SEI Discussion Cold Bent Glass Sheets in Façade Structures

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Discussion on Paper

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The authors of the paper have explored opportunities for applying plane glass sheets in curved façades. It was discovered that glass sheets display a double curvature for small deformations and a predominantly single curvature for large deformations. This phenomenon was studied by a lattice model for buckling analysis. The discussant would like to point out an inconsistency in the results and propose a few improvements.

In the paper a lattice model is proposed to derive the imposed corner deformation dZ of a square glass sheet at which transition from the double to the single curvature takes place. Three minor modifications can be made to this model in order to obtain far more accurate results. From Fig. 9 in the paper the width of the compression diagonals is measured as 1/3 of the diagonal length. The width of the tension edge stringers is 1/7 of the panel width. It can also be observed that the compression diagonals are clamped at the corners and that they are not in compression over the full length. The compression length is 7/12 of the diagonal length, therefore, the buckling length is $7/12 \times 1/2 = 7/24$ of the diagonal length. Using these assumptions it can be calculated that for a square sheet:

 $dZ_{instability} = 11,4 t$

In finite element computations [2] and [3] it was found that for a square sheet mode transition occurs at:

$$dZ_{instability} = 16,8 t$$

Agreement is far better with the proposed modifications than found by the authors. However, the difference is still too large for engineering design, therefore the lattice model is only suitable for qualitative analysis of the transition phenomenon.



Fig. 1: Lattice model of a twisted strip

In the paper the lattice model is also applied to a rectangular sheet of which the length goes to infinity. A simple experiment on a paper strip shows that the second deformation mode consists of many diagonals (*Fig. 1*). The authors used only two diagonals in their strip analysis, which cannot be correct.

Due to the repetitive nature of this deformation mode the transition deformation dZ of a strip is

$$dZ_{instability} = b\sin\frac{\alpha at}{b^2} \quad a >> b$$

where α is to be determined by finite element analysis (*Fig. 2*).

Fig. 11 of the paper shows substantial non-linearity in the maximum stresses near the corner prior to mode transition. (It is assumed that mode transition in Fig. 11 occurs at dZ = 134 mm.) On the other hand Fig. 8 shows very little non-linearity in the deformation prior to mode transition. Apparently, the non-linearity in the corner stresses is not related to the mode transition. The mentioned magnification factor of 1,6 for the stresses is very large. Accurate measurement in Fig. 11 shows that the magnification factor at mode transition is 1,75, which is even larger. The paper does not give a physical explanation for these large values and the discussant cannot find any reasonable explanation. This strongly suggests that something might have gone wrong in the finite element modelling, finite element analysis or interpretation of the results.

The glass sheet support is modelled by imposed displacements in one point at each corner. Therefore, in theory, the stresses in these points will be infinitely large. Finite stresses have been found due to the averaging effect of the finite element size. To realistically quantify the maximum stresses the sheet needs to be modelled including the specific support geometry.

In conclusion it needs be mentioned that Von Misses stresses are not particularly suitable for a brittle material such as glass. The largest principal stress would be a better value for failure.

References

[1] HERWIJNEN, F. van, STAAKS, D., EEKHOUT, M., 'Cold Bent Glass Sheets in Façade Structures', *Structural Engineering International*, Vol. 14, No. 2, (May 2004), pp. 98–101.

[2] STAAKS, D., 'Cold Bent Glass Sheets in Blobs', ('Koud torderen van glaspanelen in blobs'), M.Sc. thesis University of Technology Eindhoven, The Netherlands, 2003, (in Dutch).

[3] LAAR, H. van, 'Stability of On Site Formed Glass Panels for Double Curvature Façades', ('Stabiliteit van in het werk vervormde glaspanelen voor dubbelgekromde gevels'), M.Sc. thesis Delft University of Technology, The Netherlands, June 2004 (in Dutch).



Fig. 2: Derivation of the transition deformation of a strip

Reply by the Authors

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Our research project was carried out to gain insight into the parameters that determine the deformation patterns of twisted glass panes. We concluded that more research is necessary to analyse the influence of the length/width ratio on the moment of instability and to derive design rules for the stresses in cold bent double curved glass sheets with a rectangular shape. Delft University of Technology continued our research by an analytical and numerical research project carried out by Hein van Laar [3]. The results of this research project became available in the half year period between the completion and publication of our paper.

The lattice model we used was introduced for simple buckling analysis, and for a qualitative analysis of the transition phenomenon. By this model we discovered that the moment of instability is not material dependent, solely on the plate geometry (a,b) and the strip thickness (t). For a square lattice geometry, equation (5) could be derived: $dZ_{instability} = 3,35 t$. Later FEM analysis on a Plate Model showed another factor, as stated in the conclusions. This factor reached indeed a value of 16,8. The discussant gives a modification of the lattice model, leading to a better agreement with the plate model. The authors also examined such modifications during the research but

concluded that it should not lead to a desirable accuracy in predicting plate behaviour, while the basic model seemed equally suitable for qualitative analysis.

To check the variation of the factor 3,35 for the transition of a square model into rectangular geometries, equation (4) was also used for a >b. For a >>b the discussant showed that this is not correct, since another deformation pattern will occur in the case of a twisted strip. Since glass panes will hardly be used in a strip shape, this will however have no practical significance.

The first order stresses in Fig. 11, of the original paper are the result of a constant twisting moment in the plate according to the theory of Nadai. However, under influence of the membrane stresses the distribution of twisting moments changes and is no longer constant. At increasing twisting, the stresses at the corners of the plate will increase more than proportionally, whereas the stresses in the middle of the plate will decrease. For this reason the stresses at the corner of the plate are higher than the first order stresses The lack of non-linearity in the deformation, as mentioned by the discussant, is caused by its symmetry. Whether a twisted plate's curvature is constant or not, both diagonals will deform in the same

but opposite shape. Deformation of the midpoint will therefore show linearity towards dZ.

Again the non-linear behaviour can be explained in a qualitative way using a lattice model. In fact the mechanical scheme changes gradually from bending due to lateral loads towards bending due to in-plane loads, as shown in the figure below. Although the total deflection w is equal, the curvature/ bending along the length is not. Using the basic equations for bending and assuming an equal deflection it can be shown that the bending in the middle is approximately 1,25 times larger for the lateral load situation while the bending near the ends is 1,6 times larger for the in-plane load situation. This phenomenon explains the nonlinear increase in stress near the edges of the plate.

In the case of failure analysis for brittle materials, we also prefer to use maximum principal stresses.

We hope that this discussion has better clarified the behaviour of twisted glass sheets. The research project will be continued at the University of Technology Eindhoven by investigating the behaviour of cold bent insulated double glass units.



Fig. 1: Distribution of bending moments for the lateral load situation (left) resp. for the in-plane load situation (right).

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