Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your name and study number at the top right-hand of your work.

Exam CIE4143 Shell Analysis
Tuesday 11 August 2015, 14:00 – 17:00 hours

Problem 1 (30 minutes) 1.7 points

Explain shortly the meaning of the following eight words in relation to shell structures.

Meridional force
Lamé parameters
Middle surface
Enneper's surface
Statistical size effect
Monocoque
Sanders-Koiter equations
Kissing circle

Problem 2 (30 minutes) 1.7 points

Consider a canopy that is fixed on one end (fig.1) The canopy has a length of 12 m. Its cross-section is halve a circle with a radius of 2 m. Thickness t = 70 mm, Young's modulus E = 10^7 kN/m^2, Poisson's ratio ν = 0.15, self-weight = 24 kN/m^3. Figures 2 to 10 show the results of the linear elastic finite element analysis (8 node elements, 50 x 50 mm, Ansys SHELL281, Reissner-Mindlin theory).

a Which are the shell boundary conditions for the Sanders-Koiter equations of this canopy?

b In what way do finite element analyses use the Sanders-Koiter equations?

c Which of the boundary conditions need to be entered in the finite element program?

d Do the finite element results agree with the shell boundary conditions for the Sanders-Koiter equations? Explain your answer clearly.

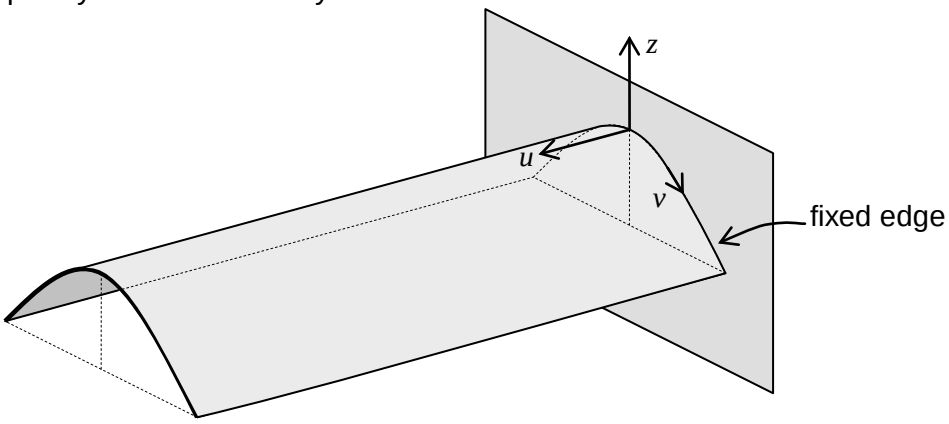


Figure 1. A canopy that is fixed on one edge

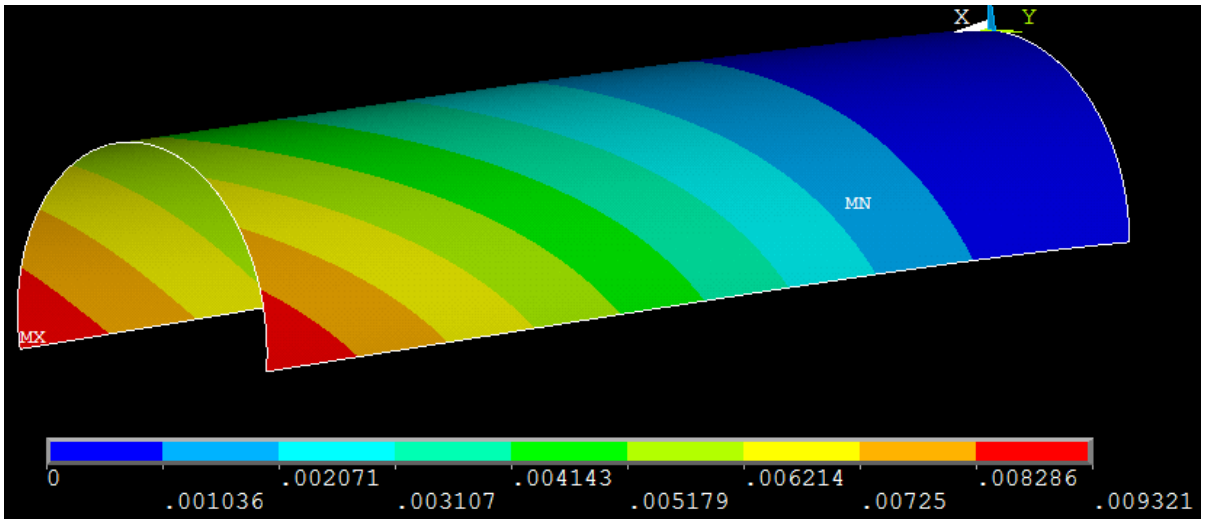


Figure 2. Displacement due to self-weight [m]

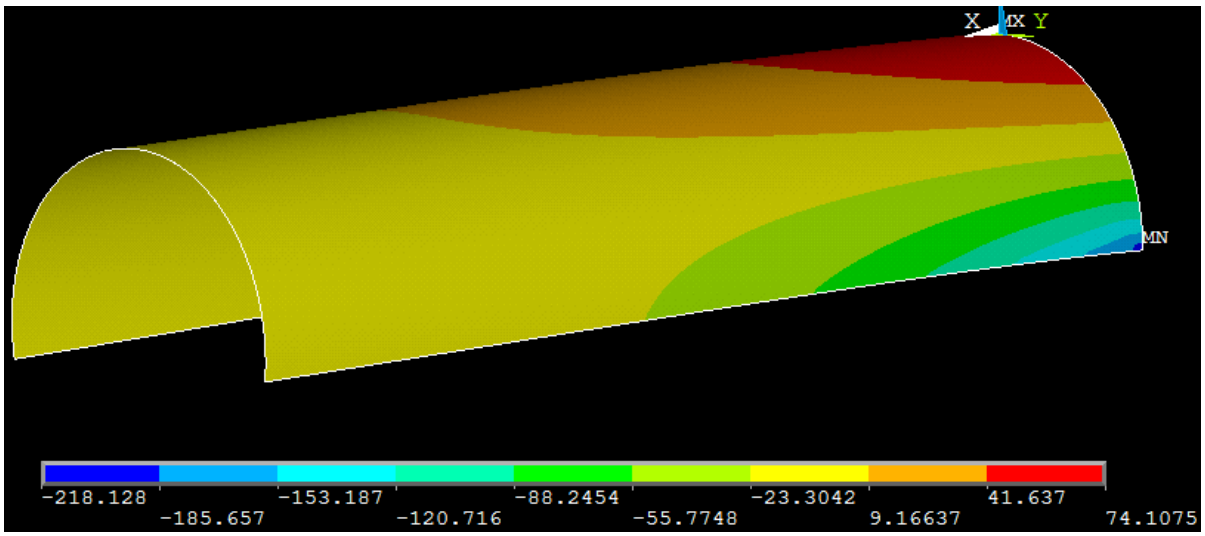


Figure 3. Normal force n_{xx} [kN/m]

