

**Exam CIE4150 Plastic Analysis of Structures**  
Thursday 29 June 2023, 13:30 – 16:30 hours

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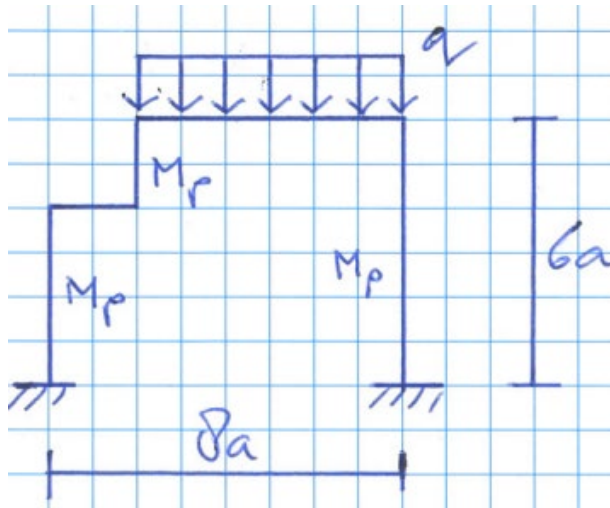


Figure 1. Frame structure

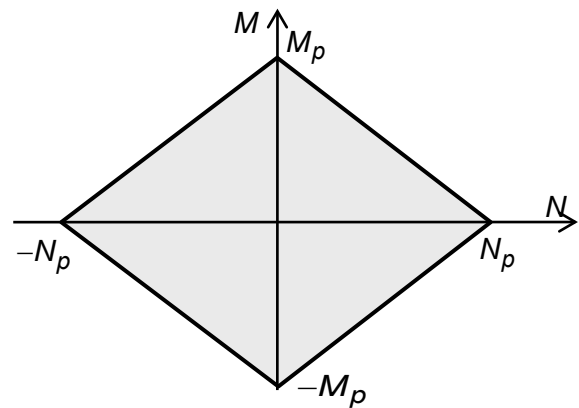


Figure 2. Yield contours

**Problem 1**

A frame consists of five members (Fig.1). All members have a strength  $M_p$ . The members are rigidly connected. The supports are fixed. The structure is loaded by an evenly distributed line load  $q$ . The relation of Figure 2 exists between the plastic moments and the plastic normal forces.

$$N_p = \beta \frac{M_p}{a}$$

The influence of shear on the yield contour is neglected. Buckling and second order effects are not considered.

- a** Assume  $\beta \rightarrow \infty$ . Determine the collapse load  $q$  for all possible mechanisms. Write the collapse loads as functions of  $M_p$  and  $a$ . What is the decisive collapse load? (1.5 point)
- b** Assume  $\beta \rightarrow \infty$ . Draw the bending moment diagram and normal force diagram for the structure at the moment of collapse. (1.5 points)
- c** Assume  $\beta = 5$ . Choose one of the following problems (You need not do both).
  - Determine the largest lower-bound for  $q$ .
  - Determine the smallest upper-bound for  $q$ .
You only need to write down the equations and not solve the equations (1.5 points).

## Problem 2

A reinforced concrete plate has simply supported edges and free edges (Fig. 3). It carries an evenly distributed load  $p$  [kN/m<sup>2</sup>]. There is no other load on the plate. The plate is homogeneous and orthotropic.

