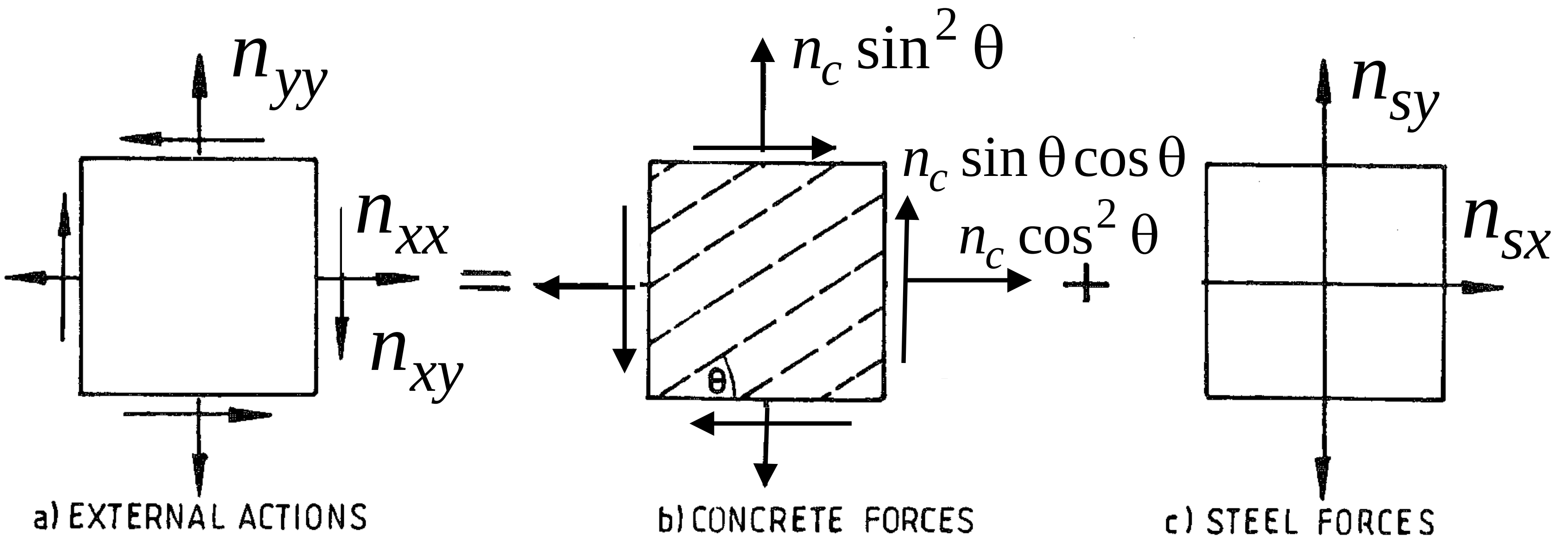
Reinforcement for Plates

Course CIE4180

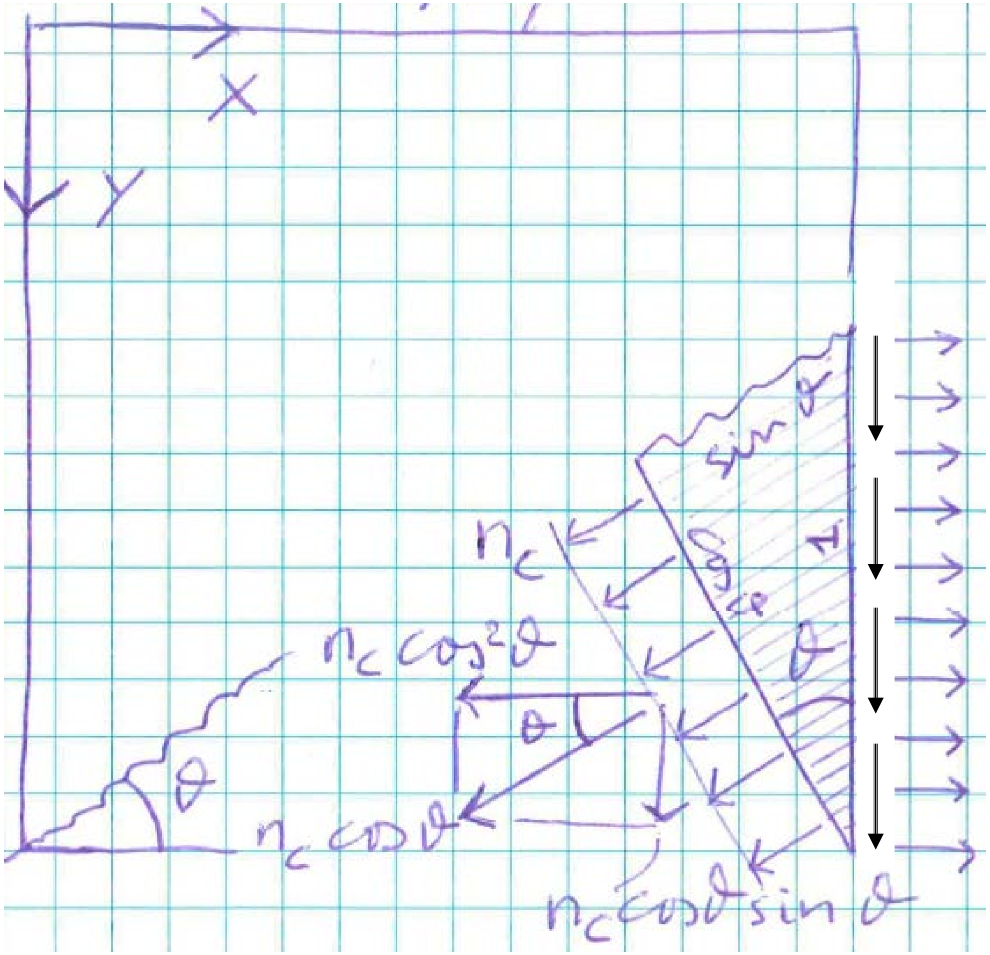
20 December 2017

Plates loaded in plane

- Only rebars in the *x* and *y* directions
- Equilibrium of a plate part



Concrete forces



- Equations

$$n_{xx} = n_{sx} + n_c \cos^2 \theta$$

$$n_{yy} = n_{sy} + n_c \sin^2 \theta$$

$$n_{xy} = -n_c \sin \theta \cos \theta$$

- For **CHECKING**: Eliminate θ and n_c .
- Solution: $(n_{sx} - n_{xx})(n_{sy} - n_{yy}) \geq n_{xy}^2$

$$n_{sx} \geq n_{xx} \quad n_{sx} \geq 0$$

$$n_{sy} \geq n_{yy} \quad n_{sy} \geq 0$$

- For **Design**: Minimise $n_{sx} + n_{sy}$ as a function of θ.

- Solution:

$$\theta = \pm \frac{\pi}{4}$$

$$n_{sx} = n_{xx} + \left| n_{xy} \right|$$

$$n_{sy} = n_{yy} + \left| n_{xy} \right|$$

$$n_c = -2 \left| n_{xy} \right|$$

EN 1992-2:2005 Annex F.1

Case	n_{sx}	n_{sy}	n_c
$n_x \geq -\lvert n_{xy}\rvert$ $n_y \geq -\lvert n_{xy}\rvert$	$n_x + \lvert n_{xy}\rvert$	$n_y + \lvert n_{xy}\rvert$	$-2\lvert n_{xy}\rvert$
$n_x < -\lvert n_{xy}\rvert$ $n_y \geq \frac{n_{xy}^2}{n_x}$	0	$n_y - \frac{n_{xy}^2}{n_x}$	$n_x + \frac{n_{xy}^2}{n_x}$
$n_x \geq \frac{n_{xy}^2}{n_y}$ $n_y < -\lvert n_{xy}\rvert$	$n_x - \frac{n_{xy}^2}{n_y}$	0	$n_y + \frac{n_{xy}^2}{n_y}$
Other	0	0	$\frac{n_x + n_y}{2} - \sqrt{\left(\frac{n_x - n_y}{2}\right)^2 + n_{xy}^2}$

Example 1

n_{xx}=1200, n_{yy}=-200, n_{xy}=-400 kN/m

f_y = 500 N/mm², f'_c= 30 N/mm²

Thickness = 100 mm

(safety factors included in the numbers)