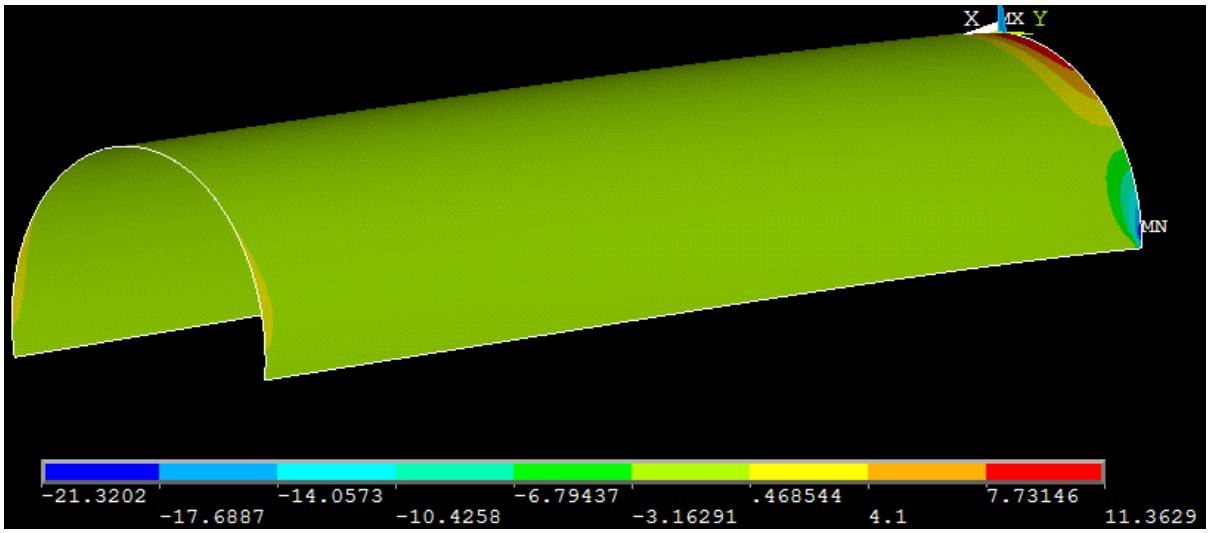


Figure 4. Normal force n_{yy} [kN/m]

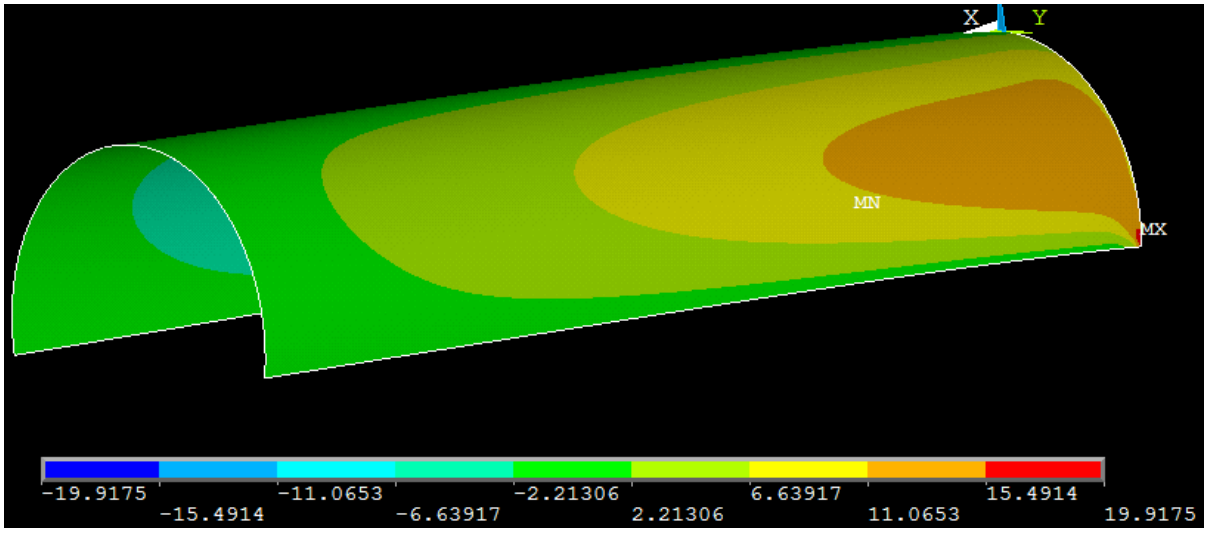


Figure 5. In plane shear force $\frac{n_{xy} + n_{yx}}{2}$ [kN/m]

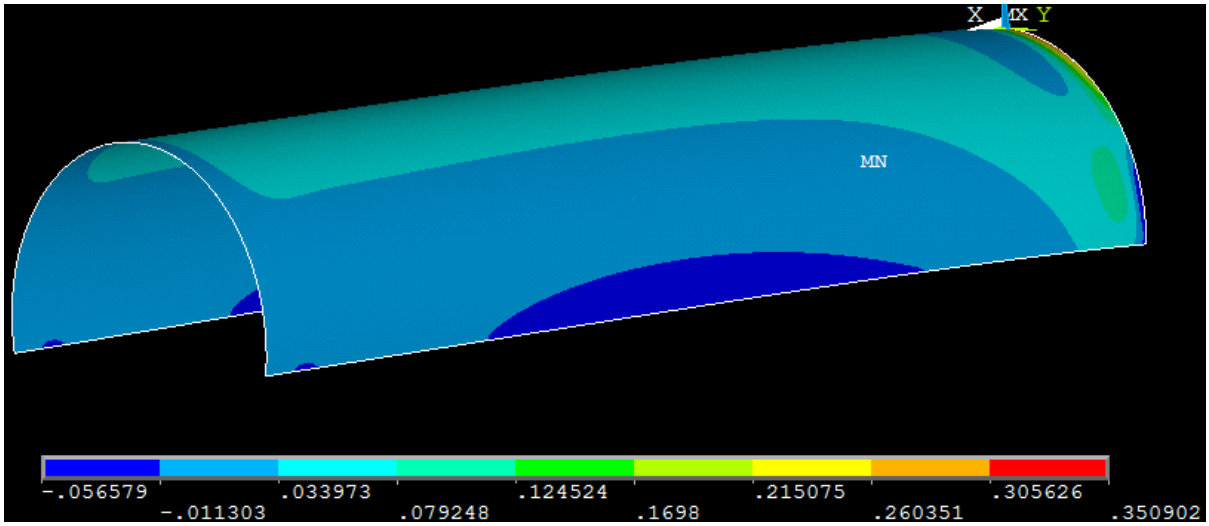


Figure 6. Bending moment m_{xx} [kNm/m]

