Plate analysis, theory and applications

Lesson: Applications of plates loaded in their plane

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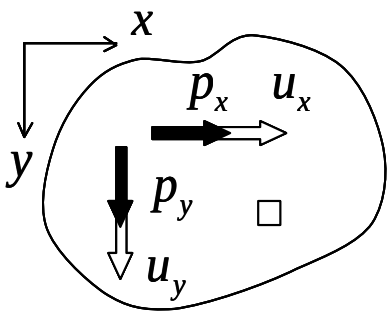
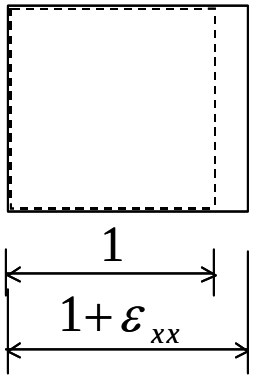
Recap

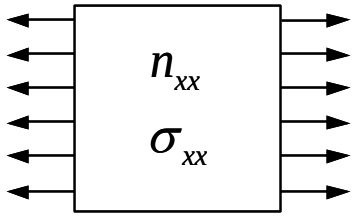
- 5 slides from the previous lecture

TU Delft

A plate loaded in-plane





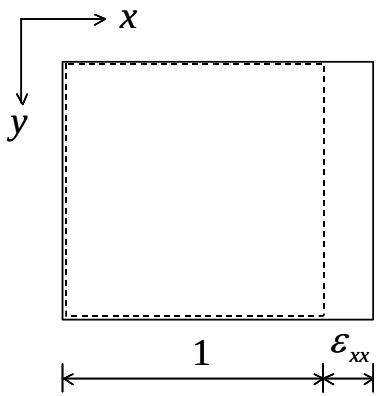


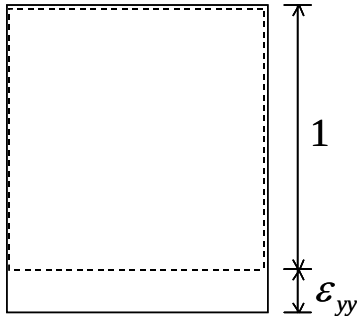
Derive:

$$\boldsymbol{u}=\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \qquad \boldsymbol{e}=\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \qquad \boldsymbol{n}=\begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} \qquad \boldsymbol{p}=\begin{Bmatrix} p_x \\ p_y \end{Bmatrix}$$

kinematic equations *constitutive equations* *equilibrium equations*

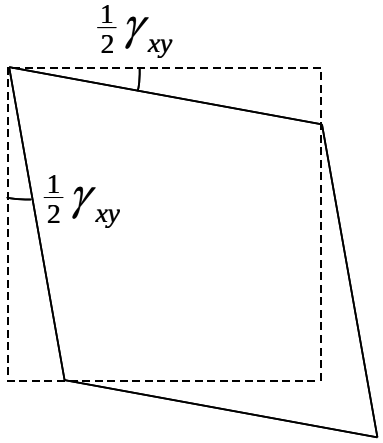
Summary of kinematic equations:





$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$



$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

Constitutive equations:

- plane stress solid

flexibility formulation:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{Et} \begin{bmatrix} 1 & -\upsilon & 0 \\ -\upsilon & 1 & 0 \\ 0 & 0 & 2(1+\upsilon) \end{bmatrix} \begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} \rightarrow \boxed{\boldsymbol{e} = \boldsymbol{C}\,\boldsymbol{n}}$$

stifness formulation:

$$\begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} = \frac{Et}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \rightarrow \boxed{\boldsymbol{n} = \boldsymbol{D}\,\boldsymbol{e}}$$

Summary equilibrium equations:

$$\sum n_x = 0 \qquad -\left(\frac{\partial n_{xx}}{\partial x} + \frac{\partial n_{yx}}{\partial y}\right) = p_x$$

$$\sum n_y = 0 \qquad -\left(\frac{\partial n_{yy}}{\partial y} + \frac{\partial n_{xy}}{\partial x}\right) = p_y$$

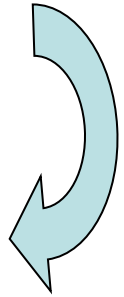
$$\boldsymbol{u} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \qquad \boldsymbol{e} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \qquad \boldsymbol{n} = \begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \end{Bmatrix} \qquad \boldsymbol{p} = \begin{Bmatrix} p_x \\ p_y \end{Bmatrix}$$

kinematic equations — *constitutive equations* — *equilibrium equations*

Displacement Method

$$-\frac{Et}{1-\upsilon^2}\left(\left(\frac{\partial^2}{\partial x^2}+\frac{1-\upsilon}{2}\frac{\partial^2}{\partial y^2}\right)u_x+\left(\frac{1+\upsilon}{2}\frac{\partial^2}{\partial x\partial y}\right)u_y\right)=p_x$$

$$-\frac{Et}{1-\upsilon^2}\left(\left(\frac{\partial^2}{\partial y^2}+\frac{1-\upsilon}{2}\frac{\partial^2}{\partial x^2}\right)u_y+\left(\frac{1+\upsilon}{2}\frac{\partial^2}{\partial x\partial y}\right)u_x\right)=p_y$$



Biharmonic equation:

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

$$-\frac{Et}{2(1+\upsilon)}\nabla^2\nabla^2 u_x=\left(\frac{\partial^2}{\partial y^2}+\frac{1-\upsilon}{2}\frac{\partial^2}{\partial x^2}\right)p_x-\left(\frac{1+\upsilon}{2}\frac{\partial^2}{\partial x\partial y}\right)p_y$$

Application of the plate theory

- The starting point is now the biharmonic equation.

- We will now try to find solutions for plates with
  - Specific dimensions
  - Specific in-plane loadings

Most of this will be shown on the blackboard.

Solutions for the biharmonic equation

- Solving the PDE's in the same way as we solved the ODE of the truss element? (see the first lecture)
- Could we use a same approach? Easy?
- From now on: consider a special case of the biharmonic equation by considering $p_x = p_y = 0$

Two approaches

- "Inverse" approach
  - Try functions $u_x(x,y)$ and $u_y(x,y)$
  - We will use polynomials for this
  - And see what applications can be modelled with this

- Consider a specific application
  - Possibly modify the application (to make the solution easier to find)
  - Find a solution

"Inverse" approach

- "Inverse" approach
  - Polynomial functions for $u_x(x,y)$ and $u_y(x,y)$

1. Consider linear polynomials
2. Consider quadratic polynomials
3. Consider cubic polynomials