Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT5141 Theory of Elasticity
Friday 5 November 2004, 9:00 – 12:00 hours

Problem 1 (3 points)

A curved beam of rectangular cross-section is loaded by a moment *M* (Figure 1). The material is laminated wood and the fibres are directed in the circumferential direction. The kinematic equations are

$$\varepsilon_{rr} = \frac{du}{dr}$$
$$\varepsilon_{\theta\theta} = \frac{u}{r}.$$

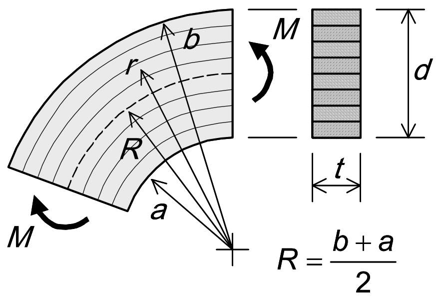


Figure 1. Curved beam of laminated wood

The constitutive equations are

$$\varepsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E_{\|}} - \frac{\nu\,\sigma_{rr}}{E_{\|}} + \varepsilon_i$$
$$\varepsilon_{rr} = \frac{\sigma_{rr}}{E_{\perp}} - \frac{\nu\,\sigma_{\theta\theta}}{E_{\|}},$$

which can be inverted to give

$$\sigma_{\theta\theta} = \frac{E_{\|}}{E_{\|} - \nu^2 E_{\perp}}\left(E_{\|}\left(\varepsilon_{\theta\theta} - \varepsilon_i\right) + \nu E_{\perp}\varepsilon_{rr}\right)$$
$$\sigma_{rr} = \frac{E_{\perp}E_{\|}}{E_{\|} - \nu^2 E_{\perp}}\left(\varepsilon_{rr} + \nu\left(\varepsilon_{\theta\theta} - \varepsilon_i\right)\right).$$

The equilibrium equation is

$$\sigma_{\theta\theta} = \sigma_{rr} + r\frac{d\sigma_{rr}}{dr}.$$

a Write the constitutive equations in matrix notation. Is this matrix symmetrical? What is the meaning of $E_{\|}$ and $E_{\perp}$?

b Derive a differential equation for solving this problem. Use either the displacement method or the force method (You do not need to do both).

c Formulate the boundary conditions that need be used to solve the differential equation.

Maple solves the differential equation and gives the following solution of the stress $\sigma_{\theta\theta}$ in the circumferential direction.

$$\sigma_{\theta\theta} = E_{||}\varepsilon_i \frac{(b^{2\alpha} - a^{2\alpha}) - \alpha(b^{\alpha+1} - a^{\alpha+1})r^{\alpha-1} - \alpha(b^{\alpha+1}a^{2\alpha} - a^{\alpha+1}b^{2\alpha})r^{-\alpha-1}}{(\alpha^2 - 1)(b^{2\alpha} - a^{2\alpha})} .$$

where

$$\alpha = \sqrt{\frac{E_{||}}{E_{\perp}}}$$

In Figure 2 this stress is plotted for $\alpha = 5$, $a = 200$ and $b = 400$.

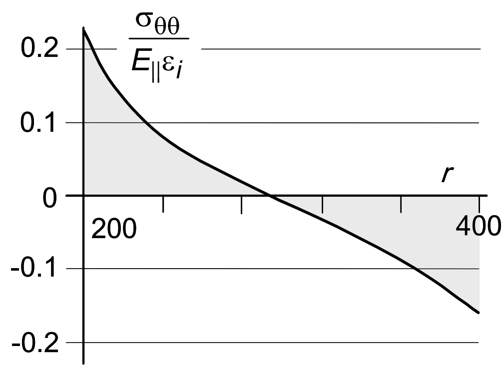


Figure 2. $\frac{\sigma_{\theta\theta}}{E_{||}\varepsilon_i}$ as a function of r

d Give the formulae for deriving the moment M and the normal force N in the cross-section (you do not need to evaluate the formulae).

