Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plasticity Theory
Friday 2 February 2007, 14:00 – 17:00 hours

Problem 1

A frame consists of three beams with strengths M_p (Fig. 1). The beams have pin connections to the foundation and each other. The frame is loaded by seven horizontal forces *F*. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

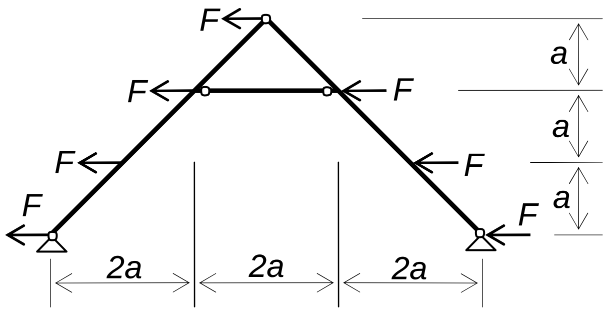


Figure 1. Frame

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

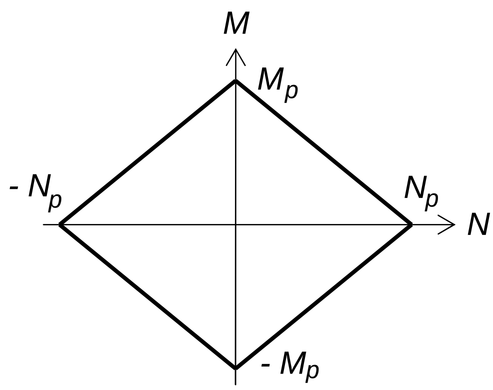


Figure 2. Yield contour

a Assume $\beta \to \infty$. Determine the collapse load *F* for possible mechanisms. Write the collapse loads as functions of M_p and *a*. What is the decisive collapse load? (1.5 points)

b Assume $\beta \to \infty$. Draw the bending moment diagram for the structure at the moment of collapse. (1 point)

c Assume $\beta = 30\sqrt{2}$. Choose one of the following problems (You need not do both).

– Determine the largest <u>lower-bound</u> for *F*.

– Determine the smallest <u>upper-bound</u> for *F*.

Use the decisive mechanism of problem **1a** and the normal force distribution of Figure 3. If you choose to do the upper-bound analysis you only need to write down the equations. (2 points)

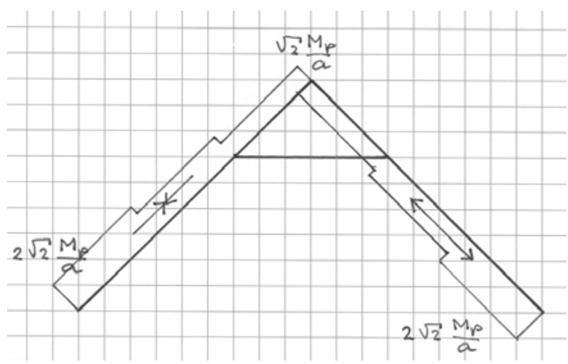


Figure 3. Normal force distribution

Problem 2

A plate is simply supported at two edges (Figure 4). The plate carries a line load *f* [kN/m] at one edge. The plate is homogeneous. The reinforcement directions *x* and *y* are not orthogonal.