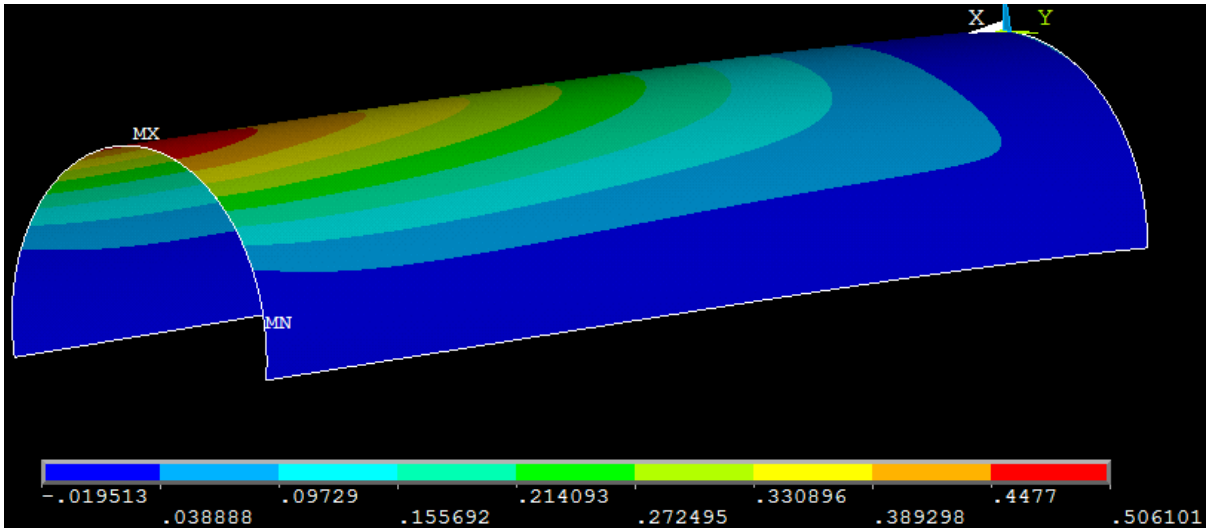


Figure 7. Bending moment m_{yy} [kNm/m]

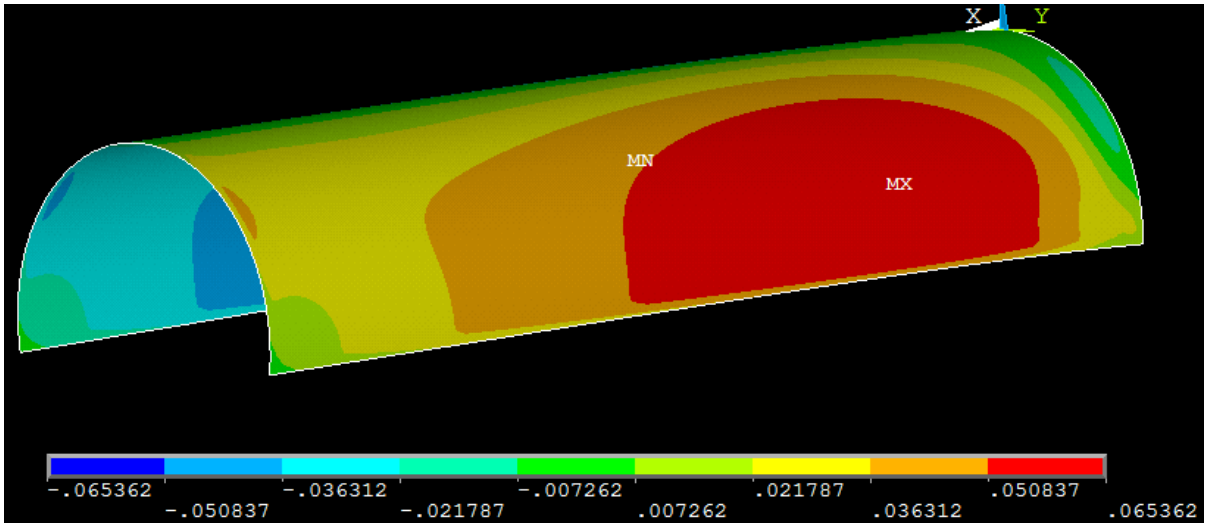


Figure 8. Torsion moment m_{xy} [kNm/m]

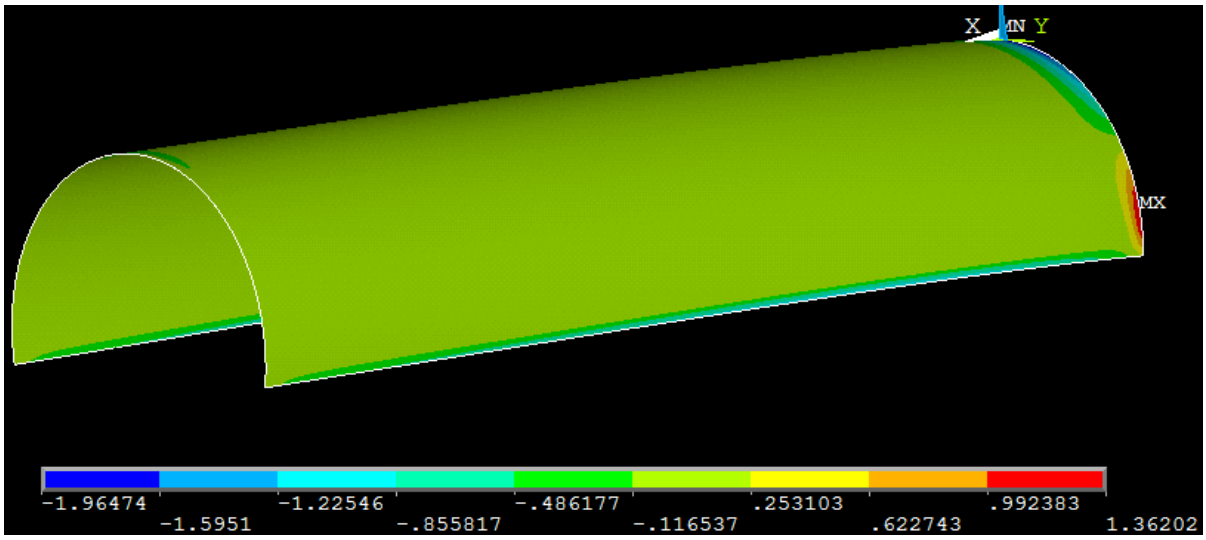


Figure 9. Out of plane shear force q_x [kNm/m]

