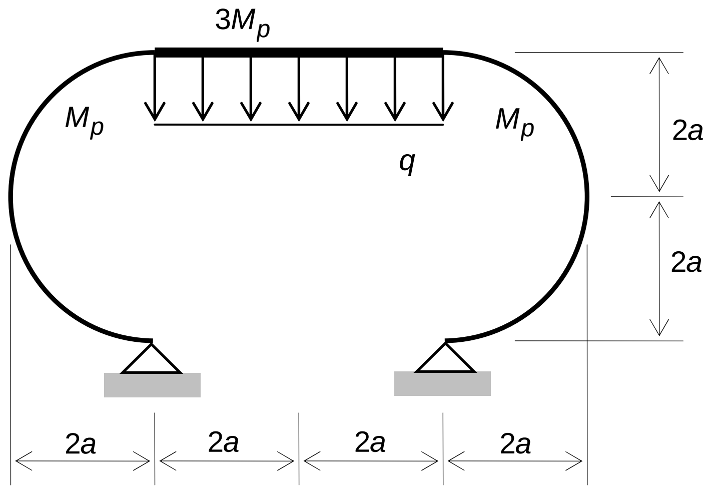
Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CT4150 Plastic Analysis of Structures
Thursday 19 August 2008, 14:00 – 17:00 hours



Problem 1

Figure 1. Curved frame

A frame consists of one straight section ($3M_p$) and two curved sections (M_p) (Fig. 1). The beams are fixed to each other and simply supported by the foundation. The frame is loaded by an evenly distributed load q. The following relation exists between the plastic moment M_p and the plastic normal force N_p (Fig. 2).

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

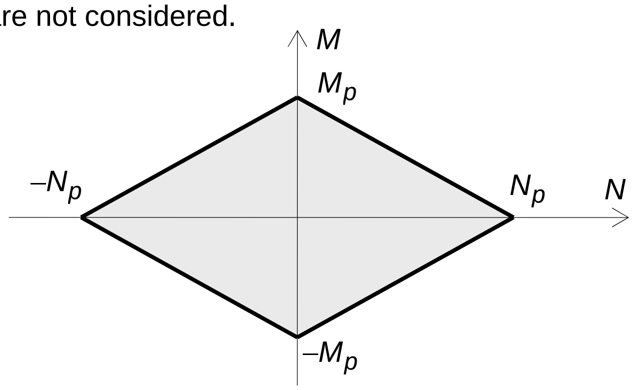


Figure 2. Yield contour

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1 point)

c Assume $\beta = 55$. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for F.
– Determine the smallest <u>upper-bound</u> for F.
If you choose the upper-bound you only need to write down the equations and not solve the equations (2 points).

Problem 2

A plate has two simply supported edges (Fig. 3). It carries an evenly distributed load q over an area of $2a$ x $2a$. The plate is homogeneous and orthotropic $m_{px} = 0$, $m'_{px} = 3m_p$, $m_{py} = m_p$, $m'_{py} = m_p$.

