Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Exam CIE4150 Plastic Analysis of Structures
Thursday 21 January 2016, 13:30 – 16:30 hours

Write your <u>name</u> and <u>study number</u> at the top of your work.

Also write whether you were a <u>member</u> of the elastic team, plastic team or no team.

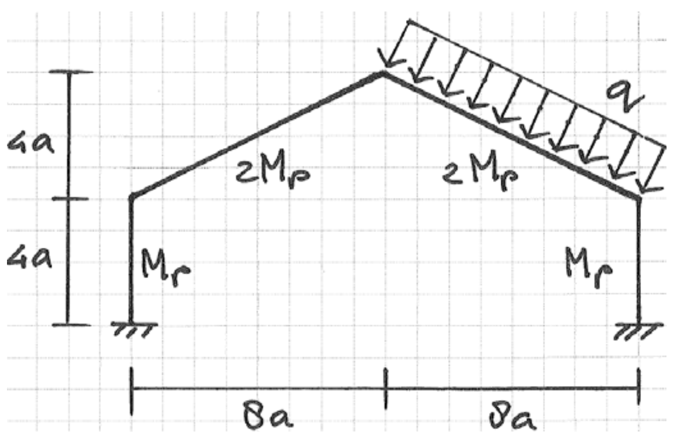


Figure 1. Frame structure

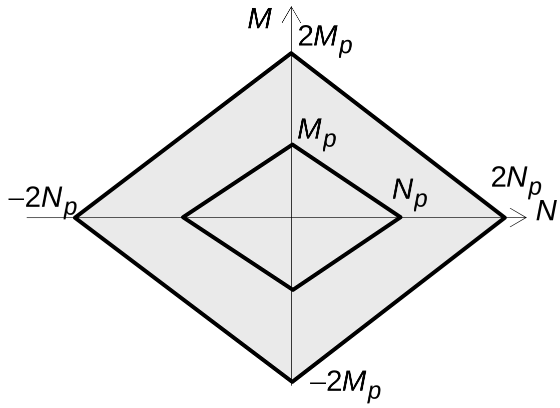


Figure 2. Yield contours

Problem 1

A frame consists of two columns and two beams (Fig.1) The columns have strength M_p and the beams have strength $2M_p$. The column, beams and foundation are rigidly connected. The structure is loaded by an evenly distributed load q. The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

a Assume $\beta \to \infty$. Determine the collapse load q for all possible mechanisms. Write the collapse loads as functions of M_p and a. What is the decisive collapse load? (1.5 point)

b Assume $\beta \to \infty$. Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)

c Assume $\beta = 7$. Choose one of the following problems (You need not do both).
– Determine the largest <u>lower-bound</u> for q.
– Determine the smallest <u>upper-bound</u> for q.
You only need to write down the equations and not solve the equations (1.5 points).

Problem 2

A reinforced concrete plate has simply supported edges (Fig. 3). It carries an evenly distributed load *p.* The plate is homogeneous and orthotropic.