Maple evaluates these formulae as

$$M = \tfrac{1}{2} t\, E_{||}\varepsilon_i \frac{(\alpha-1)^2(b^{\alpha+1} - a^{\alpha+1})^2 - a^2b^2(\alpha+1)^2(b^{\alpha-1} - a^{\alpha-1})^2}{(\alpha^2-1)^2(b^{2\alpha} - a^{2\alpha})}$$

$$N = 0 .$$

e Derive the formula for the section modulus W for calculating the extreme stress

$$\hat{\sigma}_{\theta\theta} = \frac{M}{W} .$$

Problem 2 (3 points)

A reinforced concrete barge (Dutch: ponton) is lifted at two corners (Figure 3). The barge is loaded by self-weight. The material density is ρ [kg/m³] and the gravitation is g [m/s²]. All walls of the barge and the bottom plate have a thickness t. In this problem we calculate the deflection w of the barge and the largest stress using minimum potential energy.

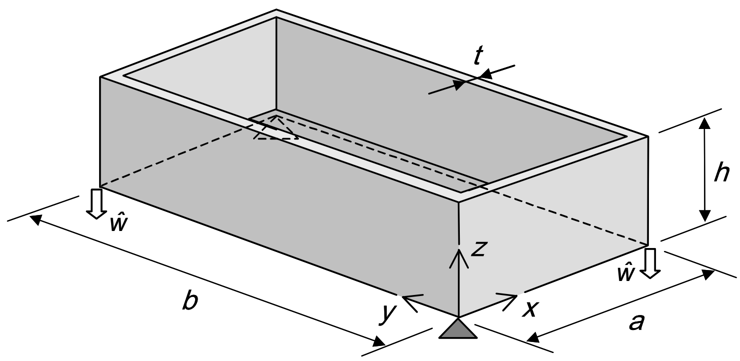


Figure 3. Barge supported at two opposite corners

We assume that the bottom plate has the following deflection (Figure 4).

$$w = \hat{w}\left(\frac{x}{a} + \frac{y}{b} - 2\frac{xy}{ab}\right)$$

a Does the barge deformation fulfil the kinematic boundary conditions? Explain your answer.

b Show that the bottom plate and each wall experience the same constant torsion

$$\rho_{xy} = \frac{4\hat{w}}{ab}.$$

c Give the formula for the potential energy of the barge. (You do not need to evaluate this formula).

This formula can be evaluated to the following result

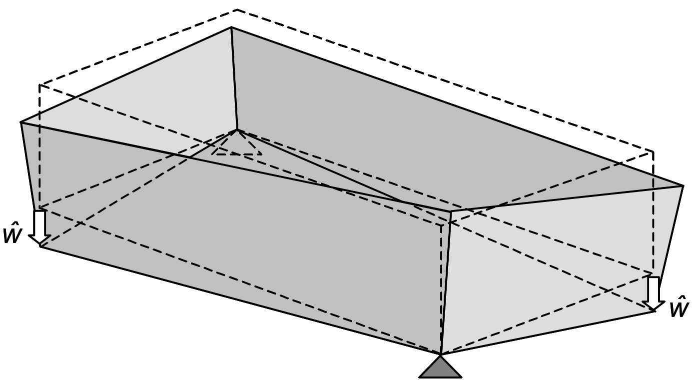


Figure 4. Barge deformation

$$E_{pot} = \tfrac{1}{3}\frac{E t^3 \hat{w}^2}{\nu+1}\frac{ab+2(a+b)h}{a^2 b^2} - \tfrac{1}{2}\rho g \hat{w} t\left(ab+2(a+b)h\right)$$

d Calculate the displacement $\hat{w}$ of the barge free corners. Explain why the height h does not appear in the formula.

e Calculate the torsion moment m_{xy}.

f Calculate the largest principal stress σ_1.

Useful formulae for plates

$$m_{xx} = D(\kappa_{xx} + \nu\kappa_{yy}) \qquad \kappa_{xx} = -\frac{\partial^2 w}{\partial x^2}$$

$$m_{yy} = D(\kappa_{yy} + \nu\kappa_{xx}) \qquad \kappa_{yy} = -\frac{\partial^2 w}{\partial y^2} \qquad D = \frac{E t^3}{12(1-\nu^2)}$$

$$m_{xy} = \tfrac{1}{2}D(1-\nu)\rho_{xy} \qquad \rho_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

Problem 3 (3 points)

A box-girder bridge has reinforced concrete flanges and corrugated steel plate webs (Figure 5, 6). The shear stiffness of concrete is G_c while the shear stiffness of steel is G_s. In this problem we derive a formula for the torsion stiffness $G\mathcal{I}_t$ of the cross-section.