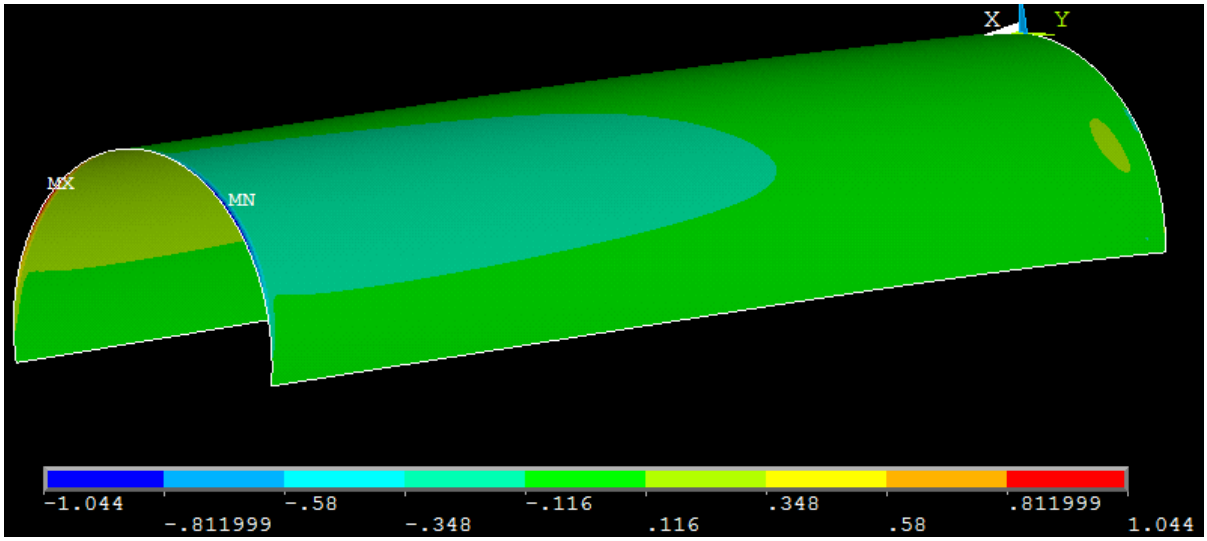


Figure 10. Out of plane shear force q_y [kN/m]

Problem 3 (30 minutes) 1.7 points

a When is it smart to include ribs in a shell design? Choose A, B, C or D.

A When its shape is cylindrical or nearly cylindrical.
B When buckling occurs before yielding.
C To remove whispering galleries.
D When it is made of steel, aluminium or copper.

b Each nodes has 6 dofs but in shell analysis 1 of these is not doing much. Why is it there? Choose A, B, C or D.

A To use the shell elements in general purpose finite element programs.
B To provide extra freedom in the shell element movement and node movements.
C To enable torsion loading to shell surfaces.
D To reduce the computation time of very large shell finite element models.

c We want to compute the curvature tensor of a shell at the location of a finite element. The only thing we know are the nodal coordinates of this element. Clearly, this computation is not possible if the element is flat.[1] How many nodes does the element need to have at least to make this computation possible? Choose A, B, C or D.

A 3
B 4
C 6
D 8

d Often when we increase the thickness of a shell the moments increase too. What is the correct explanation for this?
Choose A, B, C or D.

A imperfection sensitivity
B edge disturbance
C hardly any moments compared to arches
D in-extensional deformation

e What is your favourite way of studying shell structures?
You may choose more than one answer.

A Reading the handouts at home.
B Doing individual assignments in class.
C Doing group assignments in class.
D Listening to a teacher.

[1] Footnote: From neighbouring flat elements we can get an impression of shell curvature. However, from a computational point of view this data is not accessible because finite elements do not know who their neighbours are.

Problem 4 (30 minutes) 1.7 points

a To solve the Sanders-Koiter equations we need to know the following parameters for every point of a shell. Curvatures k_{xx}, k_{yy}, k_{xy}, Lamé parameters α_x, α_y and load p_x, p_y, p_z. In addition we need to know the boundary conditions. Which of these parameters are dependent? Explain your answer.

b Influence lengths can be related to curvature. Which influence length is related to k_{xx}, which is related to k_{yy} and which is related to k_{xy}? Which of these lengths is most important? Explain your answers.

c Consider two parallel rings (Fig. 11). The rings are connected by a soap film (not shown in the figure). Two shapes are possible; a cylinder and a catenoid. Which will occur? Explain your answer.

d On 10 June 2015 we tested 9 small shell objects. Most objects failed due to yielding or crushing. However, some buckled without yielding or crushing. What was typical about the latter objects?

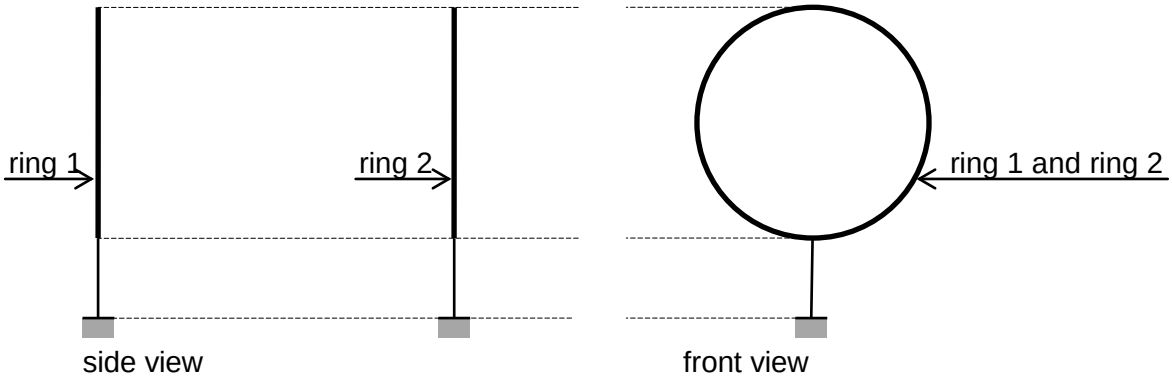


Figure 11. Two wire frame rings parallel to each other

Problem 5 (30 minutes) 1.7 points

A structure consists of twelve hypars (Fig. 12). The material is fibre reinforced concrete. The shell thickness is t = 10 mm. Snow load plus self-weight is 3 kN/m². Safety factors are included in all numbers.

a Does this design need cables? If so, add these.

b Calculate the membrane forces and related stress.

c Calculate the largest bending moment and related stress in the hypars. (Assume that the material is linear-elastic.)

d Calculate the forces in the beams, columns and cables if present.