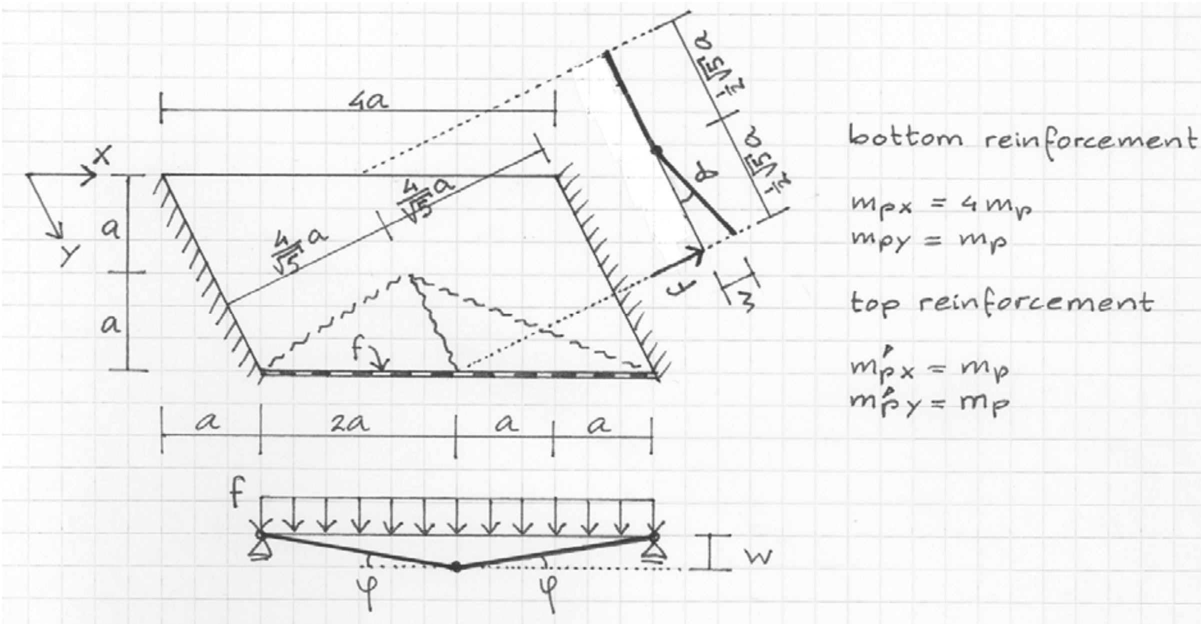


Figure 4. Inclined plate bridge

a We consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms. (1 point)

