Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4143 Shell Analysis
Tuesday 13 August 2019, 13:30 – 16:30 hours

Choose A, B, C or D.

1. k_{xx} = 1/(5 m), k_{yy} = -1/(8 m), k_{xy} = -1/(3 m). What the mean curvature?

   A 3/(80 m)
   B 3/(40 m)
   C 13/(80 m)
   D 49/(240 m)

2. What is a compatibility moment?

   A A moment that does not contribute much to carrying the load but is caused by deformation that is necessary for parts to stay connected.
   B The largest moment in a shell structure.
   C A negative bending moment in a shell structure.
   D A moment that produces stresses that can be neglected compared to the stresses due to membrane forces.

3. Consider the curvature of an ellipsoid. How many lemon umbilics does it have?

   A 0
   B 1
   C 2
   D 4

4. Is this parameterisation orthogonal? $\bar{x} = \frac{a}{2}(\cos u + \cos v)$

   $\bar{y} = \frac{a}{2}(\sin u + \sin v)$

   $\bar{z} = a \sin \frac{u - v}{2}$

   A Yes because every possible parameterisation of a sphere is orthogonal.
   B Yes because $\frac{\partial \bar{x}}{\partial u}\frac{\partial \bar{x}}{\partial v} + \frac{\partial \bar{y}}{\partial u}\frac{\partial \bar{y}}{\partial v} + \frac{\partial \bar{z}}{\partial u}\frac{\partial \bar{z}}{\partial v} = 0$.
   C No.
   D It depends on the value of a.

5. Consider a flat type quadrilateral shell finite element.
   What is an important limitation of this element?

   A Its edges need to be in the principal curvature directions.
   B It cannot describe twisting deformation.
   C It does not fit easily in general purpose finite element programs.
   D It needs an extra dof for each node.

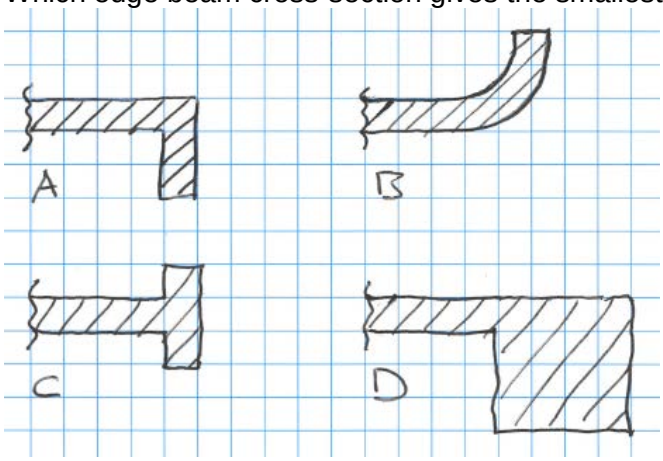
6. What is a good way of designing for edge disturbances?

   A Make the whole shell thick
   B Make the shell thicker at the edges
   C Make the shell thinner at the edges
   D Accept plastic deformation

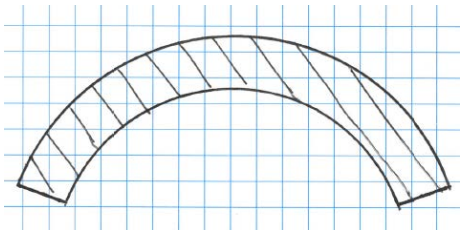
7. Consider a perpendicular point load on a thin shell of positive Gaussian curvature. Does the pressure line go through the middle third?

   A No because the point load causes moments.
   B No because the pressure line is perpendicular to the surface.
   C No because $\left|\frac{m_{xx}}{n_{xx}}\right| \geq \frac{1}{6}t$.
   D It depends on the dimensions, material and boundary conditions.

8. Which edge beam cross-section gives the smallest edge disturbance?



9. How thick is this shell cross-section?



   A very thick
   B thick
   C thin
   D Cannot be determined because dimensions are not given.

10. A chain with a weight of 10 N/m spans a 40 m wide river. The chain deflects 10 m. What is the tensile force in the middle of the chain?

