Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CT4150 Plastic Analysis of Structures
Thursday 24 January 2013, 14:00 – 17:00 hours

Write your name and study number at the top right-hand of your work.

Also write whether you were a member of the elastic team, plastic team or no team.

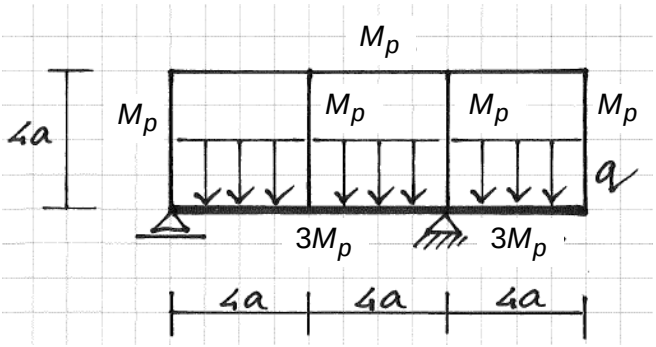


Figure 1. Frame structure

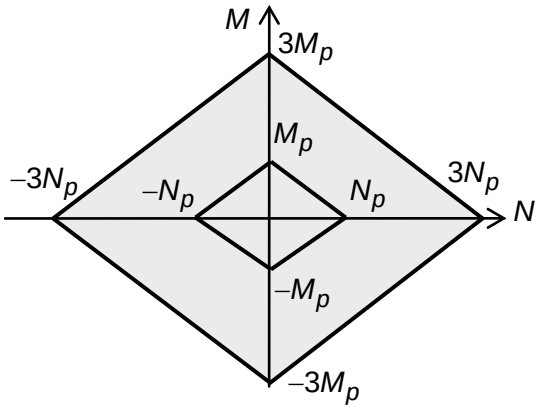


Figure 2. Yield contour

Problem 1

A frame consists of a bottom beam with a strength $3M_p$, a top beam with a strength M_p and four columns with strengths M_p (Fig. 1). The joints are fixed connections. The bottom beam is simply supported. The structure is loaded by a vertical load q on the bottom beam.
The relation of Figure 2 exists between the plastic moment M_p and the plastic normal force N_p.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 10$. Choose one of the following problems (You need not do both).
– Determine the largest lower-bound for q using Figure 3.
– Determine the smallest upper-bound for q.
You only need to write down the equations and not solve the equations (1.5 points).