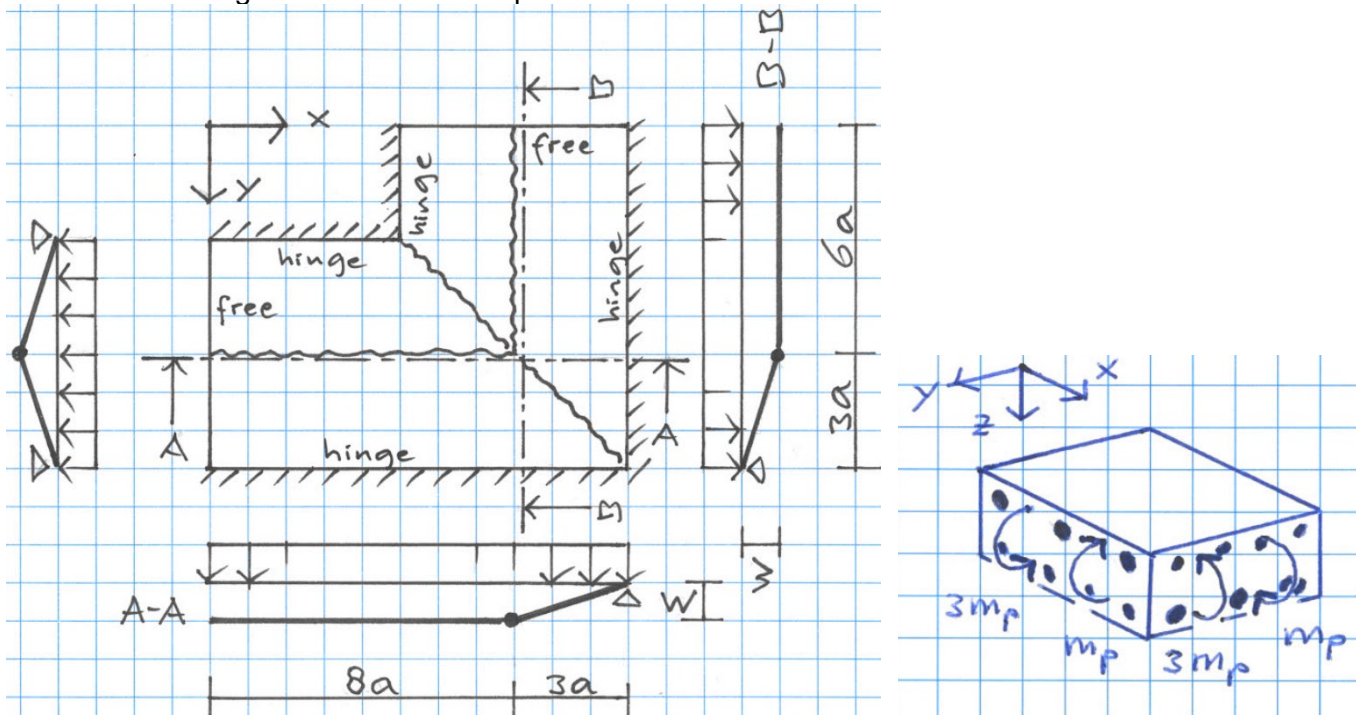


Figure 3. Plate dimensions and reinforcement

- a Consider the yield line patterns of Figure 4. Which of these patterns give kinematically possible mechanisms? (1 point)

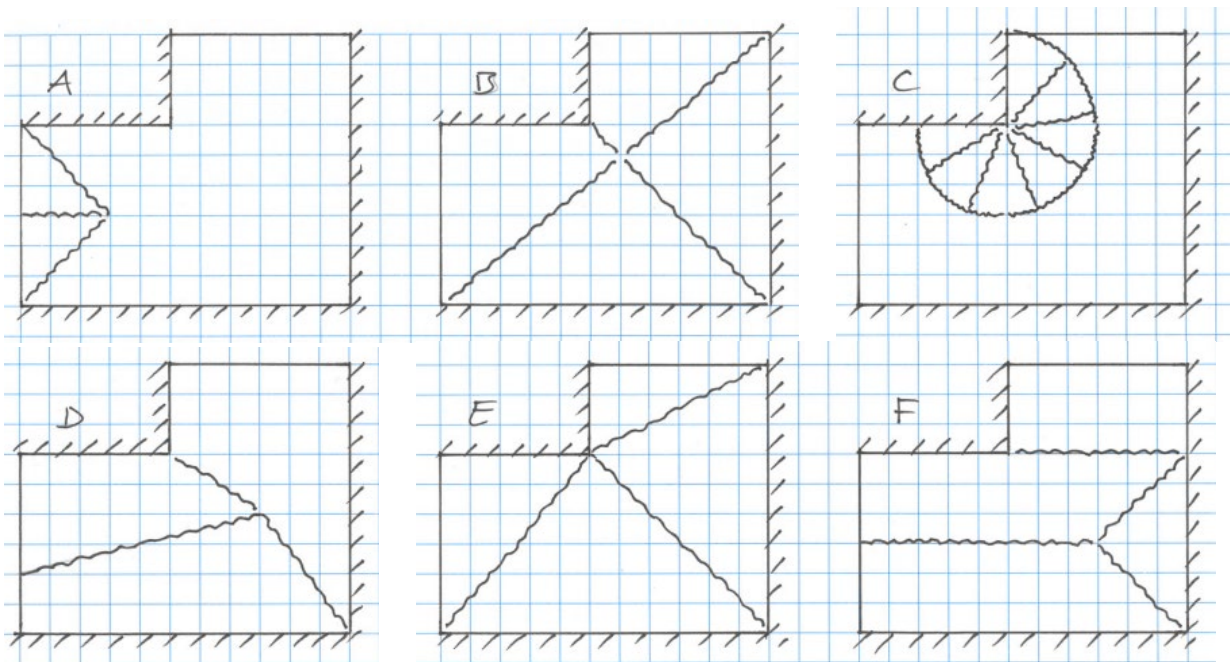


Figure 4. Yield line patterns of problem 2a

- b** Consider the yield line pattern of Figure 3. Determine an upper-bound for  $p$  expressed in  $m_p$  and  $a$  (1.5 point).
- c** Determine the largest lower-bound for  $p$  using torsion free beams ( $m_{xy} = 0$ ). You need only to write down the equations and not solve the equations. (1.5 point)

### Problem 3

- a** A statically indeterminate ductile structure is partially heated by direct solar radiation. Do we include temperature as a load case for the ultimate limit state?  
Choose A, B, C or D. (0.5 point)
- A No; expansions joints absorb the temperature strains.  
B No; temperature strains do not change the collapse load.  
C Yes; a temperature gradient may increase the moments.  
D Yes; the increased temperature reduces the compressive strength.
- b** A frame structure is statically indeterminate to the 5<sup>th</sup> degree. It has 13 locations of possible plastic hinges. An upperbound analysis is performed. How many mechanisms can occur?  
Choose A, B, C or D. (0.5 point)
- A 6  
B 13  
C 17  
D 1716
- c** The lowerbound requirements are ... Choose A, B, C or D. (0.5 point)
- A ductility, equilibrium,  $M < Mp$   
B equilibrium,  $M < Mp$ , normality  
C  $M < Mp$ , normality, ductility  
D normality, ductility, equilibrium

**Answer to problem 1a**

**Answer to problem 1b**

**Answer to problem 1c**

Upper-bound

Lower-bound

**Answer to problem 2a**

A, F

3 or less correct ..... 0.0 point

4 correct ..... 0.3 point

5 correct ..... 0.7 point

6 correct ..... 1.0 point

**Answer to problem 2b**

**Answer to problem 2c**

**Answer to problem 3**

- a     B
- b     D
- c     A