- Reinforcement

n_{sx}= 1200+400 = 1600 kN/m

n_{sy}= -200+400 = 200

- Concrete

n_c= -2x400 = -800 kN/m

a_{sx}=1600/500 = 3.20 mm = 3200 mm²/m

2∅16–120 = 2π/4x16²x1000/120 = 3351 OK

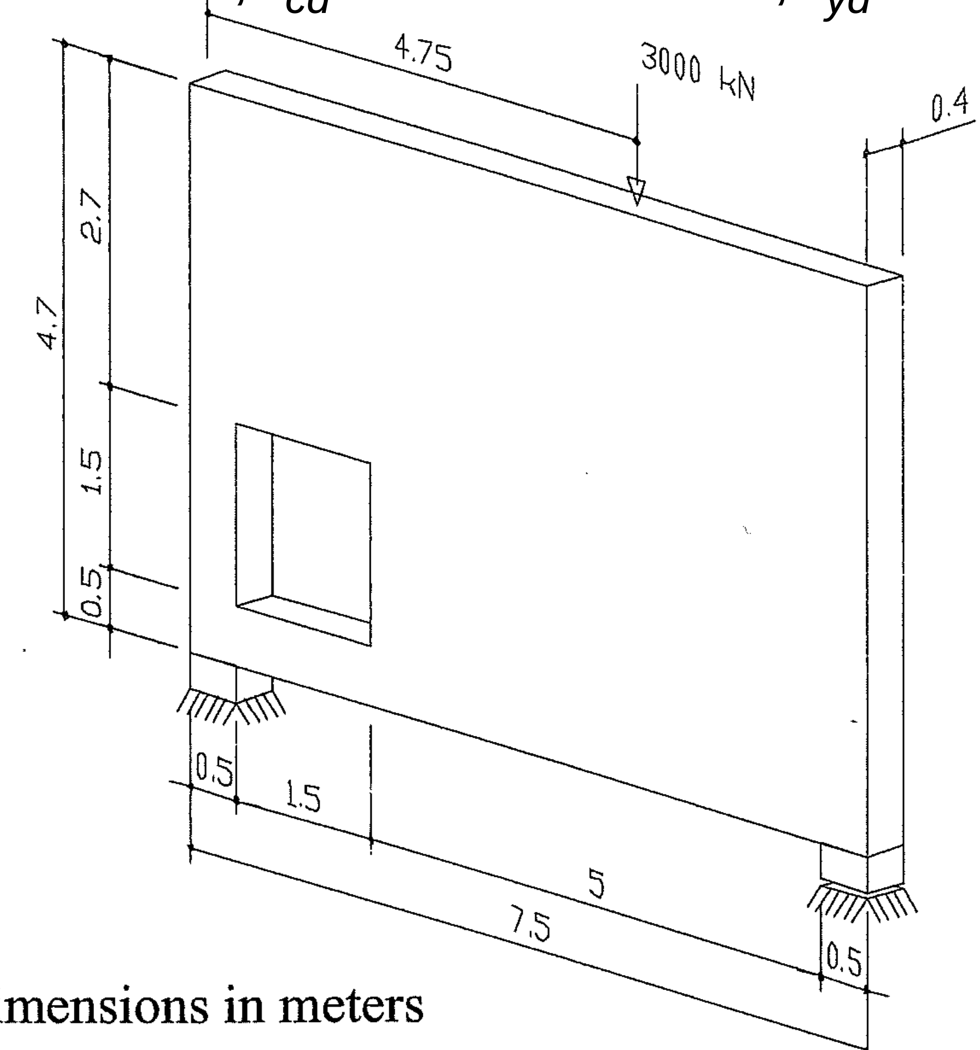
a_{sy}=200/500 = 0.40 mm = 400 mm²/m

2∅12–500 = 2π/4x12²x1000/500 = 452 OK

σ_c = 800/100 = 8 N/mm² < f'_c OK

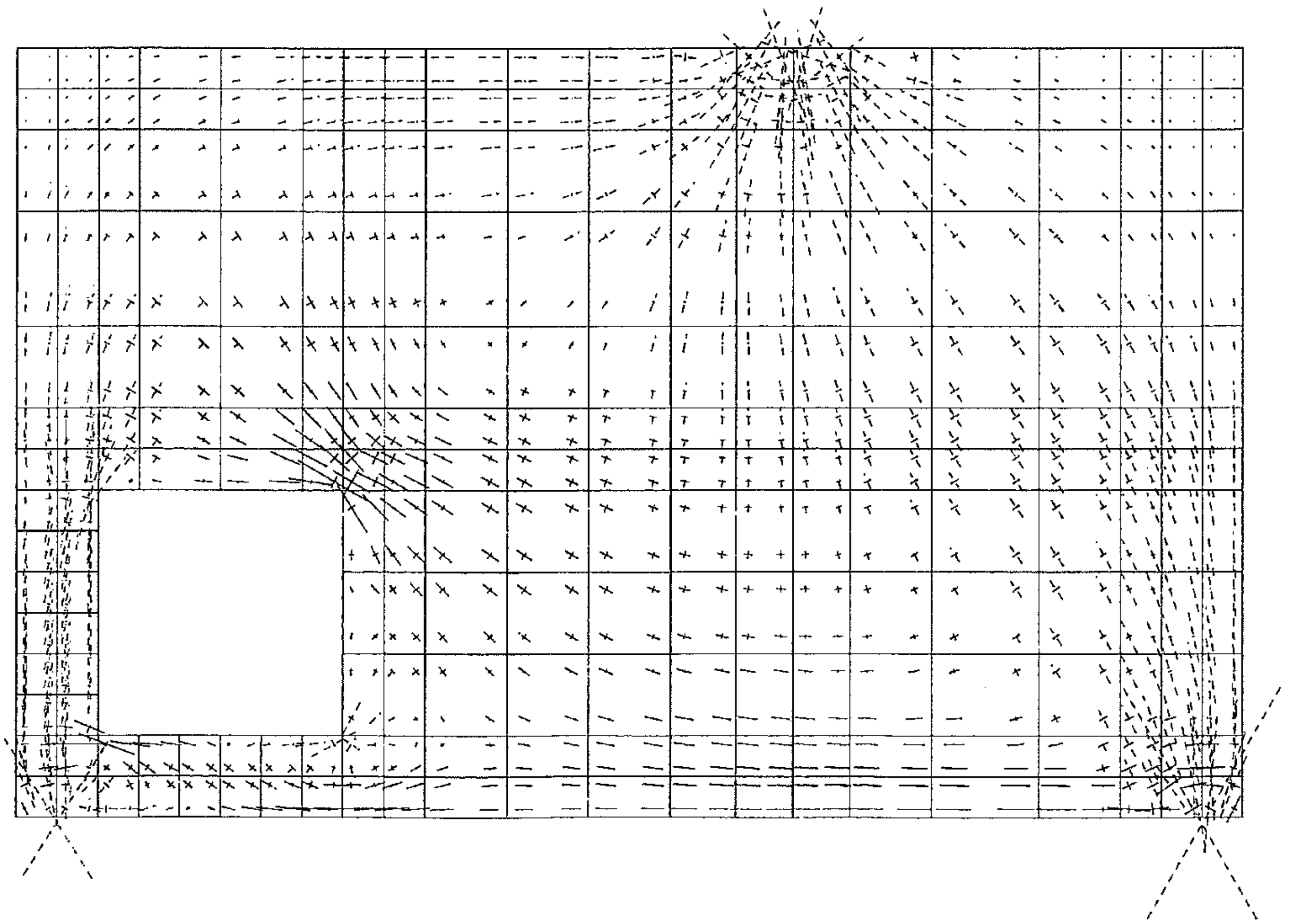
Example 2

- Deep beam 4.7x7.5 m, 2 supports, opening 1.5x1.5 m, point load 3000 kN, f_{cd} = 16.67 N/mm², f_{yd} = 435 N/mm²



