Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4180: Plates and Slabs
Thursday 2 February 2017, 13:30 – 16:30

This exam consists of 6 questions.
The respective weights are 1.8, 1.8, 1.5, 2.1, 2.0 and 0.8.

Write your name and student number **on every page of your work**.

In this exam the notation and sign conventions of the reader (part 1 and 2) and the book (Plates and FEM) have been adopted. **You are supposed to use the same notation and sign conventions**.

Question 1

Consider the plane stress (or plate membrane) problem below. The measurements are in mm. The length of the plate is 300 mm. The height is 300 mm. The plate has an uniform thickness.

Two load cases are considered. See the figure at the right-hand side. In load case 1 the plate is loaded by an equally distributed line load f ("force per length"), acting at the top of the plate. f = 100 N/mm.

In load case 2 the plate is loaded by an equally distributed line load f ("force per length"), acting at the bottom of the plate. f = 100 N/mm.

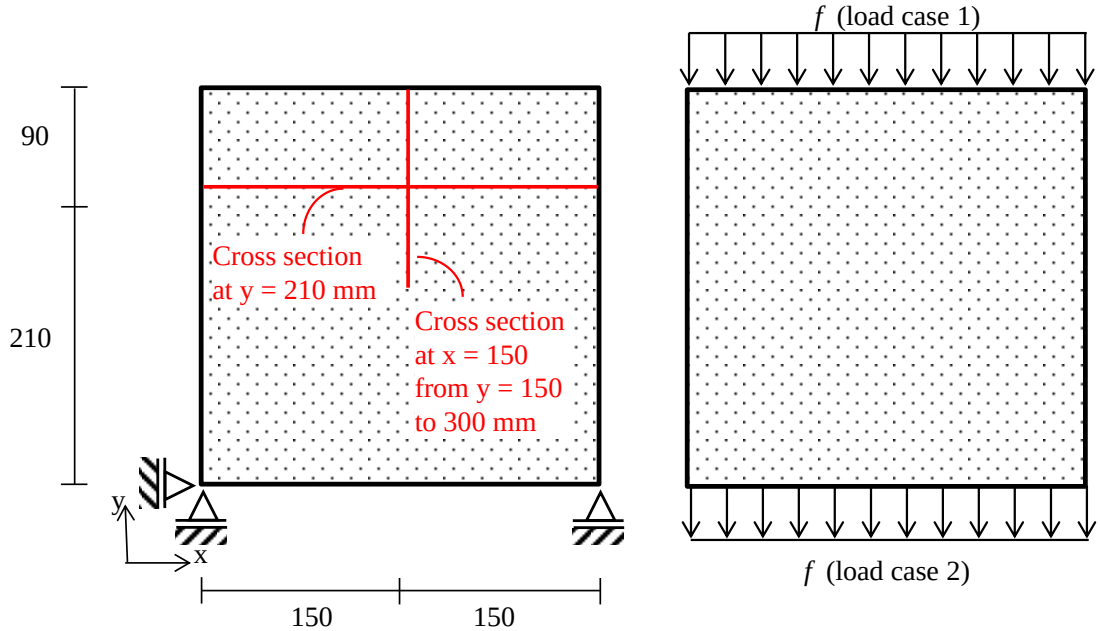
An adequate finite element analysis has been made using a regular mesh with 4-noded plane stress elements of 30×30 mm.

E = 30000 N/mm^2.

Along the two red lines (indicated as "Cross section") results are plotted (next page):

- Along the horizontal line the stress σ_{xx} is plotted (in MPa) for an unspecified load case
- Along the vertical line the stress σ_{yy} is plotted (in MPa) for load case 1.
- Along the vertical line the strain ε_{xx} is plotted for load case 1.