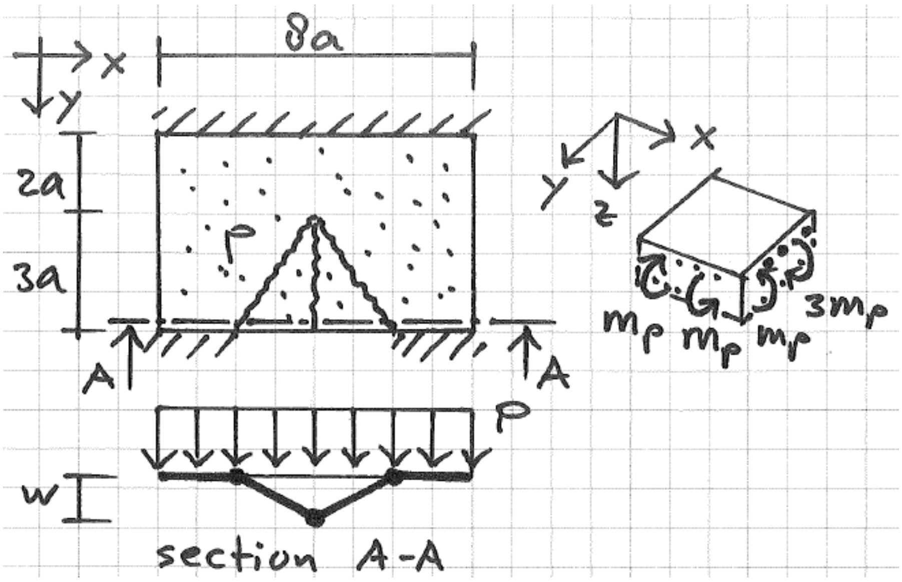


Figure 3. Plate dimensions and reinforcement

a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

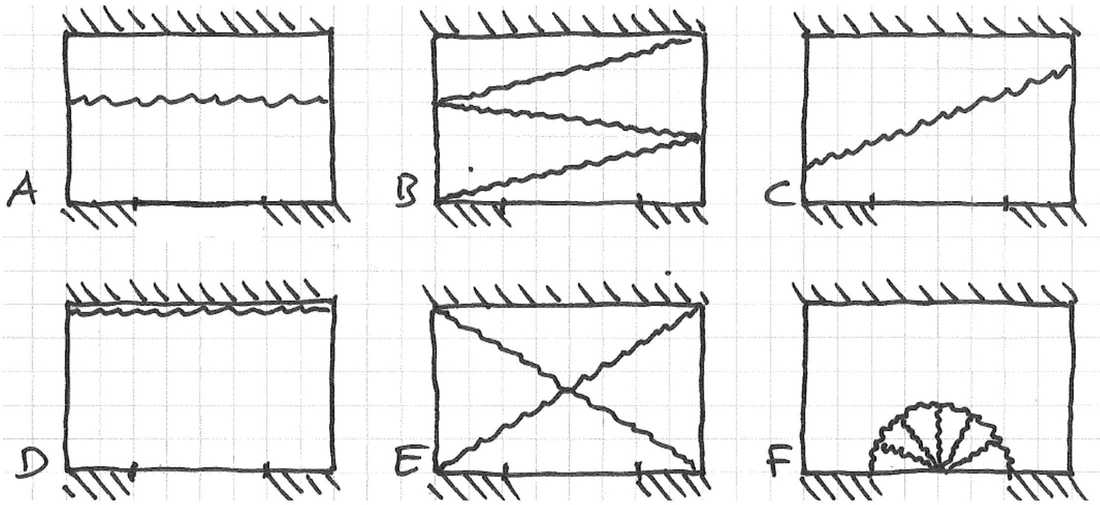


Figure 4. Yield line patterns of problem 2a

b Consider the yield line pattern of Figure 3. Determine an upper bound for *p* expressed in m_p and *a* (1.5 point).

c Determine the largest lower-bound for *p* using torsion free beams ($m_{xy} = 0$) (1.5 point).

Problem 3

a Consider a plate with orthotropic reinforcement m_p. On a free plate edge is a point load acting perpendicular to the plate. The collapse load is? Choose A, B, C or D (0.33 point).

A $2\pi m_p$

B $(2+\pi) m_p$

C $4\pi m_p$

D $6 m_p$

b Which of the following words does not belong in this list? Choose A, B, C or D (0.33 point).

A Yield contour
B Interaction diagram
C Moment-curvature diagram
D Limit state function

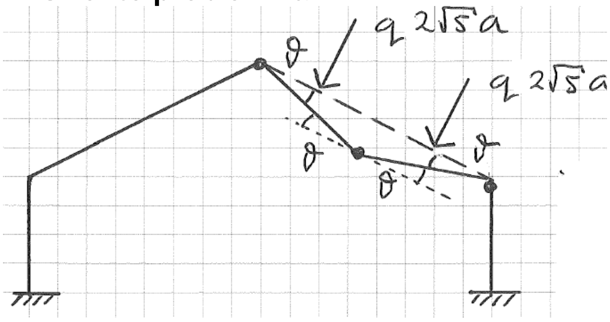
c The Von Mises yield contour is suitable for which material?
Choose from A, B, C and D (0.34 point).