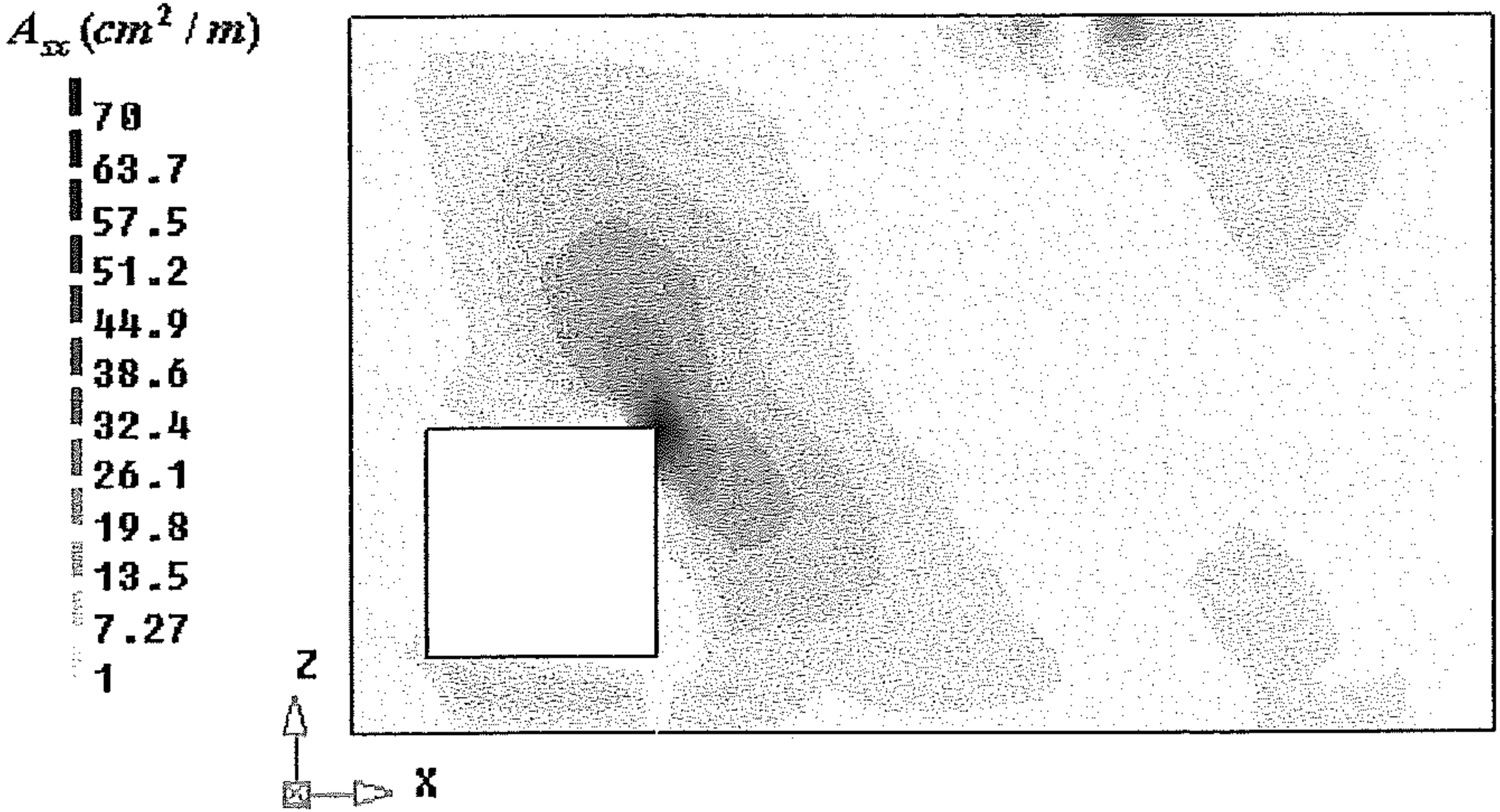
Dimensions in meters

- Forces n_{xx}, n_{yy}, n_{xy} (linear elastic analysis)
- Principal stresses



- Reinforcement requirements

Horizontal reinforcement

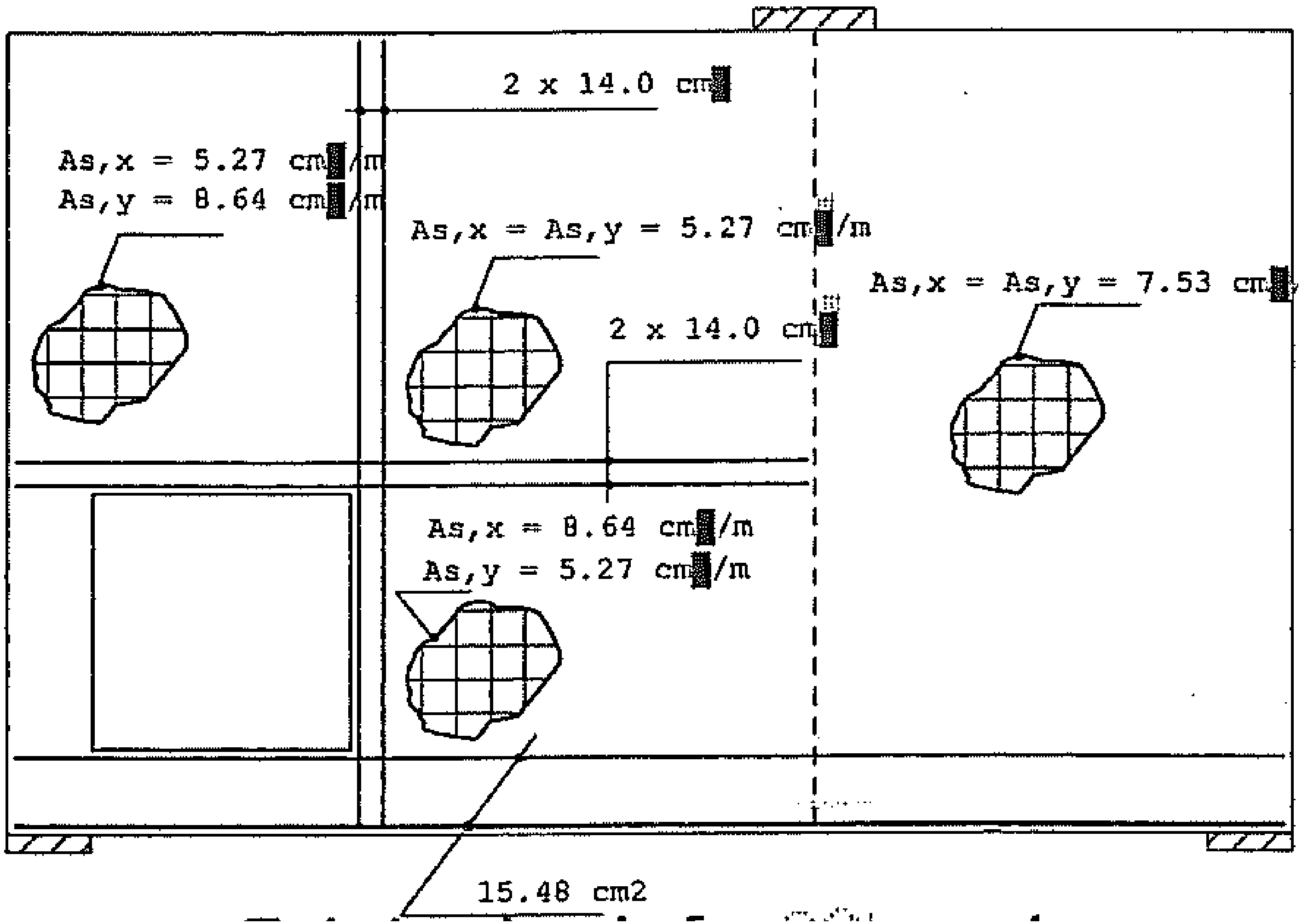


Vertical reinforcement

Note the singularities in previous figures.

Consequently, the peak value of 70 cm^2/m is mesh dependent. However, the integral of the reinforcement requirement over some width is not mesh dependent. Therefore, the designed reinforcement is not mesh dependent.

- Reinforcement (by an engineer)



Plates loaded perpendicularly

t ………. top reinforcement (neg. z)
b ……..... bottom reinforcement (pos. z)

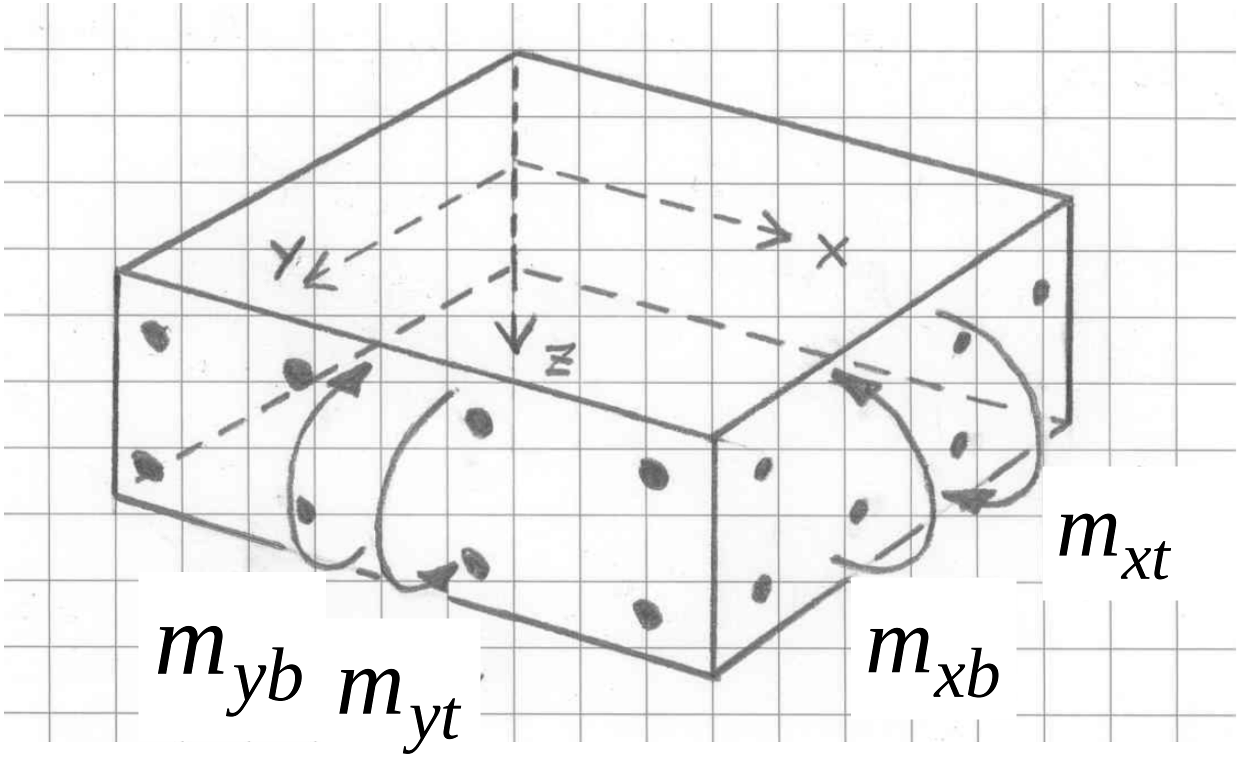
m_{xt} …….. moment capacity due to the top reinforcement in the x direction

m_{yt} ……..

m_{xb} ……..

m_{yb} ……..

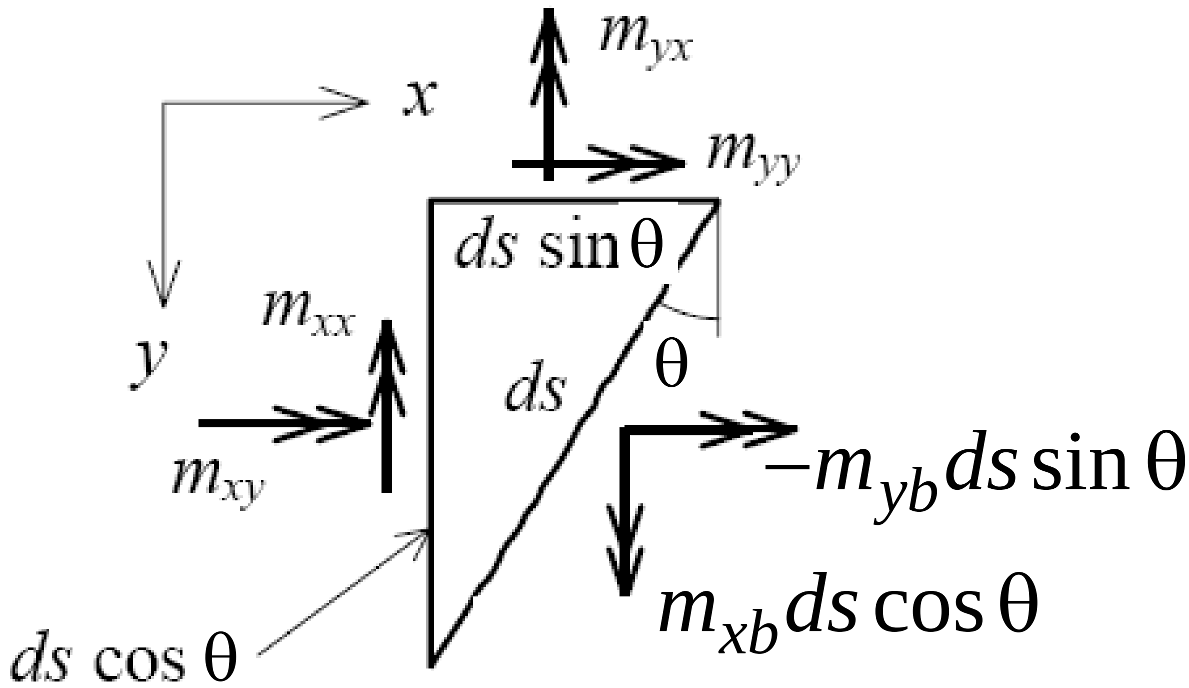
Plates loaded perpendicularly



positive directions of the moment capacities

Plates loaded perpendicularly

- Only rebars in the *x* and *y* directions
- Equilibrium of a plate part (lower bound)