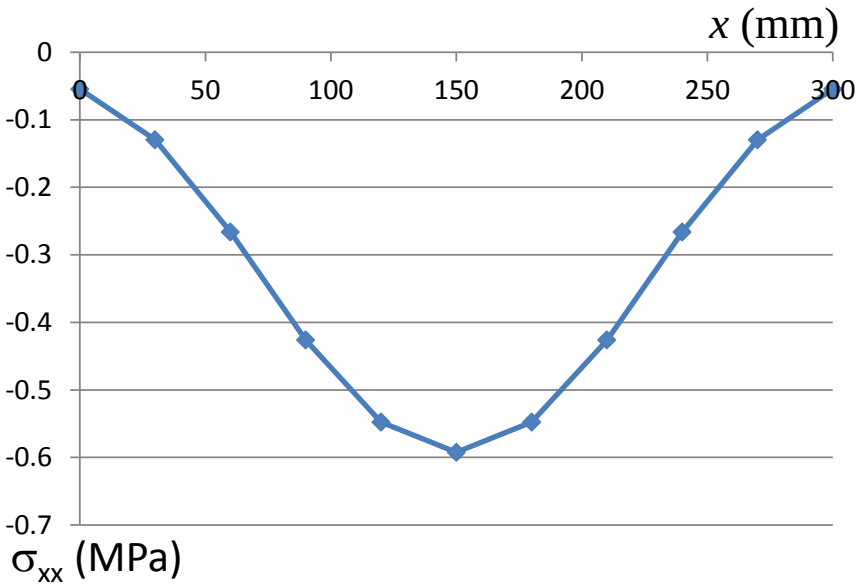
Values can be measured from the diagrams.

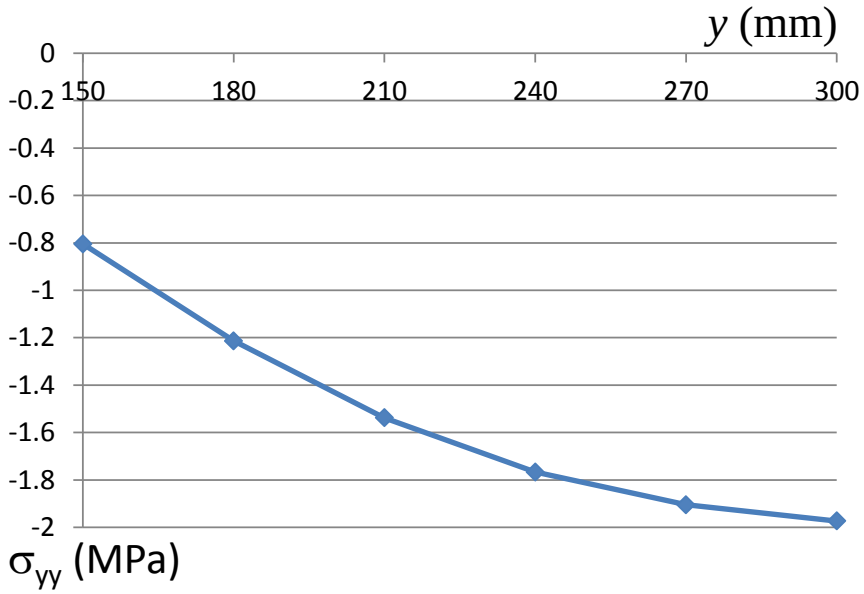


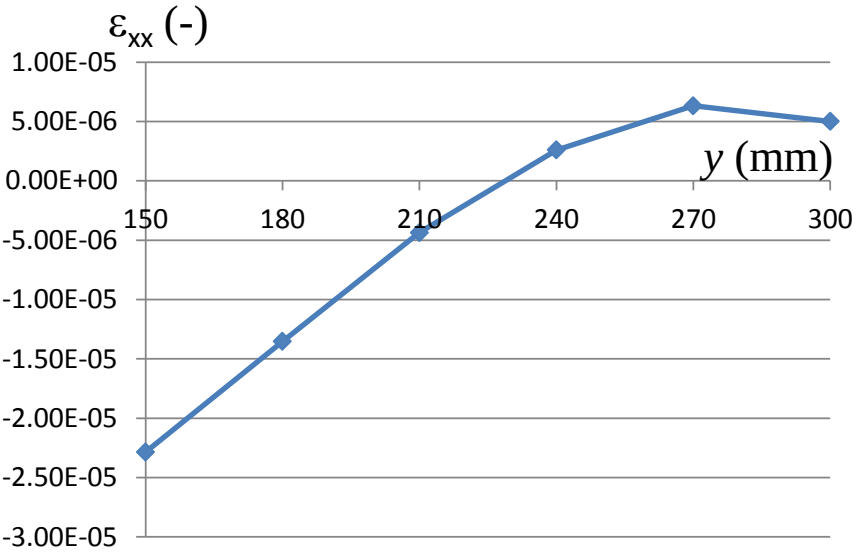
a) Could the diagram with the stress σ_{xx} be a diagram for load case 1, for load case 2, or for both load cases? Explain your answer.

b) Calculate the thickness of the plate.

c) Calculate the Poisson's ratio of the material.

