Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CT5143 Stresses in Shells
Monday 30 June 2008, 14:00 – 17:00 hours

Problem 1 (2 point, 35 minutes)

Explain shortly the meaning of the following eight words in relation to structural shells.

Pressure line
Grid shell
Integration points
Hourglass mode
Gaussian curvature
In-extensional deformation
Loof points
Reissner plate

Problem 2 (2 points, 35 minutes)

a Figure 1 shows a shell part with forces and moments in the positive directions. Add to this figure the variables for the directions, forces and moments (for example x, n_{xx} , ... Make clear which variable belongs to which arrow.)

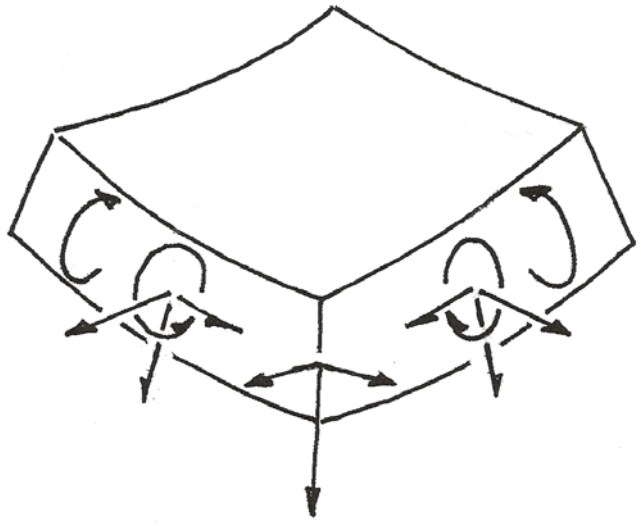

Figure 1. Representative shell part

b Which forces or moments do not occur in a tent structure?

c In the shell differential equation the *x* and *y* axis follow the shell shape. Why does not the differential equation use a global reference system like for beams, columns and plates?

d Does the shell differential equation (Sanders-Koiter equations) include in-plane shear deformation? Does it include out-of-plane shear deformation?

e Suppose that a finite element program only has one type of shell element that includes out-of-plane shear deformation. Is this program suitable for analysing thin shell structures? Explain your answer.

Problem 3 (2 points, 35 minutes)

a Figure 2 shows two deformations of a spherical cap. Which of these deformations is in-extensional? Explain your answer.



Figure 2a



Figure 2b

b What happens if snow load on a thin shell structure results in in-extensional deformation?

c What happens if a foundation settlement of a thin shell structure results in in-extensional deformation?

d Suppose that we used a linear elastic finite element program to predict the deformation of a shell structure. How can we establish whether the deformation is in-extensional?

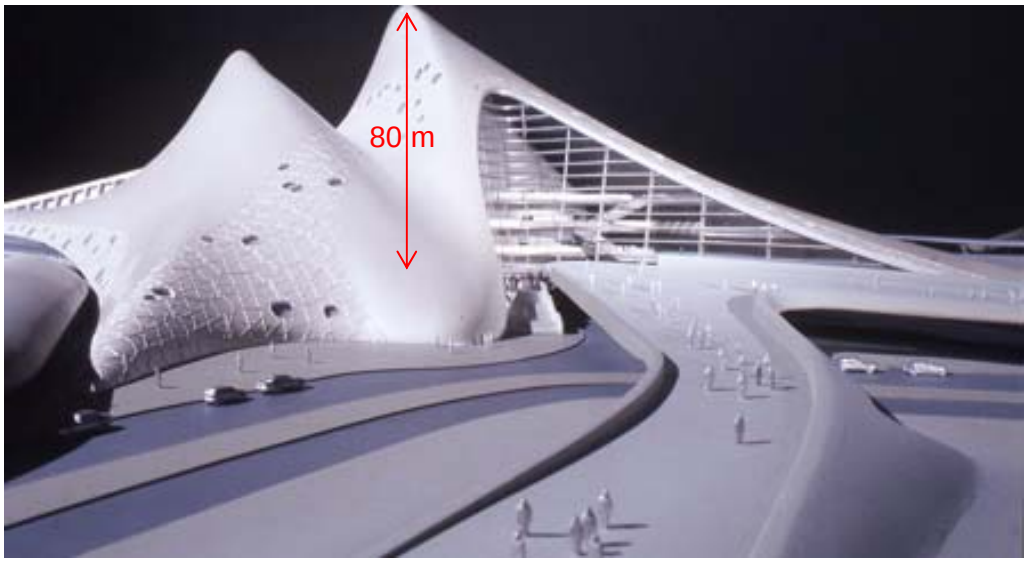

Figure 3. Opera house design for Dubai by Zaha Hadid
(http://www.dezeen.com/2008/06/06/dubai-opera-house-by-zaha-hadid/)