A Reinforced concrete
B Plain concrete
C Coper
D Kevlar

d Bottom reinforcement in a plate needs to fulfil the condition $m_{xy}^2 \leq (m_{px} - m_{xx})(m_{py} - m_{yy})$.
It is practical to write this as a unity check (... ≤ 1). There are many ways to do this. However, there is an extra condition. For example, when the unity check value is 1.2 the strength should be 20% too small. Show that this leads to the following unity check (0.5 point).

$$\frac{m_{xx}}{2m_{px}} + \frac{m_{yy}}{2m_{py}} + \sqrt{(\frac{m_{xx}}{2m_{px}} - \frac{m_{yy}}{2m_{py}})^2 + \frac{m_{xy}^2}{m_{px} m_{py}}} \leq 1$$

Answer to problem 1a



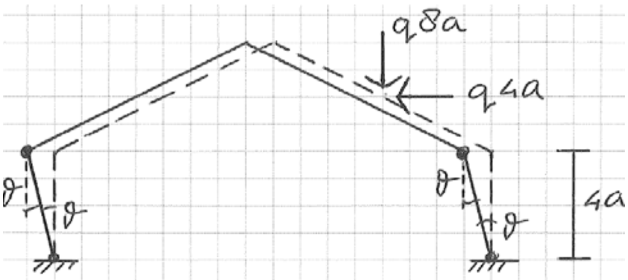

```
l:=sqrt((4*a)^2+(8*a)^2): assume(a>0): simplify(l);
```

$$4\sqrt{5}\,a\sim$$

```
E:=2*Mp*t+2*Mp*(t+t)+Mp*t:
A:=2*(q*l/2*t*l/4):
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{7\,Mp}{20\,a\sim^2}$$

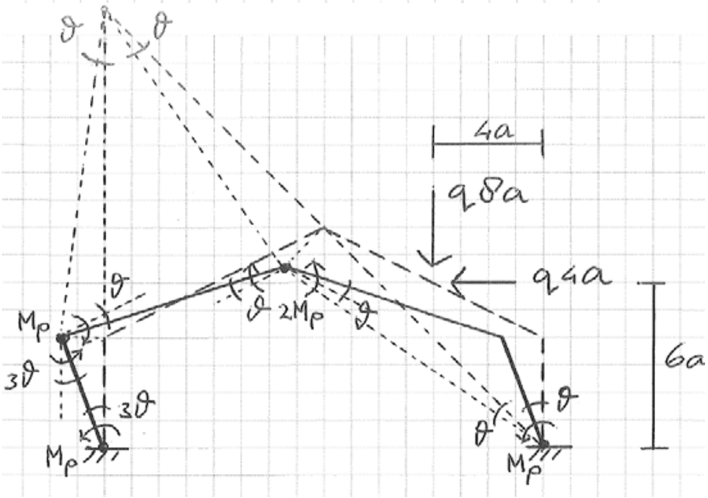
$$\frac{0.3500000000\,Mp}{a\sim^2}$$




```
E:=Mp*t+Mp*t+Mp*t+Mp*t:
A:=q*4*a*4*a*t:
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{Mp}{4\,a^2}$$