- Equations

$$m_{yy}\sin\theta + m_{yt}\sin\theta + m_{xy}\cos\theta = 0$$

$$m_{xx}\cos\theta + m_{xt}\cos\theta + m_{xy}\sin\theta = 0$$

$$m_{yy}\sin\theta - m_{yb}\sin\theta + m_{xy}\cos\theta = 0$$

$$m_{xx}\cos\theta - m_{xb}\cos\theta + m_{xy}\sin\theta = 0$$

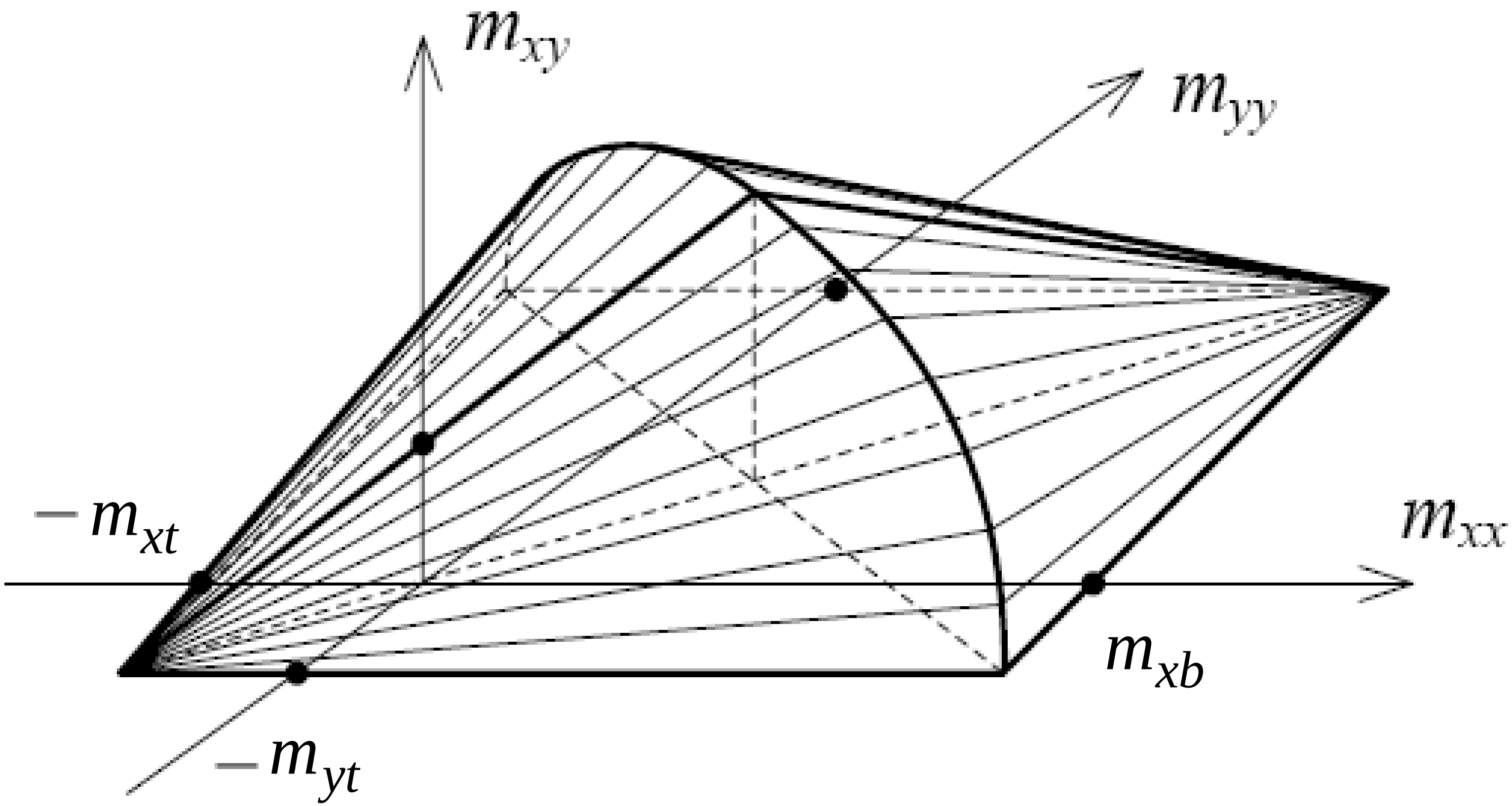
- For **CHECKING**: Eliminate θ .

- Solution:

$$(m_{xt} + m_{xx})(m_{yt} + m_{yy}) \geq m_{xy}^2$$

$$m_{xb} - m_{xx})(m_{yb} - m_{yy}) \geq m_{xy}^2$$

- Yield contour



- Only the half is shown for which $m_{xy} > 0$.

Example 3

- Moments in a point

m_{xx} = 13, m_{yy} = -8, m_{xy} = 5 kNm/m

- Moment capacities

m_{xt} =0, m_{yt} = 10, m_{xb} = 17, m_{yb} = 0

Is the capacity sufficient?

$$5^2 \leq ?(17-13)(0+8),(0+13)(10-8)$$

$$25 \leq ?32,26$$

- Yes

- For **DESIGN**: Carry the moments with the least amount of reinforcement.
- So, minimize $m_{xt} + m_{yt} + m_{xb} + m_{yb}$
- 6 constraints
- $m_{xt}, m_{yt}, m_{xb}, m_{yb} \geq 0$
- $$(m_{xt} + m_{xx})(m_{yt} + m_{yy}) \geq m_{xy}^2$$

$$(m_{xb} - m_{xx})(m_{yb} - m_{yy}) \geq m_{xy}^2$$

Solution 1 (Wood-Armer moments, 1968)

$$m_{xt} = -m_{xx} + \left|m_{xy}\right| \qquad m_{xb} = m_{xx} + \left|m_{xy}\right|$$

$$m_{yt} = -m_{yy} + \left|m_{xy}\right| \qquad m_{yb} = m_{yy} + \left|m_{xy}\right|$$

Solution 2 (when m_{xt} would be < 0)

$$m_{xt} = 0$$

$$m_{yt} = -m_{yy} + \frac{m_{xy}^2}{m_{xx}}$$

Solution 3 (when m_{yt} would be < 0)

$$m_{xt} = -m_{xx} + \frac{m_{xy}^2}{m_{yy}}$$

$$m_{yt} = 0$$

Solution 4 (when m_{xb} would be < 0)

$$m_{xb} = 0$$

$$m_{yb} = m_{yy} - \frac{m_{xy}^2}{m_{xx}}$$

Solution 5 (when m_{yb} would be < 0)

$$m_{xb} = m_{xx} - \frac{m_{xy}^2}{m_{yy}}$$

$$m_{yb} = 0$$

Solution 6 (when m_{xt} and m_{yt} would be < 0)

$$m_{xt} = 0$$

$$m_{yt} = 0$$

Solution 7 (when m_{xb} and m_{yb} would be < 0)

$$m_{xb} = 0$$

$$m_{yb} = 0$$

Example 4

- Moments in a point (as in example 3)

