CT 5143 Stresses in Shells

Exam, 27 June 2005, 14:00 – 17:00 hours

Question 1 (2 points)

Write the formulas for the characteristic lengths of the following shells and explain for each case the radius *a*.

a Circular edge of cylinder tank
b Circular edge of a cone
c Straight edge of a cylinder roof
d Straight edge of a hypar

Question 2 (2 points)

Consider the differential equation for bending of shells of arbitrary curvature.

a How are the tangential displacements u_x and u_y eliminated in derivation?

b What is an important restriction of the differential equation? How can this be handled in applications?

c Often only the homogeneous solution of the differential equation is used. Why is the inhomogeneous solution disregarded?

Question 3 (2 points)

A brick chimney has the following dimensions.

height = 40 m
radius = 2 m
wall thickness = 0.210 m

The material properties are

Young's modulus = 30000 MPa
Poisson's ratio = 0.15
mass = 2200 kg/m³

The loading consists of self-weight combined with a wind pressure of 0.001 MPa.

Calculate the largest tensile stress at the base of the chimney. Calculate the buckling stress at the base of the chimney. Could buckling occur?

Question 4 (4 points)

An LNG tank will be constructed underground. The roof of the tank is a spherical dome (Fig. 1). The dome has a radius *a* and a constant thickness *t*. Young's modulus is *E* and Poisson's ratio is approximated to zero. The soil height above the dome is large compared tot the radius of the sphere, therefore it can be assumed that the sphere is loaded by a constant vertical pressure *p*. The dome is supported by a slurry wall and the edge of the dome is prestressed in the tangential direction, therefore, the dome edge is modelled as clamped. The opening angle is $\phi_o = \frac{1}{4}\pi$.

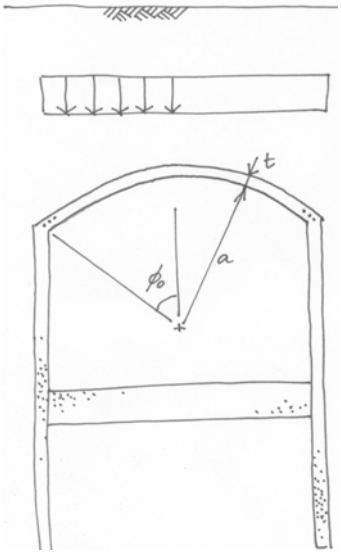


Figure 1. Underground LNG tank with a dome roof

a Formulate the membrane solution of the normal forces $n_{\phi\phi}$, $n_{\theta\theta}$. Show that the membrane edge rotation is $\varphi_\phi = -1.414\frac{pa}{Et}$ and the membrane edge displacement is $u_r = -0.0858\frac{pa^2}{Et}$ (Use the formula below).

b Determine the support reactions f_r and t_x .

c Calculate the total stress in the outer surface in the meridial direction at the dome base. Assume p = 200 kN/m², a = 15 m and t = 0.8 m

Useful formula

$$\frac{d}{d\phi}\left(\left(\frac{1}{1+\cos\phi}-\cos\phi\right)\sin\phi\right) = \frac{2+\cos\phi-2\cos^2\phi-2\cos^3\phi}{1+\cos\phi}$$

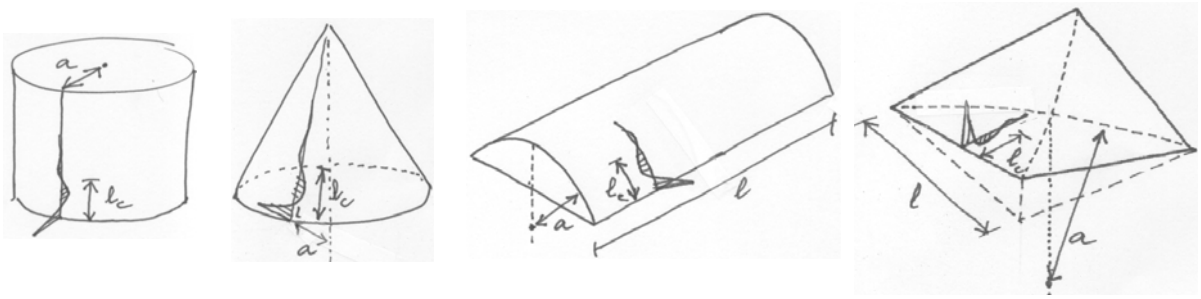
Answers to question 1

a Circular edge of cylinder tank $l_c = \mathrm{O}\left(\sqrt{at}\right), \mathrm{O}\left(\frac{a}{t}\sqrt{at}\right)$