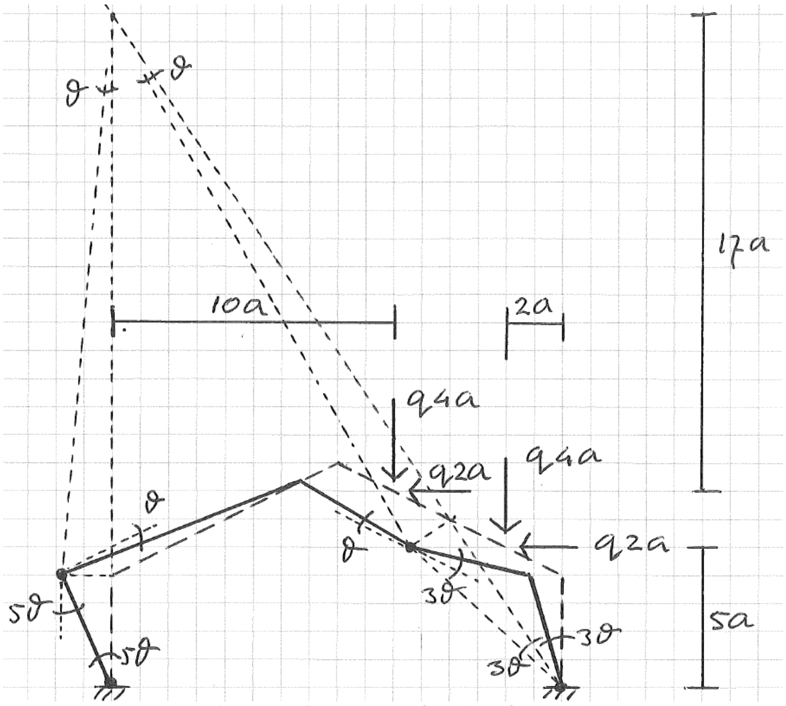
$$\frac{0.2500000000\,Mp}{a^2}$$




```
E:=Mp*3*t+Mp*(3*t+t)+2*Mp*(t+t)+Mp*t:
A:=q*4*a*t*6*a +q*8*a*t*4*a:
q:=solve(E=A,q); evalf(q);
```

$$q := \frac{3\,Mp}{14\,a^2}$$

$$\frac{0.2142857143\,Mp}{a^2}$$



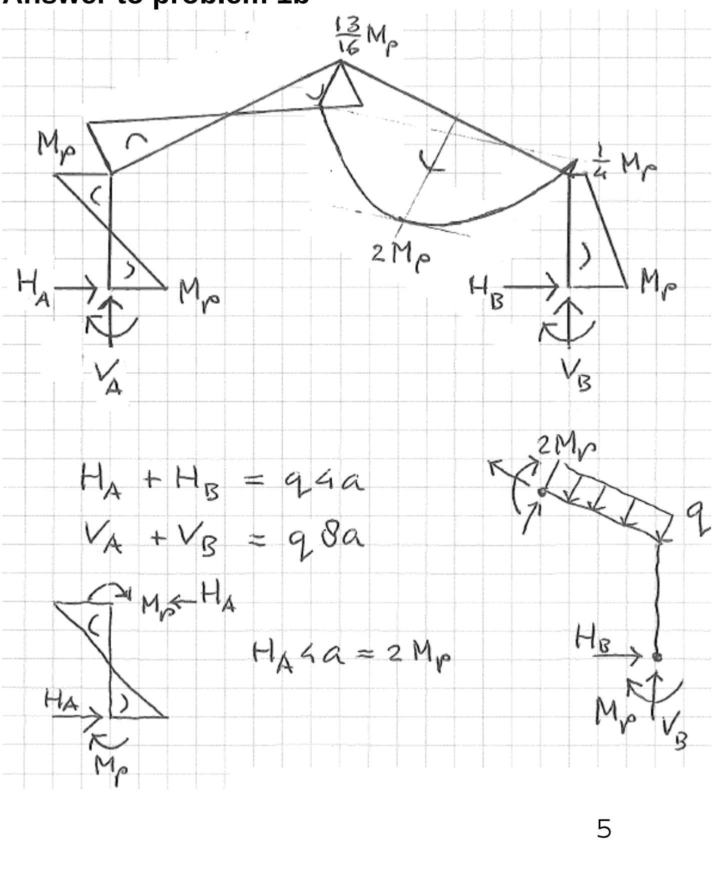

```
E:=Mp*5*t+Mp*(5*t+t)+2*Mp*(t+3*t)+Mp*3*t:
A:=q*2*a*3*t*5*a +q*4*a*3*t*2*a + q*2*a*t*17*a + q*4*a*t*10*a:
q:=solve(E=A,q); evalf(q);
```