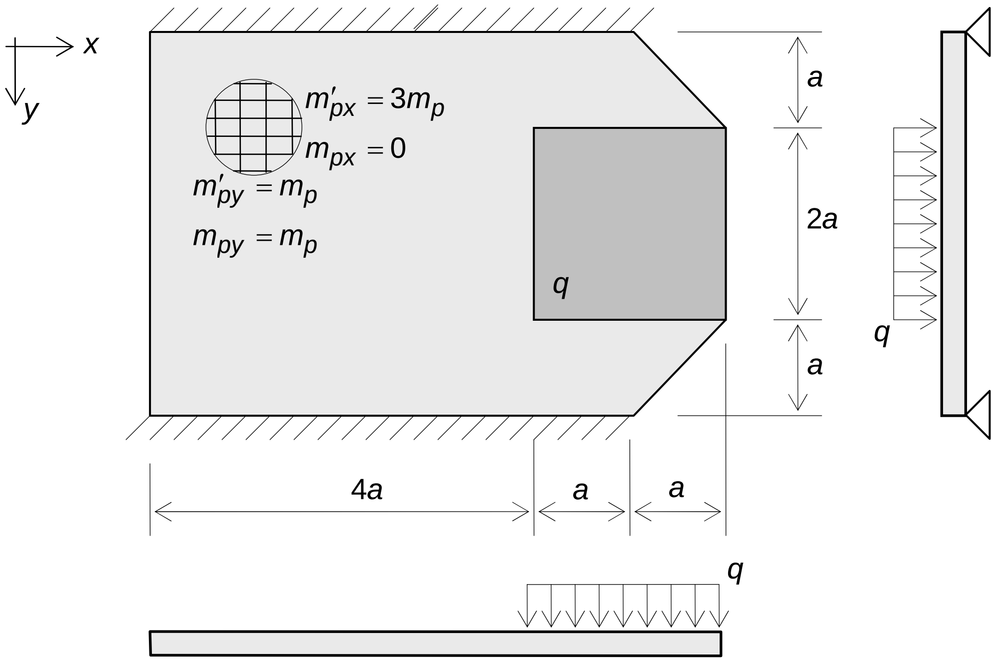


Figure 3. Plate dimensions and loading

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms. (1 point)

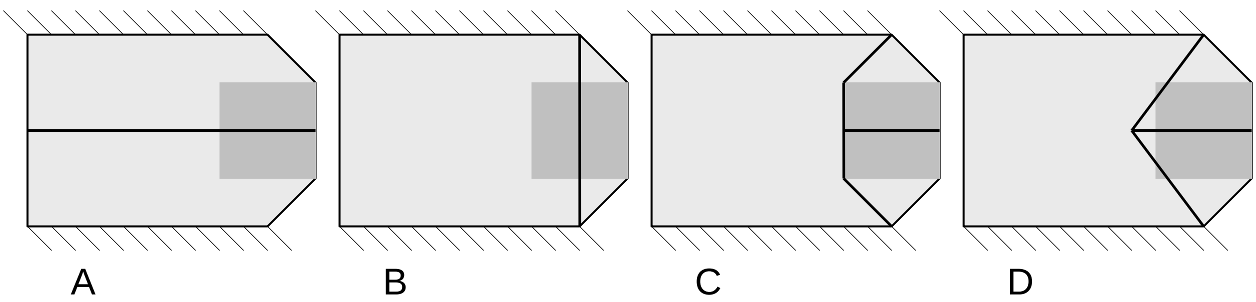


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 5. Determine an <u>upper bound</u> for q expressed in m_p and a (1.5 point).

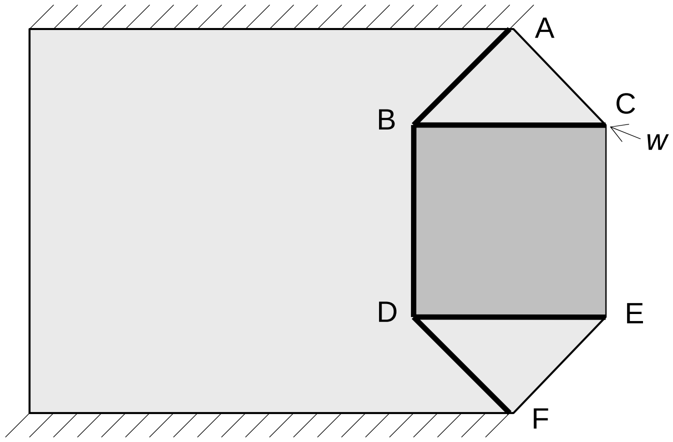


Figure 5. Yield line pattern of problem 2b

c Determine the largest <u>lower-bound</u> for q using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a A structure made of plastic materials cannot collapse due to only temperature loading. Is this correct? Choose A, B, C, or D (0.5 point).

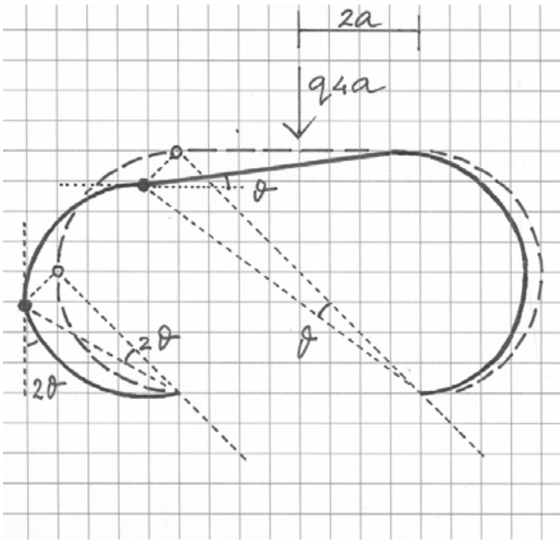
A Yes, because temperature loading does not influence upperbound calculations or lowerbound calculations.
B No, because temperature loading can cause very large stresses.
C No, because temperature loading can cause damage that can become worse due to the weather which could eventually result in collapse of the structure.
D No, otherwise we would not need dilatations in large steel structures or large reinforced concrete structures.