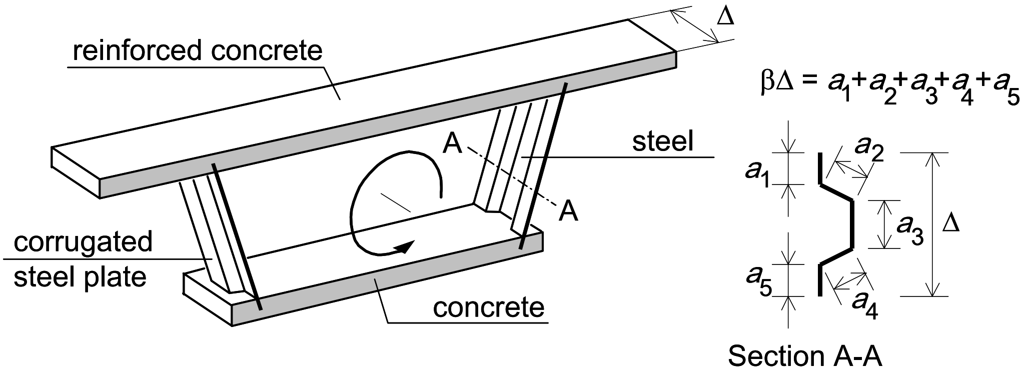


Figure 5. Section of a box-girder bridge

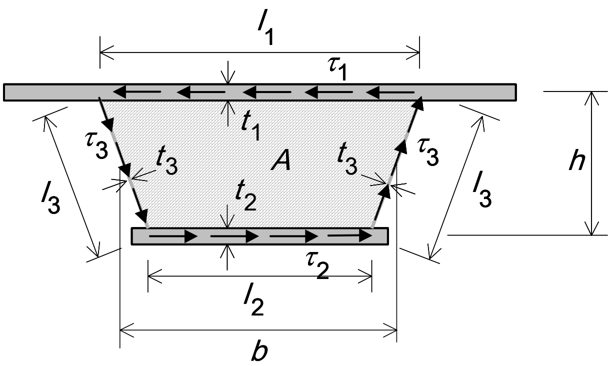


Figure 6. Cross-section dimensions of the box-girder

a Bredt's second formula states that $\tau_1 = \dfrac{M_t}{2At_1}$, $\tau_2 = \dfrac{M_t}{2At_2}$ and $\tau_3 = \dfrac{M_t}{2At_3}$. This formula is also valid for inhomogeneous cross-sections. Check this by calculating the box-girder torsion moment that the shear stresses τ_1, τ_2 and τ_3 produce.

b Formulate the complementary energy E_c of a slice of the box-girder with thickness Δ and show that this can be evaluated to the following result

$$E_c = \Delta \tfrac{1}{8}\left(\frac{M_t}{A}\right)^2 \left(\frac{l_1}{G_c t_1} + \frac{l_2}{G_c t_2} + \frac{2\beta l_3}{G_s t_3}\right).$$

c Formulate the complementary energy E_c of a wire model of the box-girder with length Δ.

d Derive the torsion stiffness GI_t.

Problem 4 (1 point)

Consider the following statement.

In thin-wall open sections the largest torsion stress occurs in the thickest wall part, while in thin-wall single-cell closed sections the largest torsion stress occurs in the thinnest wall part.

Is this statement correct? Explain your answer.

Exam CT5141, 5 November 2004
Answers to Problem 1

a Symmetry

$$\begin{bmatrix}\varepsilon_{\theta\theta}\\ \varepsilon_{rr}\end{bmatrix}=\begin{bmatrix}\dfrac{1}{E_{\|}} & -\dfrac{\nu}{E_{\|}}\\ -\dfrac{\nu}{E_{\|}} & \dfrac{1}{E_{\perp}}\end{bmatrix}\begin{bmatrix}\sigma_{\theta\theta}\\ \sigma_{rr}\end{bmatrix}+\begin{bmatrix}\varepsilon_i\\ 0\end{bmatrix}$$

The material matrix is symmetrical. $E_{\|}$ is Young's modulus in the circumferential direction or fibre direction. $E_{\perp}$ is Young's modulus in the radial direction or perpendicular to the fibres.