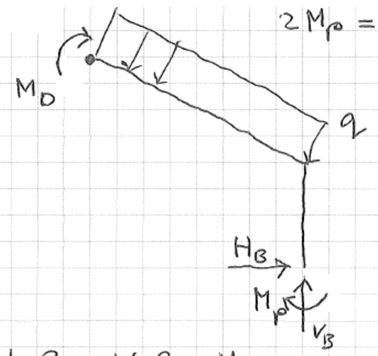
$$q := \frac{11\,Mp}{64\,a^2}$$

$$\frac{0.1718750000\,Mp}{a^2}$$

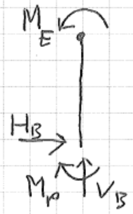
decisive

Answer to problem 1b



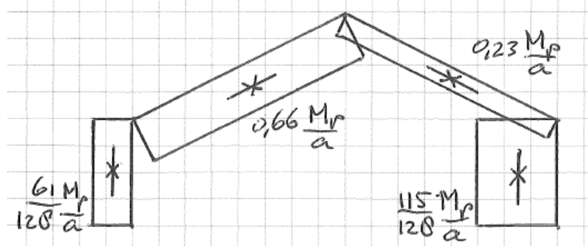


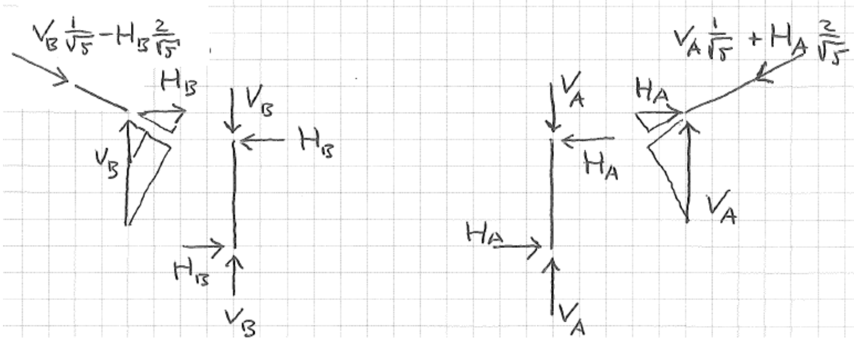
$$2M_p = H_B 6a + V_B 4a - M_p - q4a\,2a - q\,2a\,a$$



$$M_D = H_B 8a + V_B 8a - M_p - q\,8a\,4a - q\,4a\,2a$$

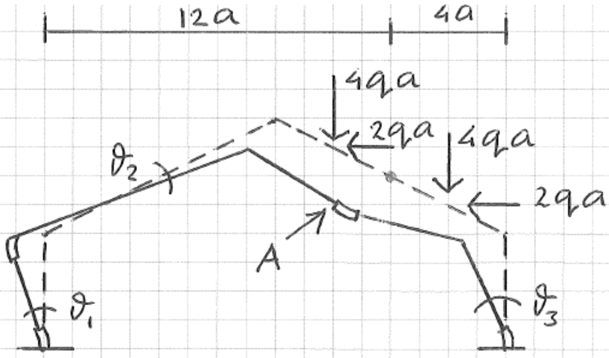
$$M_E = M_p - H_B 4a$$





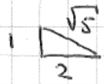
Answer to problem 1c

upperbound



$\beta = 7$

$u = \vartheta \frac{a}{\beta}$



hor. displ. of A ←

$$\vartheta_1 4a - \vartheta_2 2a = \vartheta_3 6a - (\vartheta_3 + \vartheta_2)\frac{a}{\beta}\frac{2}{\sqrt{5}}$$

vert. displ. of A ↓

$$\vartheta_1 \frac{a}{\beta} + (\vartheta_1 + \vartheta_2)\frac{a}{\beta} + \vartheta_2 12a = \vartheta_3 \frac{a}{\beta} + \vartheta_3 4a + (\vartheta_3 + \vartheta_2)\frac{a}{\beta}\frac{1}{\sqrt{5}}$$

$$E = \vartheta_1 M_p + (\vartheta_1 + \vartheta_2) M_p + (\vartheta_2 + \vartheta_3) 2M_p + \vartheta_3 M_p$$