11. What is the smallest radius of the curvature of a shell shaped as a truncated cone?

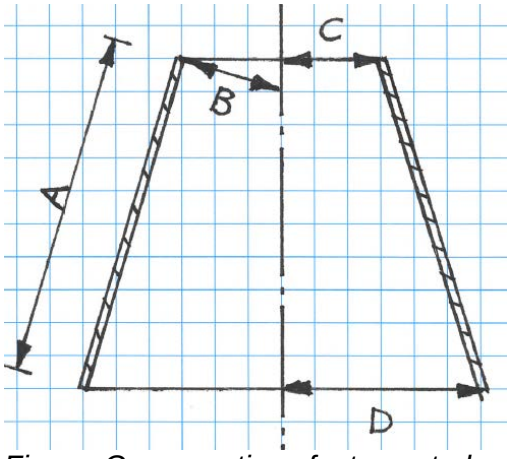


Figure: Cross-section of a truncated cone

12. What is the in absolute value largest normal force in this arch?

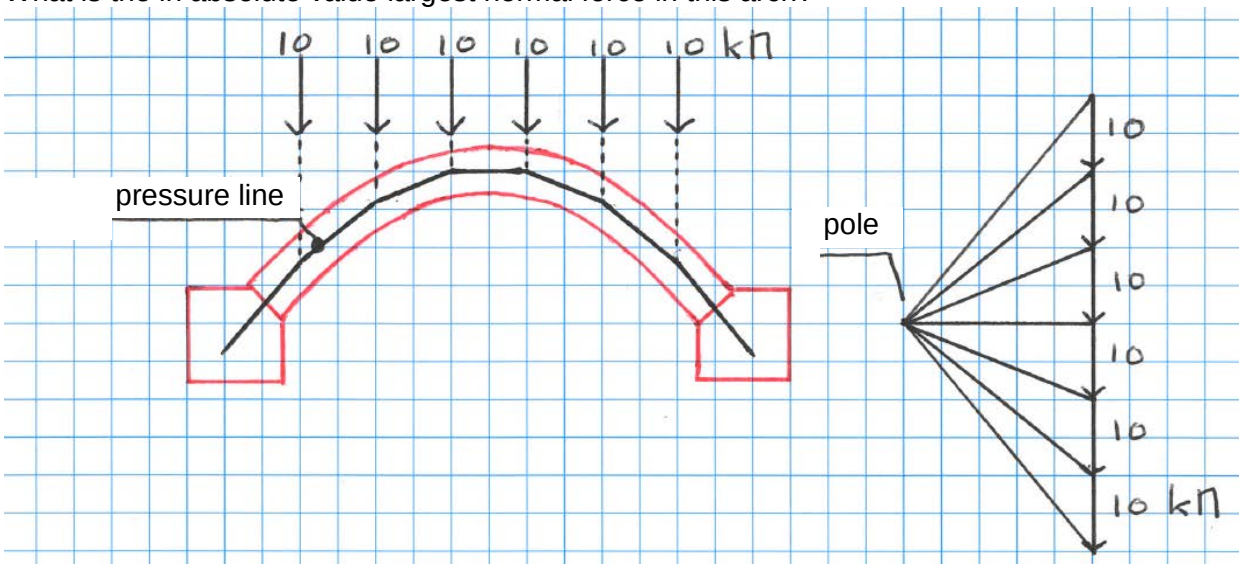


Figure: Line of action plan *Figure: magnitude plan*

A -10 kN
B -25 kN
C -32 kN
D -39 kN

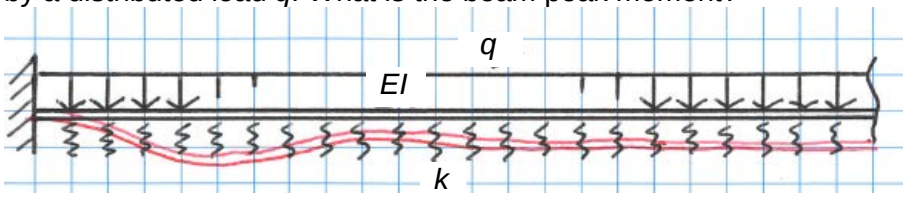
13. What is the *a*/*t* ratio in the top of Zeckendorf plaza?

