{-2})}{2(A + k_{x})}\right] \cdot d = -1.008 \times 10^{3} \cdot mm$$

$$\frac{M_{b}}{N_{b}} = -1 \times 10^{3} \cdot mm$$
Cracking moment (chapter 4.1):
$$\left(\begin{array}{c} N_{b} \end{array}\right) = 0$$

 $M_{crb} := W \cdot \left(f_{ct} - \frac{N_b}{A_{ceff}} \right) = 3.876 \times 10^8 \cdot N \cdot mm$

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Bending stiffness in a crack (chapter 4.2 equation (4-3)):

$$x_b := x1 = 157 \cdot mm$$

$$\mathrm{EI}_{\mathrm{scrb}} := \mathrm{E}_{\mathrm{c}} \cdot \left[\frac{\mathbf{b} \cdot \mathbf{x}_{\mathrm{b}}^{3}}{12} + \mathbf{b} \cdot \mathbf{x}_{\mathrm{b}} \cdot \left(\frac{\mathbf{x}_{\mathrm{b}}}{2} \right)^{2} + \alpha_{\mathrm{e}} \cdot \mathrm{A}_{\mathrm{st}} \cdot \left(\mathbf{d} - \mathbf{x}_{\mathrm{b}} \right)^{2} \right] = 1.727 \times 10^{14} \cdot \mathrm{N} \cdot \mathrm{mm}^{2}$$

Difference of the centroidal axis (chapter 4.3 equation (4-5)):

$$\eta_{st} = 2.908 \times 10^{-3}$$
 $\eta_{sc} = 6.818 \times 10^{-3}$

$$k_u = 0.111$$
 $k_d = 0.164$

A = 0.063 B = 0.024

$$\Delta x_{b} := \left[\frac{2B + k_{d}^{2}}{2(A + k_{d})}\right] \cdot d = 117.8 \cdot mm$$

Tension Stiffening value (chapter 4.4 equation (4-6)):

$$\kappa_{\text{scrb}} \coloneqq \frac{M_{\text{b}} + N_{\text{b}} \cdot \Delta x_{\text{b}}}{EI_{\text{scrb}}} = 3.064 \times 10^{-3} \frac{1}{\text{m}}$$
$$\Delta \kappa_{\text{b}} \coloneqq 0.42 \kappa_{\text{scrb}} = 1.287 \times 10^{-3} \frac{1}{\text{m}}$$

Mean stiffness (chapter 4.5):

$$\Delta M_b := N_b \cdot \Delta x_b = -7.068 \times 10^7 \cdot N \cdot mm$$

$$EI_{meanb} \coloneqq \frac{M_{b}}{\left(\frac{M_{b} + \Delta M_{b}}{EI_{scrb}}\right) - \Delta \kappa_{b}} = 3.376 \times 10^{14} \cdot N \cdot mm^{2}$$

$$E_{c} \cdot I_{c} = 9.642 \times 10^{14} \cdot N \cdot mm^{2}$$

$$EI_{scr} = 1.688 \times 10^{14} \cdot N \cdot mm^{2}$$

$$EI_{scrb} = 1.727 \times 10^{14} \cdot N \cdot mm^{2}$$

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