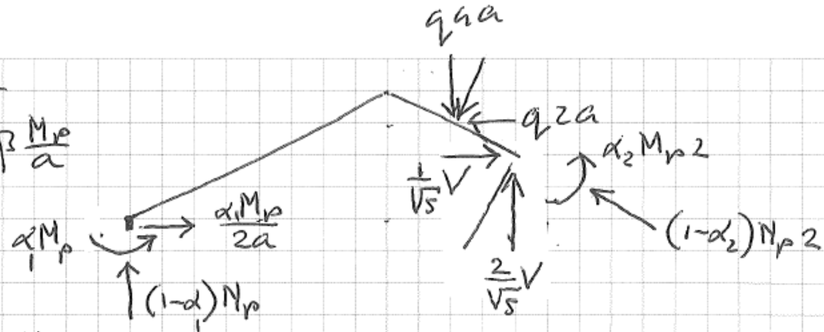
$$A = 4qa\left(\vartheta_3 \frac{a}{\beta} + \vartheta_3 2a\right) + 2qa\left(\vartheta_3 5a\right) + 4qa\left(\vartheta_1 \frac{a}{\beta} + (\vartheta_1+\vartheta_2)\frac{a}{\beta} + \vartheta_2 10a\right) + 2qa\left(\vartheta_1 4a - \vartheta_2 3a\right)$$

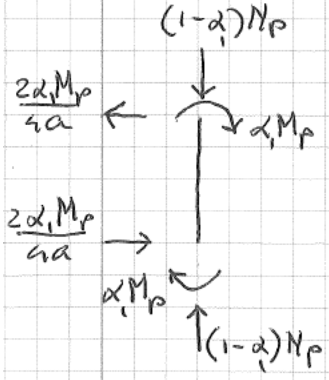
$$E = A$$

lowerbound

$$\beta = 7$$

$$N_p = \beta \frac{M_p}{a}$$

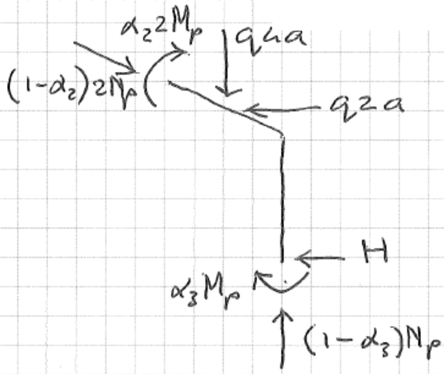




$$(1-\alpha_1)N_p - q4a + \frac{2}{\sqrt{5}}V + \frac{1}{\sqrt{5}}(1-\alpha_2)2N_p = 0$$

$$\frac{\alpha_1 M_p}{2a} - q2a + \frac{1}{\sqrt{5}}V - \frac{2}{\sqrt{5}}(1-\alpha_2)2N_p = 0$$

$$-\alpha_1 M_p + q4a\,10a - q2a\,3a + \frac{1}{\sqrt{5}}V\,2a - \frac{2}{\sqrt{5}}V\,12a - \frac{2}{\sqrt{5}}(1-\alpha_2)N_p 2\cdot 2a - \frac{1}{\sqrt{5}}(1-\alpha_2)N_p 2\,12a - \alpha_2 M_p 2 = 0$$



$$q4a - (1-\alpha_3)N_p + (1-\alpha_2)2N_p\frac{1}{\sqrt{5}} = 0$$

$$q2a + H - (1-\alpha_2)2N_p\frac{2}{\sqrt{5}} = 0$$

$$\alpha_2 2M_p + q4a\,2a + q2a\,a + H\,6a - (1-\alpha_3)N_p 4a + \alpha_3 M_p = 0$$