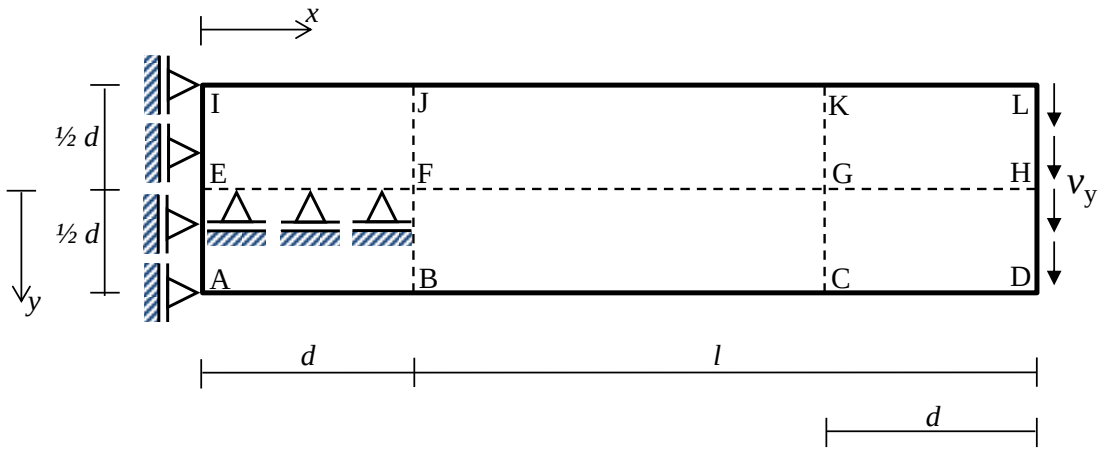

x (mm)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
50
100
150
200
250
300
σxx (MPa)


y (mm)
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
150
180
210
240
270
300
σyy (MPa)


εxx (-)
1.00E-05
5.00E-06
0.00E+00
-5.00E-06
-1.00E-05
-1.50E-05
-2.00E-05
-2.50E-05
-3.00E-05
y (mm)
150
180
210
240
270
300

Question 2

Consider a deep cantilever beam ABCDHLKJIE with dimensions $d+l$ and d :



The beam is supported in x-direction along edge IEA. The beam is supported in y-direction along midline EF. The beam is loaded by an evenly distributed (positive) shear force v_y along edge LHD. The beam has a uniform thickness t. The material properties are E and ν (= 0.3).

A finite element analysis has been made.

a) What will be the signs of the u_y displacements for the points J, F, B respectively?

For verifying and understanding the finite element results, we want to apply the plate theory for the square ABFJIE. We assume that line JFB will remain straight and will rotate around F: ϕ.

As a trial solution for the square ABFJIE polynomials terms are considered:

$$u_x(x,y) = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3$$

$$u_y(x,y) = b_1 + b_2x + b_3y + b_4x^2 + b_5xy + b_6y^2 + b_7x^3 + b_8x^2y + b_9xy^2 + b_{10}y^3 + b_{11}x^3y + b_{12}xy^3$$

Both for u_x and for u_y only 1 of the 12 terms will be selected.

b) Which of the terms would you select for u_x? Express its coefficient in d and ϕ.

c) Which of the terms would you select for u_y? Explain your answer.

Question 3

Question 3a
Does the Kirchhoff plate theory neglect out-of-plane shear deformation?
Does the Kirchhoff plate theory neglect in-plane shear deformation?
Choose answer A, B, C or D.

A yes, no
B yes, yes
C no, no
D no, yes

Question 3b
What is the reason for <u>not</u> using the Mindlin-Reissner plate theory for thin plates?
Choose A, B, C or D.