A 440
B 528
C 823
D 2112

14. What is a flying buttress?

A A free arch that carries the horizontal support reaction of a vault
B A decorative stone element of old cathedrals
C The horizontal support reaction of an arch
D An arch that is supported by columns. Consequently, at least one of the supports of this arch is free to move horizontally.

15. A long beam (stiffness *EI*) is supported by springs (stiffness *k*), fixed in one end and loaded by a distributed load *q*. What is the beam peak moment?



A $-q\sqrt{\frac{EI}{k}}$

B $-q\sqrt{kEI}$

C $-q\sqrt{\frac{k}{EI}}$

D $-\infty$

16. A shell has a surface area of 613 m^2 and a thickness of 10 mm. The radius of curvature is 30 m. The Gaussian curvature is positive. The finite element model uses 8 node elements which are all about the same size. How many dofs are in this model?

A 23 000
B 230 000
C 2 300 000
D 23 000 000

17. The bending moment in a shell part is m_{xx} = 3 kNm/m and the membrane force is n_{xx} = 1000 kN/m. The shell has a thickness *t* = 10 mm. What is the largest normal stress?

A 80 N/mm^2
B 100 N/mm^2
C 130 N/mm^2
D 280 N/mm^2

18. Suppose we want to solve the Sanders-Koiter equations for a particular shell with particular supports and a particular load. How many unknown quantities (functions) do we need to solve?

A 4
B 21
C 31
D 34

19. The three influence lengths are $2.4\sqrt{at}$, $3.4\sqrt[4]{atl^2}$ and $2.25\sqrt[3]{atl}$. Put these in order from small to large. (Assume realistic thin shell dimensions. For example, for the straight cylinder edge *l* = 6*a* and for the hypar edge *l* = *a*)

A $2.4\sqrt{at}$, $2.25\sqrt[3]{atl}$, $3.4\sqrt[4]{atl^2}$.

B $3.4\sqrt[4]{atl^2}$, $2.4\sqrt{at}$, $2.25\sqrt[3]{atl}$.

C $2.25\sqrt[3]{atl}$, $3.4\sqrt[4]{atl^2}$, $2.4\sqrt{at}$.

D $2.25\sqrt[3]{atl}$, $2.4\sqrt{at}$, $3.4\sqrt[4]{atl^2}$.

20. Which is invariant? In other words, which does not change if the directions of the local *x* and *y* axis are changed?

A $k_{xx}^2 + 2k_{xy}^2 + k_{yy}^2$

B $k_{xx}^2 - 2k_{xy}^2 + k_{yy}^2$

C $k_{xx}^2 + k_{yy}^2$

D $k_{xx}^2 - k_{yy}^2$

21. A shell design can behave like a flat plate. What name do we use for this?

A In-extensional deformation
B Middle third rule
C Gaussian curvature
D Theorema Egregium

22. What is a knockdown factor?

A 1/6
B A safety factor to prevent in-extensional deformation
C A reduction factor of the linear buckling load that accounts for imperfections
D A safety factor for nonlinear analyses of thin shell structures.

23. Suppose we build a shell structure out of thin steel plates. Each plate has a different shape and curvature. We press a plate in the right curvature with a *die*. Do we need as many *dies* as there are plates?

A Yes
B No, we need only *dies* for giving the plates the right Gaussian curvature. The mean curvature can be changed by hand.
C No, we need only one *die* because we can optimise the design to make all plates the same.
D No, we do not need *dies* because the plates are thin and can be bend by hand.

24. For thin shell structures we use larger safety factors than for similar truss, frame and plate structures. Why is this?

A Thin shell structures are difficult to analyse and somebody might make a mistake.
B Not all shell structures are very efficient in carrying load due to inextensional deformation.
C If a thin shell structure would fail it would often do this abruptly.
D The buckling load of many thin shell structures is sensitive to small shape imperfections.