Answer to problem 2a

A, B, E, F

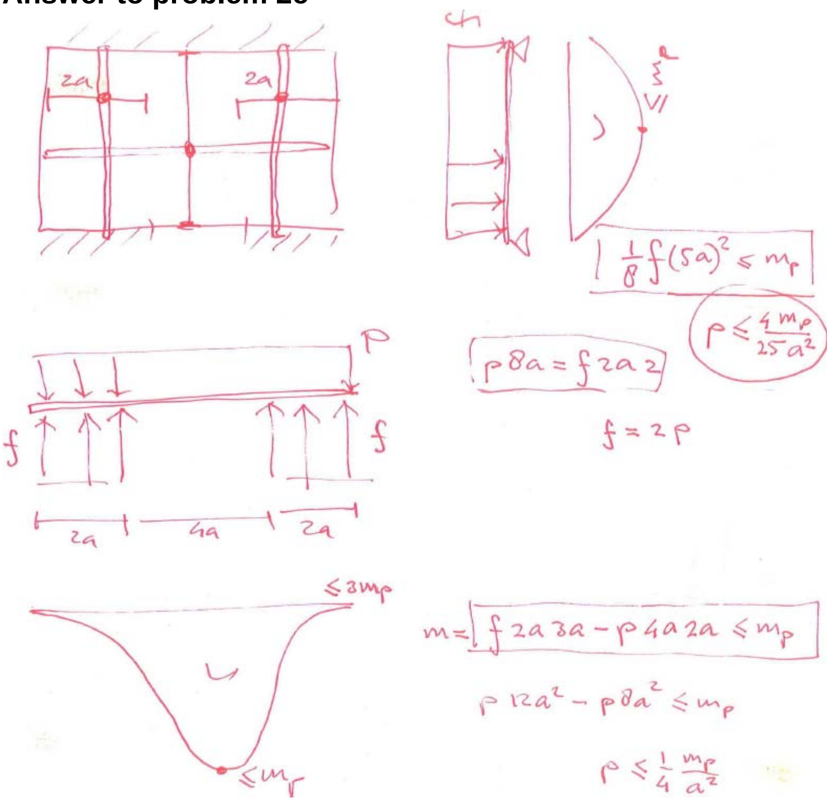
Answer to problem 2b

$$A = 2(\tfrac{1}{2}3a2ap\frac{w}{3}) = 2a^2pw$$

$$E = 2(3m_p 3a\frac{w}{2a} + m_p 2a\frac{w}{3a}) + m_p 3a\frac{w}{2a}2 = \frac{40}{3}m_p w$$

$$A = E \Rightarrow p = \frac{20}{3}\frac{m_p}{a^2} = 6.67\frac{m_p}{a^2}$$

Answer to problem 2c



Answer to problem 3

B

C

C

Answer to problem 3d

$$m_{xy}^2 \le (m_{px} - m_{xx})(m_{py} - m_{yy}) \qquad m_{px} \ge 0 \quad m_{py} \ge 0$$

$m_{xy}^2 = (u\,m_{px} - m_{xx})(u\,m_{py} - m_{yy})$ and $u \le 1$

Maple gives two solutions for u.

$$u_{1,2} = \frac{m_{xx}}{2m_{px}} + \frac{m_{yy}}{2m_{py}} \pm \sqrt{(\frac{m_{xx}}{2m_{px}} - \frac{m_{yy}}{2m_{py}})^2 + \frac{m_{xy}^2}{m_{px}m_{py}}}$$

A torsion moment m_{xy} can only increase the value of u, therefore

$$u = \frac{m_{xx}}{2m_{px}} + \frac{m_{yy}}{2m_{py}} + \sqrt{(\frac{m_{xx}}{2m_{px}} - \frac{m_{yy}}{2m_{py}})^2 + \frac{m_{xy}^2}{m_{px}m_{py}}} \quad \text{Q.E.D.}$$