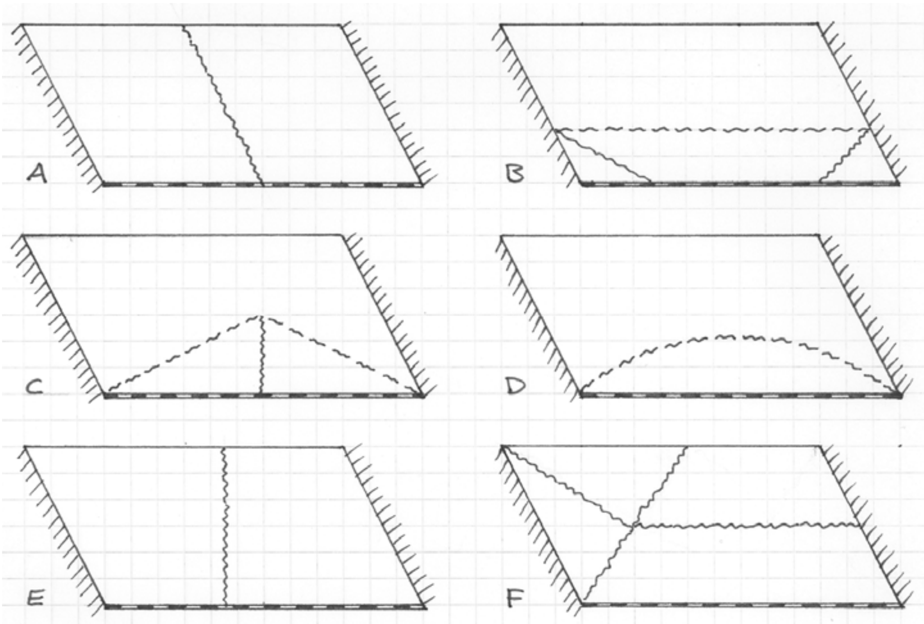


Figure 5. Yield line patterns of problem 2a

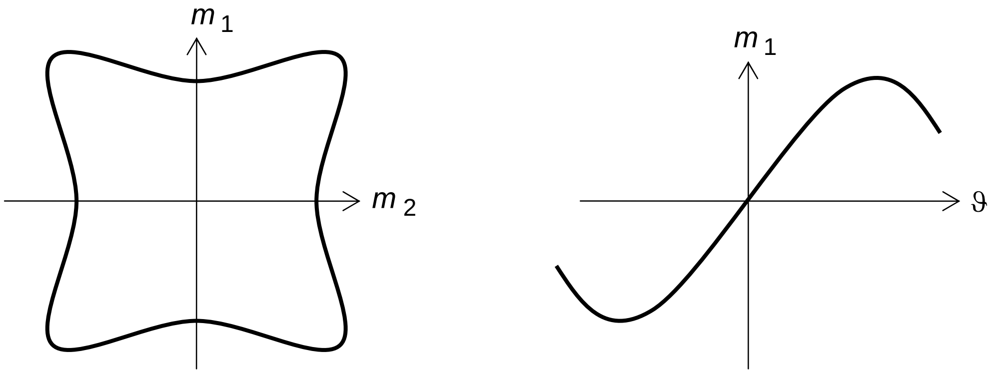
b We consider the yield line pattern of Figure 4. Determine an <u>upper bound</u> for *f* expressed in m_p and *a.* (1.5 points)

c Determine the largest <u>lower-bound</u> for *f* using torsion free beams ($m_{xy} = 0$) in the *x* direction and *y* direction. (1.5 points)

Problem 3

a A square plate needs to carry a large point load. However, this point load is two times as large as the plastic collapse load of the plate. Therefore, an engineer suggests a design change to reduce the length and width of the plate to half the original size. The thickness, supports and plastic moment capacity are not changed. Self weight can be neglected. Will this new design be able to carry the point load? (1 point)

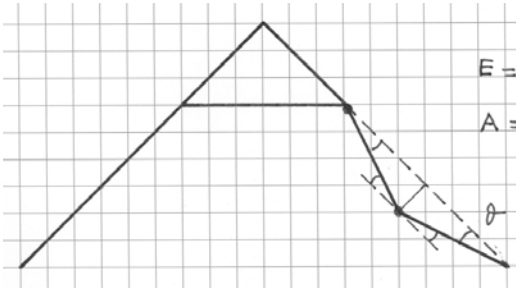
b Figure 6 shows a yield contour and a moment-rotation diagram which are experimentally established for reinforced concrete plate parts. The plates had the same reinforcement in two orthogonal directions at the top face and bottom face. How is this result idealised in upper and lower bound analysis? (0.5 points)



a) Yield contour b) Moment-rotation diagram

Figure 6. Result of experiments on plate parts

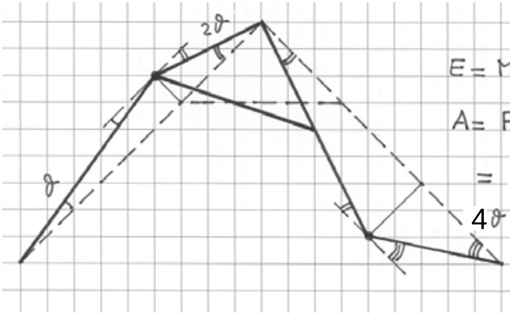
Exam CT4150, 2 February 2007
Answer to Problem 1a



$$E = M_p(\vartheta+\vartheta) + M_p\,\vartheta = 3M_p\vartheta$$

$$A = F\vartheta a$$

$$E = A \Rightarrow F = 3\frac{M_p}{a}$$

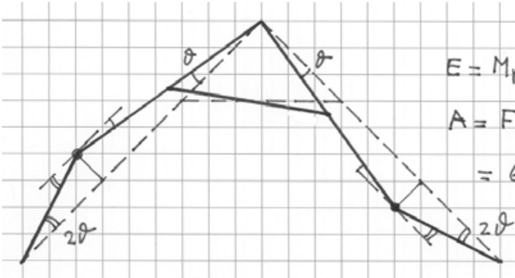


$$E = M_p(4\vartheta+2\vartheta) + M_p(\vartheta+2\vartheta) = 9M_p\vartheta$$

$$A = F4\vartheta a + F2\vartheta a + F\vartheta a + F\vartheta 2a$$
$$= 9F\vartheta a$$

