Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plates and Slabs
Wednesday 6 April 2016, 13:30 – 16:30

This exam consists of 6 questions. The respective weights are 1.8, 1.8, 1.8, 1.8, 1.0 and 1.8.

Write your name and student number **on every page of your work**.

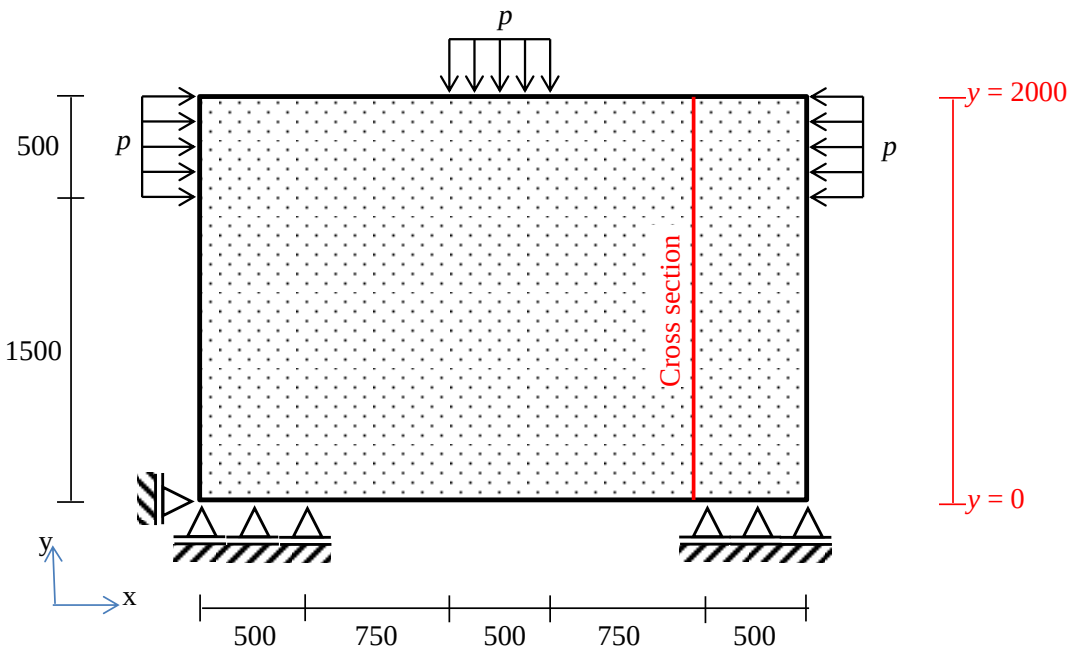
In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign conventions**.