b Displacement Method

$$\sigma_{\theta\theta}=\sigma_{rr}+r\frac{d\sigma_{rr}}{dr}$$

$$\frac{E_{\|}}{E_{\|}-\nu^2E_{\perp}}\left(E_{\|}\left(\varepsilon_{\theta\theta}-\varepsilon_i\right)+\nu E_{\perp}\varepsilon_{rr}\right)=\frac{E_{\perp}E_{\|}}{E_{\|}-\nu^2E_{\perp}}\left(\varepsilon_{rr}+\nu\left(\varepsilon_{\theta\theta}-\varepsilon_i\right)\right)+r\frac{E_{\perp}E_{\|}}{E_{\|}-\nu^2E_{\perp}}\left(\frac{d\varepsilon_{rr}}{dr}+\nu\frac{d\varepsilon_{\theta\theta}}{dr}\right)$$

$$E_{\|}\left(\varepsilon_{\theta\theta}-\varepsilon_i\right)+\nu E_{\perp}\varepsilon_{rr}=E_{\perp}\left(\varepsilon_{rr}+\nu\left(\varepsilon_{\theta\theta}-\varepsilon_i\right)\right)+rE_{\perp}\left(\frac{d\varepsilon_{rr}}{dr}+\nu\frac{d\varepsilon_{\theta\theta}}{dr}\right)$$

$$\left(E_{\|}-\nu E_{\perp}\right)\varepsilon_{\theta\theta}-(1-\nu)E_{\perp}\varepsilon_{rr}-rE_{\perp}\frac{d\varepsilon_{rr}}{dr}-\nu rE_{\perp}\frac{d\varepsilon_{\theta\theta}}{dr}=\left(E_{\|}-\nu E_{\perp}\right)\varepsilon_i$$

$$\left(E_{\|}-\nu E_{\perp}\right)\frac{u}{r}-(1-\nu)E_{\perp}\frac{du}{dr}-rE_{\perp}\frac{d^2u}{dr^2}-\nu rE_{\perp}\left(\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}\right)=\left(E_{\|}-\nu E_{\perp}\right)\varepsilon_i$$

$$\boxed{E_{\|}\frac{u}{r}-E_{\perp}\frac{du}{dr}-E_{\perp}r\frac{d^2u}{dr^2}=\left(E_{\|}-\nu E_{\perp}\right)\varepsilon_i}$$

Force Method

$$\varepsilon_{rr}=\frac{d}{dr}(r\,\varepsilon_{\theta\theta})=\varepsilon_{\theta\theta}+r\frac{d\varepsilon_{\theta\theta}}{dr}$$

$$\frac{\sigma_{rr}}{E_{\perp}}-\frac{\nu\,\sigma_{\theta\theta}}{E_{\|}}=\frac{\sigma_{\theta\theta}}{E_{\|}}-\frac{\nu\,\sigma_{rr}}{E_{\|}}+\varepsilon_i+\frac{r}{E_{\|}}\left(\frac{d\sigma_{\theta\theta}}{dr}-\nu\frac{d\sigma_{rr}}{dr}\right)$$

$$\frac{\sigma_{rr}}{E_{\perp}}-\nu\frac{\sigma_{rr}+r\dfrac{d\sigma_{rr}}{dr}}{E_{\|}}=\frac{\sigma_{rr}+r\dfrac{d\sigma_{rr}}{dr}-\nu\,\sigma_{rr}}{E_{\|}}+\varepsilon_i+\frac{r}{E_{\|}}\left(\frac{d\sigma_{rr}}{dr}+\frac{d\sigma_{rr}}{dr}+r\frac{d^2\sigma_{rr}}{dr^2}-\nu\frac{d\sigma_{rr}}{dr}\right)$$

$$\frac{\sigma_{rr}}{E_{\perp}}-\nu\frac{\sigma_{rr}}{E_{\|}}-\nu\frac{r}{E_{\|}}\frac{d\sigma_{rr}}{dr}-\frac{\sigma_{rr}}{E_{\|}}-\frac{r}{E_{\|}}\frac{d\sigma_{rr}}{dr}+\nu\frac{\sigma_{rr}}{E_{\|}}-(2-\nu)\frac{r}{E_{\|}}\frac{d\sigma_{rr}}{dr}-\frac{r^2}{E_{\|}}\frac{d^2\sigma_{rr}}{dr^2}=\varepsilon_i$$