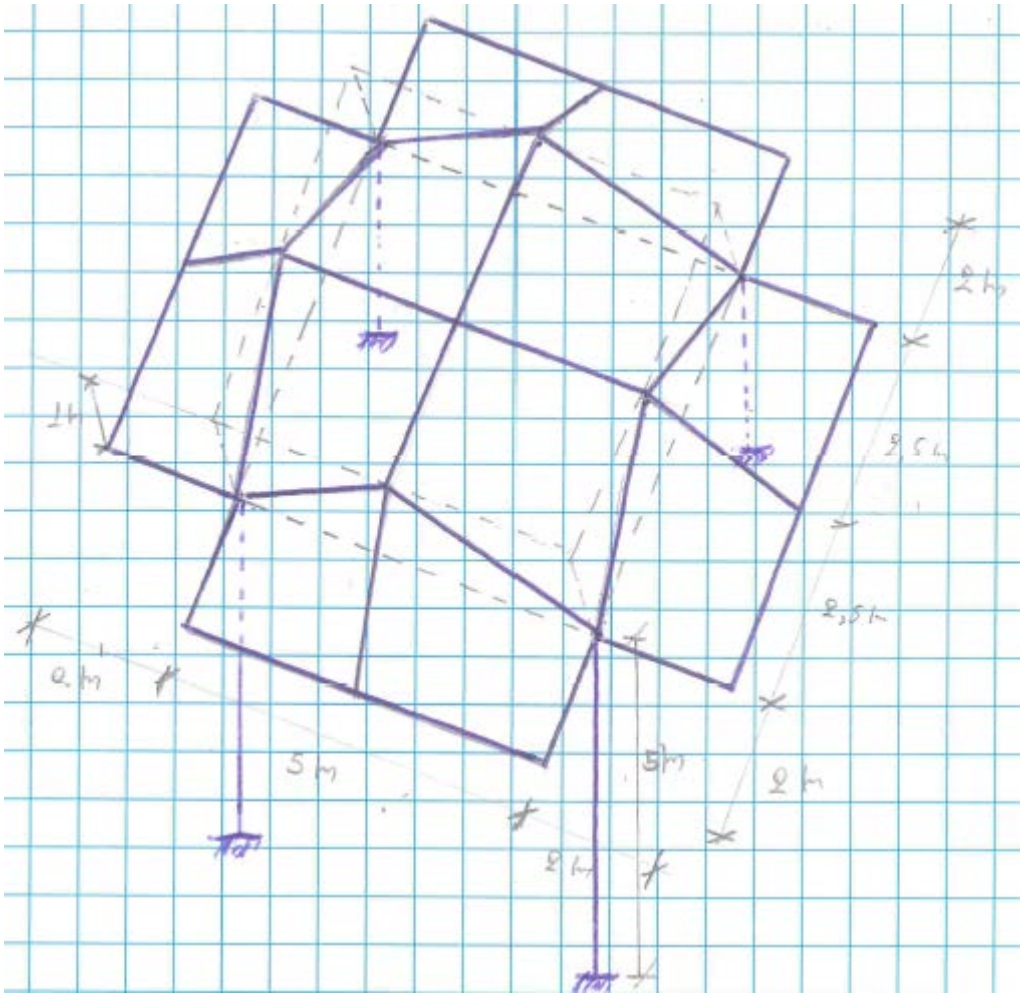


Figure 12. A twelve hypar roof (Design by Apostolos Bougioukos, shell exam July 2015)

e Check buckling. Young's modulus is 15000 N/mm².

f Calculate the required fibre reinforcement percentage. Fibre yield strength is 450 N/mm^2.

g Can this design be build? Would it be safe? Explain your answers.

Problem 6 (30 minutes) 1.5 point

Design an assembly of hypar shells to be analysed in the next Shell exam (July 2016).
It needs to be
1) different than in this exam and different than in the previous exams,
2) different than the examples in the handouts,
3) stable,
4) not extremely complicated.
You do not need to analyse this shell.

Answers to problem 1

Meridional force ..... A shell of revolution can be described by hoops and meridional lines. A normal force in the direction of a meridional line is a meridional force.

Lamé parameters ..... $\alpha_x = \frac{du}{dx}, \quad \alpha_y = \frac{dv}{dy}$

Middle surface ..... Imaginary surface halfway between the inner and outer surface of a shell

Enneper's surface ..... $x = 2au(1+v^2-\frac{1}{3}u^2),\ y = -2av(1+u^2-\frac{1}{3}v^2),\ z = 2a(u^2-v^2)$

Statistical size effect ..... Large structures fail at smaller stresses than small structures.

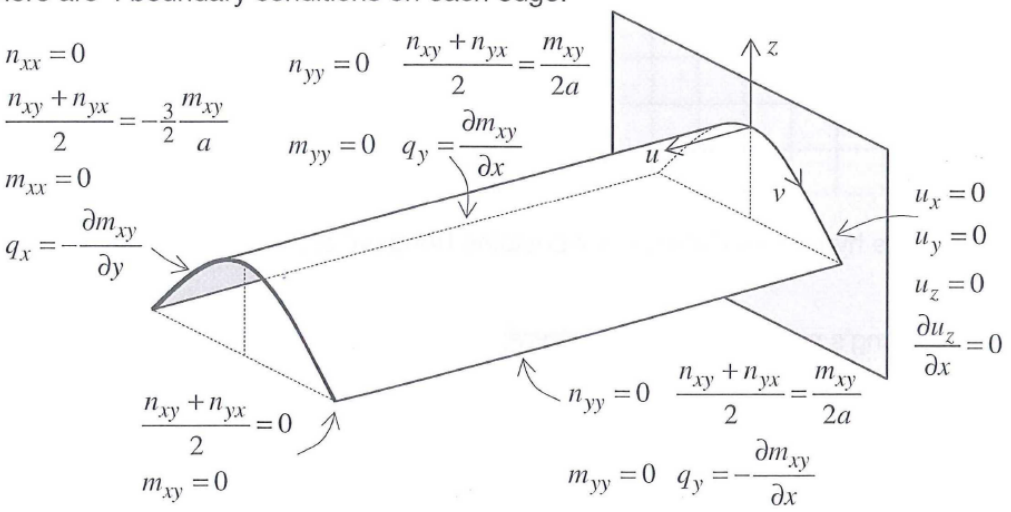
Monocoque ..... Originally the name of a fast airplane with a shell fuselage. Now used for cars that derive strength from their skin.