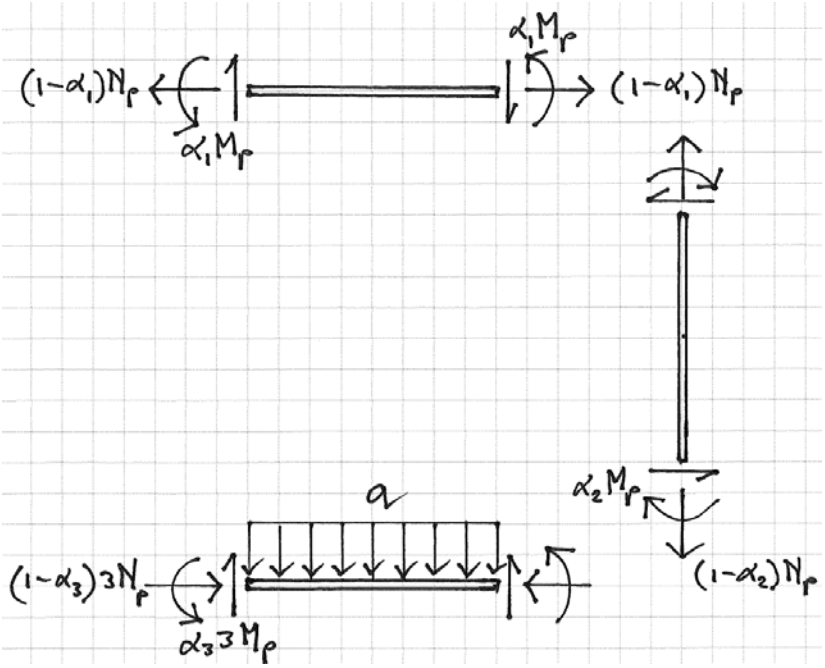
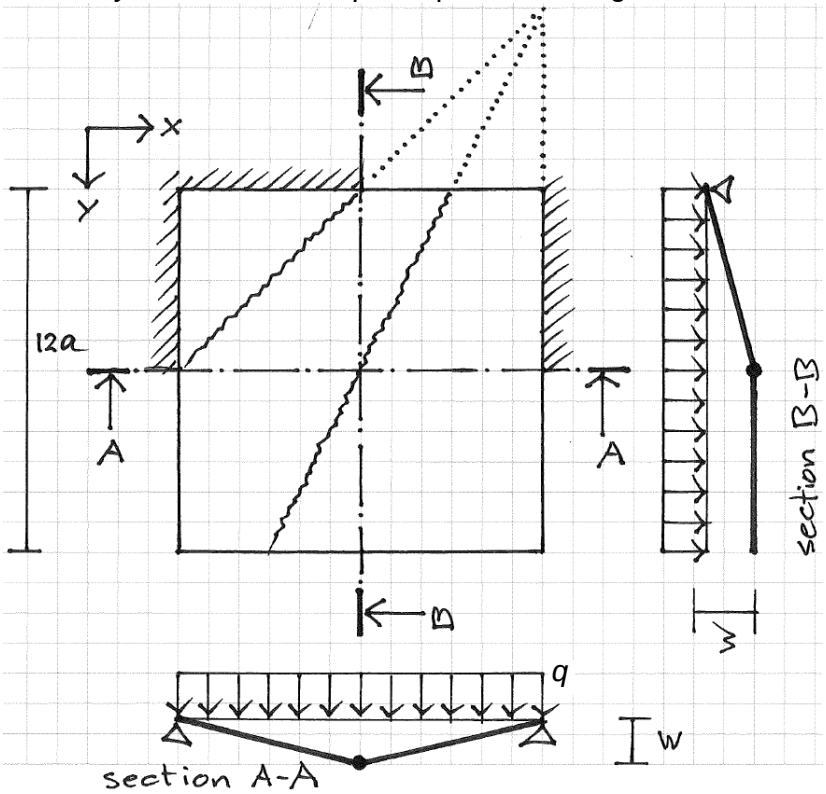


Figure 3. Forces and moments on frame members for a lower-bound analysis

Problem 2

A square reinforced concrete plate is simply supported at parts of its edges (Fig. 4). It carries an evenly distributed load *q*. The plate is homogeneous and orthotropic.



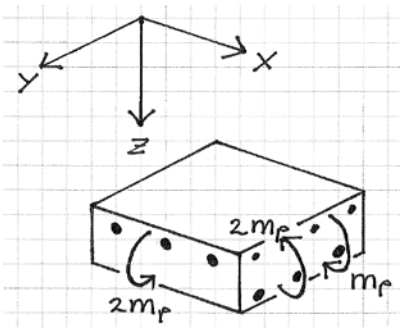


Figure 4. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 5. Which of these patterns give kinematically possible mechanisms? (1 point)