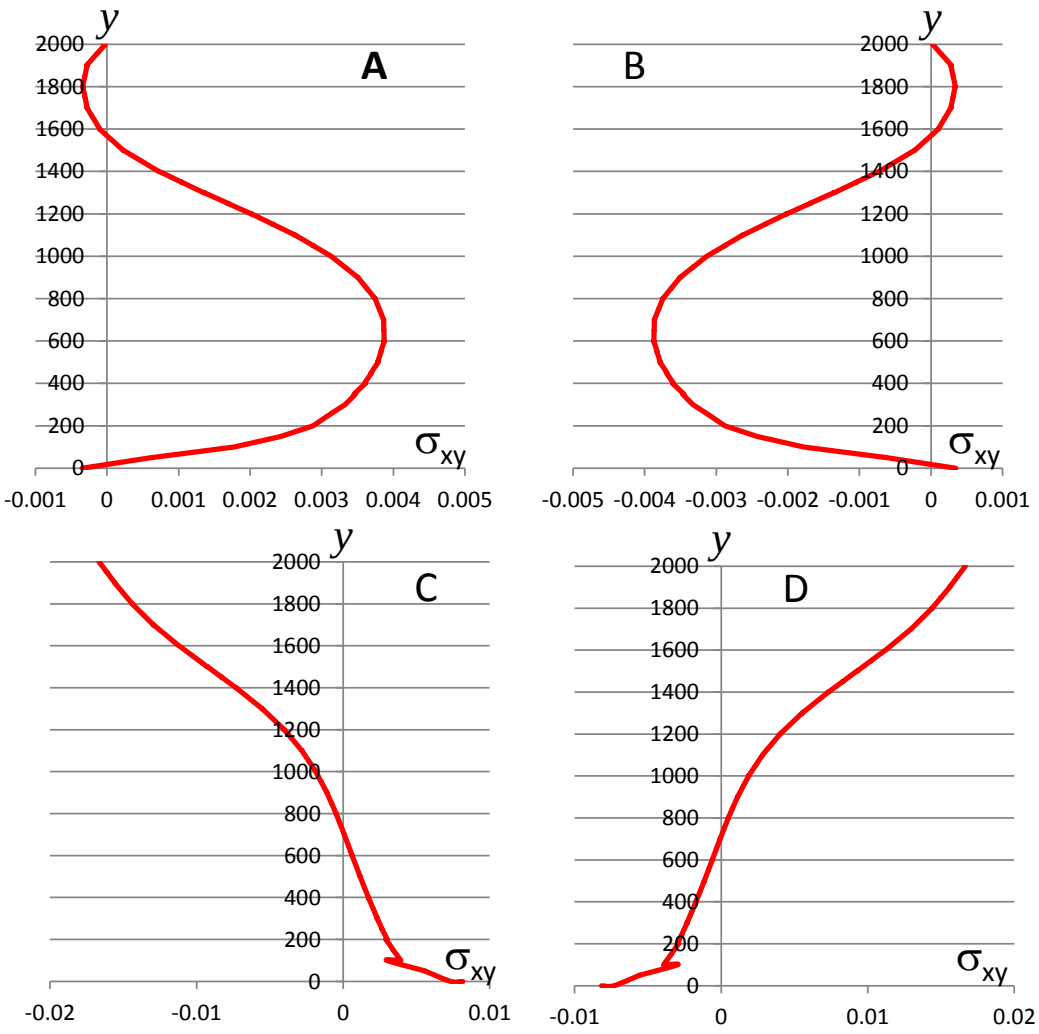
Question 1

Consider the plane stress (or plate membrane) problem of a wall below. The measurements are in mm. The length of the wall is 3000 mm. The height is 2000 mm, the thickness is 200 mm.

The wall is loaded by three equal line loads *p* ("force per length"), each acting on lengths of 500 mm. The three line loads act in different directions, as is indicated in the figure. *p* is positive.

An adequate finite element analysis has been made. Along the red line (indicated as "Cross section") the shear stress σ_{xy} is plotted (in MPa). Further below are four possible diagrams (A, B, C, D) of σ_{xy} along that line. Values can be measured from the diagrams.





a) Which of these diagrams can be correct? Multiple diagrams can be correct. Explain your answer by commenting <u>each</u> diagram. Be concise; do not use more than 50 words per diagram.

b) Select a diagram that can be correct. For this diagram, estimate the value of *p*. Clearly explain your calculations.

Question 2

Consider a square plate of unit thickness with the following displacement field:

$$u_x(x,y) = a_1 + a_2x + a_3y \quad \text{and} \quad u_y(x,y) = b_1 + b_2x + b_3y$$

The coordinates of the corners of the plate are (-1,-1), (1,-1), (1,1) and (-1,1). The displacements of <u>three</u> corners are given:

$u_x(-1,-1) = u_y(-1,-1) = 0$
$u_x(1,-1) = 0 \quad ; \; u_y(1,-1) = 0.2$
$u_x(-1,1) = 0.4 \; ; \; u_y(-1,1) = 0$

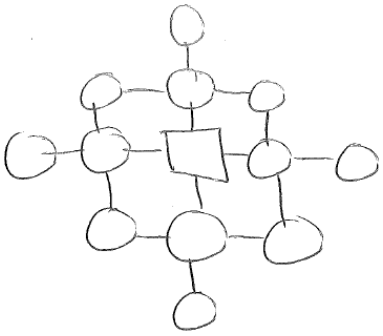
a) Does this displacement field necessarily lead to a homogeneous stress state? Explain why.

b) Express the stress components σ_{xx}, σ_{yy} and σ_{xy}, for the centre of the plate (0,0), as a function of E and ν (assuming plane stress conditions).

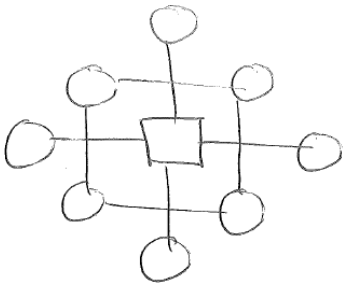
c) Calculate the rigid body displacements and rigid body rotation of the plate.