$$E = A \Rightarrow \boxed{F = \frac{M_p}{a}}$$

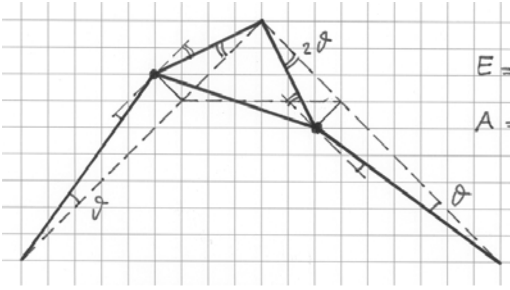
decisive



$$E = M_p(\vartheta+2\vartheta) + M_p(\vartheta+2\vartheta) = 6M_p\vartheta$$

$$A = F2\vartheta a + F\vartheta a + F2\vartheta a + F\vartheta a =$$
$$= 6F\vartheta a$$

$$E = A \Rightarrow \boxed{F = \frac{M_p}{a}}$$ decisive



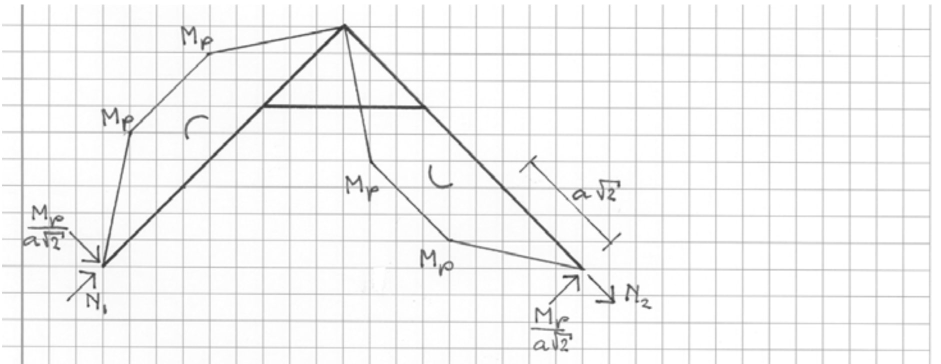
$$E = M_p(\vartheta+2\vartheta) + M_p(\vartheta+2\vartheta) = 6M_p\vartheta$$

$$A = F\vartheta 2a + F\vartheta a + F\vartheta 2a +$$
$$F\vartheta a = 6F\vartheta a$$

$$E = A \Rightarrow \boxed{F = \frac{M_p}{a}}$$ decisive

More mechanisms are possible but we will first plot the moments. If the moments are everywhere smaller than or equal to M_p then we found the correct collapse load and we do not need to study more mechanisms.

Answer to Problem 1b



$$Fa + F2a + F3a + F2a + Fa + \frac{M_p}{a\sqrt{2}}\,3a\sqrt{2} - N_1\,3a\sqrt{2} = 0 \qquad N_1 = 2\sqrt{2}\,\frac{M_p}{a}$$

$$Fa + F2a + F3a + F2a + Fa + \frac{M_p}{a\sqrt{2}}\,3a\sqrt{2} - N_2\,3a\sqrt{2} = 0 \qquad N_2 = N_1$$

Answer to problem 1c Lower-bound

We reduce the loading by a factor α. Also the moment distribution and the normal force distribution are reduced by α. Note that everything remains in equilibrium. In a section with a moment αM_p we can allow a normal force $(1-\alpha)N_p$.

$$\alpha 2\sqrt{2}\,\frac{M_p}{a} = (1-\alpha)N_p = (1-\alpha)\beta\frac{M_p}{a}$$

$$\alpha 2\sqrt{2} = (1-\alpha)\beta$$

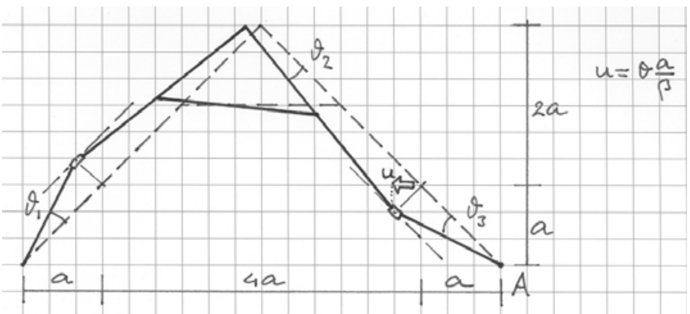
$$\alpha 2\sqrt{2} + \alpha\beta = \beta$$

$$\alpha = \frac{\beta}{2\sqrt{2}+\beta} = \frac{30\sqrt{2}}{2\sqrt{2}+30\sqrt{2}} = \frac{15}{16}$$

$$F = \alpha\frac{M_p}{a} = \frac{15}{16}\frac{M_p}{a}$$

Answer to Problem 1c Upper-bound

We consider the third mechanism because the normal force is largest in its plastic hinges.