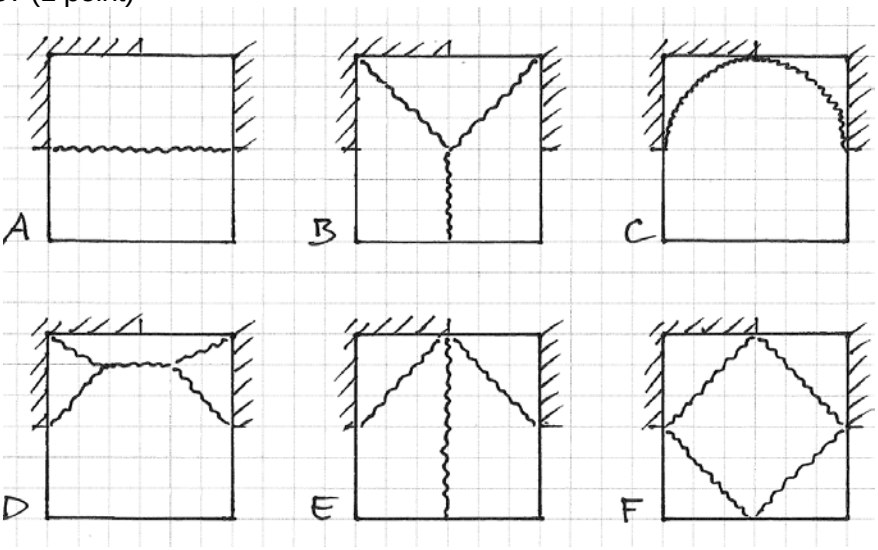


Figure 5. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 4. Determine an upper bound for q expressed in m_p and a (1.5 point).

c Determine the largest lower-bound for q using torsion free beams ($m_{xy} = 0$). You do not need to evaluate the equations (1.5 point).

Problem 3

a What is an advantage of a convex yield contour?
Choose A, B, C or D (0.5 point).

A The direction of the plastic deformation can be easily determined.
B Just a few load combinations are sufficient in design.
C It can be conveniently represented by a cylinder in the space of principal stresses.
D It can be easily implemented by a software engineer.

b What is true about upper-bound analysis compared to lower-bound analysis.
Choose A, B, C or D (0.5 point).

A It is much more accurate and much safer.