b Circular edge of a cone $l_c = \mathrm{O}\left(\sqrt{at}\right)$

c Straight edge of a cylinder roof $l_c = \mathrm{O}\left(\sqrt[4]{atl^2}\right)$

d Straight edge of a hypar $l_c = \mathrm{O}\left(\sqrt[3]{atl}\right)$



Answers to question 2

a The two tangential displacements are eliminated from the three kinematic equations. The result is a compatibility equation in which only u_z occurs.

b The differential equation is only accurate when translations are small. For large translations (rigid body motions) stresses are predicted that in reality will not occur. Therefore, load cases that produce large translations are handled separately by the membrane theory.

c The inhomogeneous solution is often replaced by the membrane solution. The latter solution can often be found conveniently and has few restrictions.

Answers to question 3

Stress due to the self-weight

$$\sigma_{xx} = -l\rho g = -40\ \mathrm{m} \times 2200\ \mathrm{kg/m^3} \times 10\ \mathrm{m/s^2} = -880000\ \mathrm{N/m^2} = -0.88\ \mathrm{MPa}$$

Largest tensile stress due to wind pressure (lecture book page 97)

$$\sigma_{xx} = 0.224\frac{l^2}{at}p_w\left(1+5.00\frac{a^2}{l^2}\frac{a}{t}\right) =$$

$$= 0.224\frac{40^2}{2\times 0.21}0.001\left(1+5.00\frac{2^2}{40^2}\frac{2}{0.21}\right) = 0.95\ \mathrm{MPa}$$

In total

$$\sigma_{xx} = -0.88 + 0.95 = 0.07\ \mathrm{MPa}$$

Buckling stress (lecture book appendix Fig. 86)

$$\sigma_{xx} = 0.234\frac{Et}{a} = 0.234\frac{30000\times 210}{2000} = 725\ \text{MPa}$$

The compressive stress due to self-weight and wind pressure (not calculated) is much smaller than the buckling stress. Consequently, there is no danger of buckling.

Answers to question 4

a The membrane normal forces are (lecture book page 147)

$$n_{\phi\phi} = -pa\frac{1}{1+\cos\phi}$$

$$n_{\theta\theta} = pa\left(\frac{1}{1+\cos\phi} - \cos\phi\right)$$

The strains are calculated with (9.9) where it is used that ν = 0.

$$\varepsilon_{\phi\phi} = \frac{1}{Et}\left(n_{\phi\phi} - \nu n_{\theta\theta}\right) = -\frac{pa}{Et}\frac{1}{1+\cos\phi}$$

$$\varepsilon_{\theta\theta} = \frac{1}{Et}\left(-\nu n_{\phi\phi} + n_{\theta\theta}\right) = \frac{pa}{Et}\left(\frac{1}{1+\cos\phi} - \cos\phi\right)$$

The rotation and displacement are (9.11)

$$u_r = \varepsilon_{\theta\theta} r = \frac{pa}{Et}\left(\frac{1}{1+\cos\phi} - \cos\phi\right) a\sin\phi = \frac{pa}{Et}\left(\frac{1}{1+\cos\frac{\pi}{4}} - \cos\frac{\pi}{4}\right) a\sin\frac{\pi}{4} = -0.0858\frac{pa^2}{Et}$$

$$\varphi_\phi = \varepsilon_{\phi\phi}\cot\phi - \frac{1}{r_1\sin\phi}\frac{d}{d\phi}(\varepsilon_{\theta\theta} r) = -\frac{pa}{Et}\frac{\cot\phi}{1+\cos\phi} - \frac{1}{a\sin\phi}\frac{d}{d\phi}\left(\frac{pa}{Et}\left(\frac{1}{1+\cos\phi} - \cos\phi\right) a\sin\phi\right) =$$

$$= -\frac{pa}{Et}\frac{\cot\phi}{1+\cos\phi} - \frac{1}{\sin\phi}\frac{pa}{Et}\left(\frac{2+\cos\phi-2\cos^2\phi-2\cos^3\phi}{1+\cos\phi}\right) =$$