A The finite elements should be extremely small at edges.

B This theory includes shear deformation.

C The theory cannot handle zero torsion moments at free and simply supported edges.

D The concentrated shear forces are not computed correctly.

Question 3c
Finite elements should be small. Why is this?
Choose A, B, C or D.

A The shape functions need to be able to approximate the real deformation.

B Small elements can be easily handled and require less space for storage.

C Small elements fit better. A mesh generator can create a good mesh quickly.

D The strain energy of the elements needs to be evenly distributed.

Question 3d
A plate has been computed with the finite element method. The coordinate system is as usual. (Cartesian; *z* axis perpendicular to the plate; *x* and *y* axes in the plate.) We change the *z* axis to its opposite direction and the *y* axis to its opposite direction. Which of the following quantities change sign?
n_{xx}, n_{yy}, n_{xy}, m_{xx}, m_{yy}, m_{xy}, v_x, v_y

Question 3e
Much of the plate lectures is about what computers should do.
Why aren't the lectures about what engineers should do?
Choose A, B, C, or D.

A Engineers have to follow the same rules as computers.

B The future job of an engineer cannot be explained in rules.

C Engineers have to program the computers.
Many engineers are needed for this.

D There is no need for engineers in the future.
Computers can analyse, design and check plate structures.

Question 4

A square plate is loaded perpendicularly. It is fixed at three edges and simply supported at one edge (Fig. 1). The plate length and width are 10000 mm. The plate thickness is 200 mm. Self-weight is -5 kN/m^2. A point load of -100 kN is applied to the middle of the plate. A linear elastic finite element model is made. E = 27100 N/mm^2, ν = 0.2. Applied are four node plate elements (Kirchhoff theory). The element size is 200 mm.