$$\left(\frac{1}{E_{\perp}}-\frac{1}{E_{\|}}\right)\sigma_{rr}-\frac{3}{E_{\|}}r\frac{d\sigma_{rr}}{dr}-\frac{r^2}{E_{\|}}\frac{d^2\sigma_{rr}}{dr^2}=\varepsilon_i$$

$$\boxed{\left(\frac{E_{\|}}{E_{\perp}}-1\right)\sigma_{rr}-3r\frac{d\sigma_{rr}}{dr}-r^2\frac{d^2\sigma_{rr}}{dr^2}=E_{\|}\varepsilon_i}$$

c Boundary Conditions in the Displacement Method

On an edge $\sigma_{rr} = 0$, therefore,

$$\left.\begin{aligned} \frac{u}{r} &= \frac{\sigma_{\theta\theta}}{E_{||}} + \varepsilon_i \\ \frac{du}{dr} &= -\frac{\nu\,\sigma_{\theta\theta}}{E_{||}} \end{aligned}\right\} \qquad \frac{u}{r} = -\frac{1}{\nu}\frac{du}{dr} + \varepsilon_i \qquad \boxed{\begin{aligned} \frac{u(a)}{a} &= -\frac{1}{\nu}\left.\frac{du}{dr}\right|_a + \varepsilon_i \\ \frac{u(b)}{b} &= -\frac{1}{\nu}\left.\frac{du}{dr}\right|_b + \varepsilon_i \end{aligned}}$$

Boundary Conditions in the Force Method

$$\boxed{\begin{aligned} \sigma_{rr}(a) &= 0 \\ \sigma_{rr}(b) &= 0 \end{aligned}}$$

d Moment and Normal Force

$$M = t\int_{r=a}^{b} \sigma_{\theta\theta}(R-r)dr$$

$$N = t\int_{r=a}^{b} \sigma_{\theta\theta}\, dr$$

e Section Modulus

According to Figure 2 the extreme stress occurs at $r = a$.

$$\begin{aligned} \hat{\sigma}_{\theta\theta} &= E_{||}\varepsilon_i \frac{(b^{2\alpha} - a^{2\alpha}) - \alpha(b^{\alpha+1} - a^{\alpha+1})a^{\alpha-1} - \alpha(b^{\alpha+1}a^{2\alpha} - a^{\alpha+1}b^{2\alpha})a^{-\alpha-1}}{(\alpha^2-1)(b^{2\alpha} - a^{2\alpha})} \\ &= E_{||}\varepsilon_i \frac{(b^{2\alpha} - a^{2\alpha}) - \alpha(a^{\alpha-1}b^{\alpha+1} - a^{2\alpha}) - \alpha(b^{\alpha+1}a^{\alpha-1} - b^{2\alpha})}{(\alpha^2-1)(b^{2\alpha} - a^{2\alpha})} \\ &= E_{||}\varepsilon_i \frac{(b^{2\alpha} - a^{2\alpha}) - \alpha(2a^{\alpha-1}b^{\alpha+1} - a^{2\alpha} - b^{2\alpha})}{(\alpha^2-1)(b^{2\alpha} - a^{2\alpha})} \end{aligned}$$

$$W = \frac{M}{\hat{\sigma}_{\theta\theta}} = \frac{\tfrac{1}{2} t E_{||}\varepsilon_i \dfrac{(\alpha-1)^2(b^{\alpha+1} - a^{\alpha+1})^2 - a^2b^2(\alpha+1)^2(b^{\alpha-1} - a^{\alpha-1})^2}{(\alpha^2-1)^2(b^{2\alpha} - a^{2\alpha})}}{E_{||}\varepsilon_i \dfrac{(b^{2\alpha} - a^{2\alpha}) - \alpha(2a^{\alpha-1}b^{\alpha+1} - a^{2\alpha} - b^{2\alpha})}{(\alpha^2-1)(b^{2\alpha} - a^{2\alpha})}}$$

$$\boxed{W = \tfrac{1}{2} t \frac{(\alpha-1)^2(b^{\alpha+1} - a^{\alpha+1})^2 - a^2b^2(\alpha+1)^2(b^{\alpha-1} - a^{\alpha-1})^2}{(\alpha^2-1)\Big((b^{2\alpha} - a^{2\alpha}) - \alpha(2a^{\alpha-1}b^{\alpha+1} - a^{2\alpha} - b^{2\alpha})\Big)}}$$

Encore (not an exam question)

Figure 7 shows that the influence of α is not large considering that for most wood species α has a value between 3 and 6. The section modulus W can be approximated by

$$\boxed{W \simeq \tfrac{1}{6} t d^2 - \tfrac{1}{18} t \frac{d^3}{R}\frac{12+\alpha}{13}}$$

It can be shown that this approximation has an error less than 3 % for $0 < d < R$ and $1 < \alpha < 6$.