m_{xx} = 13, m_{yy} = -8, m_{xy} = 5 kNm/m

- Moment capacities

m_{xt} = 0 m_{xb} = 13+5²/8 = 16.13

m_{yt} = 8+5²/13 = 9.92 m_{yb} = 0

- Amount of reinforcement is proportional to

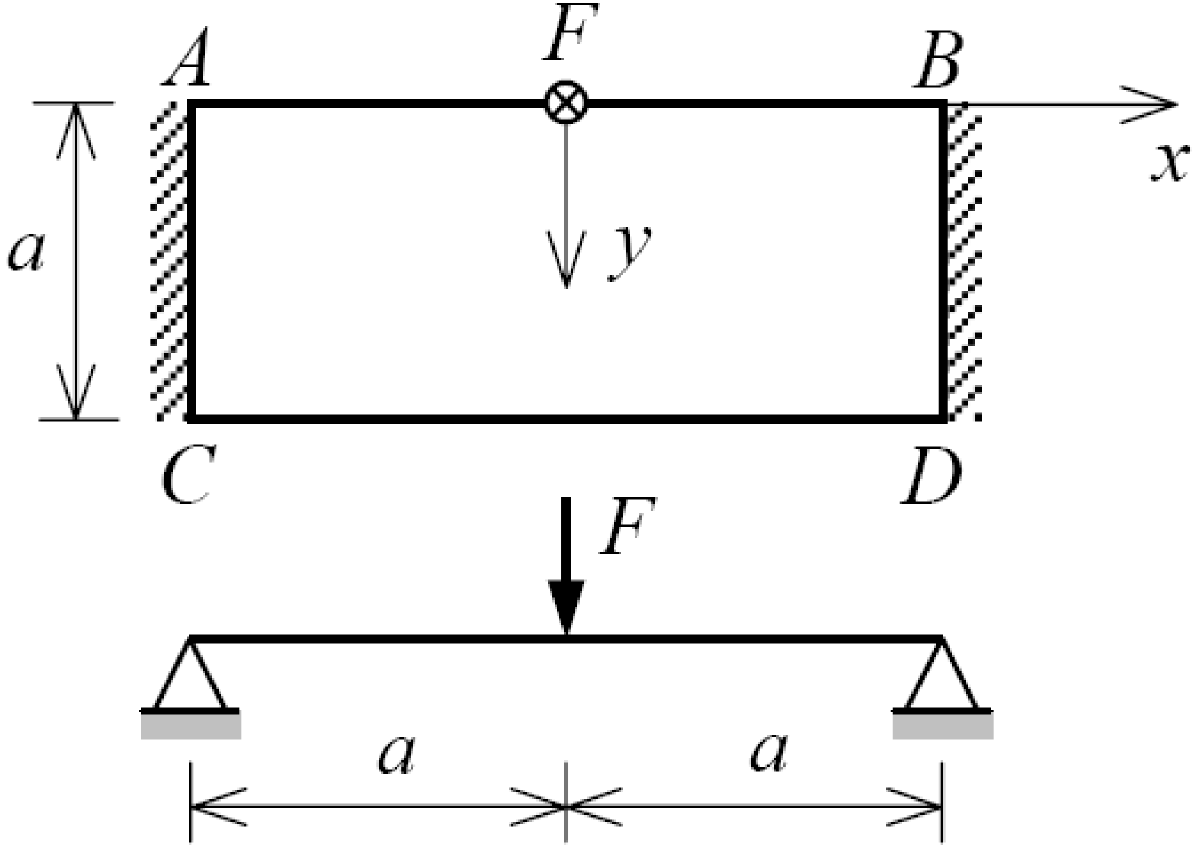
16.13+0+0+9.92 = 26

- Amount of reinforcement in example 3

17+0+0+10 = 27 (larger, so not optimal)