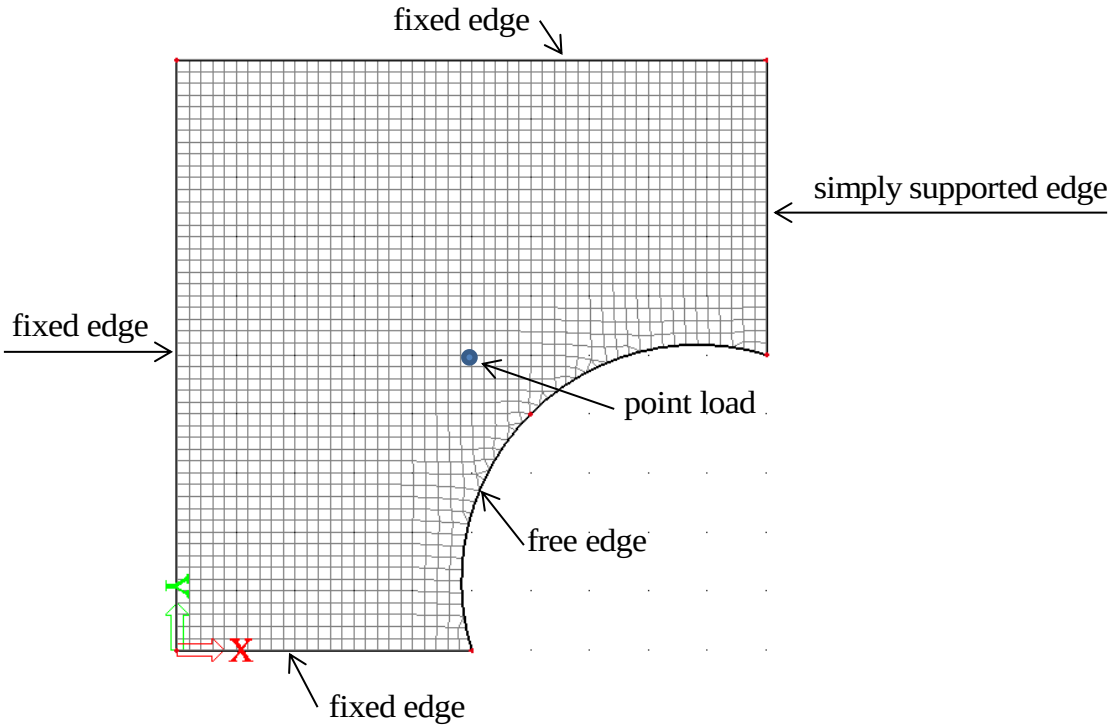


Figure 1. Plate with perpendicular load and finite element mesh

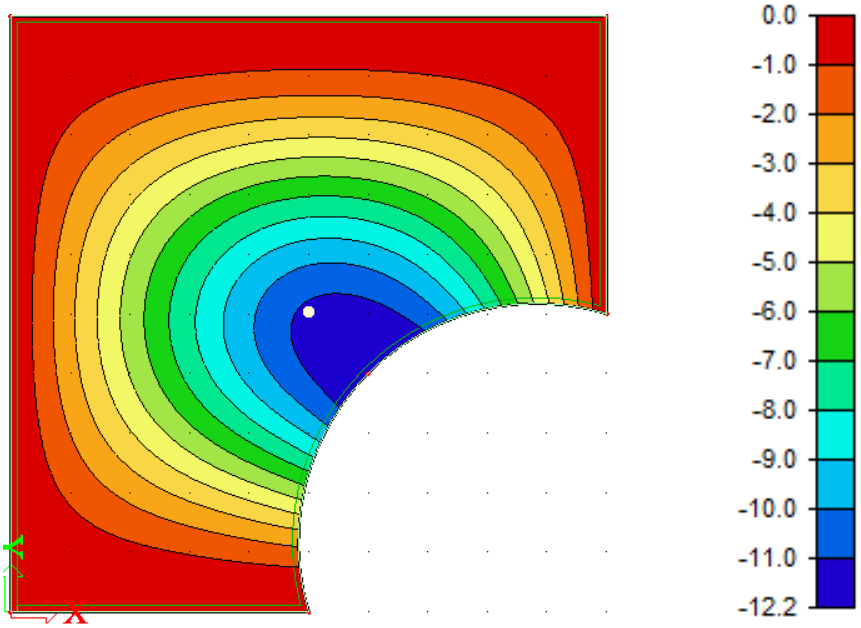


Figure 2. Contour plot of the plate deformation *w*.

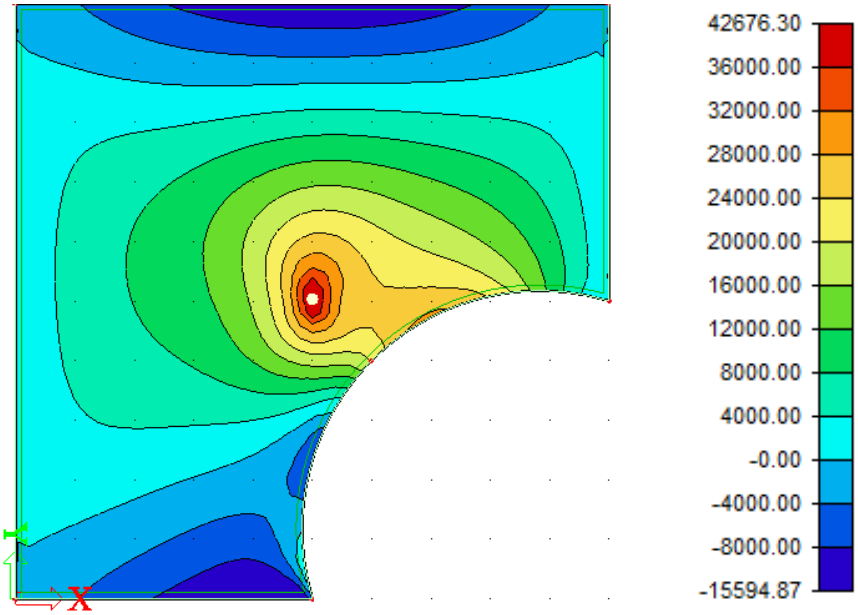


Figure 3. Contour plot of the bending moment m_{xx}

Question 4a
How much is the plate deflection?
How is the unit of the deflection determined?
Is the displayed plate deflection averaged in nodes or not averaged in nodes?

Question 4b
Do the moments agree with the specified supports? Explain your answer.