Problem 4 (2 points, 35 minutes)

a Figure 3 shows a shell design. The top has a height of approximately 80 m. Assume that the structure has no columns and that the window façade is not supporting the shell. Calculate the required shell thickness assuming a thickness/radius ratio of 1/500. (Measure dimensions from the figure.)

Problem 5 (2 points, 35 mininutes)

An assembly of four identical hypar shells is supported by one column in the middle (Fig. 4). The shell thickness is t = 75 mm. The dead weight of the shell including roofing and finishing is p = 3.5 kN/m².

a Calculate the membrane forces and the largest bending moment in the shell.

b Calculate the forces in the edge beams, interior beams, cables and column.

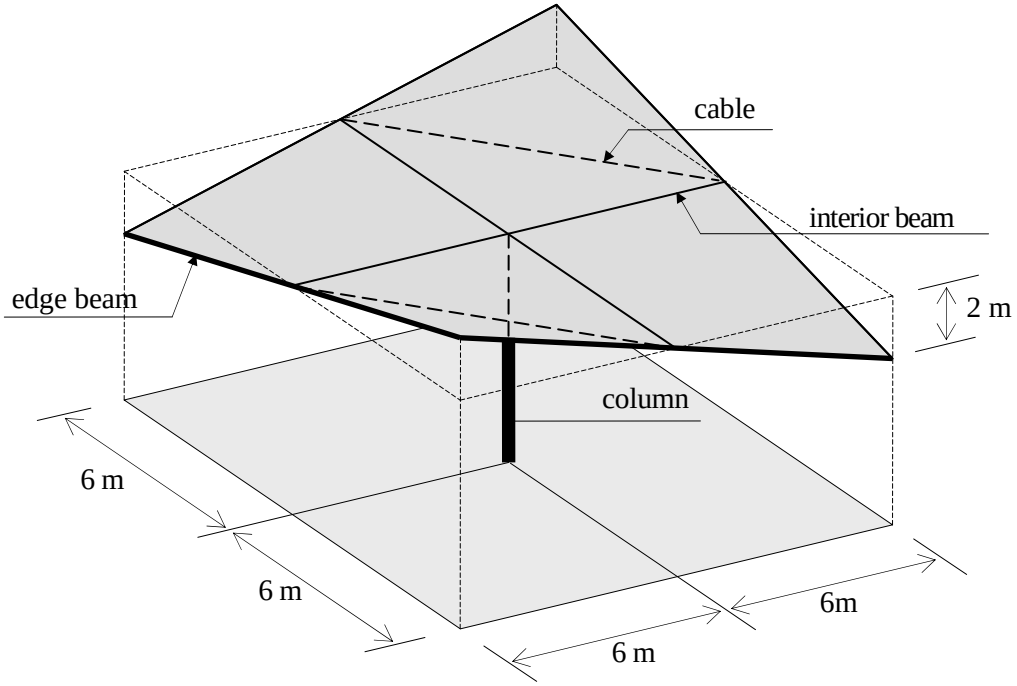

Figure 4. Four hypar shell

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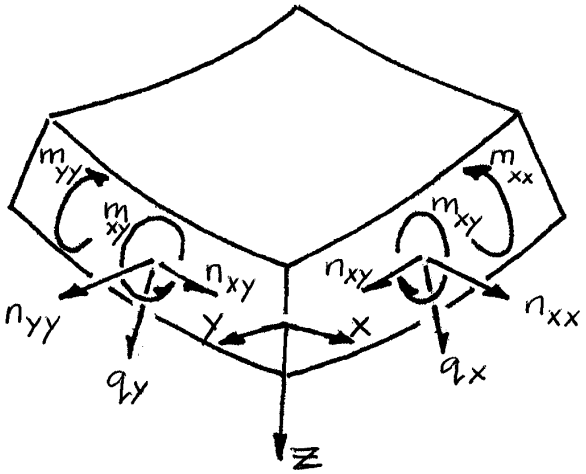
Solution to Problem 1