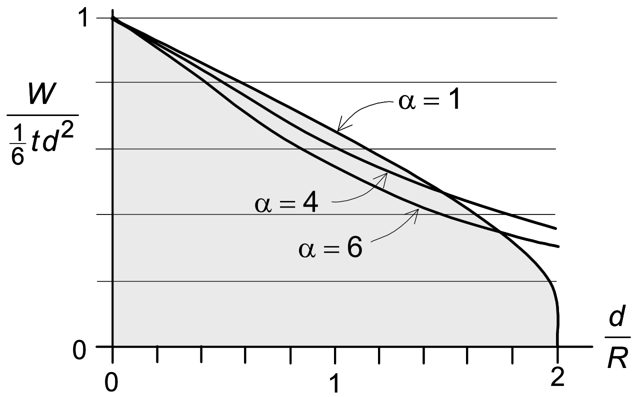


Figure 7. Section modulus *W* as function of the beam height *d* and the stiffness ratio α.

Answers to Problem 2[1]

a Kinematic Boundary Conditions
The bottom plate is correctly connected to the supports because

$$x = y = 0 \quad \rightarrow \quad w = 0$$
$$x = a \quad y = b \quad \rightarrow \quad w = 0$$

Therefore, the barge fulfils the kinematic boundary conditions.

b Torsion
The curvatures in the bottom plate are

$$\kappa_{xx} = -\frac{\partial^2 w}{\partial x^2} = -\frac{\partial^2}{\partial x^2} \hat{w} \left(\frac{x}{a} + \frac{y}{b} - 2\frac{xy}{ab} \right) = 0$$

$$\kappa_{yy} = -\frac{\partial^2 w}{\partial y^2} = -\frac{\partial^2}{\partial y^2} \hat{w} \left(\frac{x}{a} + \frac{y}{b} - 2\frac{xy}{ab} \right) = 0$$

$$\rho_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} = -2\frac{\partial^2}{\partial x \partial y} \hat{w} \left(\frac{x}{a} + \frac{y}{b} - 2\frac{xy}{ab} \right) = 4\frac{\hat{w}}{ab}$$

The curvature in the large walls is (see Figure 8)

$$\rho_{xy} = 2\frac{\frac{\hat{w}}{a}h + \frac{\hat{w}}{a}h}{h\,b} = 4\frac{\hat{w}}{ab} .$$

The curvature in the small walls is (see Figure 8.)

[1] Reinforced concrete barges are often used for constructing houseboats in The Netherlands (Dutch: woonboten).

$$\rho_{xy} = 2\frac{\frac{\hat{w}}{b}h + \frac{\hat{w}}{b}h}{ha} = 4\frac{\hat{w}}{ab}.$$