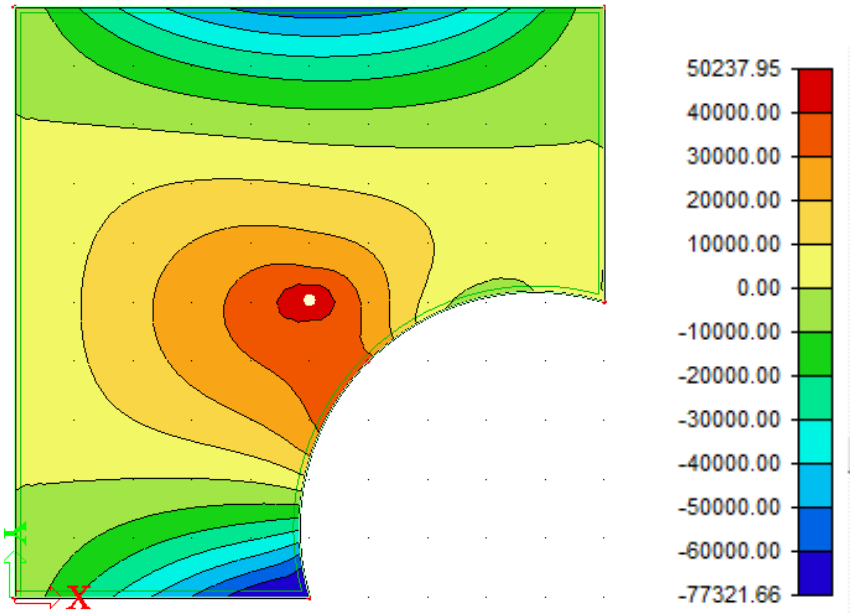


Figure 4. Contour plot of bending moment m_{yy}

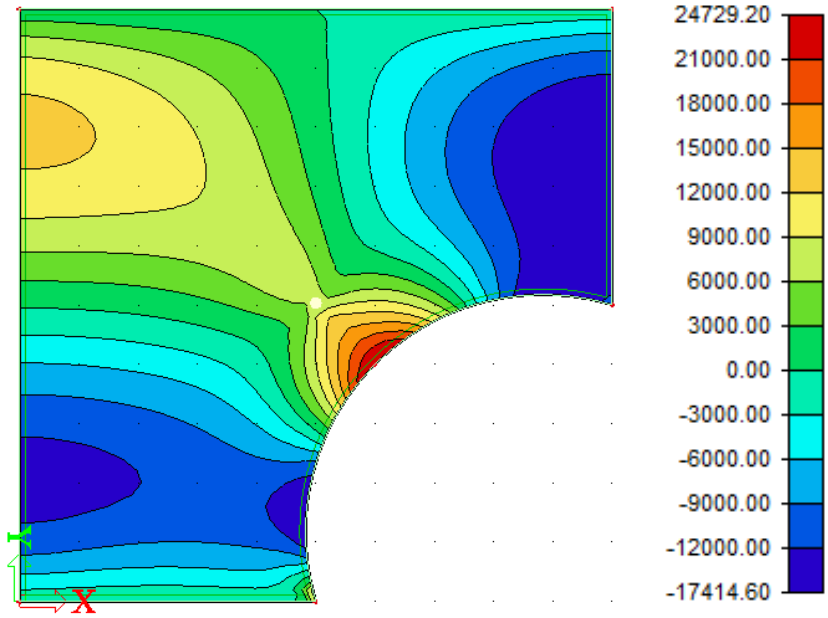


Figure 5. Contour plot of torsion moment m_{xy}

Question 4c

What is the largest value of the bending moment m_{xx} in the plate?

Which unit?

Where does it occur?

Question 4d

Why is the bending moment m_{yy} on the curved edge not equal to zero everywhere?

