Notes on the element size in assignment 2

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Finite elements approximate the exact solution to the differential equations. The smaller the elements the more accurate the solution. They also approximate the boundary conditions on moments and shear forces (dynamic boundary conditions). You decide what accuracy is sufficient. (3 digit accuracy, < 1% error, very accurate for most engineering applications).

The boundary conditions on displacements and rotations (kinematic boundary conditions) are fulfilled exactly (16 digit accuracy). You specify these boundary conditions when using the software.

Recommended element size:
For thin plate theory (Kirchhoff) use 2 times the plate thickness.
For thick plate theory (Mindlin-Reissner) use 1/10 of the plate thickness at the edges.

This recommendation is not a law!

The thin plate recommendation has been derived as follows. A thin plate was modelled in SCIA Engineer using quadrilateral elements and Kirchhoff theory. The plate was circular, simply supported and loaded by one perpendicular force in the middle. The largest stress occurs in the singularity under the force. A graph was made of the largest stress as a function of the element size. The graphs shows that this stress becomes very large for very small elements.
Timoshenko gives the exact solution to this problem based on 3D elasticity theory. In Timoshenko's analysis the load is evenly distributed over a circular area with a diameter of the plate thickness. The graph shows that the exact stress is obtained for an element size of twice the thickness. Hence the recommendation.

Triangular elements are less accurate than quadrilateral elements. (The shape functions of quads can move in more ways.) Therefore, triangular elements need to be smaller.

The exact solution is always smooth. So if the finite element result is bumpy, it is not accurate.

If you halve the element size and important results (your decision) do not change much (your decision) then you are done. Otherwise halve the element size again.

Except for singularities. In singularities smaller elements give larger moments, larger shear forces, larger membrane forces and larger stresses. If you make the elements ridiculously small then all colours of the contour plots will disappear in the peaks of the singularities. Then you need to redefine the colour scale. Which is extra work. Nonetheless, using ridiculously small elements is not wrong. It is just unpractical.