25. A thin shell is loaded by a harmonic point load. What do we design for?

A All natural frequencies must be larger than the load frequency.
B The natural frequencies must be either substantially larger or substantially smaller than the load frequency.
C The smallest natural frequency must be smaller than the load frequency.
D Add dampers to the structure.

26. Consider a hemispherical wine glass. Young's modulus of the glass is E = 70 000 N/mm^2. It has a mass density ρ = 3000 kg/m^3, a thickness t = 1.5 mm and a diameter of 70 mm. Can this class be broken by a high pitch human voice?

    A Yes, It is a normal wine class and these can be broken by human voice.
    B This cannot be determined because Poisson's ratio is missing.
    C No, its natural frequencies are too large.
    D Yes, at least one of its natural frequencies is smaller than 1000 Hz.

27. Compare the finite element error in a shell edge with this error somewhere in the shell interior. Is there a difference in the order of the errors?

    A No
    B Yes, in the edge there is no error because the boundary conditions impose the values.
    C Yes, close to the edge we use smaller elements.
    D Yes, in the edge nodes is averaged over less elements.

28. What is a shallow shell?

    A A shell with a small a/t ratio
    B A shell with a small sagitta to span ratio
    C A shell with a small radius to sagitta ratio
    D A shell that can be approximated well by the shallow shell differential equation

29. Two shells are built with the same thickness, the same curvature, the same construction method and by the same construction workers. One shell is large the other is small. We can expect that in the large shell the largest imperfection for buckling is larger than in the small shell. What causes this?

    A The buckling length of a large shell is larger.
    B The construction workers get tired when building the large shell.
    C The large shell deforms more.
    D The chance of a large imperfection increases with the area.

30. Write a good shell multiple choice question for next year's exam. The question needs to be not too easy and not too difficult. The question needs to be not just copied and not copied-and-modified-a-bit. (Unless the latter modification is substantial and original.)

Answers

1. A 100% of the answers were correct ……

> $kxx := \frac{1}{5 \cdot m} : kyy := -\frac{1}{8 \cdot m} : kxy := -\frac{1}{3 \cdot m} :$

> $\frac{kxx + kyy}{2};$

$$\frac{3}{80\ m}$$

2. A 100%
3. D 67%
4. C 89% ……

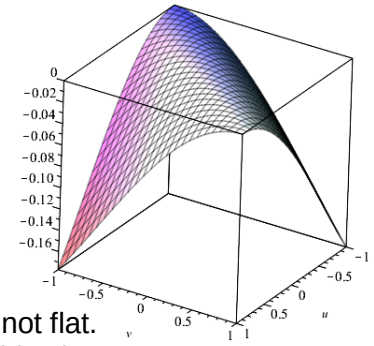
> $x := \frac{a}{2} \cdot (\cos(u) + \cos(v)) :$

> $y := \frac{a}{2} \cdot (\sin(u) + \sin(v)) :$

> $z := a \cdot \sin\left(\frac{u - v}{2}\right) :$

> $g := diff(x, u) \cdot diff(x, v) + diff(y, u) \cdot diff(y, v) + diff(z, u) \cdot diff(z, v) :$

> $a := 1 : plot3d(g, u = -1..1, v = -1..1);$



5. A 11% … kxy needs to be zero, otherwise it would be twisted and not flat.
   (B 22% is not true, C 22% is not true, D 44% is not a problem)
6. D 67%
7. C 17% … (D 72%) …

> $nxx := -\frac{\mathrm{sqrt}(3)}{8} \cdot \frac{P}{t} \mathrm{sqrt}\left(1 - \mathrm{nu}^2\right) :$

> $k1s := \mathrm{abs}(0.00725 \cdot k2 + 0.199 \cdot k1) + \mathrm{abs}(0.0529 \cdot k2 + 0.0298 \cdot k1) :$

> $mxx := 0.0388 \cdot (1 + \mathrm{nu}) \cdot P \cdot \ln\left(\frac{t}{k1s \cdot d^2}\right) :$