B It is less accurate and not safe.
C It is generally much more accurate but not on the safe side.
D It is much less accurate but always safe.

c Previous exams show that the influence of normal force ($\beta = 10$) on the collapse load of the frame structures is often very small. What causes this?
Choose A, B, C or D. (0.4 point).

A The β-values in the exams are unrealistically small.
B The β-values in the exams are unrealistically large.
C The applied yield contour is a linear approximation of the true yield contour.
D Most frame elements are slender.

d Did you understand the proof of the lower-bound theorem? (0.1 point).

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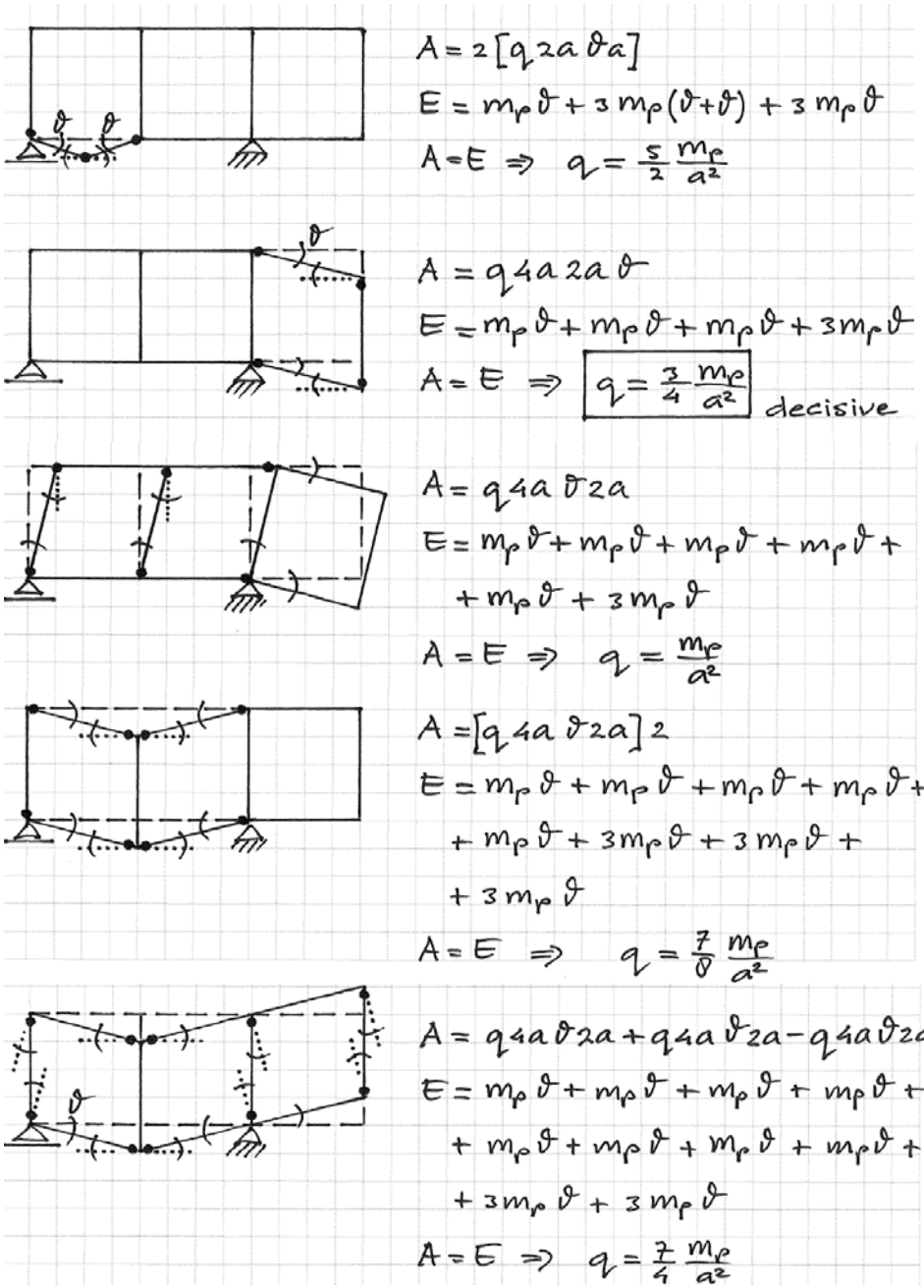
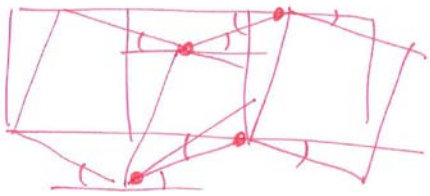
Answer to Problem 1a