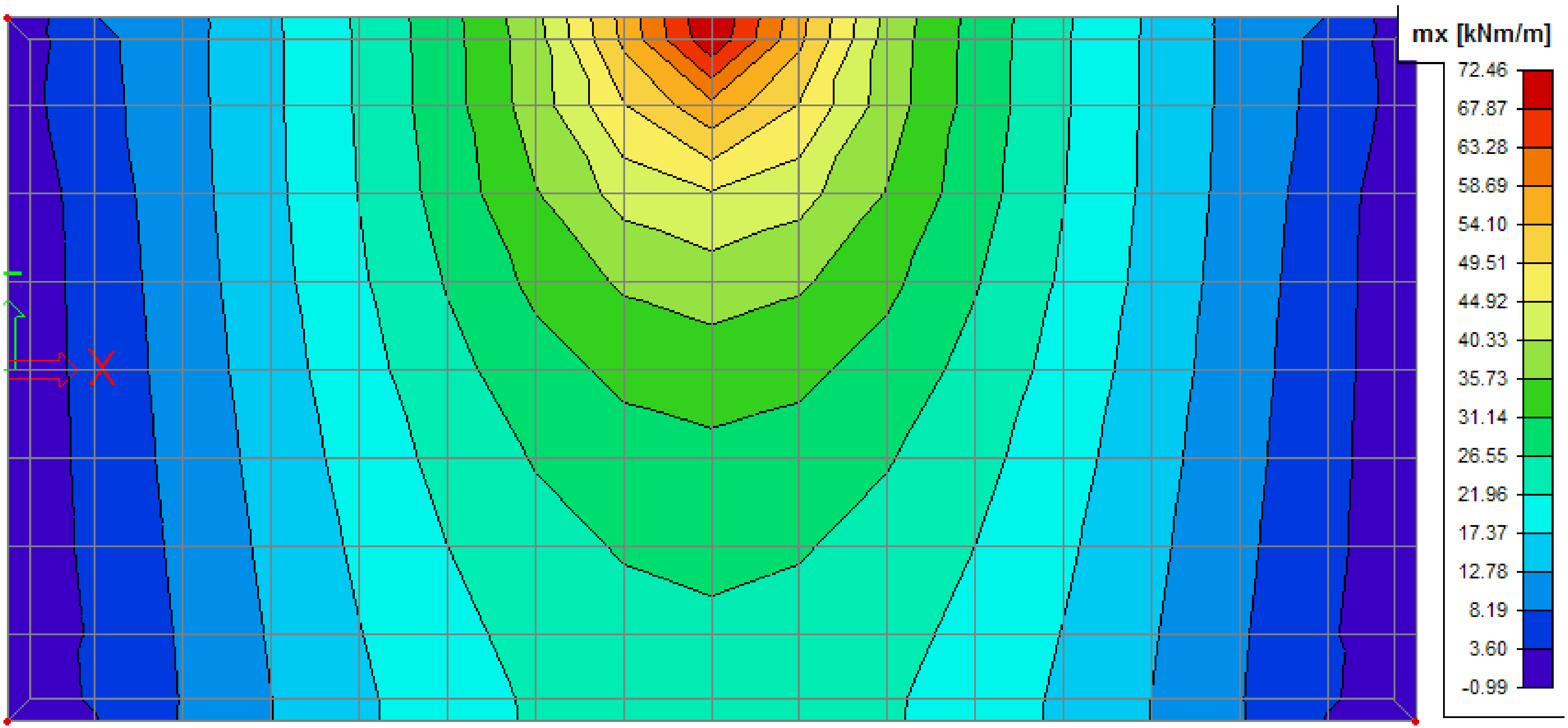
Example 5

- Plate bridge, simply supported
- 4 x 8 m, point load 80 kN, thick 0.25 m

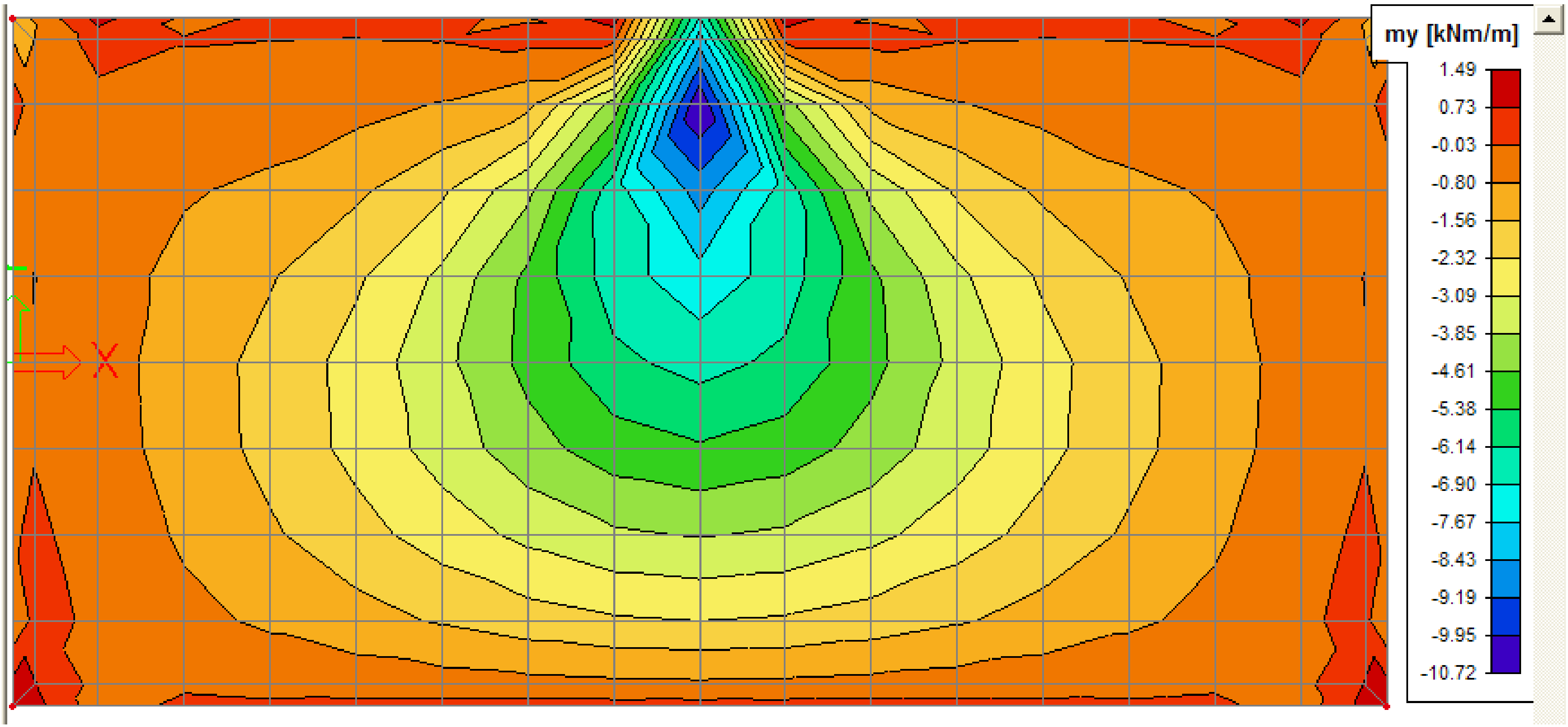


FEM moments

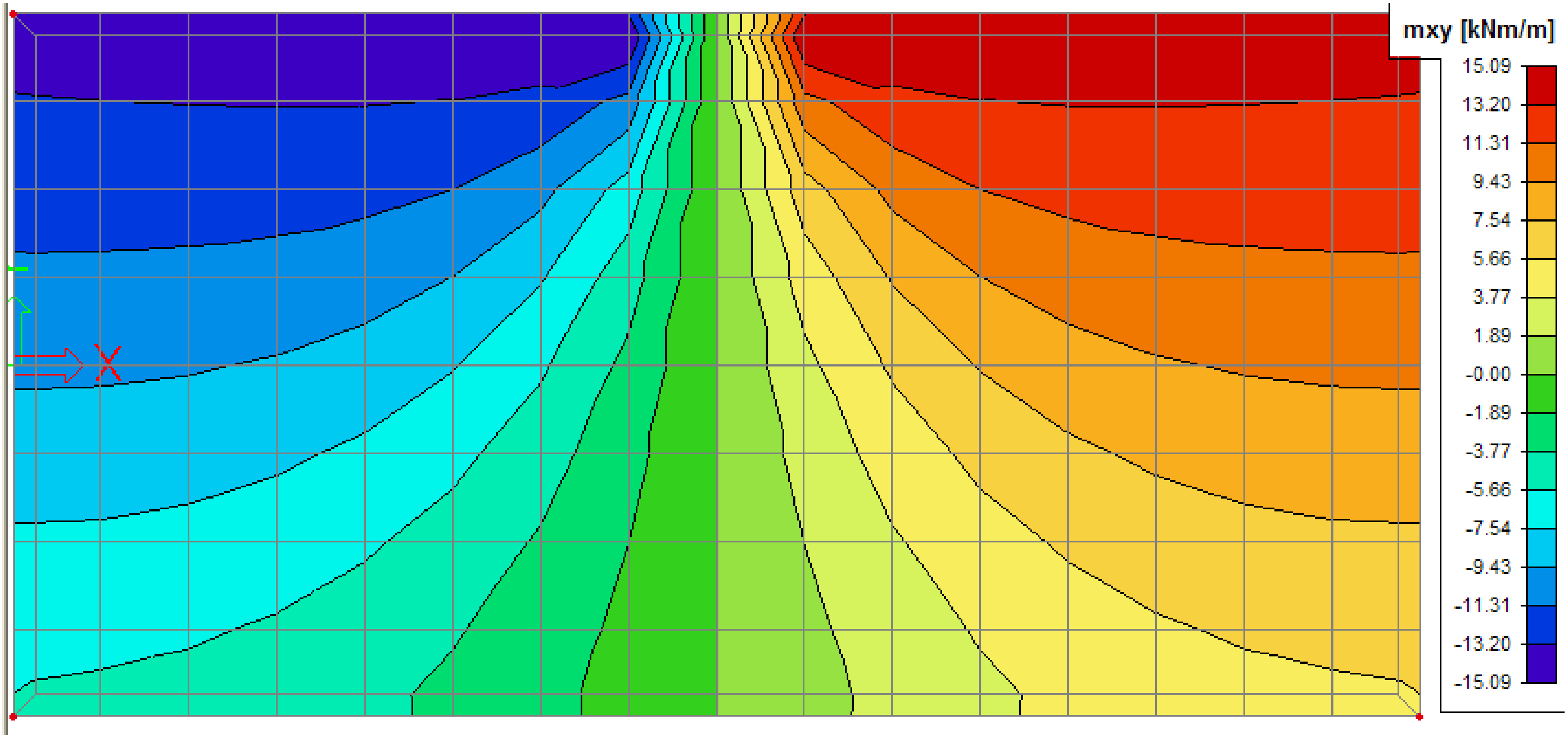
m_{xx}



m_{yy}

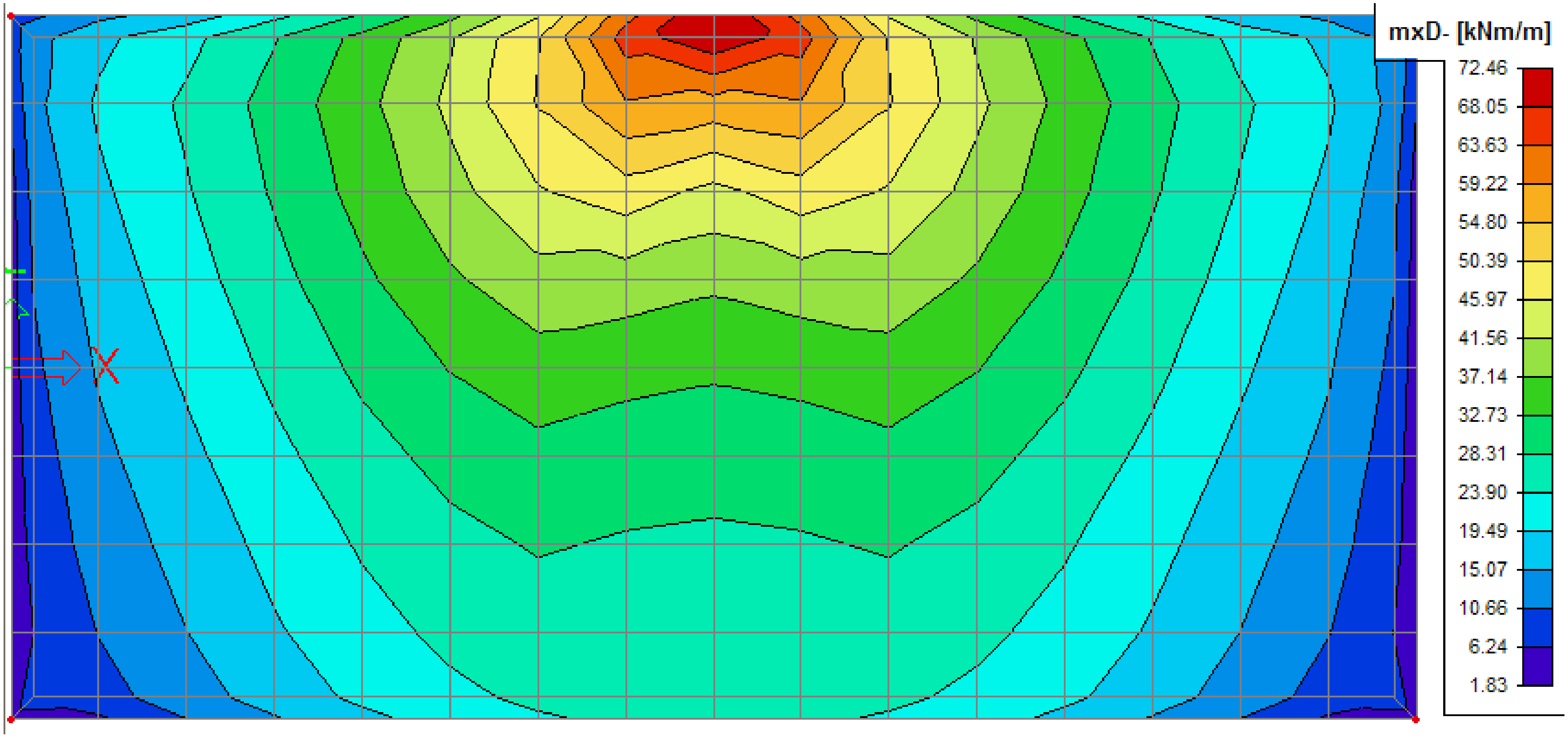

my [kNm/m]
1.49
0.73
-0.03
-0.80
-1.56
-2.32
-3.09
-3.85
-4.61
-5.38
-6.14
-6.90
-7.67
-8.43
-9.19
-9.95
-10.72
X

m_{xy}

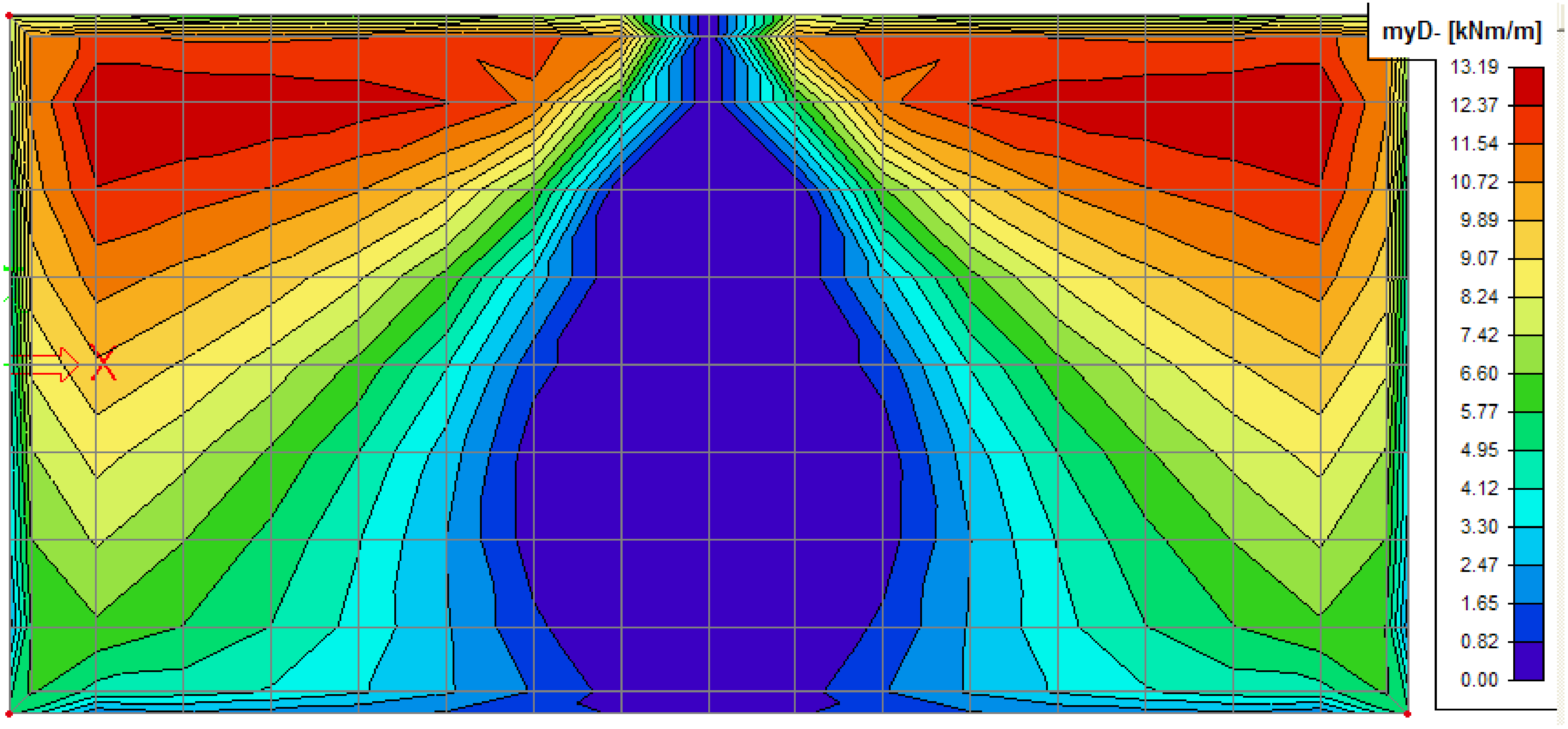

mxy [kNm/m]
15.09
13.20
11.31
9.43
7.54
5.66
3.77
1.89
-0.00
-1.89
-3.77
-5.66
-7.54
-9.43
-11.31
-13.20
-15.09
X