Sanders-Koiter equations ... 21 equations that describe the behaviour of thin shell structures

Kissing circle ..... Best approximating circle of a curved line at a point

Answers to problem 2

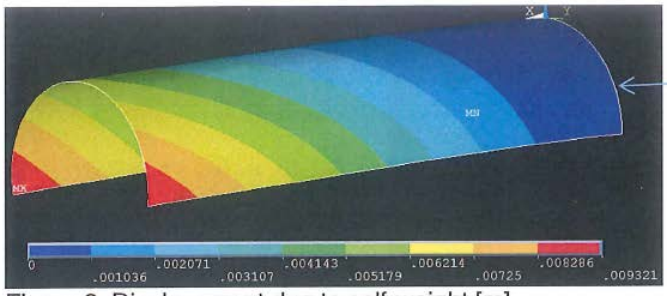
a There are 4 boundary conditions on each edge.



b Finite element analysis does not use the Sanders-Koiter equations. (except for semiloof elements.)

c The imposed displacements and rotations on boundaries need to be entered. The boundary conditions on the unsupported boundaries are fulfilled automatically.

d The finite element results agree with the shell boundary conditions:



$u_x = 0$
$u_y = 0$
$u_z = 0$
Correct
(We cannot see whether there are no rotation

Figure 2. Displacement due to self-weight [m]

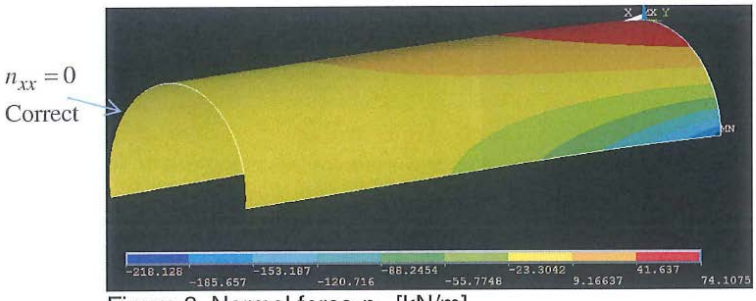
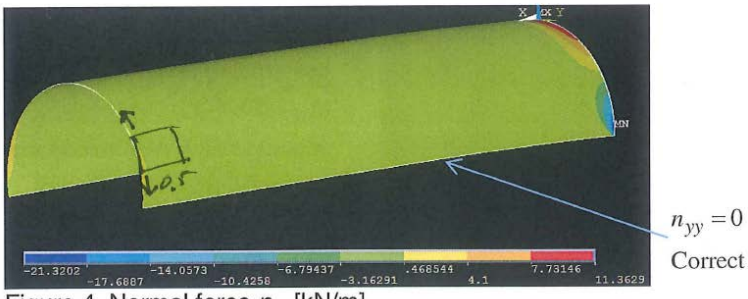


Figure 3. Normal force n_{xx} [kN/m]



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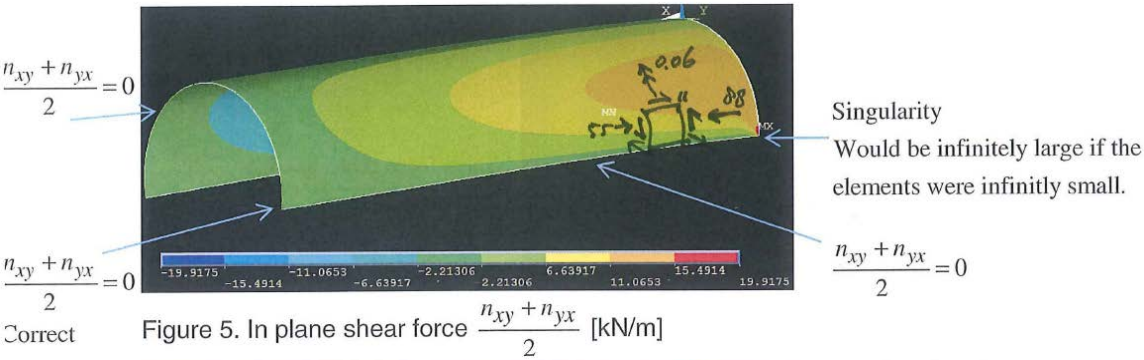


Figure 5. In plane shear force $\frac{n_{xy}+n_{yx}}{2}$ [kN/m]

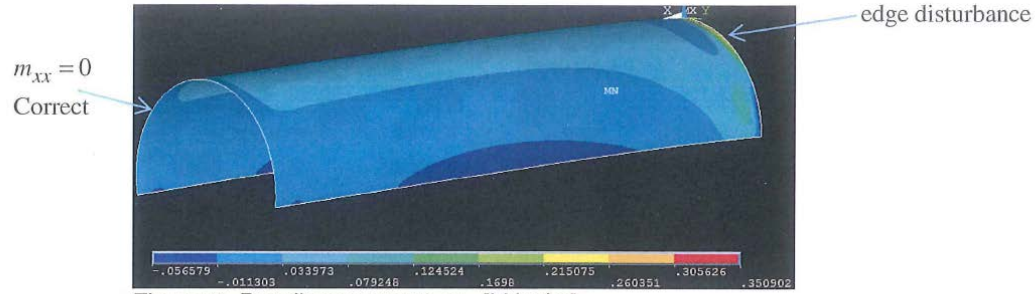


Figure 6. Bending moment m_{xx} [kNm/m]

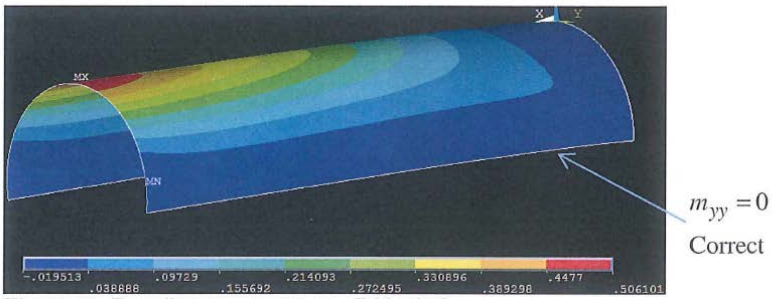


Figure 7. Bending moment m_{yy} [kNm/m]

concentrated shear force

$m_{xy} = 40$ N

$m_{xy} = 0$

Correct

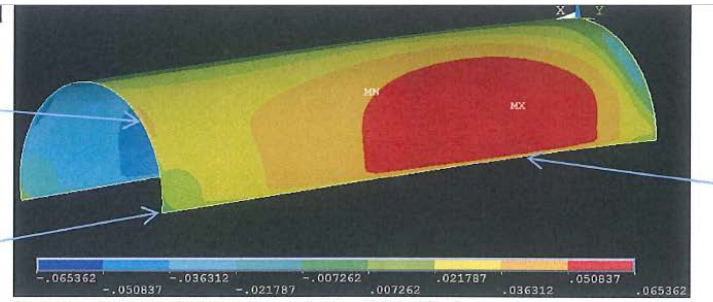


Figure 8. Torsion moment m_{xy} [kNm/m]

concentrated shear force

$m_{xy} = 50$ N

It would be 0 if the elements would very small (Reissner-Mindlin).

$$\frac{m_{xy}}{2a} = \frac{0.050}{2 \times 2} = 0.013 \text{ kN/m}$$

Can be neglected compared to the values of $\frac{n_{xy} + n_{yx}}{2}$.