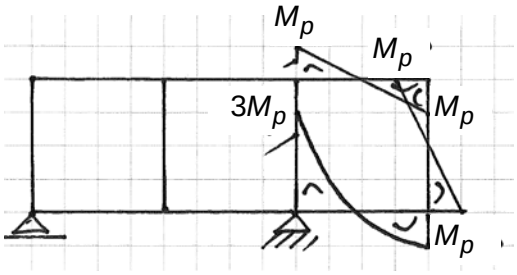
There is an even smaller upper-bound (found by 2 students during the exam).

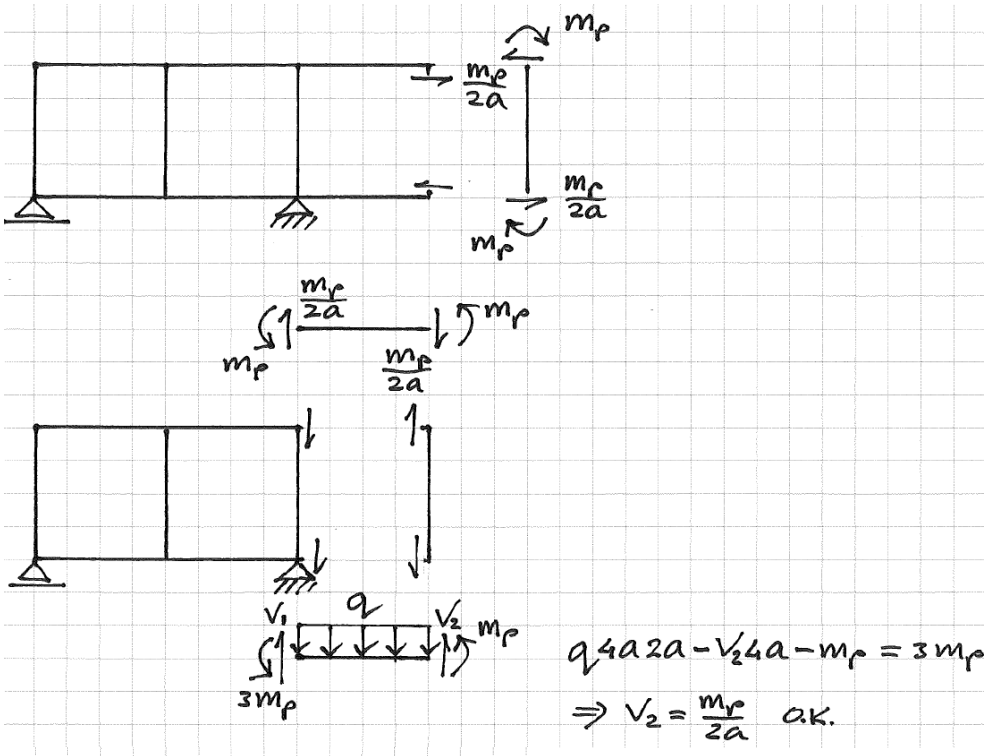


$$q = \frac{2}{3}\frac{M_p}{a^2}$$

For this mechanism there are no normal forces in the middle parts of the beams. Therefore, for this mechanism, problem 1c is trivial.

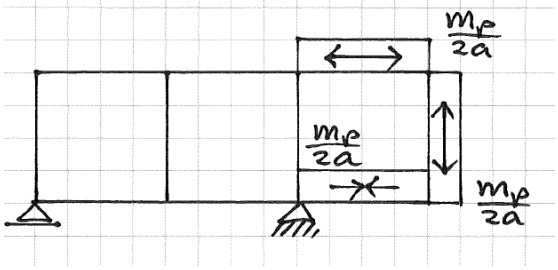
Answer to Problem 1b





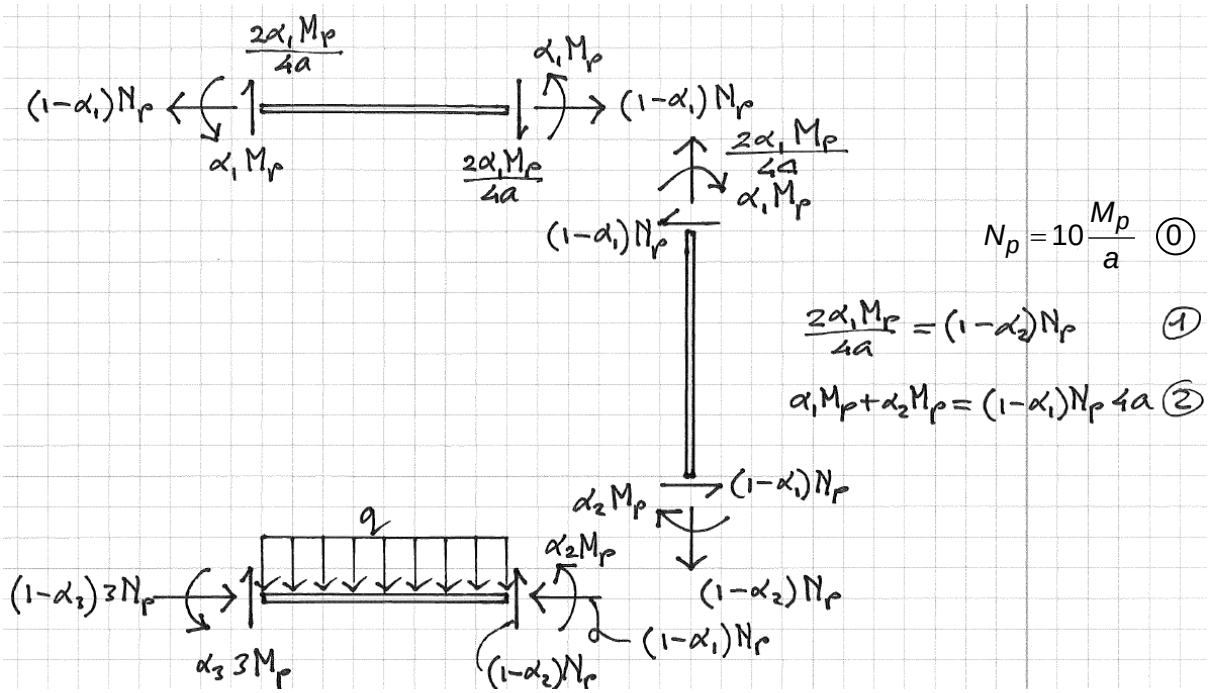
$q\,4a\,2a - V_2\,4a - m_p = 3m_p$

$\Rightarrow V_2 = \frac{m_p}{2a}$ o.k.