Pressure line ……..	Trajectory of the normal forces in an arch
Grid shell ……..	Shell made of connected curved wood laths
Integration points ……..….	Locations in a finite element where the material behaviour is computed
Hourglass mode …….	Deformation of finite elements in the shape of an hourglass
Gaussian curvature …….	Product of the principal curvatures
In-extensional deformation …	Deformation with no extension and only bending
Loof points ……..	Locations on a shell finite element edge with a rotational degree of freedom only
Reissner plate …….	Idealisation of a plate including shear deformation

Solution to Problem 2

a Variables



b Tent structure
$m_{xx} = m_{yy} = m_{xy} = q_x = q_y = 0$, $n_1 \geq 0,\ n_2 \geq 0$

c Global reference system
In theory it is possible to derive a shell differential equation in a global reference system or transform the differential equation to the global reference system. However, than it would become even more complicated than it already is.

d Shear deformation
Yes (γ_{xy} in the equations for the bending theory). No.

e Shell element
No problem. The shear deformation will just be very small and the influence on the deformation and stresses are negligible. The elements can be used as elements that do not include shear deformation. Also the results need to be interpreted as elements that do not include shear deformation.

Solution to Problem 3

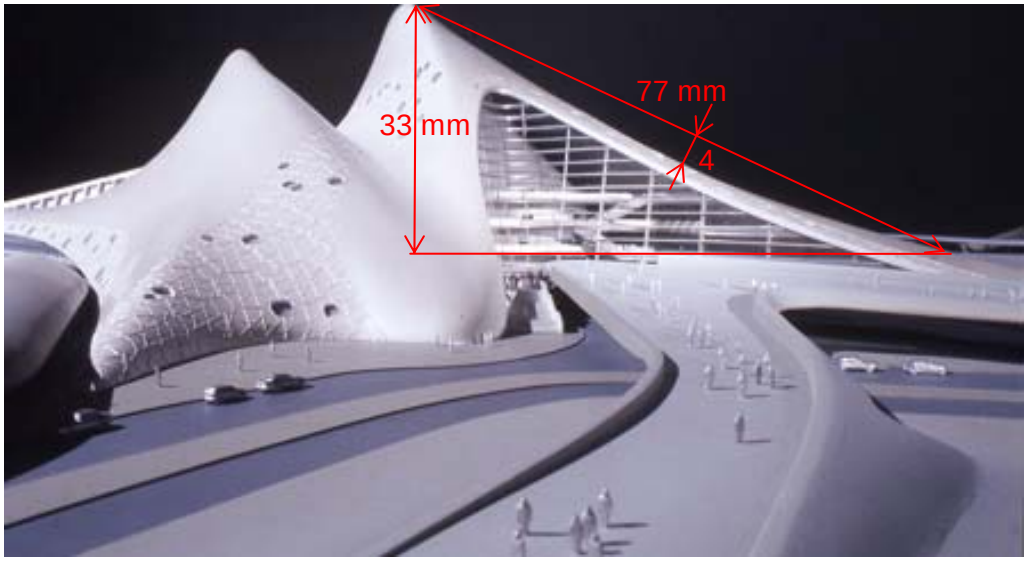
a In-extensional
Figure 2b

b Snow load
The structure would be likely to collapse.

c Foundation settlement
The structure follows the settlement without extra normal forces n_{xx}, n_{yy}, n_{xy} . Some yielding in steel or cracking in concrete might occur due to increased moments. However, the structure will be safe.

d Finite element analysis
Very large displacements due to applied forces. Very large moments.
(Programming is needed to plot the Γ or the strain energy.)

Solution to Problem 4



In the figure the radius is

$$a = \frac{1}{2}s + \frac{1}{8}\frac{l^2}{s} = \frac{1}{2}4 + \frac{1}{8}\frac{77^2}{4} = 187 \text{ mm} .$$

In reality the radius is

$$a = \frac{80 \text{ m}}{33 \text{ mm}} 187 \text{ mm} = 453 \text{ m} .$$

The thickness is

$$\frac{1}{500} = \frac{t}{a} = \frac{t}{453} \Rightarrow t = \frac{453}{500} = 0.9 \text{ m}$$

Solution to Problem 5

a Stresses

The radius of the hypar is

$$a = \frac{b^2}{h} = \frac{6^2}{2} = 18 \text{ m.}$$

The direction of the dead load is approximately perpendicular to the shell surface. Therefore the distributed membrane force is

$$n_{xy} = \tfrac{1}{2} a p_z = \tfrac{1}{2} 18 \times 3.5 = 31.5 \text{ kN/m.}$$

The length of the edge beams is $l = \sqrt{6^2 + 2^2} = 6.32$ m.