b Consider yield contours of a frame structures. Why does a distributed loading give a curved yield contour? (0.5 point).

c Consider a plastic hinge in a reinforce concrete member. When the hinge rotates it also extents. What is the name of the rule that gives the relation between the rotation and the extension? (0.5 point)

Exam CT4150, 18 June 2008

Answer to Problem 1a



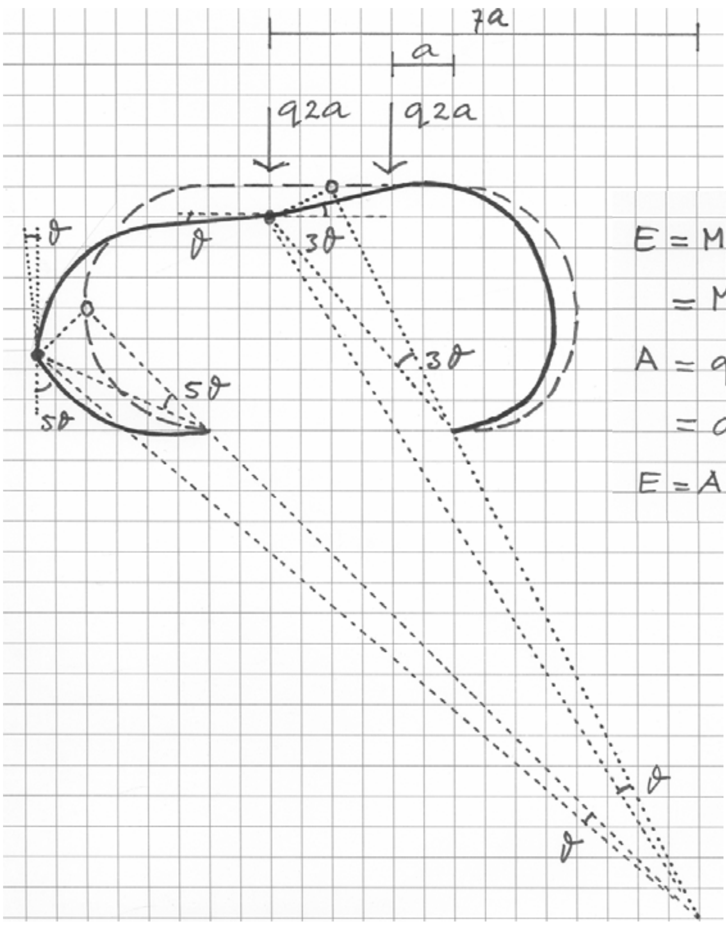
$$E = M_p\,2\vartheta + M_p\,\vartheta = M_p\,3\vartheta$$

$$A = q\,4a\,2a\,\vartheta = q\,8a^2\vartheta$$

$$E = A \Rightarrow \boxed{q = \frac{3}{8}\frac{M_p}{a^2}}$$

decisive

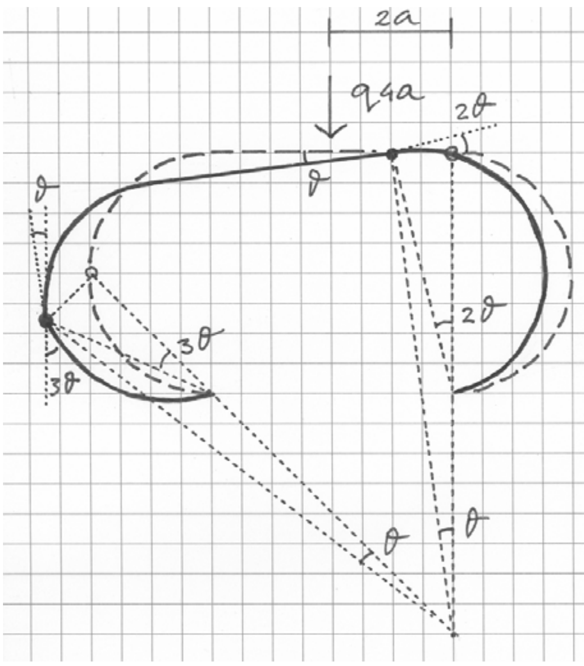
0.375



$$E = M_p(5\vartheta - \vartheta) + 3M_p(3\vartheta - \vartheta) = M_p\,10\vartheta$$