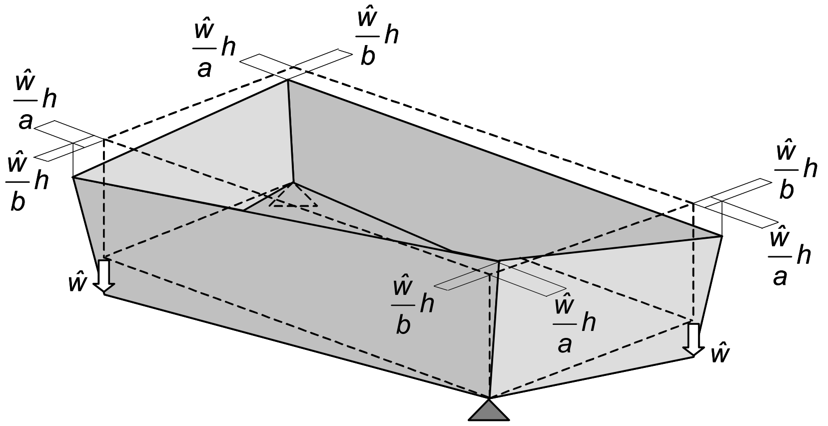


Figure 8. Deformation of the barge walls

c Potential Energy

$$E_{pot} = E_s + E_p$$

$$E_s = \tfrac{1}{2}\rho_{xy} m_{xy}\left(ab + 2ah + 2bh\right)$$

$$E_p = -\rho g t\left(\int_{x=0}^{a}\int_{y=0}^{b} w(x,y)\,dy\,dx + \int_{x=0}^{a} h\,w(x,0)\,dx + \int_{y=0}^{b} h\,w(0,y)\,dy + \int_{x=0}^{a} h\,w(x,b)\,dx + \int_{y=0}^{b} h\,w(a,y)\,dy\right)$$

d Displacement

$$\frac{dE_{pot}}{d\hat{w}} = \frac{1}{3}\frac{E\,t^3 2\hat{w}}{\nu+1}\frac{ab + 2(a+b)h}{a^2b^2} - \tfrac{1}{2}\rho g t\left(ab + 2(a+b)h\right) = 0$$

$$\hat{w} = \tfrac{3}{4}\rho\, g\, a^2 b^2 \frac{(\nu+1)}{E\,t^2}$$

The displacement does not depend on the height h. Apparently the extra stiffness is compensated by the extra load. Therefore, the formula is equally valid for just a plate (no walls) and also valid for a barge without bottom plate (only walls).

e Torsion Moment

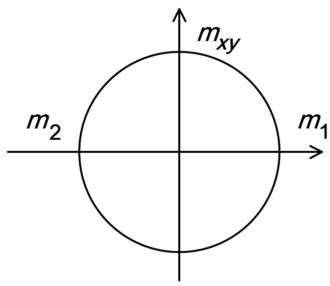
$$m_{xy} = \tfrac{1}{2}D(1-\nu)\rho_{xy} = \tfrac{1}{2}\frac{E\,t^3}{12(1-\nu^2)}(1-\nu)4\frac{\hat{w}}{ab} = \tfrac{1}{2}\frac{E\,t^3}{12(1-\nu^2)}(1-\nu)4\frac{1}{ab}\tfrac{3}{4}\rho\, g\, a^2 b^2 \frac{(\nu+1)}{E\,t^2}$$

$$\boxed{m_{xy} = \tfrac{1}{8}\rho\, g t a b}$$

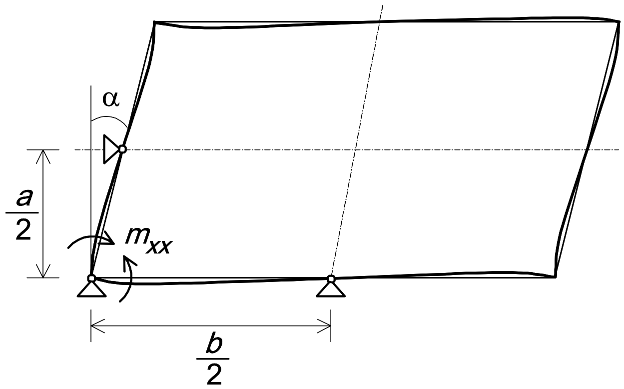
f Largest Stress

The largest principal moment is $m_1 = m_{xy}$, therefore, the largest principal stress is

$$\sigma_1 = \frac{m_1}{\frac{1}{6}t^2} = \frac{\frac{1}{8}\rho\, g t a b}{\frac{1}{6}t^2} = \frac{3}{4}\frac{\rho g a b}{t}$$



Encore (Not an exam question)
The proposed barge displacement is not completely kinematically admissible because the wall joints are not perpendicular after deformation. The joint deformation angle at the top edge of the barge is



$$\alpha = \frac{2\frac{\hat{w}}{a}h}{b} + \frac{2\frac{\hat{w}}{b}h}{a} = 4\frac{\hat{w}h}{ab}.$$

If the wall joints would remain perpendicular than bending moments in the x, y and z directions would occur. In fact these bending moments have been ignored in the present analysis. The moments would not carry much of the load but occur because the wall joints have to follow the deformation of the structure. Therefore they are called compatibility moments.