Reinforcement requirements

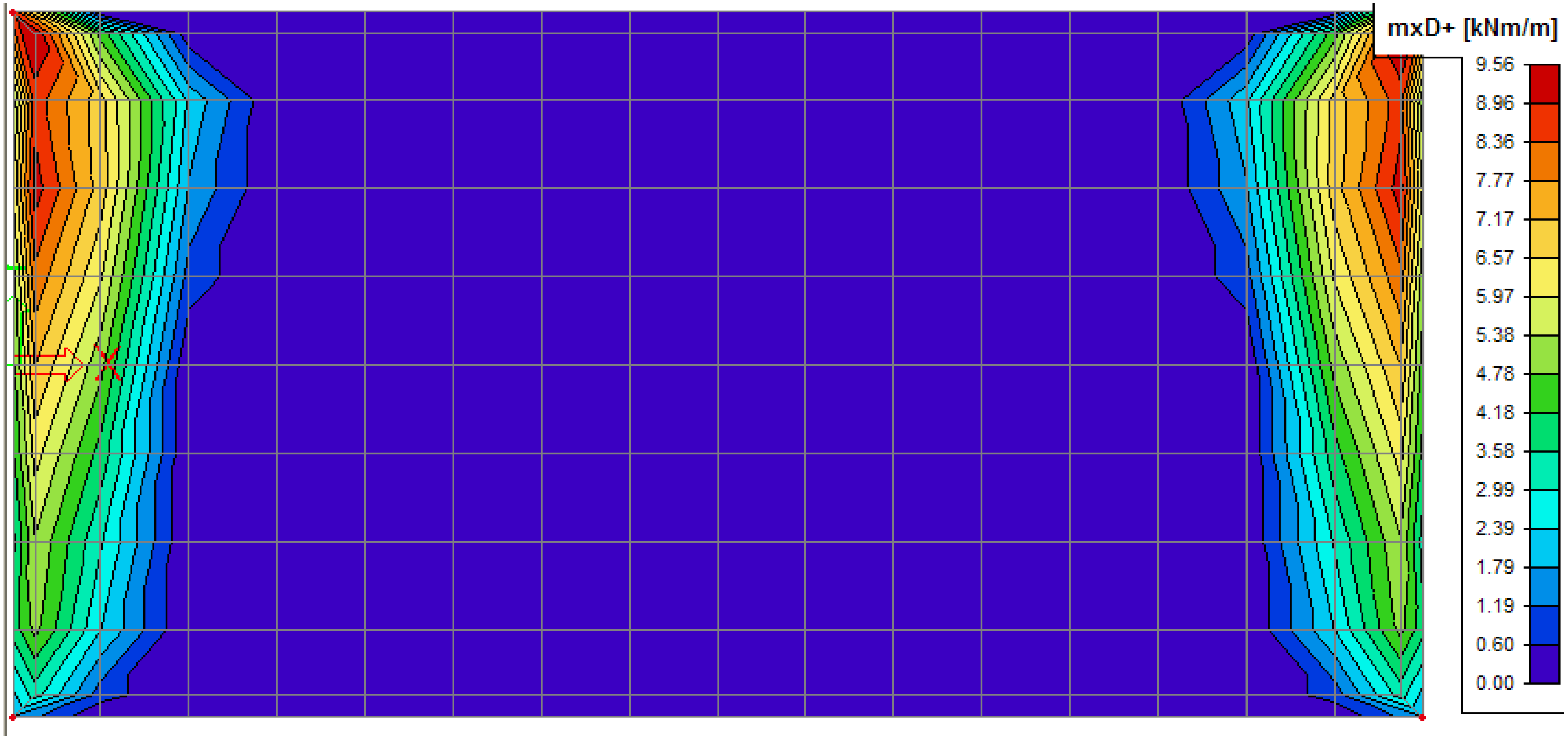
m_{xb}



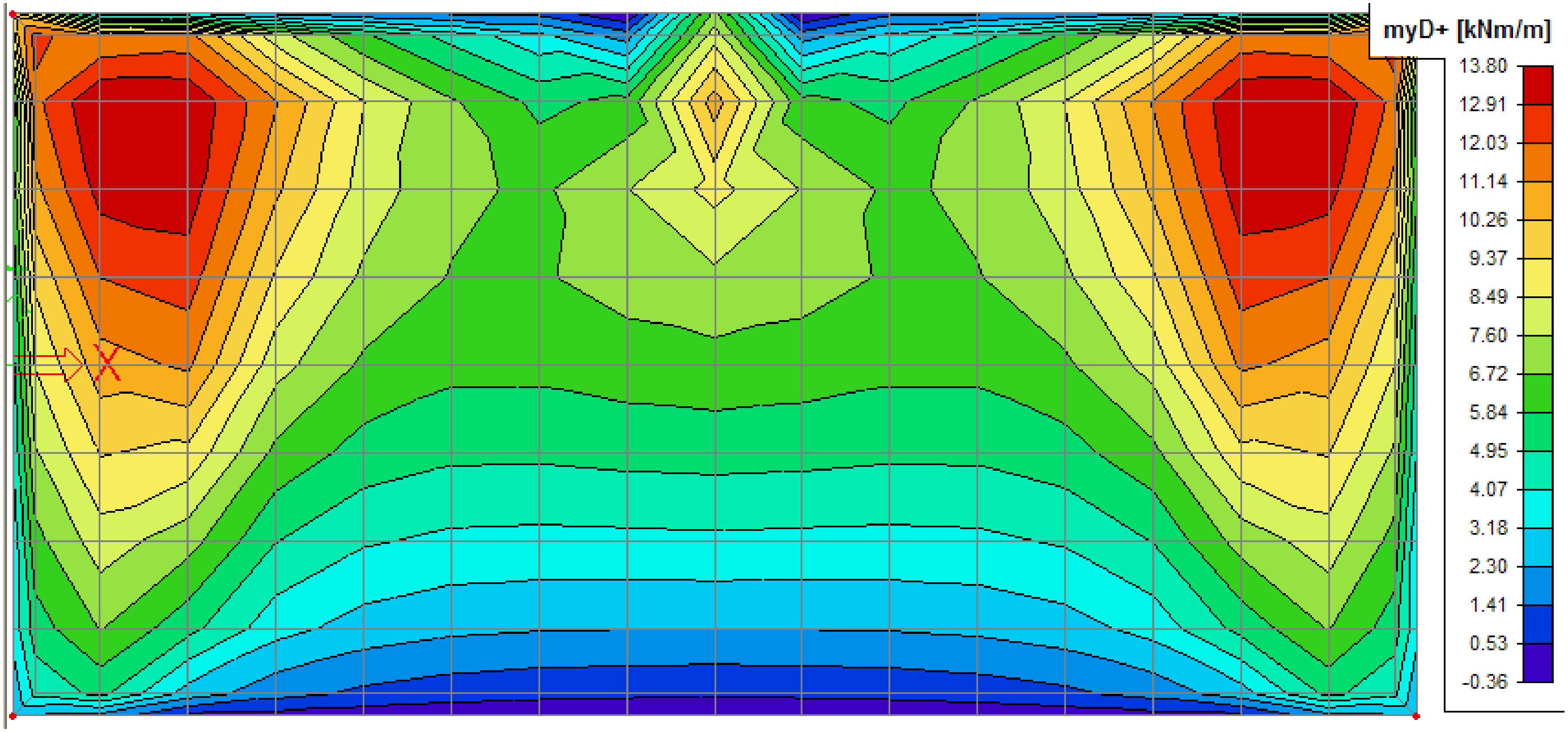
m_{yb}


myD- [kNm/m]
13.19
12.37
11.54
10.72
9.89
9.07
8.24
7.42
6.60
5.77
4.95
4.12
3.30
2.47
1.65
0.82
0.00

m_{xt}


mxD+ [kNm/m]
9.56
8.96
8.36
7.77
7.17
6.57
5.97
5.38
4.78
4.18
3.58
2.99
2.39
1.79
1.19
0.60
0.00

m_{yt}



myD+ [kNm/m]
13.80
12.91
12.03
11.14
10.26
9.37
8.49
7.60
6.72
5.84
4.95
4.07
3.18
2.30
1.41
0.53
-0.36
X

The **design** equations are conservative for

load combinations.