Question 3

Question 3a
Consider the finite difference method. Which is the main molecule for plates loaded perpendicularly to their plane? Choose A, B, C or D.



A



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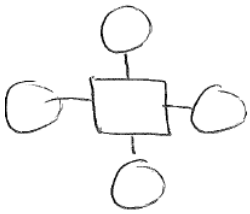
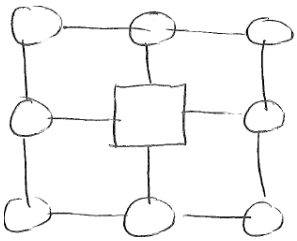


C



D

Question 3b
There are many computational ways to solve the plate equations. Under which condition do these ways produce the same result?
Choose A, B, C or D.

A No mistakes have been made and the used computer has sufficient capacity (memory, hard disk and number representation).

B No mistakes have been made and shape functions are compatible over the element edges.

C No mistakes have been made and the mesh is very fine.

D No mistakes have been made and plate is rectangular with only rectangular openings.

Question 3c
In 1850 Gustave Kirchhoff published the correct boundary conditions for plates loaded perpendicular to their plane. This completed the plate theory which had puzzled the smartest scientist for many years. Engineers pointed out to Kirchhoff that his equations show that concentrated shear forces occur in the edges of plates. To date, some engineers, who perform finite element analysis of structures, do not know about concentrated shear forces. How is it possible that they get away with this? Choose A, B, C or D.

A The software automatically checks the magnitude of concentrated shear forces (unity check).

B Concentrate shear forces are imaginary quantities related to m_{xy}. They do not show up in finite element analysis of thin plates and do not need to be checked.

C The stress due to a concentrated shear force is usually small.

D Most modern software uses the Mindlin-Reissner theory and then the stresses due to concentrated shear forces are correctly computed without interpretation and automatically checked with the other stresses.

Question 3d
Which method is used by engineers and unknown to mathematicians? Choose A, B, C or D.

A Discrete Element Method

B Finite Element Method

C Potential Energy Method

D Finite Difference Method

Question 4

A square plate is simply supported at all three edges and clamped at one edge (Fig. 1). The plate length and width are 5000 mm. The plate thickness is 250 mm. The load is self-weight of 2500 kg/m^3. The plate has an opening of 1500 x 1500 mm. A linear elastic finite element model is made. E = 27100 N/mm², ν = 0.2. Applied are eight node shell elements without shear deformation (Kirchhoff theory). The element size is 500 mm.