$\beta = 30\sqrt{2}$

displacement of point A ←

$$\vartheta_1 a + (\vartheta_1 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} - (\vartheta_2 + \vartheta_3)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_3 a = 0$$

displacement of point A ↑

$$\vartheta_1 a - (\vartheta_1 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{2}} - \vartheta_2 4a - (\vartheta_2 + \vartheta_3)\frac{a}{\beta}\frac{1}{\sqrt{2}} + \vartheta_3 a = 0$$

$$E = M_p(\vartheta_1 + \vartheta_2) + M_p(\vartheta_2 + \vartheta_3)$$

$$u = \vartheta_3 a + (\vartheta_2 + \vartheta_3)\frac{a}{\beta}\frac{1}{\sqrt{2}}$$

$$A = Fu + F(u - \vartheta_2 a) + F(u - \vartheta_2 2a) + F(u - \vartheta_2 a) + Fu$$

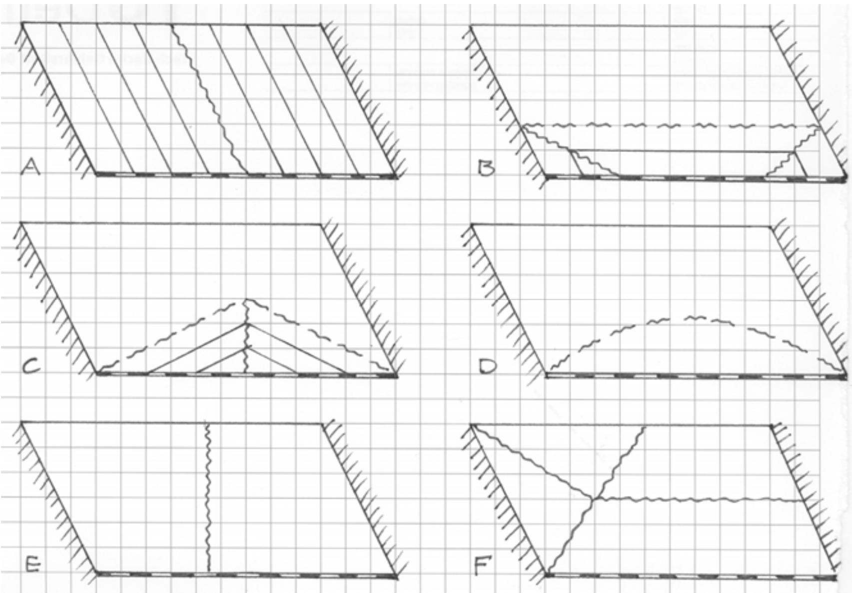
$$E = A$$

the solution is computed by Maple

$$\vartheta_1 = \frac{121}{59}\vartheta_2 \qquad \vartheta_3 = \frac{121}{59}\vartheta_2 \qquad F = \frac{15}{16}\frac{M_p}{a}$$

Answer to Problem 2a

Kinematically possible are patterns A, B and C. The figure below shows the altitude lines of the deformed mechanisms.