$$= -\frac{pa}{Et}\left(\frac{\cot\phi}{1+\cos\phi} + \frac{1}{\sin\phi}\frac{2+\cos\phi-2\cos^2\phi-2\cos^3\phi}{1+\cos\phi}\right) =$$

$$= -\frac{pa}{Et}\left(\frac{\cot\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} + \frac{1}{\sin\frac{\pi}{4}}\frac{2+\cos\frac{\pi}{4}-2\cos^2\frac{\pi}{4}-2\cos^3\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}\right) =$$

$$= -1.414\frac{pa}{Et}$$

b The support reactions f_r and t_x are calculated from the stiffness matrix (10.25)

$$\frac{1}{D_b}\begin{bmatrix} \dfrac{\sin^2\phi_o}{2\mu^3} & -\dfrac{\sin\phi_o}{2\mu^2} \\ -\dfrac{\sin\phi_o}{2\mu^2} & \dfrac{1}{\mu} \end{bmatrix}\begin{bmatrix} f_r \\ t_x \end{bmatrix} = \begin{bmatrix} u_r \\ \varphi_x \end{bmatrix}$$

where $\mu = \dfrac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{r_2 t}} = \dfrac{\sqrt[4]{3}}{\sqrt{at}}$

The inverse relation is

$$\begin{bmatrix} f_r \\ t_x \end{bmatrix} = D_b \begin{bmatrix} \dfrac{4\mu^3}{\sin^2\phi_o} & \dfrac{2\mu^2}{\sin\phi_o} \\ \dfrac{2\mu^2}{\sin\phi_o} & 2\mu \end{bmatrix}\begin{bmatrix} u_r \\ \varphi_x \end{bmatrix}$$

Using $\phi_o = \frac{\pi}{4}$ and the edge conditions we obtain

$$\begin{bmatrix} f_r \\ t_x \end{bmatrix} = \frac{Et^3}{12}\begin{bmatrix} 8\mu^3 & 2\sqrt{2}\mu^2 \\ 2\sqrt{2}\mu^2 & 2\mu \end{bmatrix}\begin{bmatrix} 0.0858\dfrac{pa^2}{Et} \\ 1.414\dfrac{pa}{Et} \end{bmatrix} = \frac{Et^3}{12}\begin{bmatrix} 0.686\mu^3 & 4\mu^2 \\ 0.243\mu^2 & 2.828\mu \end{bmatrix}\begin{bmatrix} \dfrac{pa^2}{Et} \\ \dfrac{pa}{Et} \end{bmatrix} =$$

$$= \frac{t^2 pa\mu}{12}\begin{bmatrix} 0.686\mu^2 a + 4\mu \\ 0.243\mu a + 2.828 \end{bmatrix}$$

c The total stress in the outer face in the meridial direction at the dome base.

$$\sigma_{\phi\phi} = \frac{m_{\phi\phi}}{\frac{1}{6}t^2} + \frac{n_{\phi\phi,m} + n_{\phi\phi,b}}{t}$$

$$= \frac{t_x}{\frac{1}{6}t^2} + \frac{\dfrac{-pa}{1+\cos\phi_o} + f_r\cos\phi_o}{t}$$

$$= \frac{\dfrac{t^2 pa\mu}{12}(0.243\mu a + 2.828)}{\frac{1}{6}t^2} + \frac{\dfrac{-pa}{1+\frac{1}{2}\sqrt{2}} + \dfrac{t^2 pa\mu}{12}(0.686\mu^2 a + 4\mu)\frac{1}{2}\sqrt{2}}{t}$$

$$= pa\mu(0.1215\mu a + 1.414) - \frac{pa}{(1+\frac{1}{2}\sqrt{2})t} + tpa\mu^2\frac{0.686\mu a + 4}{24}\sqrt{2}$$

p = 200 kN/m², a = 15 m, t = 0.8 m and μ = 0.38 1/m

$$\sigma_{\phi\phi} = 200\times 15\times 0.38(0.1215\times 0.38\times 15 + 1.414) - \frac{200\times 15}{(1+\frac{1}{2}\sqrt{2})\times 0.8} + 0.8\times 200\times 15\times 0.38^2\frac{0.686\times 0.38\times 15 + 4}{24}\sqrt{2}$$

$$\sigma_{\phi\phi} = 2401 - 2197 + 161 \text{ kN/m}^2$$

$$\sigma_{\phi\phi} = 366 \text{ N/mm}^2$$