$$\alpha\frac{a}{2} = \frac{m_{xx}\frac{b}{2}}{3D}\frac{a}{2} + \frac{\frac{2}{a}m_{xx}\left(\frac{a}{2}\right)^3}{3D} \quad \Rightarrow m_{xx} = 6\frac{D}{a+b}\alpha$$

$$m_{xx} = 6\frac{D}{a+b}\alpha = 6\frac{D}{a+b}4\frac{\hat{w}h}{ab} = 6\frac{\frac{Et^3}{12(1-\nu^2)}}{a+b}4\frac{h}{ab}\tfrac{3}{4}\rho g a^2 b^2 \frac{(\nu+1)}{Et^2}$$

$$m_{xx} = \tfrac{3}{2}\frac{\rho g t a b}{1-\nu}\frac{h}{a+b}$$

$$\boxed{m_{xx} = m_{xy}\frac{12}{1-\nu}\frac{h}{a+b}}$$

For practical dimensions the compatibility moments in the joints m_{xx} will often be larger than the torsion moment m_{xy}.

Answers to Problem 3

a Bredt's Formula
Moment equilibrium around the middle point gives

$$M_t = \tau_1 l_1 t_1 \tfrac{1}{2}h + \tau_2 l_2 t_2 \tfrac{1}{2}h + 2\tau_3 l_3 t_3 \frac{h}{l_3}\tfrac{1}{2}b$$

Substitution of the stresses

$$M_t = \frac{M_t}{2At_1}l_1 t_1 \tfrac{1}{2}h + \frac{M_t}{2At_2}l_2 t_2 \tfrac{1}{2}h + 2\frac{M_t}{2At_3}l_3 t_3 \frac{h}{l_3}\tfrac{1}{2}b$$

$$= \frac{M_t}{4A}h\left(l_1 + l_2 + 2b\right)$$

$$= \frac{M_t}{4A}h\left(2b + 2b\right)$$

$$= \frac{M_t}{A}hb \qquad \text{Q.E.D.}$$

b Slice

$$E_c = \frac{1}{2}\frac{\tau_1^2}{G_c}\Delta l_1 t_1 + \frac{1}{2}\frac{\tau_2^2}{G_c}\Delta l_2 t_2 + \frac{1}{2}\frac{\tau_3^2}{G_s}2\beta\Delta l_3 t_3$$

$$= \Delta\frac{1}{2}\left(\frac{\tau_1^2}{G_c}l_1 t_1 + \frac{\tau_2^2}{G_c}l_2 t_2 + \frac{\tau_3^2}{G_s}2\beta l_3 t_3\right)$$

$$= \Delta\frac{1}{2}\left(\left(\frac{M_t}{2At_1}\right)^2\frac{1}{G_c}l_1 t_1 + \left(\frac{M_t}{2At_2}\right)^2\frac{1}{G_c}l_2 t_2 + \left(\frac{M_t}{2At_3}\right)^2\frac{1}{G_s}2\beta l_3 t_3\right)$$

$$= \Delta\frac{1}{8}\left(\frac{M_t}{A}\right)^2\left(\frac{l_1}{G_c t_1} + \frac{l_2}{G_c t_2} + \frac{2\beta l_3}{G_s t_3}\right)$$

c Wire Frame

$$E_c = \frac{1}{2}\frac{M_t^2}{G\mathcal{I}_t}\Delta$$

d Torsion Stiffness

$$E_{c,\text{slice}} = E_{c,\text{wire frame}}$$

$$\frac{1}{2}\frac{M_t^2}{G\mathcal{I}_t}\Delta = \Delta\frac{1}{8}\left(\frac{M_t}{A}\right)^2\left(\frac{l_1}{G_c t_1} + \frac{l_2}{G_c t_2} + \frac{2\beta l_3}{G_s t_3}\right)$$

$$\boxed{G\mathcal{I}_t = \frac{4A^2}{\dfrac{l_1}{G_c t_1} + \dfrac{l_2}{G_c t_2} + \dfrac{2\beta l_3}{G_s t_3}}}$$

Answer to Problem 4

This statement is correct. The stress in thin-wall single-cell closed sections is calculated by Bredt's second formula

$$\tau_{\max} = \frac{M_t}{2At_{\min}}.$$

The stress in thin-wall open sections is calculated by

$$\tau_{\max} = \frac{M_t\, t_{\max}}{\frac{1}{3}\sum_i b_i t_i^3},$$