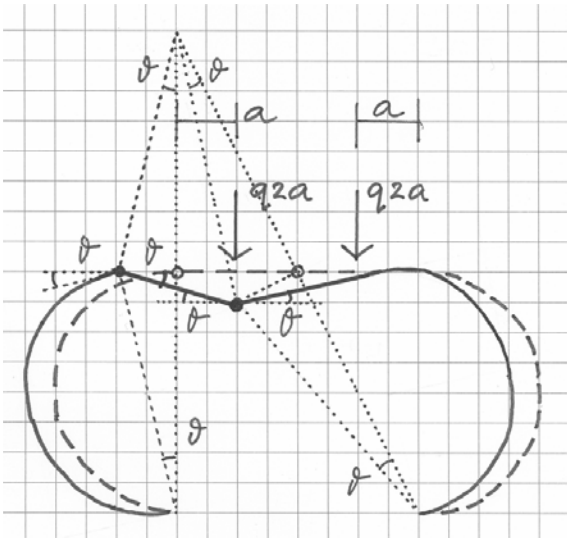
$$A = q\,2a\,a\,3\vartheta + q\,2a\,7a\,\vartheta = q\,20a^2\vartheta$$

$$E = A \Rightarrow q = \frac{1}{2}\frac{M_p}{a^2}$$



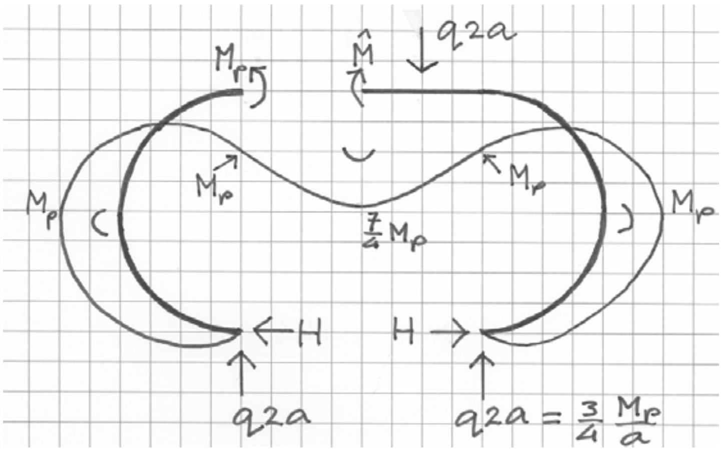
$$E = M_p(3\vartheta - \vartheta) + M_p(2\vartheta - \vartheta)$$
$$= M_p\, 3\vartheta$$
$$A = q4a\; 2a\; \vartheta$$
$$= q\, 8a^2\, \vartheta$$
$$E = A \Rightarrow \boxed{q = \frac{3}{8}\frac{M_p}{a^2}}$$

also decisive



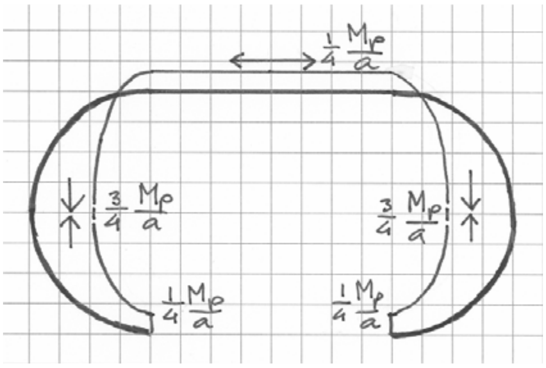
$$E = M_p(\vartheta + \vartheta) + 3M_p(\vartheta + \vartheta)$$
$$= M_p\, 8\vartheta$$
$$A = q2a\, a\, \vartheta + q2a\, a\, \vartheta$$
$$= q\, 4a^2\, \vartheta$$
$$E = A \Rightarrow q = 2\frac{M_p}{a^2}$$

Answer to Problem 1b



$$M_p = H4a \Rightarrow H = \frac{1}{4}\frac{M_p}{a}$$

$$\begin{aligned}\hat{M} &= -2qaa + H4a + q2a2a \\ &= 2qa^2 + H4a \\ &= \frac{3}{4}M_p + M_p = \frac{7}{4}M_p\end{aligned}$$



Answer to problem 1c Lower-bound

We reduce the load, moments and normal forces with a factor α. In doing so everything is still in equilibrium. We need to determine α such that everywhere the yield condition is fulfilled.