Answer to Problem 1c

lower-bound



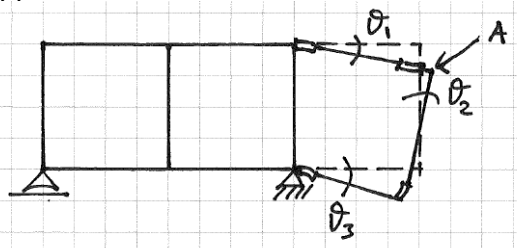
$$(1-\alpha_3)3N_p = (1-\alpha_1)N_p \quad ③$$

$$\alpha_3 3M_p = q\,4a\,2a - (1-\alpha_2)N_p\,4a - \alpha_2 M_p \quad ④$$

The solution to the equations is

$$q = \frac{61}{84}\frac{M_p}{a^2} \quad \alpha_1 = \frac{20}{21} \quad \alpha_2 = \frac{20}{21} \quad \alpha_3 = \frac{62}{63}$$

upper-bound



$$u = \vartheta\frac{a}{\beta}$$

horizontal displacement of A →

$$\vartheta_1\frac{a}{\beta} + (\vartheta_1 - \vartheta_2)\frac{a}{\beta} = -\vartheta_3\frac{a}{\beta} + \vartheta_2 4a$$

vertical displacement of A ↓

$$\vartheta_1 4a = \vartheta_3 4a - (\vartheta_3 - \vartheta_2)\frac{a}{\beta}$$

$$A = q\,4a\,\vartheta_3 2a$$

$$E = M_p\vartheta_1 + M_p(\vartheta_1 - \vartheta_2) + M_p(\vartheta_3 - \vartheta_2) + 3M_p\vartheta_3$$

$$A = E$$

The solution to the equations is

$$q = \frac{61}{84}\frac{M_p}{a^2} \quad \vartheta_1 = \frac{800}{819}\vartheta_3 \quad \vartheta_2 = \frac{59}{819}\vartheta_3$$

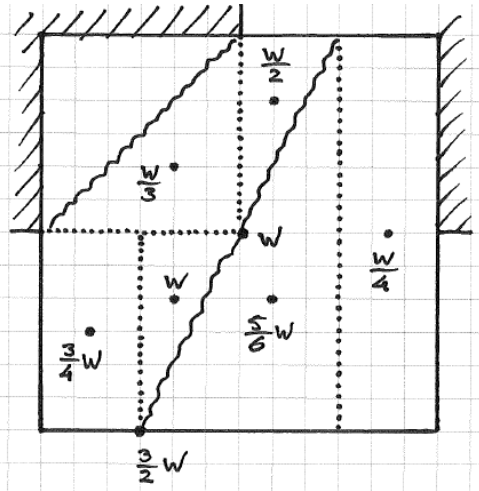
The solution to the equations is

$$q = \frac{61}{84}\frac{M_p}{a^2} \qquad \vartheta_1 = \frac{800}{819}\vartheta_3 \qquad \vartheta_2 = \frac{59}{819}\vartheta_3$$

Answer to Problem 2a

A, B, D, E, F

Answer to Problem 2b



$$A = q\tfrac{1}{2}6a\,6a\tfrac{w}{3} + q\tfrac{1}{2}6a\,3a\tfrac{w}{2} + q\,12a\,3a\tfrac{w}{4} + q\tfrac{1}{2}6a\,12a\tfrac{5}{6}w + q\tfrac{1}{2}3a\,6a\,w + q\,3a\,6a\tfrac{3}{4}w$$