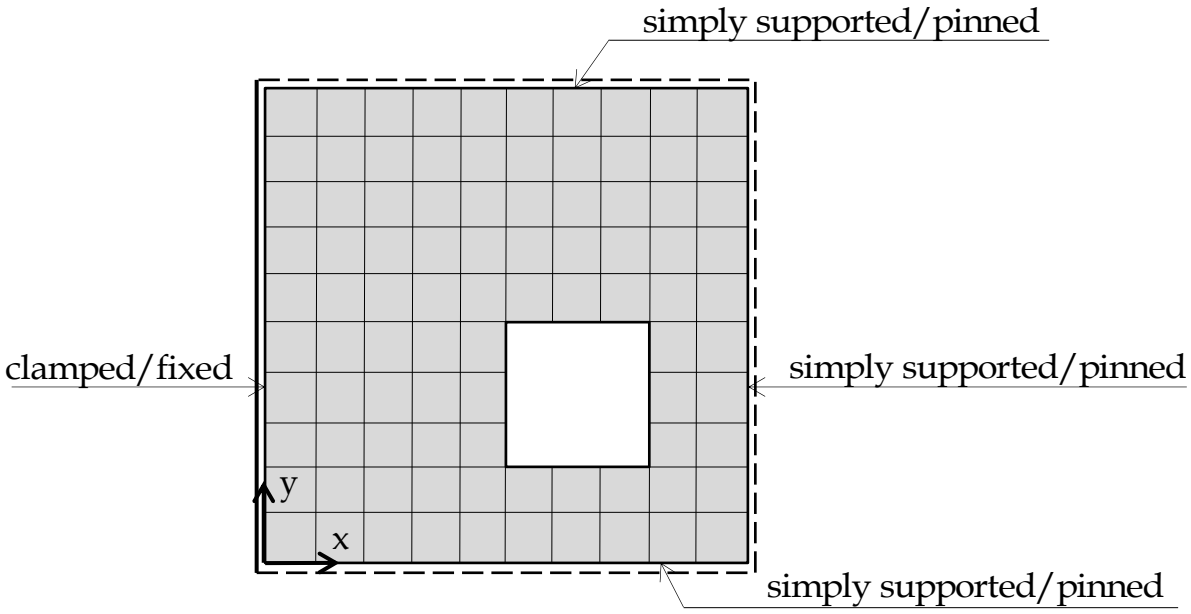


Figure 1. Simply supported plate with opening and finite element mesh

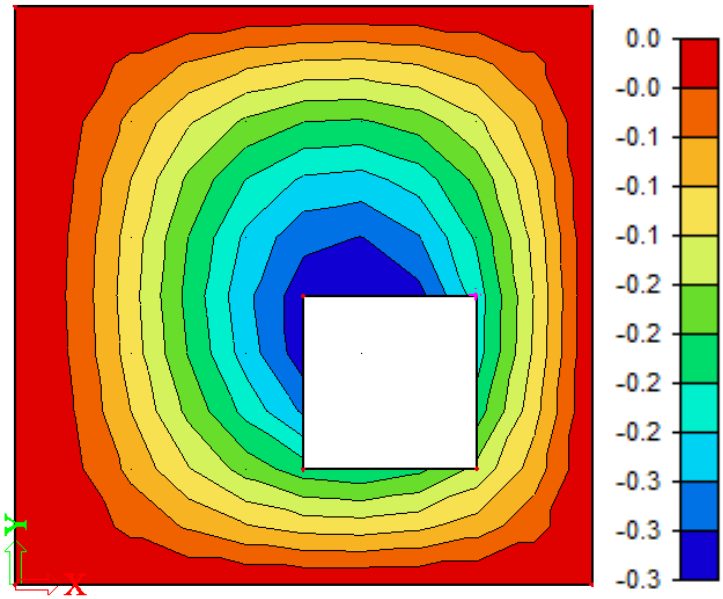


Figure 2. Contour plot of the plate deformation w

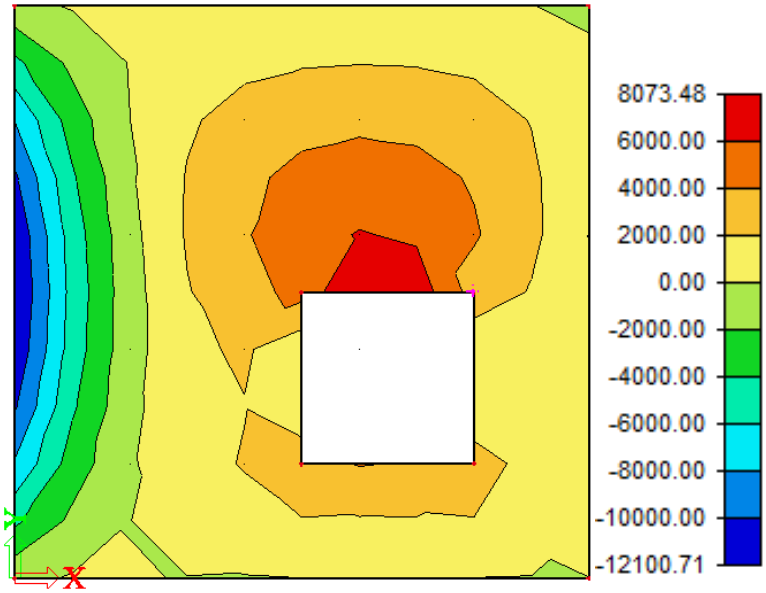


Figure 3. Contour plot of the bending moment m_{xx}

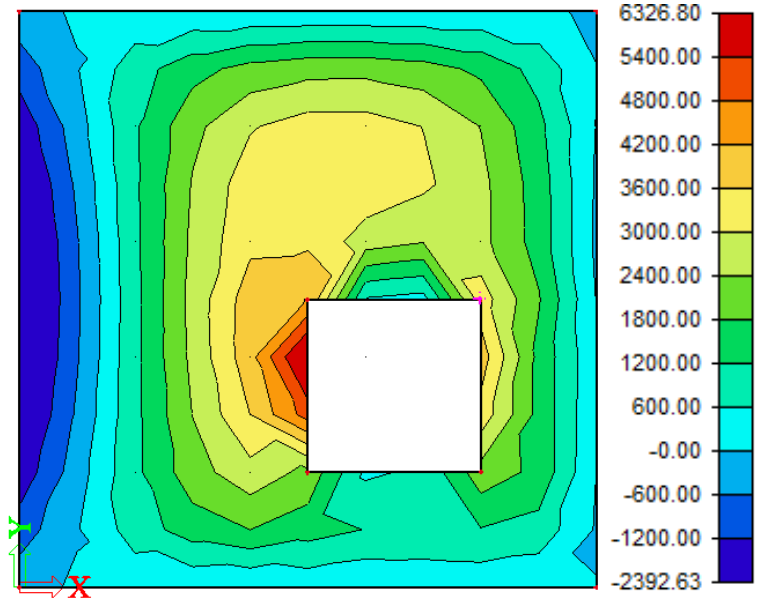


Figure 4. Contour plot of the bending moment m_{yy}

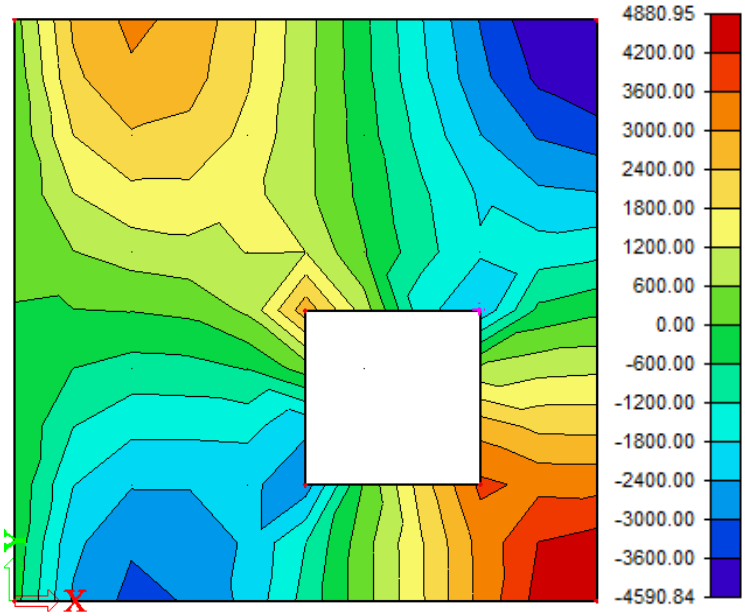


Figure 5. Contour plot of the torsion moment m_{xy}

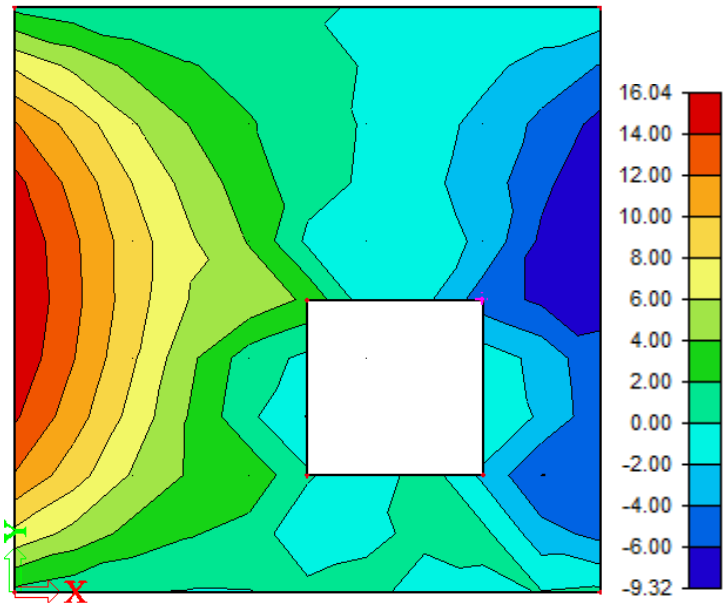


Figure 6. Contour plot of the shear force v_x

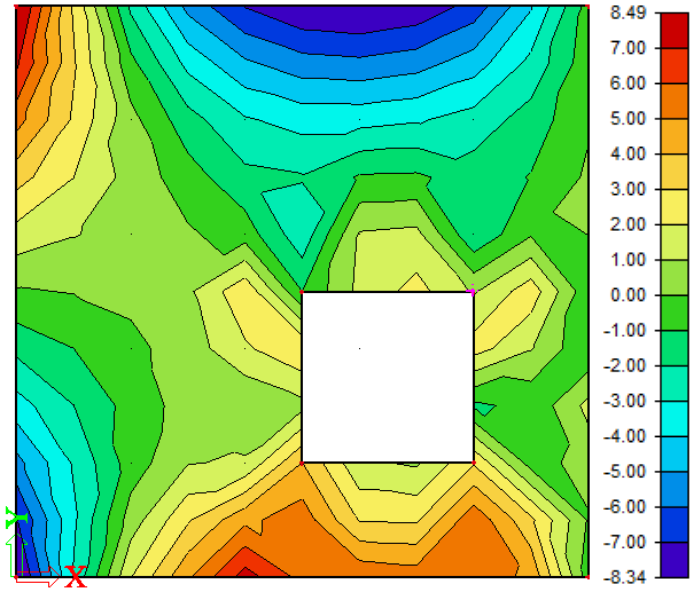


Figure 7. Contour plot of the shear force v_y

a) How much is the plate deflection? How is the unit of the deflection determined?

b) Does the deformation agree with the imposed boundary conditions?