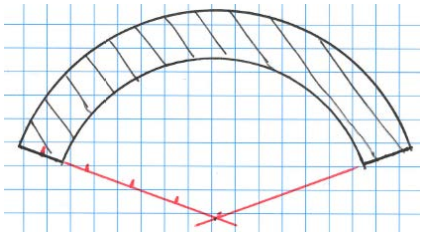
> *# After trying some numbers the numbers below give the smallest eccentriciy.*

> $\mathrm{nu} := 0 : d := 5 \cdot t : a := 30 \cdot t : k1 := \frac{1}{a} : k2 := \frac{1}{a} :$

> $assume(t > 0) : simplify\left(\frac{mxx}{nxx}\right); \frac{-1}{6.} \cdot t;$

$$-0.2551626725\ t\sim$$

$$-0.1666666667\ t\sim$$



8. B 89%
9. A 94% …… $a/t = 4 < 5$

10. 215 N 50% …… During the exam a mistake was found in the reader.
    This mistake affected question 10. Therefore, it was changed
    from a multiple choice question to an open question.

> $q := 10 \cdot \frac{N}{m} : l := 40 \cdot m : x := 0 \cdot m : T := 215 \cdot N :$

> $y := evalf\left(\frac{T}{q} \cdot \left(\cosh\left(\frac{q \cdot x}{T}\right) - \cosh\left(\frac{q \cdot l}{2 \cdot T}\right)\right)\right);$

$$y := -9.992776794\ m$$

11. B 39% ..... To find a kissing circle; push a needle through the shell, perpendicularly. Select a point on the shell next to the point where the needle was pushed through, in the direction you want to know the curvature. Take a second needle and also push this through perpendicularly. The needles cross in a point, which is the middle of the kissing circle. Its radius is the radius of curvature.

12. D 56% ..... The lines to the pole are vectors of the forces in the arch. The lengths can be measured.

13. B 61% .....

$$> b := \frac{132}{2} \cdot f : c := \frac{112}{2} \cdot f : h := 28 \cdot f : t := 3 \cdot i :$$

$$> i := \frac{f}{12} : a := \frac{b \cdot c}{h} : \frac{a}{t};$$

14. A 83%

15. A 61%

$$528$$

16. B 83% .....

$$> A := 613 \cdot m^2 : t := 10 \cdot mm : a := 30. \cdot m :$$

$$> m := 1000 \cdot mm : assume(mm > 0) :$$

$$> li := 2.4 \cdot \mathrm{sqrt}(a \cdot t) : h := \frac{li}{6} : NumberOfElements := \frac{A}{h^2} :$$

$$> NumberOfNodes := 3 \cdot NumberOfElements : \#1\ extra\ element\ needs\ 3\ extra\ nodes$$

$$> NumberOfDofs := 6 \cdot NumberOfNodes;$$

$$NumberOfDofs := 229874.9999$$

17. D 100% .....

$$> mxx := 3\ kN : nxx := 1000 \cdot \frac{kN}{m} : t := 10 \cdot mm : kN := 1000 \cdot N : m := 1000 \cdot mm :$$

$$> \mathrm{sigma} := \frac{nxx}{t} + \frac{6 \cdot mxx}{t^2};$$

$$\sigma := \frac{280\ N}{mm^2}$$

18. B 89%

19. A 89% .....

$$> a := 100. \cdot t : \ lc := 6 \cdot a : lh := a :$$

$$> 2.4 \cdot \mathrm{sqrt}(a \cdot t); 2.25 \cdot (a \cdot t \cdot lc)^{\frac{1}{3}}; 3.4 \cdot \left(a \cdot t \cdot lh^2\right)^{\frac{1}{4}};$$

$$24.00 \sqrt{t^2}$$

$$88.08452192 \left(t^3\right)^{1/3}$$

$$107.5174404 \left(t^4\right)^{1/4}$$

20. A 56% .....

$$> krr := kxx \cdot \cos(f)^2 + kyy \cdot \sin(f)^2 + kxy \cdot 2 \cdot \sin(f) \cdot \cos(f) :$$