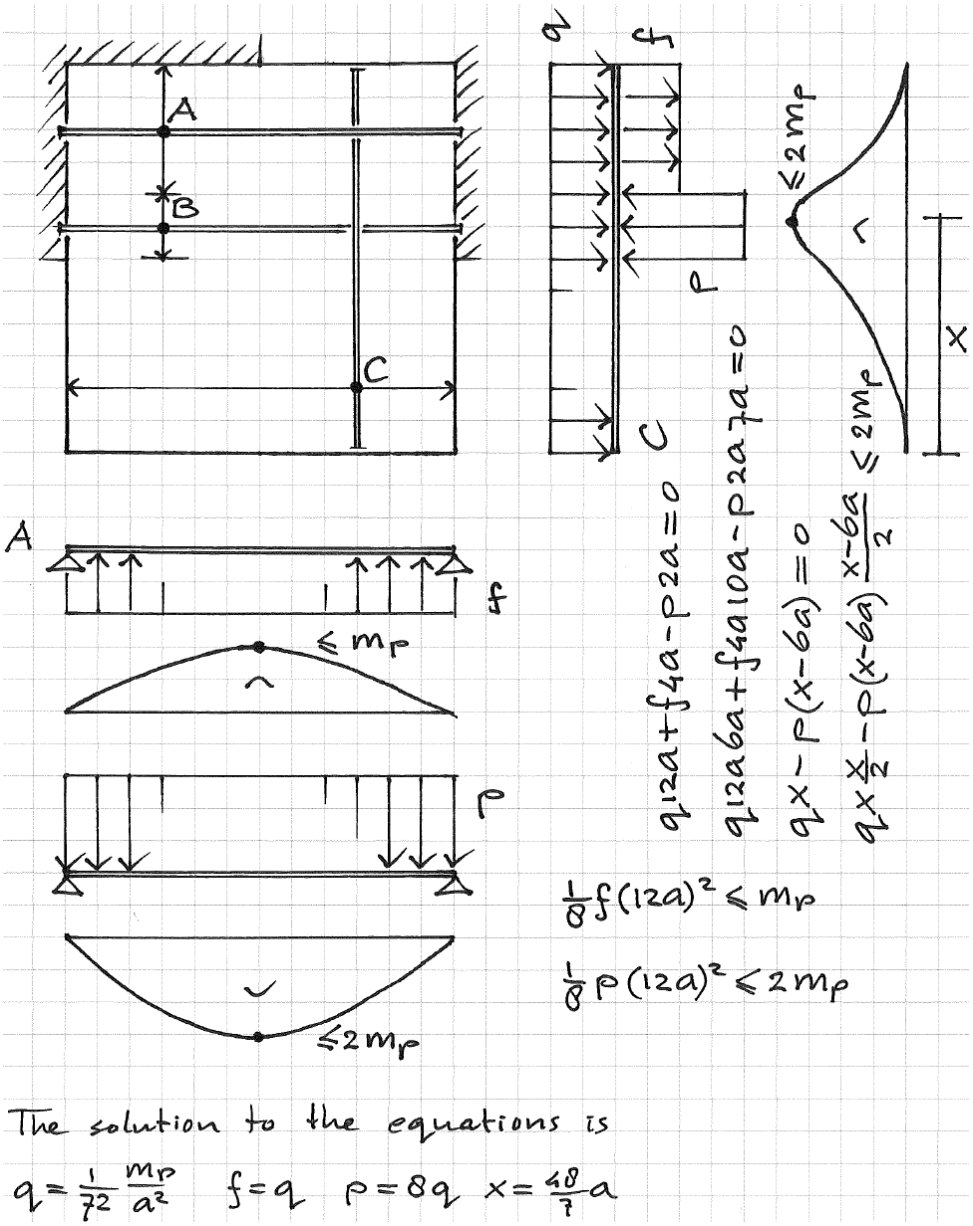
$$= 72\,q\,a^2 w$$

$$E = 2m_p\frac{w}{6a}6a + m_p\frac{w}{6a}6a + 2m_p\left(\frac{w}{6a}+\frac{w}{6a}\right)12a$$

$$= 11\,m_p\,w$$

$$A = E \Rightarrow \quad q = \frac{11}{72}\frac{m_p}{a^2}$$

Answer to Problem 2c



Answer to Problem 3a
B

Answer to Problem 3b
C

Answer to Problem 3c
D

Answer to Problem 3d
Any answer is correct. (Thank you for the information.)