Answer to Problem 2b

$$E = 4 m_p 2\varphi a + \left(m_p \varphi a + m_p \vartheta \frac{4}{\sqrt{5}} a\right) + \left(m_p \varphi a + m_p \vartheta \frac{4}{\sqrt{5}} a\right)$$

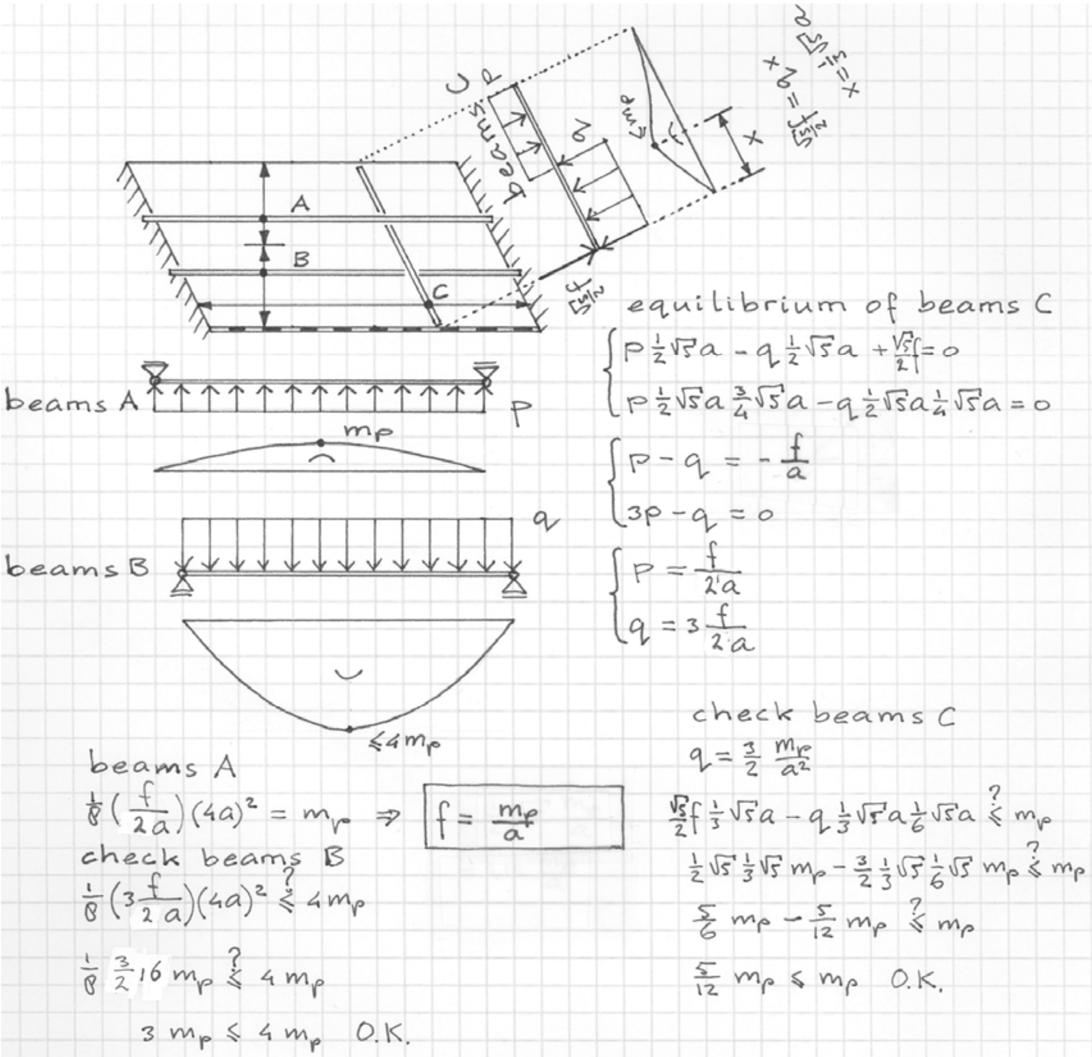
$$\varphi = \frac{w}{2a} \qquad \vartheta = \frac{w}{\frac{1}{2}\sqrt{5}\,a}$$

$$E = \frac{41}{5} m_p w$$

$$A = 2\left(f\, 2a \tfrac{1}{2} w\right) = 2faw$$

$$E = A \Rightarrow f = \frac{41}{10}\frac{m_p}{a} = 4{,}10\,\frac{m_p}{a}$$

Answer to Problem 2c



Answer to Problem 3a

No. The collapse load of any plate due to a point load does not depend on the size of the plate. If length and width are changed with the same factor the collapse load remains unchanged.

Answer to Problem 3b

Idealisation