Euler-Bernoulli beam .....bending
1750 1680

$$EIw'''' = p$$

Column .... Euler ..... buckling
1757

$$EIw'''' + Pw'' = 0$$

Timoshenko beam .....bending + shear
1909

$$-\frac{5}{6}GA(w'' + \varphi') = p$$

$$\frac{5}{6}GA(w' + \varphi) - EI\varphi'' = 0$$

Plate loaded in plane (disk) ... Navier ..... membrane forces
1821

$$-\frac{Et}{1-\nu^2}\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 u_x}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 u_y}{\partial x \partial y}\right) = p_x$$

$$-\frac{Et}{1-\nu^2}\left(\frac{\partial^2 u_y}{\partial y^2} + \frac{1-\nu}{2}\frac{\partial^2 u_y}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 u_x}{\partial x \partial y}\right) = p_y$$

Plate loaded out of plane (thick) … Mindlin-Reissner …… bending + shear

1951 1945

$$-\frac{5}{6}Gt\left(\nabla^2 w+\frac{\partial\varphi_x}{\partial x}+\frac{\partial\varphi_y}{\partial y}\right)=p$$

$$\frac{5}{6}Gt\left(\frac{\partial w}{\partial x}+\varphi_x\right)-D\left(\frac{\partial^2\varphi_x}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2\varphi_x}{\partial y^2}\right)-\frac{1+\nu}{2}D\frac{\partial^2\varphi_y}{\partial x\partial y}=0$$

$$\frac{5}{6}Gt\left(\frac{\partial w}{\partial y}+\varphi_y\right)-D\left(\frac{\partial^2\varphi_y}{\partial y^2}+\frac{1-\nu}{2}\frac{\partial^2\varphi_y}{\partial x^2}\right)-\frac{1+\nu}{2}D\frac{\partial^2\varphi_x}{\partial x\partial y}=0$$

Plate loaded out of plane (thin) … Germain, Kirchhoff … bending, no shear

1809 1850

$$D\nabla^2\nabla^2 w=p \qquad \nabla^2 w=\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}$$

Shell …. Love … Sanders-Koiter …… membrane forces + bending + curved

1888 1959

$$D\,\nabla^2\nabla^2\nabla^2\nabla^2 w+Et\,\Gamma\Gamma w=\nabla^2\nabla^2 p$$