c) What is the largest value of the bending moment m_{xx} in the plate? Which unit? Where does it occur?

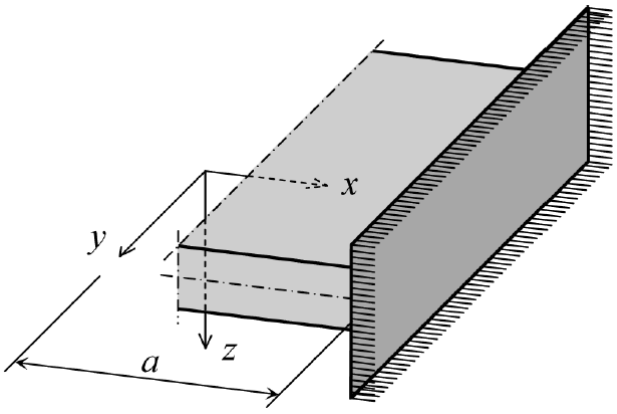
d) Why is the bending moment m_{yy} on the opening edge y = 2500 mm not equal to zero everywhere?

e) What are the values of the torsion moment m_{xy} in the corners of the opening? Which unit?

f) What is the largest value of v_x in this plate? Which unit? Where does it occur?

g) What are the values of the concentrated support reactions in the plate corners?

Question 5

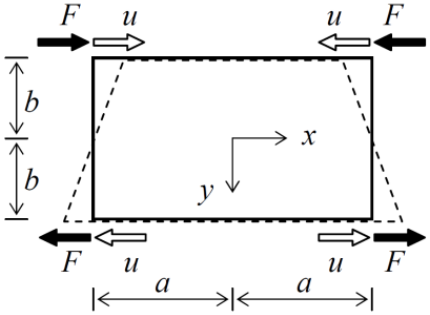


Consider the thin plate of the figure above. The plate is totally restrained along the edge $x = a$. At this clamped edge the bending moment m_{xx} will depend on coordinate y.

a) What is the twisting moment m_{xy} at the clamped edge? Express $m_{xy}(y)$ in $m_{xx}(y)$ and the material parameters. Explain your answer.

b) What is the bending moment m_{yy} at the clamped edge? Express $m_{yy}(y)$ in $m_{xx}(y)$ and the material parameters. Explain your answer.

Question 6



Consider a standard rectangular membrane element with four nodes in the corners. See the figure above. Two nodes have a displacement *u* in *x*-direction. The two other nodes have a displacement *-u* in *x*-direction. All nodes have a zero displacement in *y*-direction. The displacement field is illustrated above.

a) Give the displacement field in *x*- and *y*-directions. Express in *a*, *b*, *u*, *x* and *y*.

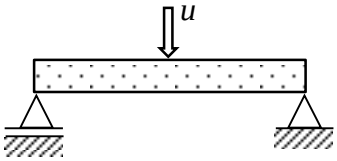
b) Give expressions for all strain components. Again, express in *a*, *b*, *u*, *x* and *y*.