Correct

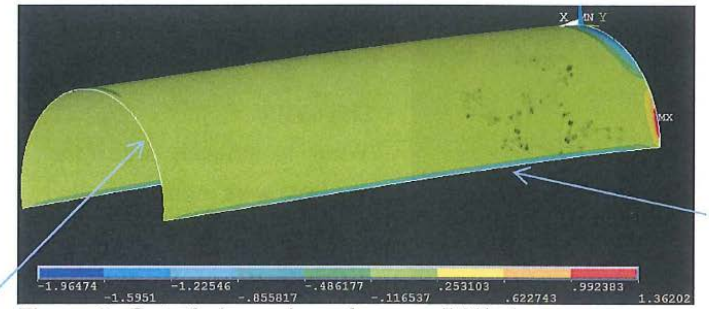


Figure 9. Out of plane shear force q_x [kN/m]

Shear force peaks at edge

$q_x = -0.48$ N/mm

It would be larger if the elements were very small (Reissner-Mindlin).

$q_x = 0$

$$-\frac{\partial m_{xy}}{\partial y} = -\frac{0.0217 - 0.0363 \text{ kN}}{2 \text{ m}} = 0.007 \text{ kN/m}$$

Can be neglected compared to the values of q_x.

Correct

shear force peaks at edge

$q_y = 1$ N/mm

It would be larger if the elements were very small (Reissner-Mindlin).

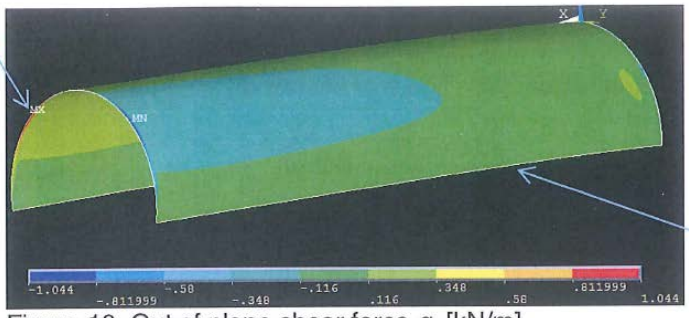


Figure 10. Out of plane shear force q_y [kN/m]

$q_y = 0$

$$-\frac{\partial m_{xy}}{\partial x} = -\frac{0.0363 - 0.0508 \text{ kN}}{3 \text{ m}} = 0.005 \text{ kN/m}$$

Can be neglected compared to the values of q_y.

Correct

Answers to problem 3

a B
b A
c C
d B
e A 37%, B 2%, C 30%, D 31%

Answers to problem 4

a The curvatures and the Lamé parameters are dependent because each can be used to calculate the Gaussian curvature.

b Assuming that y is in the direction of the discontinuity;

curved cylinder edge ..... $l_i = 2.40\sqrt{a\,t}$ ..... k_{yy} ..... $l_i = 2.40\sqrt{\dfrac{t}{k_{yy}}}$

straight cylinder edge ..... $l_i = 3.40\sqrt[4]{a\,t\,l^2}$ ..... k_{xx} ..... $l_i = 3.40\sqrt[4]{\dfrac{t\,l^2}{k_{xx}}}$

straight hypar edge ..... $l_i = 2.25\sqrt[3]{a\,t\,l}$ ..... k_{xy} ..... $l_i = 2.25\sqrt[3]{\dfrac{t\,l}{k_{xy}}}$

The most important influence length is the shortest because it determines the size of finite elements at edges.

c Catenoid. For this shape the membrane tension in all directions is in equilibrium. It has zero mean curvature.

d The objects that buckled elastically were made of soft plastic. Soft plastic has a very small Young's modulus and large yield strength. Even for thick objects ($a / t = 30$) The relative slenderness is

$$\lambda = \sqrt{0.6\frac{E}{f}\frac{a}{t}} = \sqrt{0.6\frac{2300}{70}30} = 23\,,$$

which is much larger than 1, therefore, they buckle.

Answers to problem 5

a Yes, it needs cables between the column heads. (See figure of answer **d**.)

b $a_1 = \dfrac{2.00\times 2.50}{1.00} = 5.00\text{ m}$ $\qquad$ $a_2 = \dfrac{2.50\times 2.50}{1.00} = 6.25\text{ m}$

$n_{xy1} = \frac{1}{2}a_1\,p = \frac{1}{2}5.00\times 3 = 7.50\text{ kN/m}$ $\qquad$ $n_{xy2} = \frac{1}{2}a_2\,p = \frac{1}{2}6.25\times 3 = 9.38\text{ kN/m}$

$$\sigma_{xy} = \frac{n_{xy}}{t} = \frac{9.38}{0.010} = 245\text{ kN/m}^2 = 0.938\text{ N/mm}^2$$

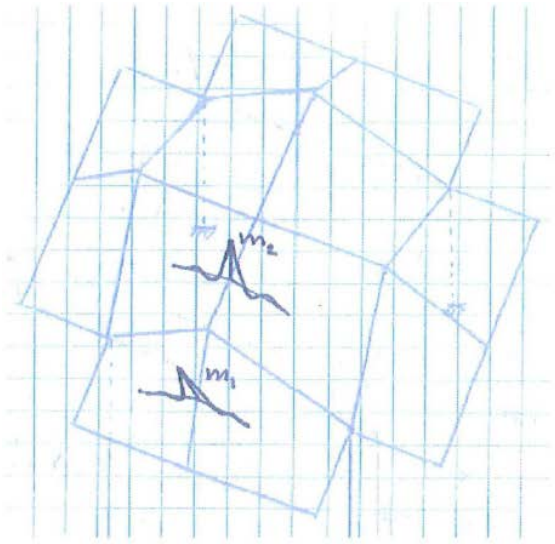
c $l_1 = \sqrt{b^2 + h^2} = \sqrt{2^2 + 1^2} = 2.24\text{ m}$ $\qquad$ $l_2 = \sqrt{c^2 + h^2} = \sqrt{2.5^2 + 1^2} = 2.69\text{ m}$