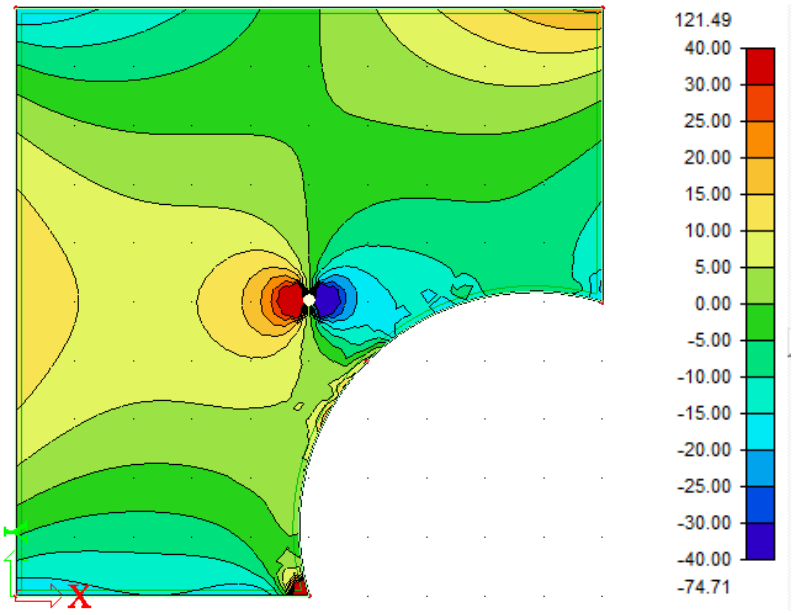


Figure 6. Contour plot of shear force v_x

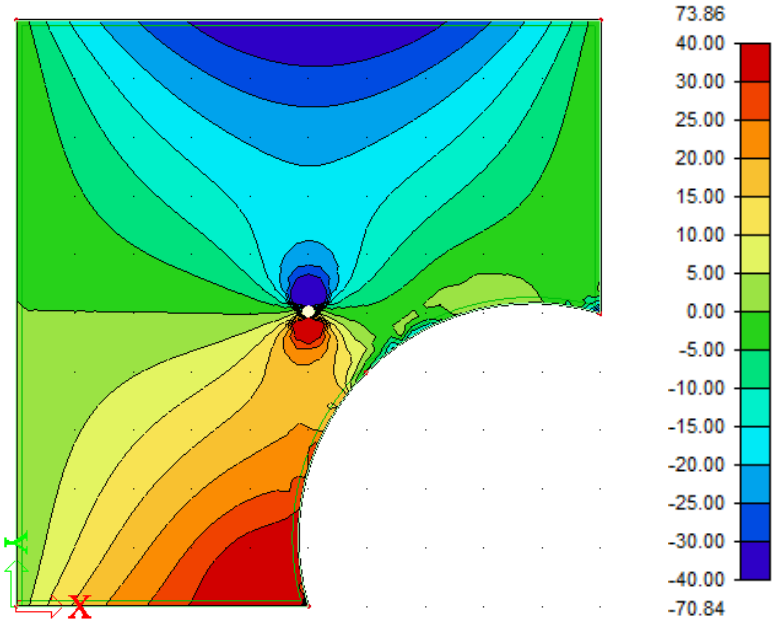


Figure 7. Contour plot of shear force v_y

Question 4e

On the right hand edge, the torsion moment m_{xy} is not zero. What causes this?

Question 4f

What is the largest value of v_x in this plate?

Which unit?

Where does it occur?

Question 4g

What are the values of the concentrated support reactions in the plate corners?

Question 5

A reinforced concrete plate is loaded in plane. At some location the membrane forces are

n_{xx} = 400 kN/m, n_{yy} = –80 kN/m, n_{xy} = –160 kN/m.

The yield stress of the reinforcement is 500 N/mm^2.
Partial safety factors are included in the values.

Question 5a
Calculate the reinforcement force [kN/m] that is minimally required in the *x* and *y* directions.

Question 5b
Somebody proposes the following reinforcement.

bars in *x* direction	bars in *y* direction
diameter = 12 mm	diameter = 12 mm
spacing = 250 mm	spacing = 250 mm
two layers	two layers

Is this reinforcement sufficient? Explain your answer.

Question 5c
Calculate the reinforcement force [kN/m] that is minimally required in the principal directions.

Question 5d
Which reinforcement requirement (5a, 5b or 5c) is smallest? Explain your answer.

Question 6

For a standard rectangular 4-noded plane stress element the stiffness matrix can be calculated according to the formula:

$$\boldsymbol{K} = \iint_A \boldsymbol{B}^T(x,y)\, \boldsymbol{D}(x,y)\, \boldsymbol{B}(x,y)\, dA$$

a) What are the matrix dimensions of ***B*** and ***D***?