The left hinge has the largest normal force, therefore this hinge will be decisive. In the left hinge there is a moment αM_p and a normal force $\alpha \frac{3}{4}\frac{M_p}{a}$. According to the yield contour (Fig. 2) we can allow a normal force $(1-\alpha)N_p$. Therefore,

$$\alpha \frac{3}{4}\frac{M_p}{a} = (1-\alpha)N_p = (1-\alpha)\beta\frac{M_p}{a}$$

$$\alpha \frac{3}{4} = (1-\alpha)\beta$$

$$\alpha \frac{3}{4} = \beta - \alpha\beta$$

$$\alpha \frac{3}{4} + \alpha\beta = \beta$$

$$\alpha = \frac{\beta}{\frac{3}{4}+\beta} = \frac{55}{\frac{3}{4}+55} = 0.987$$

We check the other hinge;

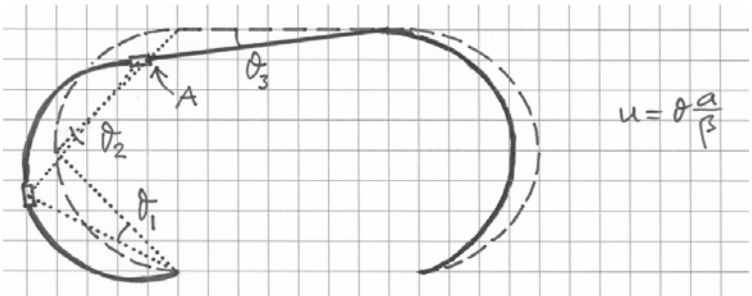
Present $\alpha \frac{1}{4}\frac{M_p}{a} = 0.247\frac{M_p}{a}$

Allowed $(1-\alpha)N_p = (1-\alpha)\beta\frac{M_p}{a} = 0.740\frac{M_p}{a}$ OK

The ultimate load is

$$q = \alpha\frac{3}{8}\frac{M_p}{a^2} \qquad \boxed{q = 0.370\frac{M_p}{a^2}}$$

Answer to Problem 1c Upper-bound



horizontal displacement of A ←

$\vartheta_1 2a + \vartheta_2 2a - (\vartheta_3 - \vartheta_2)\frac{a}{\beta} = \vartheta_3 4a$

vertical displacement of A ↓

$\vartheta_1 2a + (\vartheta_1 - \vartheta_2)\frac{a}{\beta} - \vartheta_2 2a = \vartheta_3 4a$

$E = M_p(\vartheta_1 - \vartheta_2) + M_p(\vartheta_3 - \vartheta_2)$

$A = q 4a \vartheta_3 2a$

$A = E$

The solution to the equations is

$\vartheta_1 = 148\,\vartheta_2, \quad \vartheta_3 = 74{,}1\,\vartheta_2, \quad q = 0{,}371 \frac{M_p}{a^2}$

Answer to Problem 2a

Kinematically possible are pattern A, B and D.