Now consider that it is a constant shear element, and we repeat the questions:

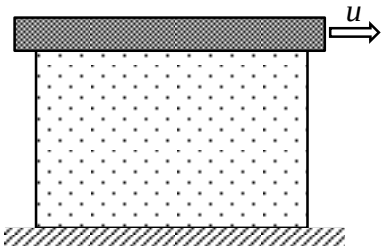
c) Give the displacement field in *x*- and *y*-directions.

d) Give expressions for all strains components.

e) Consider the application below. A beam is loaded in 3-point bending. The beam will be modelled with membrane elements Is the standard rectangular element preferable to the constant shear element? Explain in words. Be concise; do not use more than 50 words.



f) Consider the application below. A wall is clamped at the bottom. Via a relatively stiff beam the top of the wall is displaced in horizontal direction. The beam will not tilt. The wall will be modelled with membrane elements. Is the standard rectangular element preferable to the constant shear element? Explain in words. Be concise; do not use more than 50 words.



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For two questions the division of sub-points are indicated between […] brackets.

Question 1

a) "A" can be correct. Integrating the shear stresses over the depth gives a positive shear force. "B" cannot be correct. The resulting shear force is negative. "C" and "D" cannot be correct. Shear stresses should be zero at the edges.
b) Roughly integrating $\int_0^{2000} \sigma_{xy}\, dy$ of diagram "A" as a rectangle of 0.003×1300 yields 3.9 N/mm. Multiplying with the thickness gives a shear force of 780 N. This force should be in equilibrium with ½ p 500. Consequently p = 780/250 = 3.12 N/mm (or kN/m).

Question 2

a) Yes. Applying the kinematic equations gives strains $\varepsilon_{xx} = a_2$, $\varepsilon_{yy} = b_3$ and $\gamma_{xy} = a_3 + b_2$ which are constant over the plate domain. Consequently, also the stresses are constant.
b) Here $a_1 = 0.2$, $a_2 = 0$, $a_3 = 0.2$, $b_1 = 0.1$, $b_2 = 0.1$ and $b_3 = 0$. Applying the constitutive law we find $\sigma_{xx} = \frac{E}{1-\nu^2}(a_2 + \nu b_3) = 0$, $\sigma_{yy} = \frac{E}{1-\nu^2}(b_3 + \nu a_2) = 0$ and $\sigma_{xy} = \frac{E}{2(1+\nu)}(a_3 + b_2) = 0.3\frac{E}{2(1+\nu)}$ (for any point of the square plate).
c) $u_x = a_1 = 0.2$, $u_y = b_1 = 0.1$ and $\omega_{xy} = ½\,(-a_3 + b_2) = -0.05$

Question 3

a) A
b) C
c) C
d) A

[1.8 for 4 correct answers, 1.2 for 3 correct answers, 0.6 for 2 correct answers, 0.0 for 1 or no corrects answers]

Question 4

a) 0.3 mm; the unit follows the units of the input unless specified otherwise [0.1]
b) yes, there are no inconsistencies [0.2]
c) -12100 Nm/m at the fixed edge; 8073 Nm/m in the field [0.2]
d) Element size; For very small elements the m_{yy} would be zero on that edge [0.3]
e) Infinitely large; Nmm/mm [0.4]
f) 16.04 N/mm in the fixed edge; -9.32 N/mm in the edge x = 5000 mm [0.2]
g) (0,0) → 0; (5000,0) → -9182 N; (5000,5000) → -9762 N; (0,5000) → 0 [0.4]

Question 5

a) Clamped, so along the edge $\partial w/\partial x$ is zero for every y. Consequently $\partial^2 w/\partial x\,\partial y$ is zero. Therefore the torsion deformation is zero. From the constitutive equations we see that m_{xy} cannot occur: $m_{xy}(y) = 0$
b) From the constitutive equations $m_{xx} = D(\kappa_{xx} + \nu\kappa_{yy})$ and $m_{yy} = D(\nu\kappa_{xx} + \kappa_{yy})$ with $\kappa_{yy} = 0$ it follows that $m_{yy}(y) = \nu m_{xx}(y)$.

Question 6

a) $u_x(x,y) = \frac{x}{a}\frac{y}{b}u$ and $u_y(x,y) = 0$.
b) $\varepsilon_{xx}(x,y) = \frac{y}{ab}u$, $\varepsilon_{yy}(x,y) = 0$ and $\gamma_{xy}(x,y) = \frac{x}{ab}u$.
c) Same as answer a)
d) Same as answer b), except $\gamma_{xy}(x,y) = 0$.
e) The constant shear elements are to be preferred. This is a bending dominated application. The standard elements will be too stiff.
f) Both element types are suitable. This is a shear dominated application.