Example

Combination 1: $m_{xx} = 4 \quad m_{yy} = 5 \quad m_{xy} = 3 \quad \text{kNm/m}$

Combination 2: $m_{xx} = 5 \quad m_{yy} = 4 \quad m_{xy} = 3 \quad \text{kNm/m}$

Design $m_{xb} = m_{xx} + \left| m_{xy} \right| = 4 + 3 = 7 \text{ or } 5 + 3 = 8 \text{ so } 8$

$$m_{yb} = m_{yy} + \left| m_{xy} \right| = 5 + 3 = 8 \text{ or } 4 + 3 = 7 \text{ so } 8$$

Check

$$(m_{xb} - m_{xx})(m_{yb} - m_{yy}) \geq m_{xy}^2 \qquad (8-4)(8-5) = 12 \geq ?\, 3 \times 3 = 9$$

$$(8-5)(8-4) = 12 \geq ?\, 9$$

OK … but 7.6 would do too.

$$(7.6-4)(7.6-5) = 9.4 \geq ?\, 9$$

$$(7.6-5)(7.6-4) = 9.4 \geq ?\, 9$$

The **check** equations can be rewritten as **unity checks**.

Example

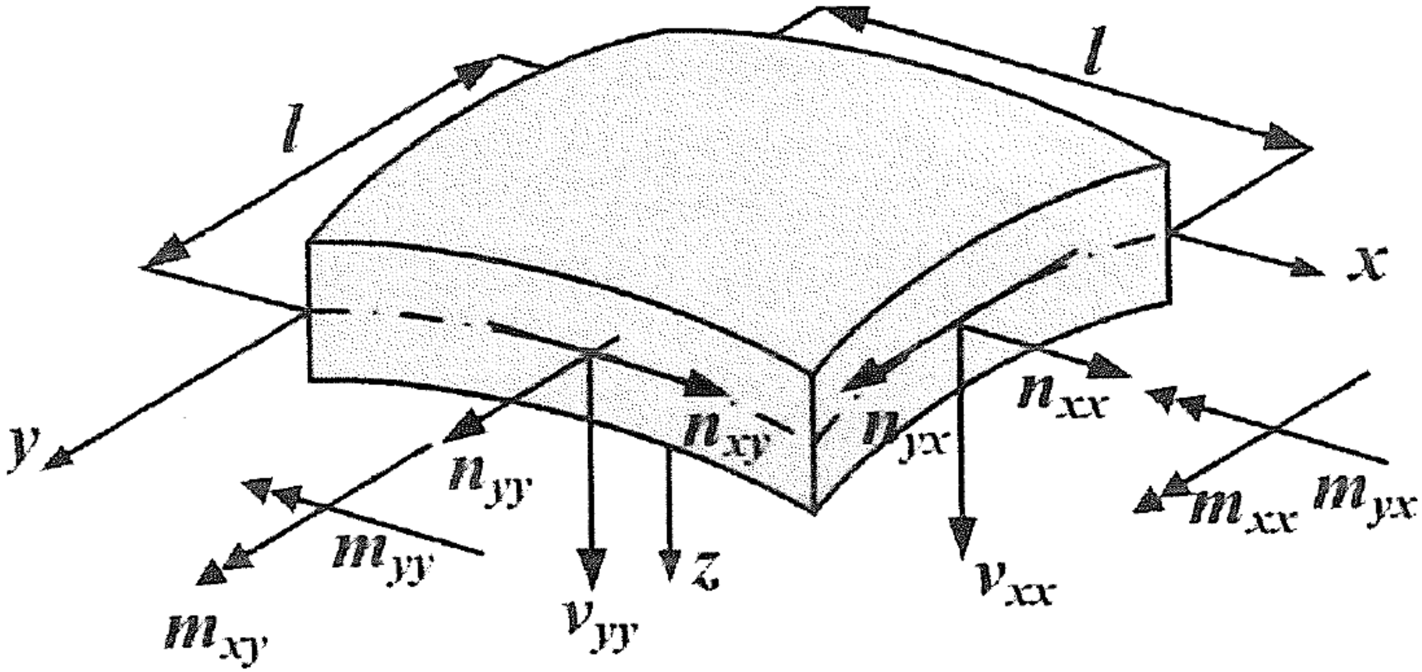
$$(m_{xb} - m_{xx})(m_{yb} - m_{yy}) \geq m_{xy}^2 \quad \Rightarrow$$

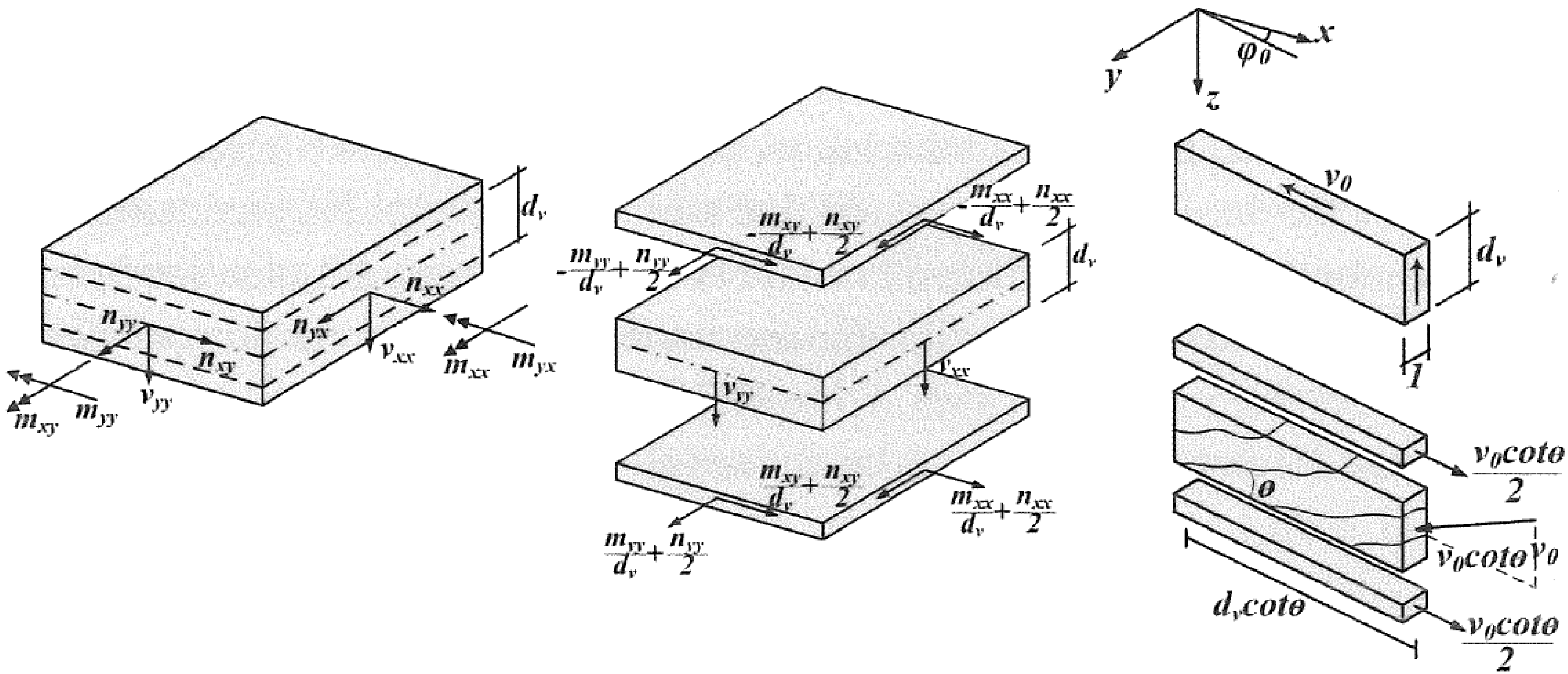
$$(\mu\, m_{xb} - m_{xx})(\mu\, m_{yb} - m_{yy}) = m_{xy}^2 \qquad \mu \leq 1 \quad \Rightarrow$$

$$\mu = \frac{1}{2}\left(\frac{m_{xx}}{m_{xb}} + \frac{m_{yy}}{m_{yb}}\right) + \sqrt{\frac{1}{4}\left(\frac{m_{xx}}{m_{xb}} - \frac{m_{yy}}{m_{yb}}\right)^2 + \frac{m_{xy}^2}{m_{xb} m_{yb}}} \leq 1$$

- Interpretation: Principal value of a tensor
- Advantage: Clear how much extra reinforcement is needed
- Disadvantage: Division by zero is possible.

For **shells** the software can design or check with a three layer **sandwich model**.





d_v
n_{xx}
n_{yy}
n_{yx}
n_{xy}
v_{xx}
v_{yy}
m_{xx}
m_{yx}
m_{xy}
m_{yy}
$-\frac{m_{xy}}{d_v}+\frac{n_{xy}}{2}$
$-\frac{m_{xx}}{d_v}+\frac{n_{xx}}{2}$
$-\frac{m_{yy}}{d_v}+\frac{n_{yy}}{2}$
$\frac{m_{xy}}{d_v}+\frac{n_{xy}}{2}$
$\frac{m_{xx}}{d_v}+\frac{n_{xx}}{2}$
$\frac{m_{yy}}{d_v}+\frac{n_{yy}}{2}$
x
y
z
φ_0
v_0
1
θ
$\frac{v_0 \cot\theta}{2}$
$v_0 \cot\theta$
$d_v \cot\theta$

Conclusions

We can ask the computer to **CHECK** the reinforcement.

$$(n_{sx} - n_{xx})(n_{sy} - n_{yy}) \geq n_{xy}^2$$

$$(m_{xt} + m_{xx})(m_{yt} + m_{yy}) \geq m_{xy}^2$$

$$(m_{xb} - m_{xx})(m_{yb} - m_{yy}) \geq m_{xy}^2$$

We can ask the computer for help in **DESIGNING** reinforcement.

$$n_{sx} = n_{xx} + |n_{xy}| \qquad m_{xt} = -m_{xx} + |m_{xy}| \qquad m_{xb} = m_{xx} + |m_{xy}|$$

$$n_{sy} = n_{yy} + |n_{xy}| \qquad m_{yt} = -m_{yy} + |m_{xy}| \qquad m_{yb} = m_{yy} + |m_{xy}|$$