Answer to Problem 2b

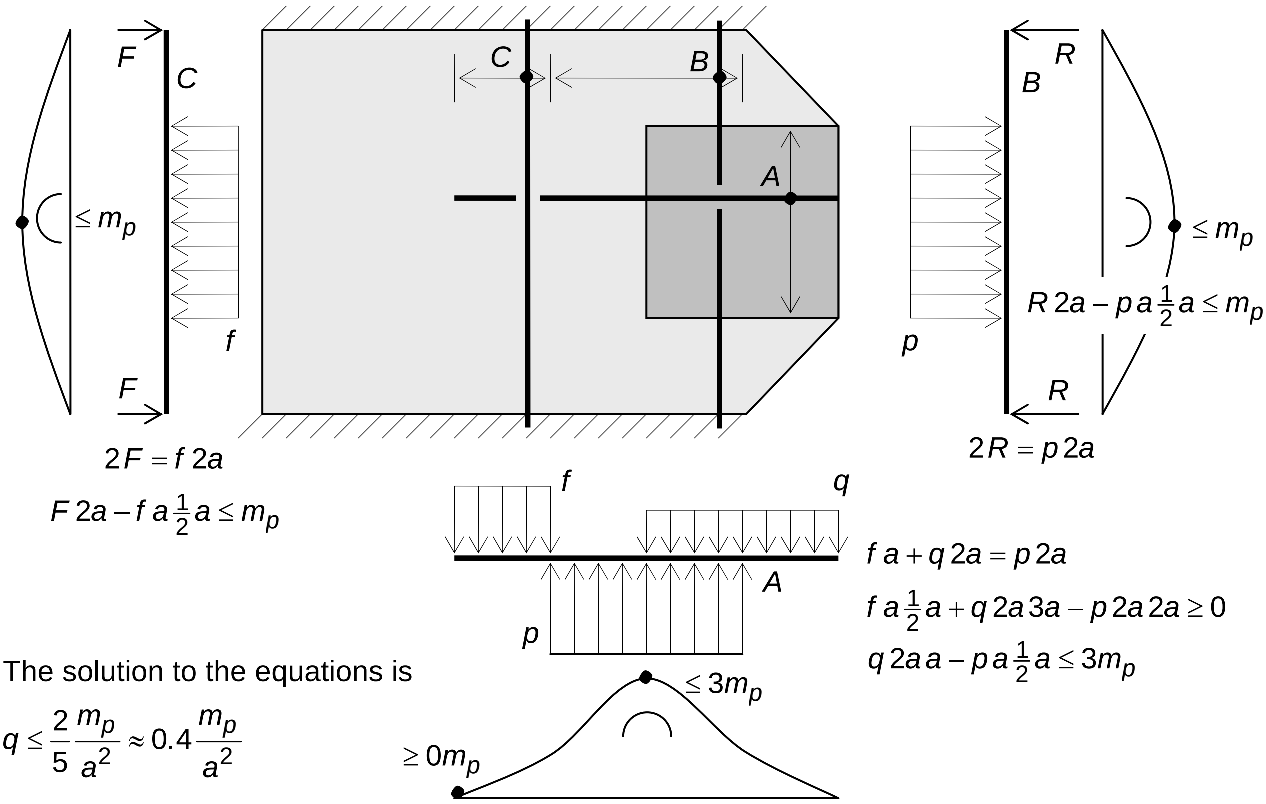
yield line	l_x	l_y	$\lvert\varphi_x\rvert$	$\lvert\varphi_y\rvert$	$\lvert m_{yy}\rvert$	$\lvert m_{xx}\rvert$	$l_x\,\varphi_x\,m_{yy} + l_y\,\varphi_y\,m_{xx}$
BA	a	a	$\frac{1}{2}w/a$	$w/2a$	m_p	$3m_p$	$2w\,m_p$
BC	2a	0	$\frac{1}{2}w/a$	0	m_p	0	$w\,m_p$
BD	0	2a	0	$w/2a$	m_p	$3m_p$	$3w\,m_p$
DE	2a	0	$\frac{1}{2}w/a$	0	m_p	0	$w\,m_p$
DF	a	a	$\frac{1}{2}w/a$	$w/2a$	m_p	$3m_p$	$2w\,m_p$
						sum	$9w\,m_p$

$$E = 9w\,m_p$$

$$A = q\,A\,u = q \times (2a)^2 \times \tfrac{1}{2}w = 2q\,a^2 w$$

$$E = A \Rightarrow \quad q = \frac{9}{2}\frac{m_p}{a^2} = 4.50\frac{m_p}{a^2}$$

Answer to Problem 2c



The solution to the equations is

$$q \leq \frac{2}{5}\frac{m_p}{a^2} \approx 0.4\frac{m_p}{a^2}$$

Answer to Problem 3a

Answer A is correct. (With a little bit of yielding all temperature stresses are gone and then we can load the structure like it has never been loaded before.)
(B These are elastic stresses. In a plastic model the stresses are never larger than the yield stress.)
(C This can happen in some structures. Maintenance can reduce the deterioration. However, al of this is not important for structural calculations and it is not part of the plastic material model.)
(D The dilatations prevent ugly cracks. They do not make the structure stronger.)

Answer to Problem 3b

Because the position of the plastic hinge is not fixed.

Answer to Problem 3c

Normality rule