In a tapered element the thickness of the element is not uniform.

Assume that the formulation of the standard rectangular 4-noded plane stress element has been modified such that it can handle a tapered shape of the element.
Assume that the number of nodes and the degrees of freedom remain unchanged.
Assume that the element remains a rectangular element.

Also for the tapered element the stiffness matrix can be calculated according to the above formula, but possibly with different expressions or notions for the variables *A*, ***B*** and ***D***.

b) Discuss (i.e. in words) the possible consequence of the modification of the element for the variables *A*, ***B*** and ***D***.

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Question 1

a) Could be for both (stronger: both load cases will lead to the same σ_{xx})
b) $t = n_{yy}/\sigma_{yy}$. For load case 1, $n_{yy} = -f = -100$ N/mm at y =300 and $\sigma_{yy} = -2$ MPa at (x=150, y =300). Hence *t* = 50 mm.
c) At (x=150, y =210) for load case 1 we have $\sigma_{xx} = -0.59$ MPa, $\sigma_{yy} = -1.55$ MPa, $\varepsilon_{xx} = -4.5\ 10^{-6}$. With $\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy})$ and E = 30000 MPa we find ν ≈ 0.3.

Question 2

a) Positive, zero, positive. Poisson effect.
b) $u_x = -{}^{\varphi}/_{d}\, xy$ (verify by successively substituting the coodinates of A, B, F, J, I and E.
c) $u_y = b_6 y^2$. See also a).

Question 3

a) B
b) A
c) A (Answer C has some truth in it because in plates with complicated shapes, small elements need to be distorted only a little for them to fit. However, this meaning is not formulated well in answer C.)
d) Not changed: n_{xx}, n_{yy}, v_y. Changed: n_{xy}, m_{xx}, m_{yy}, m_{xy}, v_x. (8 correct = full score, 5 or less correct = zero score)
e) Subject to debate: B

Question 4a
The largest plate deflection is 12.2 mm.
The unit of this output follows from the units of the input.
Deflections cannot be averaged in nodes.
(All elements attached to a node already have exactly the same deflection in this node.)

Question 4b
No, the left boundary seems to be simply supported instead of fixed.

Question 4c
Infinitely large (singularity in the linear elastic model).
Nmm/mm or Nm/m

Under the point load.

Question 4d
Because it is curved.
myy would be zero everywhere if it were straight and in the x direction.

Question 4e
It only looks like there is a torsion moment on the edge. In reality the torsion stresses stay in the plate and go round. A torsion moment m_{xy} on an edge needs to be interpreted as a concentrated shear force in the edge.

Question 4f
Infinitely large (singularity in the linear elastic model); N/mm; Under the point load.

Question 4g
They are almost zero except for 2 x 17414 Nmm/mm = 34828 N = 35 kN between the right hand edge and the curved edge.

Question 5a
nsx = 400 + 160 = 560 kN/m
nsy = –80 + 160 = 80 kN/m

Question 5b
nsx = 2 x 1/4 x 3.14 x 12^2 x 500 / 250 = 452 kN/m
nsy = 2 x 1/4 x 3.14 x 12^2 x 500 / 250 = 452 kN/m
(452 – 400) x (452 + 80) = 27800 >? 160^2 = 25600, is sufficient.

Question 5c
n1 = (400 – 80)/2 + sqrt($((400 + 80)/2)^2 + 160^2$) = 448 kN/m
n2 = (400 – 80)/2 – sqrt($((400 + 80)/2)^2 + 160^2$) = –128 kN/m
ns1 = 448 kN/m
ns2 = 0

Question 5d
5a: 560 + 80 = 640 kN/m
5b: 452 + 452 = 904 kN/m
5c: 448 + 0 = 448 kN/m
Requirement 5c is smallest.

Question 6a
B has 3 rows and 8 columns. ***D*** has 3 rows and 3 columns.

Question 6b
A still is the area of the element. If the interpolation has been modified, also ***B*** is modified. ***D*** includes the local thickness: ***D*** will now be a function of *x* and *y* (it used a be a constant function).