The interior beams do not rotate. There the largest shell bending moments occur. In addition, the shortest beam gives the largest moment. The largest edge shell moments are

$$m_{xx1} = 0.551\frac{p}{l_1^2}(a_1\,t\,l_1)^{\frac{4}{3}} = 0.551\frac{3}{2.24^2}(6.25\times 0.010\times 2.24)^{\frac{4}{3}} = 0.0165\text{ kN.}$$

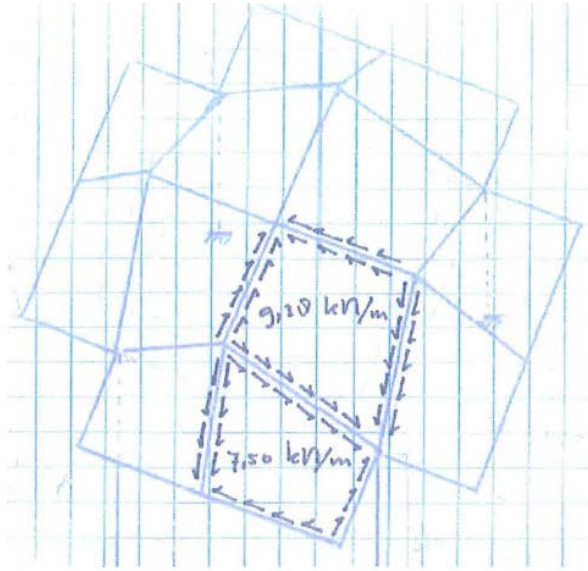
The largest middle shell moments are

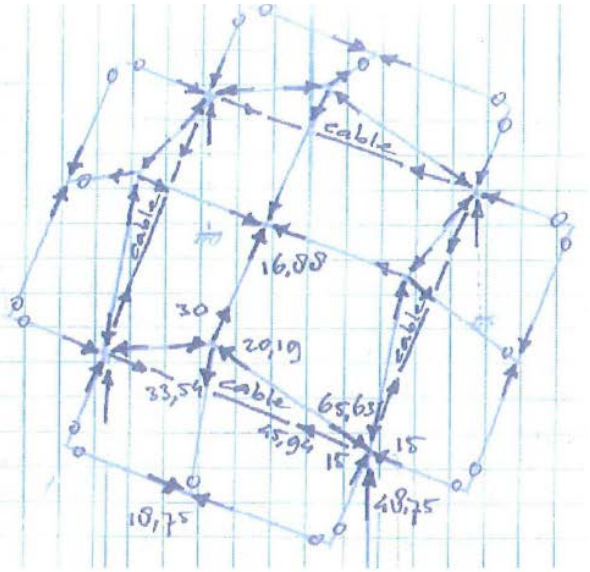
$$m_{xx2} = 0.551\frac{p}{c^2}(a_2\,t\,c)^{\frac{4}{3}} = 0.551\frac{3}{2.50^2}(5.00\times 0.010\times 2.50)^{\frac{4}{3}} = 0.0206\text{ kN.}$$

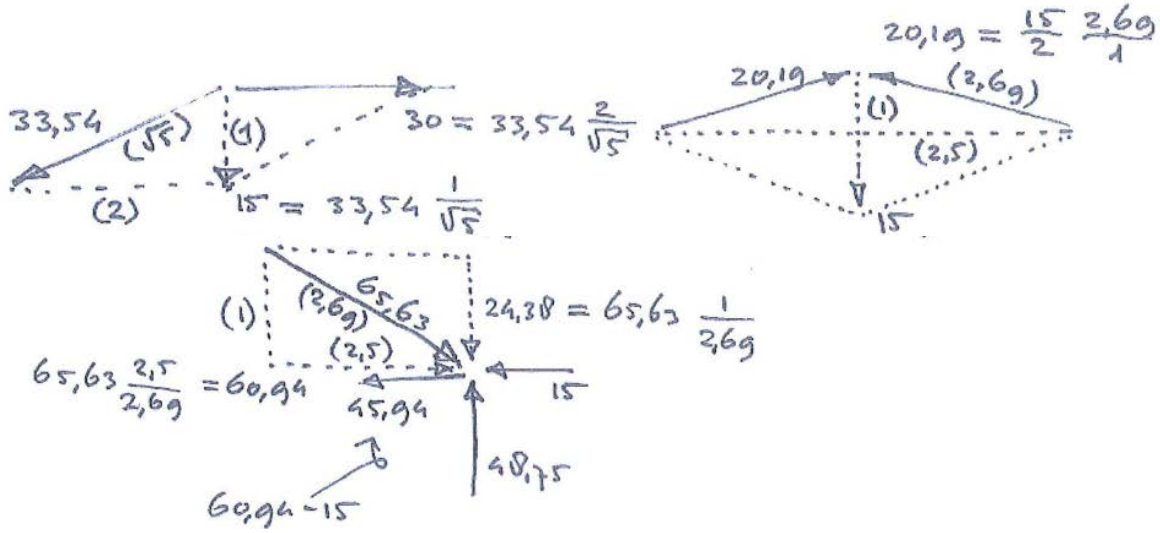


$$\sigma_{xx} = \frac{m_{xx}}{\frac{1}{6}t^2} = \frac{0.0206}{\frac{1}{6}0.010^2} = 1238 \text{ kN/m}^2 = 1.24 \text{ N/mm}^2$$

d







Check column force: p (4 x 2.5 x 2.5 +8 x 2.5 x 2) / 4 = 48.75 kN OK

e $n_{cr} = 0.6\frac{E t^2}{a} = 0.6\frac{15\times10^6 \times 0.010^2}{6.25} = 144$ kN/m No buckling because 9.38 < 144.

f $\rho = 3\frac{\sigma_1 + \sigma_1}{f_y} = 3\frac{0.94\ \text{N/mm}^2 + 1.24\ \text{N/mm}^2}{450\ \text{N/mm}^2} = 1.44\%$

The factor 3 is because the fibres are not just directed in the direction of the tensile force but they are randomly directed in 3 dimensions.

g Yes, it can be build. The panels can be prefabricated on two formworks. The panels can be transported by truck to the construction site and lifted in place by a crane. The beams and columns can be cast in place.
Yes, it would be safe. The stresses are not too large for fibre reinforced concrete, it will not buckle and the reinforcement ratio is not too large. However, it can be necessary to brace the columns against storms and earthquakes.