The largest moment in the shell occurs at the interior beam because there rotation will not occur due to symmetry. The moment is

$$m_{xx} = 0.511 \frac{p}{l^2} \sqrt[3]{(a\,t\,l)^4} = 0.511 \frac{3.5}{6^2} \sqrt[3]{(18 \times 0.075 \times 6)^4} = 0.81 \text{ kNm/m.}$$

b Force flow

The normal force in the middle of the edge beams is

$N_e = 6.32 \times 31.5 = 199$ kN compression or tension.

The horizontal and vertical component of $2N_e$ are

$$N_h = 2 \times 199 \frac{6}{6.32} = 378 \text{ kN.} \quad N_v = 2 \times 199 \frac{2}{6.32} = 126 \text{ kN.}$$

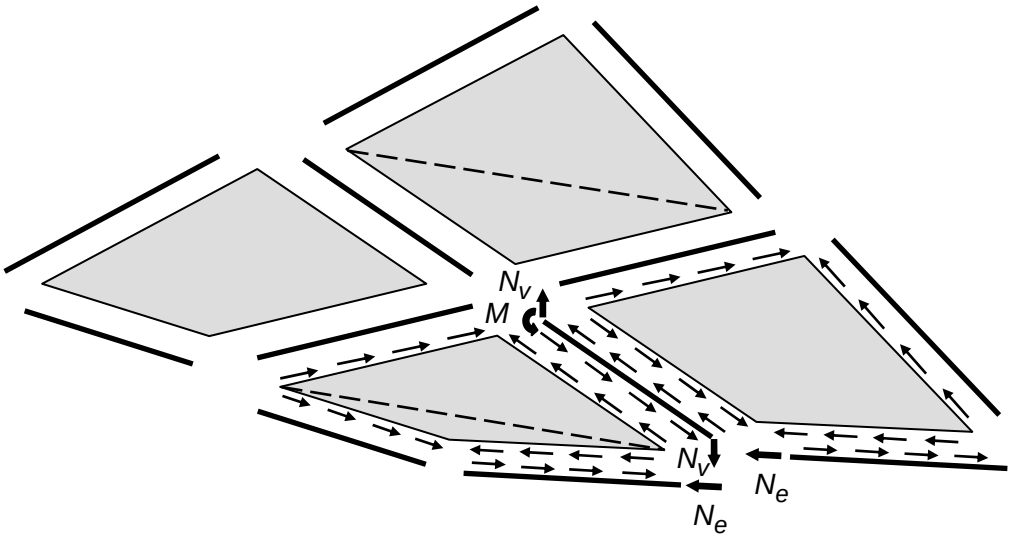
The normal force in the interior beams is
$N_i = N_h = 378$ kN compression.

The force in the cable is
$T = N_h \sqrt{2} = 378\sqrt{2} = 535$ kN tension.

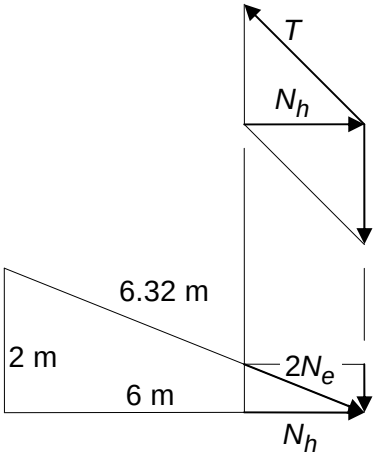
The moment in the interior beams at the column is
$M = 6 \times N_v = 6 \times 126 = 756$ kNm.

The force in the column is $R = 4N_v = 4 \times 126 = 504$ kN.

As a check we calculate the total load $G = 12 \times 12 \times 3.5 = 504$ kN which is equal to the column force.



T



top view of the forces at the node

front view of the forces at the node