$$> kss := kxx \cdot \sin(f)^2 + kyy \cdot \cos(f)^2 - kxy \cdot 2 \cdot \sin(f) \cdot \cos(f) :$$

$$> krs := (kyy - kxx) \cdot \sin(f) \cdot \cos(f) + kxy \cdot \left(\cos(f)^2 - \sin(f)^2\right) :$$

$$> simplify\left(krr^2 + 2 \cdot krs^2 + kss^2\right);$$

$$kxx^2 + 2\ kxy^2 + kyy^2$$

$$> simplify\left(krr^2 - 2 \cdot krs^2 + kss^2\right);$$

$$4\ (-2\ kxy + kxx - kyy)\ (2\ kxy + kxx - kyy) \cos(f)^4 + 16\ kxy \sin(f)\ (kxx - kyy) \cos(f)$$
$$- kyy)\ (2\ kxy + kxx - kyy) \cos(f)^2 - 8\ kxy \sin(f)\ (kxx - kyy) \cos(f) + kxx^2 - 2\ k$$

$$> simplify\left(krr^2 + kss^2\right);$$

$$2\ (-2\ kxy + kxx - kyy)\ (2\ kxy + kxx - kyy) \cos(f)^4 + 8\ kxy \sin(f)\ (kxx - kyy) \cos(f)^3$$
$$- kyy)\ (2\ kxy + kxx - kyy) \cos(f)^2 - 4\ kxy \sin(f)\ (kxx - kyy) \cos(f) + kxx^2 + kyy$$

$$> simplify\left(krr^2 - kss^2\right);$$

$$(kxx + kyy)\ (4\ kxy \sin(f) \cos(f) + 2\ kxx \cos(f)^2 - 2\ kyy \cos(f)^2 - kx$$

21. A 94%
22. C 94%
23. B 39% … (A 39%, C 22% would be true if we optimise into a cylinder, D 0%)
24. C 61% … (D 39% imperfection sensitivity is taken into account by a knockdown factor)
25. A 72% … (B 17% is wrong because shell natural frequencies are closely spaced.)
26. C 22% … (A 11%, B 28%, D 39%) ……. for any Poisson ratio between 0 and 0.5.

$$> E := 70000 \cdot \frac{N}{mm^2} : t := 1.5 \cdot mm : \mathrm{nu} := 0.50 : \mathrm{rho} := 3000 \cdot \frac{kg}{m^3} : a := \frac{70 \cdot mm}{2} :$$

$$> m := 1000 \cdot mm : N := kg \cdot \frac{m}{s^2} : assume(mm > 0) : assume(s > 0) :$$

$$> fn := 0.2407 \cdot \frac{t}{a^2} \cdot \mathrm{sqrt}\left(\frac{E}{\mathrm{rho} \cdot (1 + \mathrm{nu})} \right);$$

$$fn := \frac{1162.449309}{s\sim}$$

27. D 11% …. (A 33% would be right if nodal averaging would be switched off, which is hardly ever done in engineering computations, B 6% is wrong because membrane forces and moments in edges are approximated in the finite element method, C 50% is wrong because the order, for example O(h^2), does not depend on the element size h.)
28. B 72% …. (D 28% is wrong because shallow shells exist independently of the shallow shell differential equation.)
29. D 50% …. (A 17% is wrong because a and t are the same so the buckling lengths are the same, B 6% is wrong because workers have breaks, C 28% is wrong because in general deformation does not look like a buckling imperfection. Anyway, deformation is included in nonlinear finite element analysis and we do not add it to the model.)
30. 78%

The grade is $\frac{40s - 280}{89}$ rounded to the closest halve, where s is the number of correct answers.

The grade formula above is derived as follows. Question 27 was too difficult, perhaps controversial, and could not be known or deduced. Therefore, a student with grade g knows 29/10 g answers. This student guesses 28 - 27/10 g answers. (A student with grade 10 guesses 1 answer and a student with grade 0 guesses 28 answers because there are 28 multiple choice questions.) The expected number of correct answers is s = 29/10 g + (28 – 27/10 g)/4, from which g can be solved.

